1

DUAL LANGUAGE PROGRAMME

KSSM
KURIKULUM STANDARD SEKOLAH MENENGAH
KEMENTERIAN PENDIDIKAN MALAYSIA

Form

MATHEMATICS 1

MATHEMATICS Form 1

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KEMENTERIAN PENDIDIKAN MALAYSIA

RUKUN NEGARA

BAHAWASANYA NEGARA KITA MALAYSIA

mendukung cita-cita hendak:

Mencapai perpaduan yang lebih erat dalam kalangan seluruh masyarakatnya;

Memelihara satu cara hidup demokrasi;

Mencipta satu masyarakat yang adil di mana kemakmuran negara akan dapat dinikmati bersama secara adil dan saksama;

Menjamin satu cara yang liberal terhadap tradisi-tradisi kebudayaannya yang kaya dan pelbagai corak;

Membina satu masyarakat progresif yang akan menggunakan sains dan teknologi moden;

MAKA KAMI, rakyat Malaysia,
berikrar akan menumpukan
seluruh tenaga dan usaha kami untuk mencapai cita-cita tersebut
berdasarkan prinsip-prinsip yang berikut:

KEPERCAYAAN KEPADA TUHAN
KESETIAAN KEPADA RAJA DAN NEGARA
KELUHURAN PERLEMBAGAAN
KEDAULATAN UNDANG-UNDANG
KESOPANAN DAN KESUSILAAN

(Sumber Jabatan Penerangan Kementerian Komunikasi dan Multimedia Malaysia)

KURIKULUM STANDARD SEKOLAH MENENGAH

MATHEMATICS
FORM 1

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CONTENTS

INTRODUCTION

This Form 1 Mathematics Textbook is prepared based on the *Dokumen Standard Kurikulum dan Pentaksiran* (*DSKP*) which incorporates six fundamental strands in the framework of KSSM, integrating knowledge, skills and values as well as encompassing explicitly of the 21st Century Skills and Higher Order Thinking Skills (HOTS) so as to produce balanced and harmonious individuals as enshrined in the National Philosophy of Education.

This book contains 13 chapters arranged systematically and presented in an interesting manner so as to cultivate the students' interest for mathematics. Every chapter begins with a stimulating page of interesting photographs and questions to encourage discussions and understanding among students in the context of daily living.

The special features of this textbook are:

Why study this chapter?	Poses open-ended questions for interaction between teacher and students.
Walking through Time	Recounts the history and development of mathematics and the contributions of great mathematicians.
Exploration Activity Individual Pair Group Learning Outside the Classroom	Prepares various types of activities aimed at involving students individually, in pairs or in groups inside or outside the classroom.
Exploration Activity Flip Classroom	Encourages students to explore the learning contents by themselves first before the lesson begins and involves students in active discussions during the lesson.
Self Practice 1.1a	Tests the understanding of the students on the concepts that they have learnt.
Mastery Q 1.1	Includes questions to test various skills of students.
Let's PRACTISE	Prepares more diversified exercises comprising Test Yourself, Self Mastery and Challenge Yourself which incorporate the elements of LOTS and HOTS as well as the elements of TIMSS and PISA assessment.
	Indicates HOTS questions to help in developing students' higher order thinking skills.
Did You Know	Imparts general knowledge and interesting applications of mathematics.

Application	Enables students to apply mathematical concepts in real-life context.
Try This	Tests the understanding and mastery of students.
	Poses questions to test students' minds.
SMART TIPS	Provides important and beneficial tips and extra input.
Think Smart	Presents mind-stimulating questions for students to think critically and creatively.
Let's Discuss	Poses questions for discussions whereby students share their ideas.
Communication Corner	Develops communication skills in mathematics.
Creative & Innovative	Generates students' thinking skills to produce creative ideas and to invent.
Flashback	Helps students to remember what they have learnt.
Smart	Shows the use of scientific calculators in calculations.
Smart Technology	Exposes students to the use of technological tools in the learning of mathematics.
Career in Mathematics	Introduces careers that involve the concepts of mathematics that students have learnt.
ASSIGNMENT	Enables students to carry out assignments and then present their completed work in class.
Exploring MATHEMATICS	Provides additional activities to effectively enhance students' understanding.
	Enables students to scan QR Code using mobile devices to watch videos, perform activities and obtain extra information. (Websites in the public domain proposed in this book may be subject to change from time to time.)

Every chapter ends with a summary in the form of a thinking map or a concept map. In general, we hope this textbook can be fully used by all students as well as educators.

SYMBOLS AND FORMULAE

SYMBOLS

$\sqrt{\ }$	square root	Δ	triangle
$\sqrt[3]{\ }$	cube root	$\angle$	angle
$a : b$	ratio of a to b	$\{\ \}, \phi$	empty set
$=$	is equal to	ξ	universal set
$\approx$	is approximately equal to	$\in$	is an element of
$\neq$	is not equal to	$\notin$	is not an element of
$>$	is greater than	$\subset$	is a subset of
$\geqslant$	is greater than or equal to	$\not\subset$	is not a subset of
$<$	is less than	$n(A)$	number of elements of set A
$\leqslant$	is less than or equal to	A'	complement of set A

FORMULAE

Area of a triangle = $\frac{1}{2}$ × length of base × height

Area of a parallelogram = length of base × height

Area of a kite = $\frac{1}{2}$ × product of two diagonals

Area of a trapezium = $\frac{1}{2}$ × sum of two parallel sides × height

SCAN

https://goo.gl/UxjF6c

Download the free application from Google Play or other platforms to your mobile devices. Scan the QR Code using the application or visit the website as shown on the left to download files of audio of Word Link, GeoGebra, spreadsheets, and extra questions of Mastery Q. Then, save the downloaded files in your own folder for offline use.

Note: Students can download the free GeoGebra software program to open the related files.
http://www.geogebra.org/

CHAPTER 1

Rational Numbers

What will you learn?

- Integers
- Basic Arithmetic Operations Involving Integers
- Positive and Negative Fractions
- Positive and Negative Decimals
- Rational Numbers

Why study this chapter?

Numbers play an important role in business and commerce, banking, engineering and others. Discuss the importance of numbers in these fields.

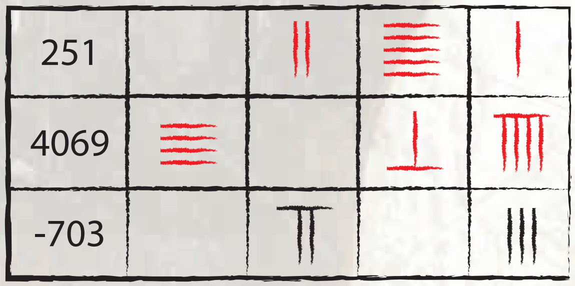
The World Climate Summit in Paris in 2015 discussed the issue of global warming. Global warming has caused the global average temperature to rise and consequently the glaciers in the North and South Poles have melted.

Today, the average temperature at the North and South Poles is below 0°C.

How is the value of temperature below 0°C represented as a number?

If one day, the temperature at the North and South Poles rises until 0°C, predict the impact on the Earth.

Walking through Time



As early as 200 B.C., the Chinese had used bamboo rods to represent positive numbers and negative numbers. They used red rods to represent positive numbers and black rods to represent negative numbers. Negative numbers did not appear in Europe until the 15th century.

For more information:

http://goo.gl/X7Kr81

Word Link

• Identity Law	• *Hukum Identiti*
• Distributive Law	• *Hukum Kalis Agihan*
• Associative Law	• *Hukum Kalis Sekutuan*
• Commutative Law	• *Hukum Kalis Tukar Tertib*
• integer	• *integer*
• rational number	• *nombor nisbah*
• fraction	• *pecahan*
• decimal	• *perpuluhan*
• zero	• *sifar*

Open the folder downloaded from page vii for the audio of Word Link.

1.1 Integers

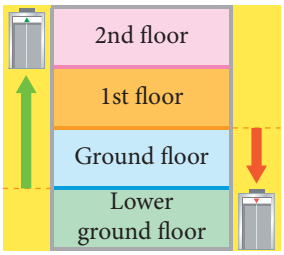
What are positive numbers and negative numbers?

LEARNING STANDARDS

Recognise positive and negative numbers based on real-life situations.

In our daily lives, we often encounter situations involving two opposite changes like moving towards right or towards left; going up or going down; working with values that are more than or less than zero; and adjusting to an increase or decrease of a value.

These situations can be represented using positive numbers and negative numbers. For instance,



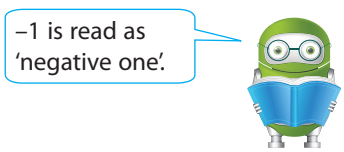
- A lift going up two floors is written as +2 or 2.
- A lift going down one floor is written as –1.

- The temperature of 45°C at a desert is written as +45 or 45.

- The temperature of a glacier which is 10°C below 0°C is written as –10.

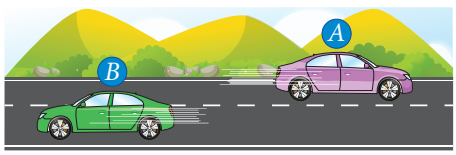
Numbers written with the '+' sign or without any sign like +2, +45 or 2, 45 are known as **positive numbers**.

Numbers written with the '–' sign like –1, –10 are known as **negative numbers**.



Example 1

Car *A* moves 40 m towards the right while car *B* moves 50 m towards the left. Represent the movements of car *A* and car *B* using a positive number or a negative number.



Solution

Assume that moving towards the right is represented by a positive number and moving towards the left is represented by a negative number.

Thus, the movement of car *A*, 40 m towards the right, is represented as 40 or +40; the movement of car *B*, 50 m towards the left, is represented as –50.

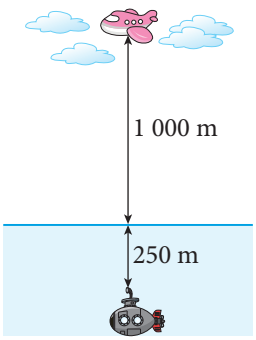
Think Smart

Can the movement towards left be represented with a positive number whereas the movement towards right be represented with a negative number? Explain.

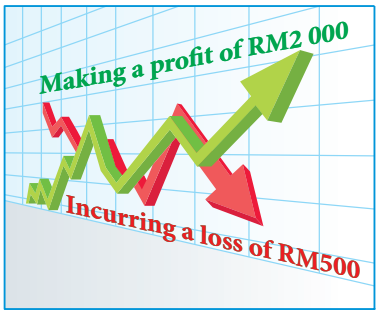
Self Practice 1.1a

1. For each of the following situations, represent the two opposite changes using a positive number and a negative number.

(a)



(b)



What are integers?

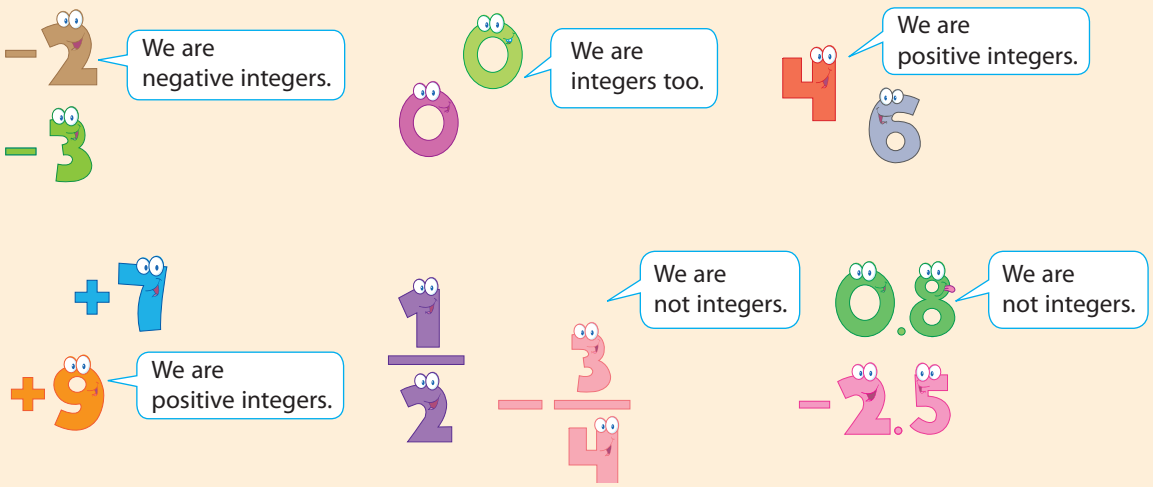
Exploration Activity 1

Group

LEARNING STANDARDS

Recognise and describe integers.

Aim: To recognise and describe integers.
Instruction: Perform the activity in groups of four.



1. Study the diagrams above carefully.

2. Discuss with your friends and explain how you would describe the meaning of integers.

From the results of Exploration Activity 1, it is found that **integers** are groups of numbers which include positive and negative whole numbers as well as zero.

Example 2

Determine whether each of the following numbers is an integer.

$15,\ 23,\ -3.4,\ -76,\ 0,\ \frac{1}{2},\ 6,\ 0.88,\ -\frac{4}{5},\ 301,\ -239$

Solution

Integer: $15, 23, -76, 0, 6, 301, -239$ Non-integer: $-3.4, \frac{1}{2}, 0.88, -\frac{4}{5}$

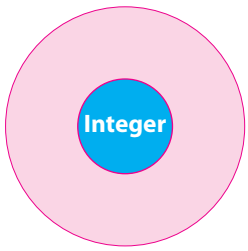
Self Practice 1.1b

1. In the following table, mark '✓' for numbers which are integers and mark '✗' for numbers which are non-integers.

$\frac{3}{4}$	-24	35	6.7	-29	900	-4.34	72	$-\frac{1}{2}$	0

2. Copy the diagram on the right. Select integers from the following list of numbers and write them in the diagram.

$0.25,\ \frac{1}{3},\ 48,\ -12,\ -2.8,\ -\frac{2}{7},\ 0,\ 59,\ 458,\ -6$



How do you represent integers on a number line?

Exploration Activity 2

Flip Classroom

Aim: To explore the representation of integers on a number line.

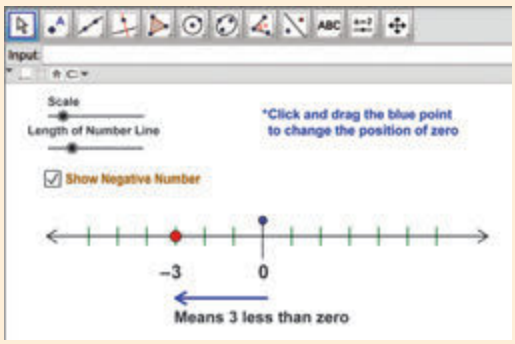
Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

LEARNING STANDARDS

Represent integers on number lines and make connections between the values and positions of the integers with respect to other integers on the number line.

1. Open the file *Integer number line.ggb* using *GeoGebra*.
2. Click and drag the red point on the number line to define an integer on the number line.
3. Observe the position of the defined integer on the number line in relation to the position of zero.

4. Answer the following questions based on the information obtained from the screen displayed.
   (a) Describe the value of integer 10 as compared to zero.
   (b) How do you determine the positions of negative numbers like –1, –2 and –3 on a number line?
   (c) How do you represent 1, 2, 3, –1, –2 and –3 on a number line?



From the results of Exploration Activity 2, it is found that **positive integers** are integers more than zero whereas **negative integers** are integers less than zero.

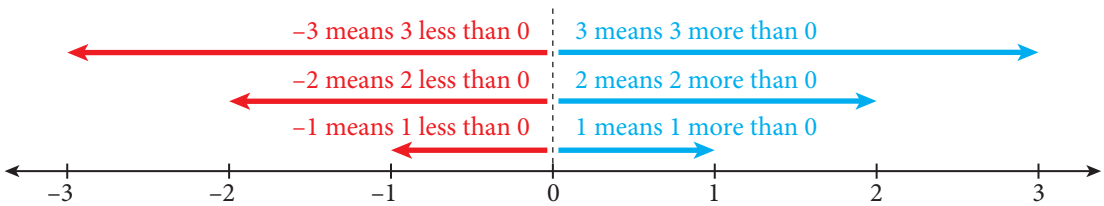
If the direction towards right is assumed as positive and the direction towards left is assumed as negative, thus we can conclude that

(i) for a positive integer, the more the number is greater than 0, the further its position to the right on the number line and the greater will be its value.

(ii) for a negative integer, the more the number is less than 0, the further its position to the left on the number line and the lesser will be its value.

Flashback

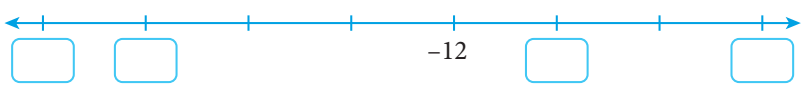
On a number line, the numbers in the positive direction are always greater than the numbers in the negative direction.



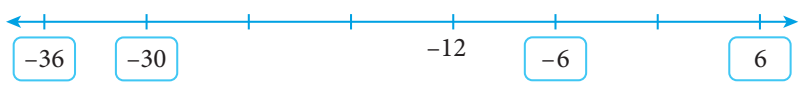
Example 3

Complete the number line using the following numbers.

–30, 6, –6, –36



Solution



–36, –30, –12 and –6 are negative integers. Thus, –36 has the least value and it is the furthest towards the left.

6 is a positive integer. Thus, 6 is the furthest towards the right.

Did You Know?

The lowest temperature ever recorded was about –93°C in the Antarctic in August, 2010.

Self Practice 1.1c

1. For each of the following, determine and mark the positions of the given integers on a number line.

(a) –5, 5, 1 and –3 (b) 0, –8, 2 and –10

2. Complete each of the following number lines.

(a)

(b)

How do you compare and arrange integers in order?

Based on the positions of integers given on a number line, we can compare the respective values of the integers and hence arrange the integers in ascending order or descending order.

LEARNING STANDARDS

Compare and arrange integers in order.

Example 4

(a) Compare and arrange –3, 4, 2, –5, 6, 0, –1 in ascending order.

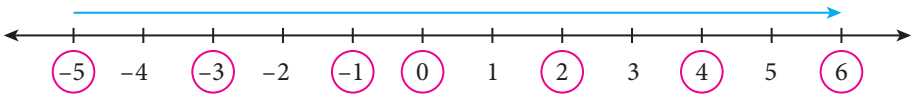
(b) Compare and arrange –4, 3, 2, 5, –2, –1, –5 in descending order.

SMART TIPS

A positive number always has a larger value than a negative number.

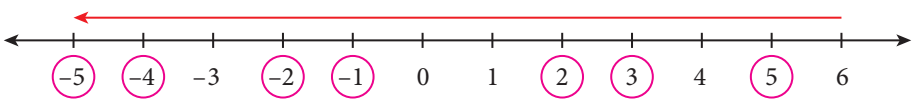
Solution

(a)



Ascending order: –5, –3, –1, 0, 2, 4, 6

(b)



Descending order: 5, 3, 2, –1, –2, –4, –5

Self Practice 1.1d

1. Compare and arrange –4, 3, 1, –6, 5, 0, –2 in ascending order.

2. Compare and arrange –5, –3, 3, 4, –4, 2, –1 in descending order.

Mastery Q 1.1

Open the folder downloaded from page vii for extra questions of Mastery Q 1.1.

1. (a) If +20 represents 20 m above sea level, then –20 represents ☐.

(b) If +90 represents a movement of 90 m to the north, then –90 represents ☐.

(c) If +800 represents RM800 being credited into a savings account, then RM800 being debited into a savings account is represented as _____.

(d) If +1 000 represents a profit of RM1 000, then a loss of RM1 000 is represented as _____.

2. State the following numbers using '+' or '–'.
(a) 80 less than zero (b) 76 more than zero

3. List all integers
(a) from –8 to 4 (b) from –12 to –2

4. Determine whether each of the following numbers is an integer.
–14, 3.9, 12, –26, 85, 0, –2

5. Compare and arrange the following values of temperature in the order beginning from the coldest temperature.
–3°C, 2°C, –4°C, 1°C, 4°C

1.2 Basic Arithmetic Operations Involving Integers

How do you add and subtract integers?

LEARNING STANDARDS

Add and subtract integers using number lines or other appropriate methods. Hence, make generalisation about addition and subtraction of integers.

Exploration Activity 3

Flip Classroom

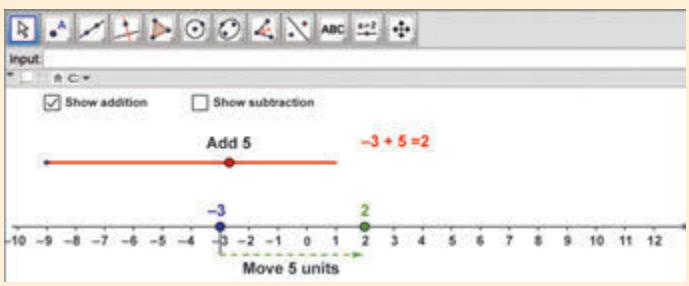
Aim: **To explore addition and subtraction of integers on a number line.**

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Add subtract integers.ggb* using *GeoGebra*. The screen shows the following display.

2. Click and drag the red slider and the blue point displayed on the screen.

3. Observe the movement of other points on the display in relation to addition and subtraction of integers.



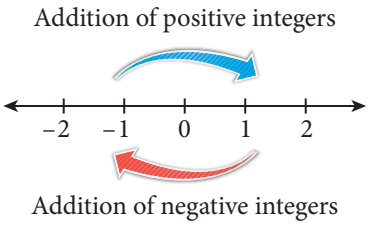
4. Present and discuss the findings with your friends during the lesson.

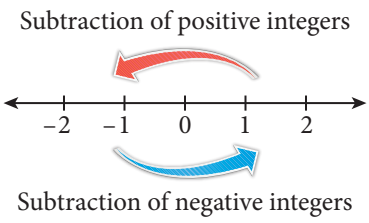
5. Make a generalisation regarding the addition and subtraction of integers.

From the results of Exploration Activity 3, it is found that on a number line,

(a) addition of
  (i) positive integers is represented by moving towards the right
  (ii) negative integers is represented by moving towards the left

(b) subtraction of
  (i) positive integers is represented by moving towards the left
  (ii) negative integers is represented by moving towards the right





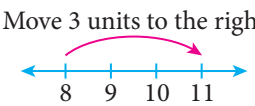
Example 5

Solve

(a) 8 + (+3)
(b) 5 + (–2)
(c) 2 – (+4)
(d) –1 – (–4)

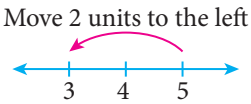
Solution

(a) 8 + (+3)
= 8 + 3
= 11



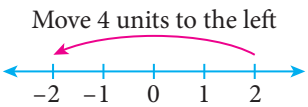
Addition of positive integers is represented by moving towards the right.

(b) 5 + (–2)
= 5 – 2
= 3



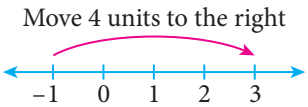
Addition of negative integers is represented by moving towards the left.

(c) 2 – (+4)
= 2 – 4
= –2



Subtraction of positive integers is represented by moving towards the left.

(d) –1 – (–4)
= –1 + 4
= 3



Subtraction of negative integers is represented by moving towards the right.

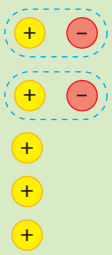
Scan the QR Code or visit https://youtu.be/q5ogKyt0cYA to learn about other examples of addition and subtraction of integers.

Alternative Method

Coloured chips method

Yellow chips, (+), represent positive integers and red chips, (–), represent negative integers.

For Example 5(b):



Thus, 5 + (–2) = 3

Self Practice 1.2a

1. Solve each of the following:

(a) 6 + (+2)
(b) –4 + (–3)
(c) 3 – (+2)
(d) –2 – (–4)
(e) –8 + (–2)
(f) 6 – (+3)
(g) 9 + (+4)
(h) –5 – (–3)

How do you multiply and divide integers?

LEARNING STANDARDS

Multiply and divide integers using various methods. Hence make generalisation about multiplication and division of integers.

Exploration Activity 4

Flip Classroom

Aim: To explore the multiplication and division of integers.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open and print the file *Multiplication of integers table.pdf* as shown in the diagram.
2. Complete the purple region for the multiplication of positive integers that you have learnt.
3. Complete the other regions in the table according to the patterns of the numbers shown.
4. Present your findings about the patterns of multiplication of integers shown.
5. Discuss with your friends about the patterns of division of integers.
6. Make a generalisation regarding the multiplication and division of integers.

×	−5	−4	−3	−2	−1	0	1	2	3	4	5
−5						0					
−4						0					
−3						0					
−2						0				−8	
−1						0				−4	−5
0	0	0	0	0	0	0	0	0	0	0	0
1					−1	0	1	2	3	4	5
2				−4	−2	0	2	4	6	8	10
3						0					15
4						0					
5						0					

From the results of Exploration Activity 4, it is found that

Operation	Sign of the product
(+) × (+)	+
(+) × (−)	−
(−) × (+)	−
(−) × (−)	+

Operation	Sign of the quotient
(+) ÷ (+)	+
(+) ÷ (−)	−
(−) ÷ (+)	−
(−) ÷ (−)	+

In general, the rules of multiplication and division of integers can be summarized as follows.

The product or quotient of two integers with the same signs is a positive integer.

The product or quotient of two integers with different signs is a negative integer.

SMART TIPS

Other than addition and subtraction of integers, coloured chips can also be used to perform multiplication and division of integers. Visit the site below for further information.

www.goo.gl/7j6CTd

Example 6

Evaluate each of the following:

(a) $-5 \times (-4)$ (b) -6×4 (c) $6 \div (-2)$ (d) $-12 \div (-2)$

Solution

(a) $-5 \times (-4)$
$= +(5 \times 4)$
$= 20$

(b) -6×4
$= -(6 \times 4)$
$= -24$

(c) $6 \div (-2)$
$= -(6 \div 2)$
$= -3$

(d) $-12 \div (-2)$
$= +(12 \div 2)$
$= 6$

Self Practice 1.2b

1. Solve each of the following:

(a) $-6 \times (-3)$ (b) -7×2 (c) $4 \times (-8)$ (d) $8 \times (-6)$
(e) $-12 \div 3$ (f) $-18 \div (-6)$ (g) $15 \div (-5)$ (h) $-20 \div 4$

How do you perform computations involving combined basic arithmetic operations of integer?

When performing a computation involving combined operations of integers, follow the order of operations below.

Brackets ( ) → × or ÷ → + or –

From left to right (× or ÷)

From left to right (+ or –)

LEARNING STANDARDS

Perform computations involving combined basic arithmetic operations of integers by following the order of operations.

Example 7

Solve each of the following:

(a) $-8 \times (-2 + 3)$ (b) $7 + 2(-3)$

(c) $4 - 12 \div (-2) + (-1)$ (d) $\dfrac{-12 + (-16)}{-22 - (-24)}$

Solution

(a) $-8 \times (-2 + 3)$
$= -8 \times (1)$
$= -8$

(b) $7 + 2(-3)$
$= 7 + 2 \times (-3)$
$= 7 + (-6)$
$= 7 - 6$
$= 1$

(c) $4 - 12 \div (-2) + (-1)$
$= 4 - (-6) - 1$
$= 4 + 6 - 1$
$= 9$

(d) $\dfrac{-12 + (-16)}{-22 - (-24)}$
$= \dfrac{-12 - 16}{-22 + 24}$
$= \dfrac{-28}{2}$
$= -14$

SMART TIPS

Brackets are also a notation for multiplication.

Smart

For Example 7(a),

press [(–)] [8] [×] [(] [(–)] [2] [+] [3] [)] [=]

For Example 7(d),

press [(] [(–)] [1] [2] [+] [(–)] [1] [6] [)] [÷] [(] [(–)] [2] [2] [–] [(–)] [2] [4] [)] [=]

Self Practice 1.2c

1. Evaluate each of the following:

(a) $-9 \times (-4 + 6)$ (b) $8 + (-4) \times 8$ (c) $4 - 15 \div (-3) + (-8)$

(d) $\dfrac{-14 + (-22)}{-23 - (-35)}$ (e) $-12 - 15 \times (-3) - (-6)$ (f) $\dfrac{-6 + (-8) \times (-5)}{-27 - (-38)}$

How do you describe the laws of arithmetic operations?

Exploration Activity 5

Flip Classroom

Aim: To explore the rules of arithmetic operations.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Laws of arithmetic.pdf* and print the printout as shown in the diagram.

2. Complete the given table.

3. Compare your results and discuss with the members from the other groups.

4. Make a conclusion about the rules of arithmetic operations.

LEARNING STANDARDS

Describe the laws of arithmetic operations which are Identity Law, Commutative Law, Associative Law and Distributive Law.

From the results of Exploration Activity 5, it is found that

(i) for all values of a and b,

$$a + b = b + a$$
$$a \times b = b \times a$$

Addition and multiplication are said to obey the **Commutative Law**.

(ii) for all values of a, b and c,

$$(a + b) + c = a + (b + c)$$
$$(a \times b) \times c = a \times (b \times c)$$

Addition and multiplication are said to obey the **Associative Law**.

(iii) for all values of a, b and c,

$$a \times (b + c) = a \times b + a \times c$$
$$a \times (b - c) = a \times b - a \times c$$

Addition and subtraction are said to obey the **Distributive Law**.

(iv) for all values of a,

$$a + 0 = a \qquad a + (-a) = 0$$
$$a \times 0 = 0 \qquad a \times \frac{1}{a} = 1$$
$$a \times 1 = a$$

These statements are known as the **Identity Law**.

How do you perform computations efficiently?

LEARNING STANDARDS

Perform efficient computations using the laws of basic arithmetic operations.

The laws of arithmetic operations you have just learnt can be used to perform computations more efficiently.

Example 8

Solve each of the following using efficient computations.
(a) $29 + 38 + 2$ (b) $2 \times 24 \times 5$ (c) 7×3040

Solution

(a) $29 + 38 + 2$
$= 29 + (38 + 2)$ ← Associative Law
$= 29 + 40$
$= 69$

(b) $2 \times 24 \times 5$
$= 24 \times 2 \times 5$ ← Commutative Law
$= 24 \times (2 \times 5)$ ← Associative Law
$= 24 \times 10$
$= 240$

(c) 7×3040
$= 7 \times (3000 + 40)$
$= 7 \times 3000 + 7 \times 40$ ← Distributive Law
$= 21000 + 280$
$= 21280$

Self Practice 1.2d

1. Using laws of arithmetic operations that you have learnt, solve each of the following using efficient computations.
(a) $356 + 61 + 9$ (b) $20 \times 567 \times 5$ (c) 89×5080
(d) $6 \times 200 + 6 \times 25 + 6 \times 5$ (e) $26 \times 3 - 24 \times 3$ (f) 899×5

How do you solve problems?

LEARNING STANDARDS

Solve problems involving integers.

MATHEMATICS APPLICATION ZONE

Luqman's credit card account showed a balance of debts of RM230 at one time. He had used his credit card to pay for three books each costing RM120. A week later, his credit card account was charged an interest of RM3 and Luqman made a payment of RM400 to his account. Explain whether Luqman had cleared his debts.

Solution

Total account balance = –230 ← Account balance means debt and is written using the '–' sign.

Total payment for books using credit card = $3 \times (-120)$
$= -360$

Interest charged = –3
Payment to account = +400

Check

Account balance = 230
Total expenses
= 3(120)
= 360
Interest charged = 3
Payment = 400
Account balance
= 230 + 360 + 3 – 400
= RM193

Final credit card account balance $= -230 + (-360) + (-3) + 400$
$= -230 - 360 - 3 + 400$
$= -193$

Luqman had not cleared his debts because his credit card account still showed a balance of debts of RM193.

Self Practice 1.2e

1. A shop made a profit of RM16 800 in the first year and incurred a loss of RM6 500 each year for the next two consecutive years. In the fourth year, the loss incurred was twice the loss incurred in the second year. How much was the profit or loss of the shop at the end of those four years?

2. From 7:00 p.m. to 5:00 a.m. of the next day, the temperature in Kuching dropped by 4°C. The temperature then rose by 8°C at 11:00 a.m. and continued rising by 2°C three hours later. If the temperature in Kuching at 11:00 a.m. was 30°C, calculate the temperature at
(a) 7:00 p.m. of the first day
(b) 2:00 p.m. of the second day

Mastery Q 1.2

Open the folder downloaded from page vii for extra questions of Mastery Q 1.2.

1. Using the following numbers, write five calculations that give an answer of –14.
–12, 6, 2, –3, –2, 8, 11, 5, 15

2. For each of the following, fill in the empty boxes with suitable operations '+', '–', '×' or '÷'.
(a) $-8 \square (-6) = -3 \times (-6 \square 10)$ (b) $5 + (-9) \square 3 = -5 \square (-7)$

3. Complete each of the following number patterns.
(a) –9, –7, ☐, –3, ☐, 1, ☐ (b) –2, 4, ☐, 16, –32, ☐, ☐

4. The temperature in a town at a certain time was 12°C. The temperature dropped until –6°C. The temperature then rose by 3°C and finally dropped by 8°C. Determine
(a) the change in temperature of the town,
(b) the final temperature of the town.

5. A diver was at 50 m below sea level. The diver swam up 2 m every 5 seconds. Explain whether the diver would have reached the sea surface after 2 minutes.

6. The current account of Encik Hafidz showed a balance of RM1 238. He signed two payment cheques of RM890 and RM1 730 respectively.
(a) Determine whether the RM890 cheque or the RM1 730 cheque would bounce when the cheques were credited.
(b) How much would Encik Hafidz have to top up in his account so that both cheques that he signed would not bounce when they are credited?

1.3 Positive and Negative Fractions

How do you represent positive and negative fractions on a number line?

LEARNING STANDARDS

Represent positive and negative fractions on number lines.

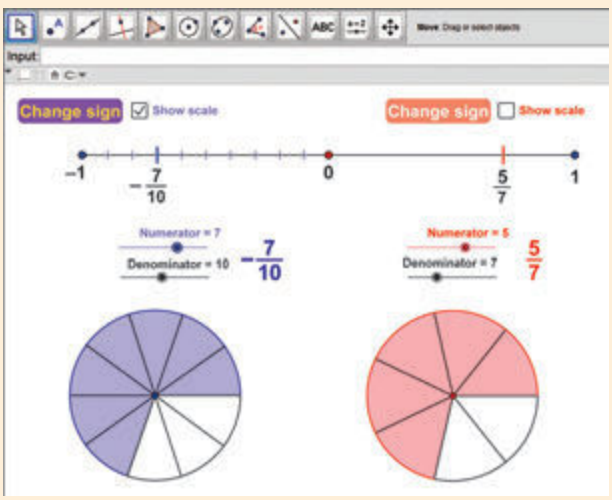
Exploration Activity 6

Flip Classroom

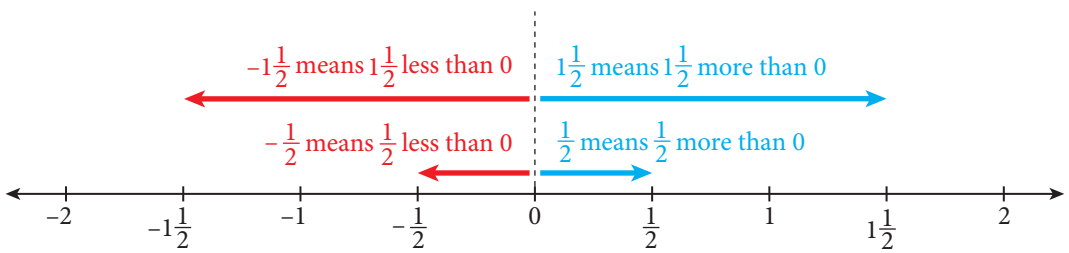
Aim: To explore the representation of positive and negative fractions on a number line.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Fraction number line.ggb* using *GeoGebra*.
2. Click and drag the slider 'Numerator' and 'Denominator' to define a fraction.
3. Click at the button 'Change sign' to interchange between positive and negative.
4. Observe the position of the fraction defined on the number line.
5. Discuss the methods used to determine the position of a fraction on a number line.



Representation of fractions on a number line is the same as that of integers. **Positive fractions** are fractions more than zero whereas **negative fractions** are fractions less than zero.



Example 9

Represent the following fractions on a number line.

$\frac{1}{2}, \frac{1}{4}, -\frac{3}{4}, -\frac{1}{2}$

SMART TIPS

- Positive fractions are on the right of zero.
- Negative fractions are on the left of zero.

Solution

Self Practice 1.3a

1. Represent the following fractions on a number line.

(a) $\frac{1}{10}, -\frac{3}{5}, \frac{1}{2}, -\frac{1}{5}$

(b) $-\frac{1}{3}, \frac{1}{6}, \frac{1}{2}, -1\frac{2}{3}$

How do you compare and arrange positive and negative fractions in order?

LEARNING STANDARDS

Compare and arrange positive and negative fractions in order.

The values of two or more fractions can be compared by equating the denominator first. Subsequently the fractions can be arranged in ascending order or descending order.

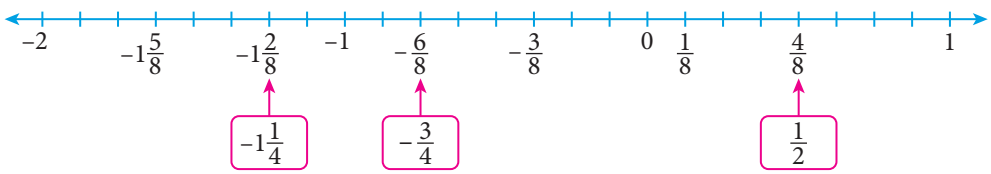
Example 10

Compare and arrange the following fractions in ascending order.

$$\frac{1}{8}, -\frac{3}{4}, -1\frac{1}{4}, \frac{1}{2}, -1\frac{5}{8}, -\frac{3}{8}$$

Solution

$$\frac{1}{8}, -\frac{3}{4}, -1\frac{1}{4}, \frac{1}{2}, -1\frac{5}{8}, -\frac{3}{8}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$\frac{1}{8}, -\frac{6}{8}, -1\frac{2}{8}, \frac{4}{8}, -1\frac{5}{8}, -\frac{3}{8}$$



Hence, the fractions arranged in ascending order are

$-1\frac{5}{8}, -1\frac{1}{4}, -\frac{3}{4}, -\frac{3}{8}, \frac{1}{8}, \frac{1}{2}$

Self Practice 1.3b

1. Compare and arrange the following fractions in ascending order.

(a) $-\frac{5}{6}, -\frac{1}{4}, \frac{3}{8}, -\frac{5}{12}, \frac{7}{24}$ (b) $\frac{1}{3}, -\frac{5}{6}, \frac{5}{8}, -\frac{2}{3}, -\frac{13}{18}, -\frac{15}{24}$

2. Compare and arrange the following fractions in descending order.

(a) $\frac{3}{5}, -\frac{7}{20}, -\frac{5}{12}, -\frac{1}{8}, \frac{5}{6}$ (b) $-\frac{1}{2}, \frac{2}{9}, -\frac{5}{9}, -\frac{7}{12}, -\frac{7}{18}, \frac{11}{18}$

How do you perform computations involving combined basic arithmetic operations of positive and negative fractions?

LEARNING STANDARDS

Perform computations involving combined basic arithmetic operations of positive and negative fractions by following the order of operations.

Example 11

Solve

(a) $1\frac{2}{3} \times \left(\frac{2}{5} - \frac{5}{6}\right)$ (b) $\frac{5}{8} + 1\frac{1}{3} \div \left(-\frac{5}{6}\right)$

Solution

(a) $1\frac{2}{3} \times \left(\frac{2}{5} - \frac{5}{6}\right)$

$= \frac{5}{3} \times \left(\frac{12 - 25}{30}\right)$ ← Calculation in the brackets is performed first.

$= \frac{5^{1}}{3} \times \left(-\frac{13}{\cancel{30}_{6}}\right)$

$= -\frac{13}{18}$

(b) $\frac{5}{8} + 1\frac{1}{3} \div \left(-\frac{5}{6}\right)$ ← Division is performed first.

$= \frac{5}{8} + \frac{4}{\cancel{3}_{1}} \times \left(-\frac{\cancel{6}^{2}}{5}\right)$ ← Change ÷ to × and the reciprocal of $-\frac{5}{6}$ is $-\frac{6}{5}$.

$= \frac{5}{8} + \left(-\frac{8}{5}\right)$

$= \frac{25}{40} - \frac{64}{40}$

$= -\frac{39}{40}$

Follow the order of operations

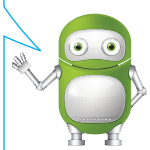
( )

↓

× or ÷

↓

+ or −



Self Practice 1.3c

1. Evaluate each of the following.

(a) $1\frac{1}{6} \times \left(\frac{3}{4} + \frac{1}{5}\right)$ (b) $-\frac{5}{6} + 1\frac{2}{3} \div \left(-\frac{3}{7}\right)$ (c) $-2\frac{1}{2} \div \left(-3\frac{1}{3}\right) + \left(-\frac{1}{6}\right)$

(d) $-6 \times \left(3\frac{2}{7} - 4\frac{1}{2}\right)$ (e) $-\frac{1}{3} + 2\frac{5}{6} - \frac{3}{8} \times 1\frac{2}{3}$ (f) $-\frac{1}{4} + \left(-\frac{4}{5}\right) \times 2\frac{1}{6} - \frac{5}{16}$

How do you solve problems?

MATHEMATICS APPLICATION ZONE

A mathematics quiz contains 20 questions. A score of 2 marks is awarded for every correct answer and a score of $-\frac{1}{2}$ mark is given for every incorrect answer. Mei Ling participated in the quiz and answered all the questions. Her score for incorrect answers was -4. What was the total score Mei Ling obtained in the quiz?

LEARNING STANDARDS

Solve problems involving positive and negative fractions.

Solution

Understanding the problem

- A score of 2 marks is awarded for every correct answer.
- A score of $-\frac{1}{2}$ mark is given for every incorrect answer.
- Score for incorrect answers $= -4$
- Find the total score obtained.

Think Smart

$-\frac{1}{2}$ is given for every incorrect answer.
What does it mean by $-\frac{1}{2}$?

Devising a plan

- $+2$ represents the score for every correct answer.
- $-\frac{1}{2}$ represents the score for every incorrect answer.
- Find the total number of incorrect answers using division.
- Find the total score using multiplication and addition.

Implementing the strategy

$$\text{Number of incorrect answers} = -4 \div \left(-\frac{1}{2}\right)$$
$$= 8$$

$$\text{Total score} = (20 - 8) \times 2 + (-4)$$
$$= 12 \times 2 - 4$$
$$= 24 - 4$$
$$= 20$$

Doing reflection

$$\text{Total score for correct answers} = 12 \times 2$$
$$= 24$$

$$\text{Total score for incorrect answers} = \overset{4}{\cancel{8}} \times \left(-\frac{1}{\cancel{2}_{1}}\right)$$
$$= -4$$

$$\text{Total score obtained} = 24 + (-4)$$
$$= 20$$

SMART TIPS

Problem-solving steps:

Understanding the problem
↓
Devising a plan
↓
Implementing the strategy
↓
Doing reflection

Self Practice 1.3d

1. A baker usually uses $3\frac{3}{4}$ cups of sugar to bake a sponge cake. He reduces the amount of sugar by $1\frac{1}{2}$ cups for a less-sweet sponge cake. A customer orders 3 sponge cakes and 5 less-sweet sponge cakes. How many cups of sugar are required to bake the cakes the customer has ordered?

2. Adam had RM40. Susan gave $\frac{1}{3}$ of her money to Adam. After giving $\frac{1}{8}$ of his money to Gopal, Adam still had RM350. Find the total amount Susan had originally.

Mastery Q 1.3

Open the folder downloaded from page vii for extra questions of Mastery Q 1.3.

1. Using **three** different fractions and **two** different operations (+, –, ×, ÷), write three calculations which give an answer of $-\frac{1}{2}$.

2. For each of the following number patterns, complete it using a suitable fraction.

(a) $-\frac{1}{3}, \frac{1}{6}, \square, 1\frac{1}{6}$

(b) $-\frac{5}{8}, \frac{5}{24}, -\frac{5}{72}, \square$

3. For each of the following, complete it using a suitable fraction.

(a) $-\frac{1}{3} - \frac{5}{6} = \frac{1}{2} \times \left(\square\right)$

(b) $-\frac{2}{3} + \frac{5}{8} = \frac{1}{3} \div \left(\square\right)$

4. The water level in a tank was $2\frac{2}{5}$ m at 4:00 p.m. The water level dropped by $\frac{1}{6}$ m every hour for 5 subsequent hours. When it was towards 12:00 midnight, the water level rose by $1\frac{2}{3}$ m. Calculate the water level at midnight.

5. Container A contains 60 ml of water. $\frac{3}{8}$ of the water in container B is poured into container A. $\frac{5}{12}$ of the water in container A is then poured into an empty container C. If container C contains 45 ml of water now, find the volume of water in container B.

1.4 Positive and Negative Decimals

How do you represent positive and negative decimals on a number line?

LEARNING STANDARDS

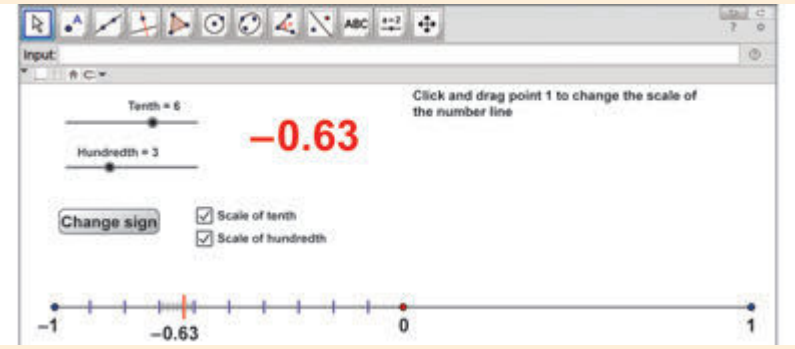
Represent positive and negative decimals on number lines.

Exploration Activity 7

Flip Classroom

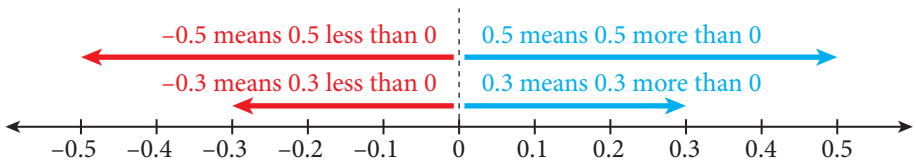
Aim: To explore the representation of positive and negative decimals on a number line.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.



1. Open the file *Decimal number line.ggb* using *GeoGebra*.
2. Click and drag the slider 'Tenth' and 'Hundredth' to define a decimal.
3. Click at the button 'Change Sign' to interchange between positive and negative.
4. Observe the position of the decimal defined on the number line.
5. Discuss the position of a decimal on a number line.

Representation of decimals on a number line is the same as that of integers and fractions. **Positive decimals** are decimals more than zero whereas **negative decimals** are decimals less than zero.



Example 12

Represent the following decimals on a number line.

0.7, –0.5, –0.8, 0.2, –1.3

Solution

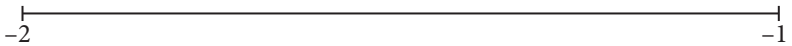
Self Practice 1.4a

1. For each of the following, represent the decimals on a number line.

(a) 0.6, –0.7, 0.2, –0.3 (b) 0.7, –0.4, 0.3, –1.3

2. The diagram below shows a straight line of length 10 cm representing a number line. Copy the diagram and represent the following decimals on the number line.

(a) –1.46, –1.84, –1.20, –1.62



(b) –0.25, –0.08, –0.39, –0.17

How do you compare and arrange positive and negative decimals in order?

LEARNING STANDARDS

Compare and arrange positive and negative decimals in order.

The values of two or more decimals can be compared and arranged in ascending order or descending order.

Example 13

Compare and arrange the following decimals in descending order.

–1.6, 0.5, –0.3, 1.4, –0.7

Solution

The decimals arranged in descending order are 1.4, 0.5, –0.3, –0.7, –1.6

Self Practice 1.4b

1. Compare and arrange each of the following in ascending order.

(a) –1.23, –1.48, 0.34, –0.034, 1.034

(b) –1.456, –1.546, 1.456, –1.654, 1.564

2. Compare and arrange each of the following in descending order.

(a) –2.005, –2.505, –2.052, 2.522, 2.452

(b) 0.065, –0.647, –0.639, –0.068, 0.621

How do you perform computations involving combined basic arithmetic operations of positive and negative decimals?

LEARNING STANDARDS

Perform computations involving combined basic arithmetic operations of positive and negative decimals by following the order of operations.

Example 14

Evaluate each of the following:

(a) $3.5 - (-6.5) \times 0.2$

(b) $(7.23 + 2.77) \div (-0.8)$

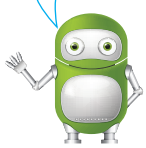
(c) $-3.7 + (4.25 + 2.85) \times 0.3$

Solution

(a) $3.5 - (-6.5) \times 0.2 = 3.5 - (-1.3)$
$= 3.5 + 1.3$
$= 4.8$

Multiplication is performed first.

Follow the order of operations
( )
↓
× or ÷
↓
+ or –



(b) $(7.23 + 2.77) \div (-0.8) = 10.0 \div (-0.8)$
$= -12.5$

Calculation in the brackets is performed first.

(c) $-3.7 + (4.25 + 2.85) \times 0.3 = -3.7 + (7.1 \times 0.3)$
$= -3.7 + 2.13$
$= -1.57$

Calculation in the brackets is performed first followed by multiplication.

Self Practice 1.4c

1. Evaluate each of the following:

(a) $4.7 - 7.8 \times 0.3$

(b) $(8.36 + 3.89) \div (-0.28)$

(c) $-3.48 + (7.36 + 1.24) \times 0.6$

(d) $0.36 - (-8.67) \div (-0.3) + 0.82$

(e) $-2.65 - \dfrac{1.44}{-1.2} + 3.22$

(f) $2.34 + 3.1 \times (-0.1) + 0.2$

How do you solve problems?

LEARNING STANDARDS

Solve problems involving positive and negative decimals.

MATHEMATICS APPLICATION ZONE

The price of the stock of a company was RM2.05 at a certain time. The price hiked by RM0.32, then subsequently dropped RM0.28 every hour for the next three hours. Calculate the final price of the stock.

Solution

Understanding the problem

- Price of stock was RM2.05.
- Price of stock hiked by RM0.32.
- Price of stock dropped RM0.28 every hour for the next three hours.
- Calculate the final price of the stock.

Devising a plan

- Increase in price is written as +0.32.
- Decrease in price is written as –0.28.
- Use multiplication and addition.

Implementing the strategy

The final price of the stock $= 2.05 + 0.32 + 3 \times (-0.28)$
$= 2.37 + (-0.84)$
$= 2.37 - 0.84$
$= 1.53$

The final price of the stock was RM1.53.

Doing reflection

RM2.05 + RM0.32 – 3 × RM0.28
= RM2.37 – RM0.84
= RM1.53

Communication Corner

Explain the importance of negative numbers in monetary finance.

Self Practice 1.4d

1. Aisah bought a shirt for RM19.90 and two pairs of long trousers of the same price. When she paid RM55 to the cashier, she was told that the amount was not enough. Aisah then paid another RM10 and received a change of RM5.40. Calculate the price of a pair of long trousers that she bought.

2. The average temperature in Kuala Lumpur was 30.5°C on a certain day. The average temperature then rose by 1.8°C every day for two consecutive days and then dropped by 1.3°C every day for another three consecutive days. Calculate the average temperature in Kuala Lumpur during those five days.

Mastery Q 1.4

Open the folder downloaded from page vii for extra questions of Mastery Q 1.4.

1. Using **three** different decimals and **two** different operations (+, –, ×, ÷), write three calculations which each gives an answer of –2.5.

2. For each of the following number patterns, complete it using suitable decimals.

(a) $-1.2, -0.9, \square, -0.3, \square$

(b) $-2.1, \square, -8.4, 16.8, \square$

3. For each of the following, complete it using a suitable decimal.

(a) $3.2 \times (-2.1) + 5.8 = 0.5 \times (\square)$

(b) $-5.12 - (-2.4) \div (-0.5) = 1.6 \times (\square)$

4. Ramesh bought 63 oranges for RM34.65. The oranges were packed in small packets with 3 oranges in each packet. Calculate the price Ramesh sold for each packet of oranges if he had

(a) incurred a loss of RM19.95

(b) made a profit of RM51.45

after he sold all the oranges.

5. A fish is at 1.34 m below sea level while a bird is at 4.32 m above sea level. A turtle is below sea level at a vertical distance that is twice the distance between the fish and the bird. Calculate the vertical distance between the bird and the turtle.

1.5 Rational Numbers

LEARNING STANDARDS

Recognise and describe rational numbers.

What are rational numbers?

Exploration Activity 8

Group

Aim: To recognise and describe rational numbers.
Instruction: Perform the activity in groups of four.

1. You are given some number cards as follows.

0.3	-1.25	$1\frac{1}{3}$	5	-8	$-2\frac{2}{5}$

2. Discuss how you would write these numbers in the form of $\frac{p}{q}$ such that p and q are integers.

3. What are your conclusions?

Numbers that can be written in fractional form, that is $\frac{p}{q}$, such that p and q are integers, $q \neq 0$, are known as **rational numbers**.

Think Smart

Explain why a rational number $\frac{p}{q}$ is subjected to the condition $q \neq 0$.

Example 15

Determine whether $1\frac{4}{5}$, $\frac{3}{4}$, -9 and 3.5 are rational numbers.

Solution

$1\frac{4}{5} = \frac{9}{5}$, $\frac{3}{4}$, $-9 = \frac{-9}{1}$, $3.5 = 3\frac{5}{10}$

$= 3\frac{1}{2}$

$= \frac{7}{2}$

All the numbers can be written in the form of $\frac{p}{q}$.

Thus, $1\frac{4}{5}$, $\frac{3}{4}$, -9 and 3.5 are rational numbers.

Think Smart

Is 3.141592654… a rational number? Explain.

Self Practice 1.5a

1. Determine whether the following numbers are rational numbers. Explain your answer.

$\frac{-2}{4}$, $\frac{8}{7}$, $\frac{-1.2}{1.5}$, 7.65, $2\frac{2}{5}$, -4.2

How do you perform computations involving combined basic arithmetic operations of rational numbers?

LEARNING STANDARDS

Perform computations involving combined basic arithmetic operations of rational numbers by following the order of operations.

Example 16

Solve each of the following:

(a) $-0.4 + 1\frac{1}{2} \times \left(-\frac{1}{8}\right)$ (b) $\left[18 \times \left(-\frac{7}{12}\right) + 1.5\right] \div 0.3$

Solution

(a) $-0.4 + 1\frac{1}{2} \times \left(-\frac{1}{8}\right)$

$= -\frac{4}{10} + \frac{3}{2} \times \left(-\frac{1}{8}\right)$ ← Convert decimals into fractions first.

$= -\frac{4}{10} + \left(-\frac{3}{16}\right)$

$= -\frac{32}{80} - \frac{15}{80}$

$= -\frac{47}{80}$

Follow the order of operations

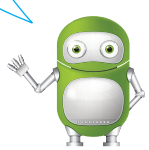
( )

↓

× or ÷

↓

+ or −



(b) $\left[18 \times \left(-\frac{7}{12}\right) + 1.5\right] \div 0.3$

$= \left[\frac{\overset{3}{\cancel{18}}}{1} \times \left(-\frac{7}{\cancel{12}}\right) + \frac{3}{2}\right] \div \frac{3}{10}$

$= \left(-\frac{21}{2} + \frac{3}{2}\right) \div \frac{3}{10}$

$= -\frac{18}{2} \div \frac{3}{10}$

$= -\frac{\overset{3}{\cancel{9}}}{1} \times \frac{10}{\cancel{3}_{1}}$

$= -30$

Self Practice 1.5b

1. Evaluate each of the following:

(a) $-0.6 + \frac{3}{4} \times \left(-1\frac{3}{5}\right)$

(b) $\left(-\frac{17}{20} + 0.8\right) \div \left(\frac{1}{2} - 1.3\right)$

(c) $1.125 + 1\frac{2}{3} - 2\frac{5}{6} \times \left(-\frac{8}{27}\right)$

(d) $-3.25 \div \frac{13}{15} - \left(-2\frac{1}{6}\right) \times 0.25$

How do you solve problems?

MATHEMATICS APPLICATION ZONE

Noriah has a savings of RM120. She donates $\frac{3}{8}$ of her savings to flood victims. She then buys a pair of school shoes for RM25.60. Calculate the balance she still has.

Solution

Amount donated $= \frac{3}{8} \times$ RM120.00

$=$ RM45.00

Balance she still has $=$ RM120.00 – RM45.00 – RM25.60

$=$ RM49.40

LEARNING STANDARDS

Solve problems involving rational numbers.

Career in Mathematics

An accountant uses the knowledge of rational numbers to do efficient computations.

Self Practice 1.5c

1. A company donates to charity every year as its contribution to society. If the company makes a profit in that year, $\frac{2}{9}$ of the profit will be allocated for donation.

If the company incurs a loss, the company will also allocate 0.05 of the loss for donation. If the company makes a profit of RM43.2 million in a certain year and incurs a loss of RM2.5 million and RM6.5 million respectively in two consecutive years, calculate the total donations the company would have allocated for charity in those three years.

2. A roll of ribbon is used to tie 12 gifts which will be given to teachers on Teacher's Day. Every gift requires a length of 1.85 m. After tying all the gifts, it was found that $\frac{2}{3}$ of the ribbon had been used. The remaining ribbon was cut into 12 pieces of the same length. Calculate the length of each piece of ribbon that had been cut.

Mastery Q 1.5

Open the folder downloaded from page vii for extra questions of Mastery Q 1.5.

1. Evaluate each of the following.

(a) $2.5 + (-8) \div \frac{6}{5} \times 3.5$

(b) $\left(\frac{1}{4} + 3.2 \times 2\right) - \left(5.4 - \frac{2}{3} \div 0.04\right)$

2. For each of the following number patterns, complete it using suitable rational numbers.

(a) $-2.4, -\frac{7}{2}, -4.6, \square, \square$

(b) $-\frac{1}{2}, -0.25, \square, -\frac{1}{16}, -0.03125$

3. For each of the following, complete it using a suitable rational number.

(a) $6.8 \div \frac{2}{5} - 4.62 = \square \times 0.01$

(b) $3.76 + \frac{3}{4} \times (-4.5) = \square \times 0.5$

4. Ishak, Jia Kang and Suresh went mountain-climbing together. At a certain instance, Ishak was at a level 1.45 m higher than Jia Kang while Suresh was at a level $2\frac{1}{3}$ m lower than Jia Kang. Ishak, Jia Kang and Suresh had climbed 1.25 m, 0.5 m and $3\frac{3}{4}$ m respectively. Find the positions of Jia Kang and Suresh now with reference to the position of Ishak.

SUMMARY

RATIONAL NUMBERS

Integers

- Positive integers 1, 2, 3, 4, …
- Zero, 0
- Negative integers …, –4, –3, –2, –1

Fractions

- Positive fractions
  Example: $\frac{1}{2}, \frac{7}{4}, 1\frac{1}{5}$
- Negative fractions
  Example: $-\frac{1}{3}, -\frac{9}{2}, -4\frac{1}{2}$

Decimals

- Positive decimals
  Example: 0.5, 4.3, 3.24
- Negative decimals
  Example: –0.1, –5.5, –7.65

At the end of this chapter, I can...	Very good	Work harder
recognise positive and negative numbers based on real-life situations.		
recognise and describe integers and rational numbers.		
represent integers, positive fractions, negative fractions, positive decimals and negative decimals on number lines.		
compare and arrange integers, positive fractions, negative fractions, positive decimals and negative decimals in order.		
add and subtract integers using number lines or other appropriate methods. Hence, make generalisation about addition and subtraction of integers.		
multiply and divide integers using various methods. Hence make generalisation about multiplication and division of integers.		
perform computations involving combined basic arithmetic operations of integers, positive and negative fractions, positive and negative decimals and rational numbers by following the order of operations.		
describe the laws of arithmetic operations which are Identity Law, Commutative Law, Associative Law and Distributive Law.		
perform efficient computations using the laws of basic arithmetic operations.		
solve problems involving integers, positive and negative fractions, positive and negative decimals and rational numbers.		

Let's PRACTISE

Test Yourself

1. From the following, choose the correct calculation step for

$$5(-3 + 10) \times 2.4 \div \frac{3}{4}$$

A $5(-7) \times 3.2$

B $35 \times 2.4 \times \frac{4}{3}$

C $35 \times 2.4 \times \frac{3}{4}$

D -35×3.2

2. Determine the number with a larger value without making any calculations.

(a) $-\frac{1}{2}, \frac{1}{100}$ (b) $-4.3, -4.5$ (c) $2\frac{2}{5}, 2.5$

3.

Team	Goals scored	Goals conceded	Goal difference
Tiger	20	17	3
Eagle	16	18	

The table shows the number of goals scored and the number of goals conceded for two soccer teams. Find the goal difference for the Eagle Team.

4. A treasure chest hidden in the year 56 B.C. was found in the year 292 A.D. For how many years was the treasure chest hidden?

Self Mastery

5. When doing charity work, Ali gives rice, sugar and biscuits to 80 fire victims. If each victim gets 2 kg of rice, $\frac{1}{2}$ kg of sugar and 0.4 kg of biscuits and these alms are equally transported using three vans, explain how you would find the mass of alms transported by a van. Give your answer correct to two decimal places.

6. Fill in the boxes with '+' or '–' so that the answer obtained has the largest value.

(a) $\frac{1}{2}$ ☐ 5 ☐ 4.3 (b) -4.2 ☐ $\frac{1}{2}$ ☐ -4

7. The temperature of a place at sea level is 8°C. The temperature will drop by 3°C for every km above sea level. Calculate the temperature of the place at 5 km above sea level.

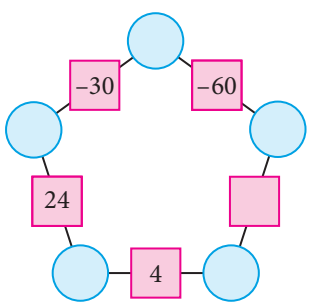
8. Sheila was at point *O* at a certain instance. She moved 1.85 m towards the left and then moved 4 steps measuring 0.65 m each towards the right. Calculate the position of Sheila now from point *O*.

Challenge Yourself

9. Jasmin moves 9.5 m towards east, then she moves 10.7 m towards west and then 6.8 m towards east. Describe the movement of Jasmin so that she can go back to her original position.

10. A lift was at level H at a certain instance. The lift went up two floors for a height of 9.8 m. The lift then went down 5 floors. Calculate the distance of the lift from level H now.

11. In the diagram, numbers in two collinear circles will multiply to give the product in the collinear middle square box. Complete the empty spaces with suitable rational numbers.



ASSIGNMENT

Negative numbers are used in the making of glasses to correct eyesight problems. Get information from valid sources such as conducting interviews with an optician, referring to reading materials or exploring other suitable sources. Write a report to explain how negative numbers are used in making the lenses of glasses.

Exploring MATHEMATICS

Do you know that positive and negative numbers can be generated using a spreadsheet program? Follow the steps as follows to generate a particular list of numbers.

	A	B	C	D	E	F	G	H
1	–6	2	10	18	26	34		
2	–8							
3	–10							
4	–12							
5	–14							
6	–16							
7								

1. Run a suitable spreadsheet program.

2. Type –6 into cell A1, as shown in the diagram.

3. Select cell A2 and type =A1-2.

4. Select cell A2 and drag the fill handle at the lower right corner down. Describe what you observe.

5. Select cell B1 and type =A1+8.

6. Select cell B1 and drag the fill handle at the lower right corner to the right. Describe what you observe.

7. Change the number in cell A1 to another number. Describe what you observe.

8. Explain how you can generate other lists of numbers.

CHAPTER 2

Factors and Multiples

What will you learn?

- Factors, Prime Factors and Highest Common Factor (HCF)
- Multiples, Common Multiples and Lowest Common Multiple (LCM)

Why study this chapter?

Factors and multiples are used in daily situations which involve numbers that are multiplied or numbers that are obtained from arrangement of rows or groups. Discuss daily situations that involve factors and multiples.

How would the knowledge of highest common factor help a construction worker determine the biggest size of square tiles required to fully cover the entire floor?

How does the knowledge of lowest common multiple help a musician understand the rhythm of music better?

Walking through Time

Eratosthenes

Eratosthenes (276 B.C. – 194 B.C.) was a Greek mathematician who introduced the Sieve of Eratosthenes which is used to filter out all prime numbers within a given range of numbers.

For more information:

https://goo.gl/WiMswX

Word Link

• factor	• *faktor*
• prime factor	• *faktor perdana*
• common factor	• *faktor sepunya*
• highest common factor/ greatest common divisor	• *faktor sepunya terbesar*
• multiple	• *gandaan*
• common multiple	• *gandaan sepunya*
• lowest common multiple	• *gandaan sepunya terkecil*
• prime factorisation	• *pemfaktoran perdana*

Open the folder downloaded from page vii for the audio of Word Link.

2.1 Factors, Prime Factors and Highest Common Factor (HCF)

What are factors?

LEARNING STANDARDS

Determine and list the factors of whole numbers, hence make generalisation about factors.

Exploration Activity 1

Individual

Aim: To identify the factors of a number.

1. Copy and complete the following boxes.

$12 \div 1 =$ ☐	$12 \div 2 =$ ☐	$12 \div 3 =$ ☐
$12 \div 4 =$ ☐	$12 \div 6 =$ ☐	$12 \div 12 =$ ☐

Thus, 12 can be divided completely by ☐

2. Explain the conclusion you arrive at.

From the results of Exploration Activity 1, it is found that a number can be divided completely by certain numbers.
For example, 12 can be **divided completely** by 1, 2, 3, 4, 6 and 12.
Therefore, numbers 1, 2, 3, 4, 6 and 12 are known as **factors** of 12.

Factors of a number are whole numbers that can divide the number completely.

Example 1

Determine whether
(a) 12 is a factor of 36.
(b) 9 is a factor of 30.

Solution

(a) $36 \div 12 = 3$
Thus, 12 is a factor of 36.

(b) 30 cannot be divided completely by 9.
Thus, 9 is not a factor of 30.

Example 2

List all the factors of 18.

Solution

$18 = 1 \times 18$ Thus, 1 and 18 are factors of 18.
$18 = 2 \times 9$ Thus, 2 and 9 are factors of 18.
$18 = 3 \times 6$ Thus, 3 and 6 are factors of 18.

Therefore, factors of 18 are 1, 2, 3, 6, 9 and 18.

SMART TIPS

1 is a factor of all numbers.

Self Practice 2.1a

1. Determine whether each of the following numbers is a factor of the number in the brackets.

(a) 12 (40) (b) 5 (50) (c) 8 (24) (d) 6 (90)
(e) 3 (45) (f) 14 (56) (g) 15 (60) (h) 18 (98)

2. List all the factors of each of the following numbers.

(a) 15 (b) 32 (c) 40 (d) 48
(e) 51 (f) 87 (g) 98 (h) 124

What are prime factors?

LEARNING STANDARDS

Determine and list the prime factors of a whole number, hence express the number in the form of prime factorisation.

Flashback

Which of the following numbers are prime numbers?

2, 9, 23, 41, 57, 69

Factors of 18 are 1, 2, 3, 6, 9 and 18. Among these factors, 2 and 3 are prime numbers. Thus, 2 and 3 are known as **prime factors** of 18.

A number can be expressed in the form of **prime factorisation** in which the number is written as the product of its prime factors.
For instance, $18 = 2 \times 3 \times 3$

Prime factorisation of a number can be done by **repeated division** by prime numbers or by using a **factor tree**.

Scan the QR Code or visit https://youtu.be/V4tiNgMPYsU to learn about the use of Eratosthenes method in determining prime numbers.

Example 3

Determine whether each of the following numbers is a prime factor of 84.

(a) 2 (b) 3 (c) 5

Solution

(a) $84 \div 2 = 42$
2 is a prime number. Thus, 2 is a prime factor of 84.

(b) $84 \div 3 = 28$
3 is a prime number. Thus, 3 is a prime factor of 84.

(c) $84 \div 5 = 16$ remainder 4
5 is a prime number but 84 cannot be divided completely by 5.
Thus, 5 is not a prime factor of 84.

Example 4

List all the prime factors of 72.

Solution

Factors of 72 are 1, (2), (3), 4, 6, 8, 9, 12, 18, 24, 36, 72

Prime numbers

1. List all the factors of 72.
2. Identify the prime numbers from the list.

Thus, prime factors of 72 are 2 and 3.

Example 5

Express 60 in the form of prime factorisation.

Solution

Method of repeated division

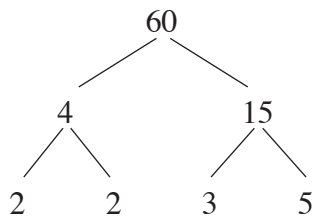
2	60
2	30
3	15
5	5
	1

1. Perform division repeatedly by dividing with the smallest prime number.
2. Division is continued until the quotient is 1.

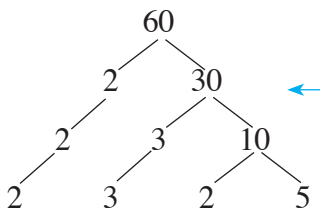
Scan the QR Code or visit https://youtu.be/eJLjA6uMm4g to learn more about prime factorisation of a number.

Thus, 60 = $2 \times 2 \times 3 \times 5$

Method of drawing a factor tree



or



60 is written as the product of two factors continuously until all the factors at the bottom level are prime factors.

Thus, 60 = $2 \times 2 \times 3 \times 5$

Self Practice 2.1b

1. Determine whether
(a) 3, 5 and 7 are prime factors of 30.
(b) 3, 5 and 9 are prime factors of 54.

2. List all the prime factors of each of the following numbers.
(a) 36 (b) 48 (c) 58 (d) 99

3. Express each of the following numbers in the form of prime factorisation.
(a) 42 (b) 96 (c) 120 (d) 135

What are common factors?

LEARNING STANDARDS

Explain and determine the common factors of whole numbers.

Exploration Activity 2

Pair

Aim: To explain and determine common factors.
Instruction: Perform the activity in pairs.

1. List all the factors of 18 and 24.
2. Are there numbers in the list which are factors of 18 as well as 24?
3. Discuss with your friends and explain the properties of these numbers in relation to 18 and 24.

From the results of Exploration Activity 2, it is found that 1, 2, 3 and 6 are factors of both 18 and 24. It means that both 18 and 24 can be divided completely by 1, 2, 3 and 6. Therefore, 1, 2, 3 and 6 are known as **common factors** of 18 and 24.

Example 6

Determine whether
(a) 6 is a common factor of 24 and 36
(b) 4 is a common factor of 30, 40 and 60
(c) 8 is a common factor of 16, 32, 48 and 72

Solution

(a) $24 \div 6 = 4$
$36 \div 6 = 6$
24 and 36 can be divided completely by 6.
Thus, 6 is a common factor of 24 and 36.

(b) $30 \div 4 = 7$ remainder 2
$40 \div 4 = 10$
$60 \div 4 = 15$
30 cannot be divided completely by 4.
Thus, 4 is not a common factor of 30, 40 and 60.

(c) $16 \div 8 = 2$
$32 \div 8 = 4$
$48 \div 8 = 6$
$72 \div 8 = 9$
16, 32, 48 and 72 can be divided completely by 8.
Thus, 8 is a common factor of 16, 32, 48 and 72.

Example 7

List all the common factors of 20, 44, 56 and 64.

SMART TIPS

1 is a common factor of all numbers.

Solution

Factors of 20 : (1), (2), (4), 5, 10, 20
Factors of 44 : (1), (2), (4), 11, 22, 44
Factors of 56 : (1), (2), (4), 7, 8, 14, 28, 56
Factors of 64 : (1), (2), (4), 8, 16, 32, 64

Thus, common factors of 20, 44, 56 and 64 are 1, 2 and 4.

Self Practice 2.1c

1. Determine whether each of the following numbers is a common factor of the list of numbers in the brackets.

(a) 6 (12, 24, 42) (b) 8 (8, 32, 72) (c) 3 (12, 18, 20)
(d) 12 (48, 96, 120, 144) (e) 7 (28, 35, 91, 105) (f) 9 (18, 36, 108, 119)

2. List all the common factors of each of the following lists of numbers.

(a) 12, 30 (b) 24, 48 (c) 35, 105
(d) 12, 84, 98 (e) 35, 90, 150 (f) 12, 72, 96
(g) 42, 102, 144, 200 (h) 84, 132, 300, 360 (i) 110, 210, 360, 540

What is highest common factor (HCF)?

LEARNING STANDARDS
Determine the HCF of two and three whole numbers.

We have learnt that common factors of 20, 44, 56 and 64 are 1, 2 and 4. Among these common factors, 4 is the greatest number. Therefore, the number 4 is known as the **highest common factor (HCF)** of 20, 44, 56 and 64.

HCF can be determined by using the methods of
- listing the common factors
- repeated division
- prime factorisation

Example 8

Determine the highest common factor of
(a) 18 and 24 (b) 36, 60 and 72 (c) 48, 64 and 80

Solution

(a) **Method of listing the common factors**

Factors of 18 : (1), (2), (3), (6), 9, 18

Factors of 24 : (1), (2), (3), 4, (6), 8, 12, 24

Thus, the common factors of 18 and 24 are 1, 2, 3 and 6.
Therefore, the HCF of 18 and 24 is 6.

(b) **Method of repeated division**

2	36	,	60	, 72
2	18	,	30	, 36
3	9	,	15	, 18
	3	,	5	, 6

Divide the given numbers by a common factor repeatedly. Then determine the HCF by multiplying all the divisors.

Thus, the HCF of 36, 60 and 72 is $2 \times 2 \times 3 = 12$

(c) **Method of prime factorisation**

$48 = 2 \times 2 \times 2 \times 2 \times 3$
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$80 = 2 \times 2 \times 2 \times 2 \times 5$

Find the prime factorisation of each given number. Then determine the HCF by multiplying all the common prime factors.

Thus, the HCF of 48, 64 and 80 is $2 \times 2 \times 2 \times 2 = 16$

Self Practice 2.1d

1. By using a suitable method, find the highest common factor of each of the following lists of numbers.

(a) 24, 48 (b) 18, 54 (c) 36, 96 (d) 30, 72
(e) 36, 90, 108 (f) 42, 54, 84 (g) 140, 256, 348 (h) 27, 45, 60

How do you solve problems?

MATHEMATICS APPLICATION ZONE

LEARNING STANDARDS

Solve problems involving HCF.

The Boy Scouts of a school held a charity activity. A total of 252 shirts, 180 pairs of trousers and 108 pairs of socks were donated by members to an orphanage. All the items were divided equally in each pack. What would be the maximum number of packs that were prepared?

Solution

Understanding the problem

252 shirts, 180 pairs of trousers and 108 pairs of socks were divided equally in each pack. Find the maximum number of packs that were prepared.

Devising a plan

Find the HCF of 252, 180 and 108.

Implementing the strategy

$252 = 2 \times 2 \times 3 \times 3 \times 7$
$180 = 2 \times 2 \times 3 \times 3 \times 5$
$108 = 2 \times 2 \times 3 \times 3 \times 3$

The HCF of 252, 180 and 108 is $2 \times 2 \times 3 \times 3 = 36$.
The maximum number of packs that were prepared would be 36.

Doing reflection

$252 \div 36 = 7$
$180 \div 36 = 5$
$108 \div 36 = 3$

Self Practice 2.1e

1. The Police Cadets in a school donated 60 sacks of rice, 78 bottles of cooking oil and 90 bottles of mineral water to some flood victims in a district. All the things were divided equally in each box for the flood victims. What would be the maximum number of boxes that were prepared?

2. At a *Hari Raya* party, Mak Minah prepared 45 pieces of *kuih koci*, 75 pieces of *kuih tako* and 90 pieces of curry puffs. She wished to serve all the food on some plates. What would be the maximum number of plates needed if all the food was distributed equally on all the plates?

Mastery Q 2.1

Open the folder downloaded from page vii for extra questions of Mastery Q 2.1.

1. Find the greatest number that is a factor of both 45 and 75.

2. Write the prime factorisation of 1 968 and 4 968. Hence, find the highest common factor of 1 968 and 4 968.

3. The highest common factor of n and 36 is 4. If n is less than 36, what are the possible values of n?
4. Three pieces of string have a length of 192 cm, 242 cm and 328 cm respectively. Aishah wishes to cut the strings so that every piece is cut into the same number of segments with no extra string left. What would be the maximum number of segments that each piece of string can be cut into?
5. Anita wishes to cut a piece of paper measuring 260 cm in length and 20 cm in width into several square pieces. What would be the measurement of the square pieces so that there is no paper left?

2.2 Multiples, Common Multiples and Lowest Common Multiple (LCM)

LEARNING STANDARDS

Explain and determine the common multiples of whole numbers.

What are common multiples?

Note that $9 \times 1 = 9$
$9 \times 2 = 18$
$9 \times 3 = 27$

The number 9 is multiplied by 1, 2, 3, … to give the products of 9, 18, 27, …
9, 18, 27, … are known as **multiples** of 9.

Exploration Activity 3

Pair

Aim: To explain and determine common multiples.
Instruction: Perform the activity in pairs.

1. List the first five multiples of 6 and 8.
2. Are there numbers in the list which are multiples of 6 as well as 8?
3. Discuss with your friends and explain the properties of these numbers in relation to 6 and 8.

From the results of Exploration Activity 3, it is found that 24 is a multiple of 6 as well as 8. It means that 24 can be divided completely by 6 and 8. Therefore, the number 24 is known as a **common multiple** of 6 and 8.

Let's Discuss

How many common multiples of 6 and 8 can be written? Discuss with your friends.

Example 9

Determine whether
(a) 32 is a common multiple of 2 and 8.
(b) 60 is a common multiple of 5, 15 and 24.
(c) 72 is a common multiple of 8, 9, 18 and 24.

Solution

(a) $32 \div 2 = 16$
$32 \div 8 = 4$
← 32 can be divided completely by 2 and 8.
Thus, 32 is a common multiple of 2 and 8.

(b) $60 \div 5 = 12$
$60 \div 15 = 4$
$60 \div 24 = 2$ remainder 12 ← 60 cannot be divided completely by 24.
Thus, 60 is not a common multiple of 5, 15 and 24.

(c) $72 \div 8 = 9$
$72 \div 9 = 8$
$72 \div 18 = 4$
$72 \div 24 = 3$
← 72 can be divided completely by 8, 9, 18 and 24.
Thus, 72 is a common multiple of 8, 9, 18 and 24.

Exploration Activity 4

Group

Aim: **To explore the relationship between the first common multiple and the subsequent common multiples.**

Instruction:
- Perform the activity in groups of four.
- Open the folder downloaded from page vii.

1. Open the file *Common multiples.xls* with an electronic spreadsheet. The screen display shows a list of multiples of 3 (red) and a list of multiples of 5 (blue).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78
2	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
3																										
4																										

2. Observe the lists of multiples displayed, then write the first common multiple of 3 and 5.
3. Write three subsequent common multiples of 3 and 5.
4. Explain the relationship between the first common multiple and the three subsequent common multiples.
5. Repeat Steps **2** to **4** by changing the number in the cells A1 and A2 respectively to the following numbers:
   (a) 6 and 8 (b) 4 and 10
6. Discuss with your friends and draw a conclusion about common multiples.

From the results of Exploration Activity 4, it is found that the first common multiple of 3 and 5 is 15 and all other common multiples of 3 and 5 are multiples of 15.

Example 10

List the first five common multiples of 2, 3 and 4.

Solution

Multiples of 2 : 2, 4, 6, 8, 10, (12), 14, …
Multiples of 3 : 3, 6, 9, (12), 15, …
Multiples of 4 : 4, 8, (12), 16, …

Determine the first common multiple of 2, 3 and 4. Then determine the multiples of the first common multiple.

The first common multiple of 2, 3 and 4 is 12.
Thus, the first five common multiples of 2, 3 and 4 are 12, 24, 36, 48 and 60.

Self Practice 2.2a

1. Determine whether each of the following numbers is a common multiple of the list of numbers in the brackets.

(a) 72 (9, 12) (b) 168 (2, 9) (c) 360 (4, 6, 24)
(d) 2250 (2, 5, 9) (e) 9720 (4, 5, 10, 20) (f) 16416 (6, 9, 12, 24, 36)

2. List the first five common multiples of each of the following lists of numbers. (Use an electronic spreadsheet to help you)

(a) 2, 5 (b) 3, 5 (c) 18, 22 (d) 3, 5, 8
(e) 2, 3, 6, 8 (f) 4, 8, 12, 15 (g) 6, 15, 20, 24 (h) 4, 9, 12, 15, 18

What is lowest common multiple (LCM)?

LEARNING STANDARDS

Determine the LCM of two and three whole numbers.

We have learnt that the common multiples of 6 and 8 are 24, 48, 72, 96, … . Among these common multiples, the first common multiple, 24, is the least number. The number 24 is known as the **lowest common multiple (LCM)** of 6 and 8.

LCM can be determined by using the methods of
- listing the common multiples
- repeated division
- prime factorisation

Let's Discuss

Explain how you would use LCM to solve $\frac{1}{6} + \frac{1}{8}$.

Example 11

Determine the lowest common multiple of
(a) 2 and 3 (b) 3, 6 and 9 (c) 3, 8 and 12

Solution

(a) **Method of listing the common multiples**

Multiples of 2: 2, 4, (6), 8, 10, …
Multiples of 3: 3, (6), 9, 12, …
The lowest common multiple is 6.

Choose the lowest common multiple.

(b) **Method of repeated division**

3	3 , 6 , 9
2	1 , 2 , 3
3	1 , 1 , 3
	1 , 1 , 1

1. Divide the numbers repeatedly by a divisor that can divide completely at least one of the numbers.
2. Numbers that cannot be divided completely by the selected divisor are brought down for subsequent divisions.
3. Division is continued until all the quotients become 1.

Thus, LCM of 3, 6 and 9 is $3 \times 2 \times 3 = 18$.

(c) **Method of prime factorisation**

$3 = 3$

$8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$

$2 \times 2 \times 2 \times 3$

Find the prime factorisation of each of the given numbers.

Thus, LCM of 3, 8 and 12 is $2 \times 2 \times 2 \times 3 = 24$.

Alternative Method

3, 8, (12) ← 1. Identify the greatest number.

12, 24, 36, … ← 2. List the multiples of the greatest number. 3. Determine the lowest multiple that can be divided completely by all the other given numbers.

24 can be divided completely by 3 and 8.

Thus, LCM of 3, 8 and 12 is 24.

Self Practice 2.2b

1. By using a suitable method, find the lowest common multiple of each of the following lists of numbers.

(a) 16, 18 (b) 10, 15 (c) 6, 8 (d) 18, 60

(e) 6, 15, 18 (f) 8, 14, 32 (g) 9, 12, 21 (h) 4, 15, 60

How do you solve problems?

MATHEMATICS APPLICATION ZONE

Canned coffee is sold in 6 cans per box and canned tea is sold in 9 cans per box. Ainun wishes to buy the same number of canned coffee and canned tea for her sister's birthday party. What is the minimum number of boxes of each type of canned drinks she needs to buy?

LEARNING STANDARDS

Solve problems involving LCM.

Solution

Understanding the problem	Devising a plan	Implementing the strategy	Doing reflection
Number of cans of coffee = 6 cans per box Number of cans of tea = 9 cans per box Find the minimum number of boxes of each type of canned drinks Ainun needs to buy.	Find the LCM of 6 and 9 to determine the same number of cans. Use division to find the number of boxes of canned coffee and canned tea.	Multiples of 6: 6, 12, (18), 24, 30, … Multiples of 9: 9, (18), 27, 36, 45, … Thus, LCM of 6 and 9 = 18 Number of boxes of canned coffee = 18 ÷ 6 = 3 Number of boxes of canned tea = 18 ÷ 9 = 2 Therefore, the minimum number of boxes Ainun needs to buy is 3 boxes of canned coffee and 2 boxes of canned tea.	Number of canned coffee = 3 × 6 = 18 Number of canned tea = 2 × 9 = 18

Self Practice 2.2c

1. A neon light blinks every 9 seconds whereas another neon light blinks every 12 seconds. If both the lights blink together when the switch is on, after how many seconds will both the neon lights blink together again?

2. Vijaya wishes to pack some souvenirs consisting of a key chain and a refrigerator magnet in each pack for her friends. Key chains are sold in 10 pieces per packet whereas refrigerator magnets are sold in 6 pieces per packet. What is the minimum number of key chains Vijaya has to buy so that the number of refrigerator magnets she buys can be packed exactly with all the key chains?

Mastery Q 2.2

Open the folder downloaded from page vii for extra questions of Mastery Q 2.2.

1. Find the smallest number that can be divided completely by 8 and 20.

2. The lowest common multiple of n and 6 is 72. Find the value of n.

3. Sharif likes exercising. He goes hill-climbing every 12 days and goes swimming every 8 days. If he goes hill-climbing and swimming today, how many days later will he go hill-climbing and swimming on the same day again?

4. Nurul, Hui Ling and Gopal were each given a piece of ribbon of the same length. They cut their own ribbons into several pieces of the same length. The pieces of ribbon cut by Nurul, Hui Ling and Gopal measured 15 cm, 25 cm and 30 cm long respectively. All the ribbons were cut and there were no remainders left. What was the shortest possible length of the ribbon given to them?

SUMMARY

Factors

Factors of 12:
1, 2, 3, 4, 6, 12
Factors of 18:
1, 2, 3, 6, 9, 18

↓

Common factors

Common factors of 12 and 18:
1, 2, 3, 6

↓

Highest common factor

Highest common factor
of 12 and 18:
6

Multiples

Multiples of 6:
6, 12, 18, 24, 30, …
Multiples of 8:
8, 16, 24, 32, 40, …

↓

Common multiples

Common multiples of 6 and 8:
24, 48, 72, 96, 120, …

↓

Lowest common multiple

Lowest common multiple
of 6 and 8:
24

At the end of this chapter, I can...	Very good	Work harder
determine and list the factors of a whole number, and subsequently make a generalisation about factors.		
determine and list the prime factors of a whole number and subsequently express the number in the form of prime factorisation.		
explain and determine common factors of a whole number.		
determine the HCF of two or three whole numbers.		
solve problems involving HCF.		
explain and determine common multiples of a whole number.		
determine the LCM of two or three whole numbers.		
solve problems involving LCM.		

Let's PRACTISE

Test Yourself

1. Find the LCM of 2, 3, 4, 5 and 6.

2. Find the HCF of 36, 42, 56, 72 and 96.

Self Mastery

3. A number can be divided completely by 12 and 30. What is the smallest value of the number?

4. The HCF of m and 54 is 6. Find the largest value of m such that the value of m is less than 54.

5. The LCM of 36, 56 and n is 1 512. What is the smallest value of n?

Challenge Yourself

6. Explain with examples how the LCM of two numbers is the same as the product of the two numbers.

7. The LCM of two numbers is 60 and the HCF of the two numbers is 6. Find the two possible numbers.

8. A clock is set to ring every 15 minutes whereas another clock is set to ring every 25 minutes. If both the clocks ring together at 4:00 p.m., find the time at which both clocks will subsequently ring together again.

9. A room has a measurement of 7.5 m × 9.6 m. If Encik Zaki wishes to lay square tiles on the floor of the room, what is the biggest size, in cm, of the tiles so that the tiles will fully cover the entire floor?

10. A cafeteria serves groundnut cakes every 4 days and burgers every 6 days. If groundnut cakes and burgers are served on Monday, on which day will both types of food be served on the same day again at the cafeteria?

11. Ai Lin wants to produce a history scrapbook using 24 photographs and 42 newspaper cuttings. She wishes to use all the photographs and newspaper cuttings such that every page of the scrapbook contains the same number of photographs and newspaper cuttings.

(a) What is the maximum number of pages she can have for the scrapbook?

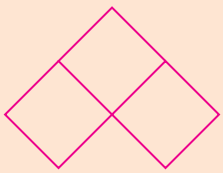
(b) For each page of the scrapbook, how many photographs and newspaper cuttings will there be?

ASSIGNMENT

A bakery received an order for 7 similar cakes. The customer requested that the cakes be cut and packed equally in 12 similar small boxes with each box containing two slices of cake. The shopkeeper sought help through social media. Write a report to explain how you would use the knowledge of factors and multiples to help the shopkeeper solve the problem.

Exploring MATHEMATICS

A The diagram shows three identical adjoining squares. Considering the common multiples of 3 and 4, explain how you would divide the three squares into four identical parts.



Open the file Exploring Chapter 2.ggb from the folder downloaded from page vii for help if needed.

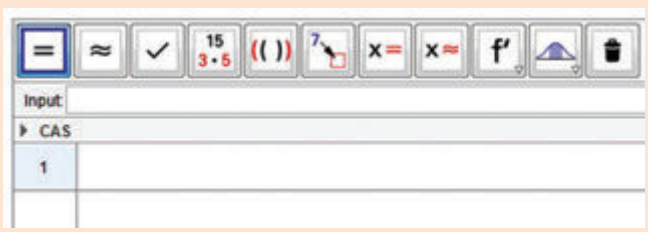
B *GeoGebra* can help you to determine the LCM, HCF, prime factorisation, listing of factors and others.

1. Open the software of *GeoGebra*.

2. Click *View* → *CAS*. The following screen is displayed.

3. Type each of the following commands into cell 1 to cell 7 and press Enter in the CAS view.

(i) PrimeFactors[60]
(ii) DivisorsList[60]
(iii) GCD[12,56]
(iv) LCM[12,56]
(v) *A*:={8,12,18,20}
(vi) GCD[A]
(vii) LCM[A]



4. Explain what can be observed.

5. Try other numbers.

CHAPTER 3

Squares, Square Roots, Cubes and Cube Roots

What will you learn?

- Squares and Square Roots
- Cubes and Cube Roots

Why study this chapter?

As the basic knowledge in fields that require the concepts of the area of squares and the volume of cubes. Discuss the fields that involve both the concepts mentioned.

The national silat squad had won three gold medals in the 2015 SEA Games. The squad had shown incredible performances when they defeated the competitors from the other countries. In the silat competition, our team performed on a square court with an area of 100 m^2. How do you determine the length of the sides of the court?

Scientists use the idea of volume to explain the structure of some cube-shaped crystals. What is the relationship between the length of the sides of a salt crystal and its volume?

Walking through Time

Christoff Rudolff

René Descartes

In the year 1637, the symbols of squares and cubes were used by a French mathematician, René Descartes, in his book, *Geometrie*. The symbols of square roots and cube roots were introduced by a German mathematician, Christoff Rudolff, in the year 1525 in his book *Die Coss*.

For more information:

http://goo.gl/fBrPNI

http://goo.gl/9flVlm

Word Link

• estimation	• *anggaran*
• square	• *kuasa dua*
• perfect square	• *kuasa dua sempurna*
• cube	• *kuasa tiga*
• perfect cube	• *kuasa tiga sempurna*
• square root	• *punca kuasa dua*
• cube root	• *punca kuasa tiga*

Open the folder downloaded from page vii for the audio of Word Link.

3.1 Squares and Square Roots

What are squares and perfect squares?

LEARNING STANDARDS

Explain the meaning of squares and perfect squares.

About 2 500 years ago, a group of academics created patterns of numbers in the shape of squares by arranging pebbles in equal number of rows and columns. Are you able to determine the next pattern of square?

Flashback

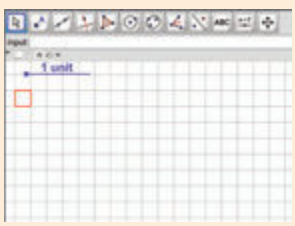
The area of a square with sides of 1 unit is $1 \times 1 = 1$ unit^2.

Exploration Activity 1

Flip Classroom

Aim: To explore the formation of squares.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.



1. Open the file *squares.ggb* using *GeoGebra*. The screen displayed shows a square with sides of 1 unit.
2. Click and drag the blue slider on the screen displayed to change the length of the sides of the square and determine the corresponding area of each square.
3. Copy and complete the table below for the area of the squares.

Length of the sides of a square (unit)	Area of the square in the form of repeated multiplication (unit^2)	Area (unit^2)
1	1×1	

4. What is the relationship between the area of the square and the length of the sides of the square?

From the results of Exploration Activity 1, it is found that a square with
a side length of (unit) 1, 2, 3, 4, …
has an area of (unit^2) 1, 4, 9, 16, …

For example, for a square with a side length of 4 units,
area = 4 × 4
= 16 unit^2

We can state that the square of 4 is 16.
The square of 4 is written as 4^2.
Thus, we write $4^2 = 16$.

4^2 is read as 'four squared' or 'square of four'.

Exploration Activity 2

Group

Aim: To explain the meaning of perfect squares.

Instruction:
- Perform the activity in groups of four.
- Open the folder downloaded from page vii.

1. Open the file *grid.pdf* and print the file on a piece of paper.
2. Cut the grid into small pieces of paper with size 1 unit × 1 unit.
3. Arrange the pieces of paper starting from one piece, two pieces, three pieces and so on to form a square (if possible).
4. Copy and complete the table below.

Number of pieces of paper with size 1 unit × 1 unit	1	2	3	4	5	6	7	8	9	10
Can the arrangement form a square? (Mark ✓ or ✗)	✓	✗	✗							

5. Write the numbers that represent the number of pieces of paper with size 1 unit × 1 unit that can be arranged to form a square.
6. What is the relationship between the numbers that represent the number of pieces of paper and the formation of a square?

From the results of Exploration Activity 2, it is found that only a certain number of pieces of paper with size 1 unit × 1 unit can be arranged to form a square.

The numbers of pieces of paper that can form a square in this activity are 1, 4, 9, 16, …

The numbers 1, 4, 9, 16, … are known as **perfect squares**.

Try This *State the subsequent perfect squares.*

Let's Discuss

Open and print the file *multiplication table.xls* from the folder downloaded from page vii. Circle all the perfect squares. Discuss how the multiplication table can be used to identify other perfect squares.

How do you determine whether a number is a perfect square?

LEARNING STANDARDS

Determine whether a number is a perfect square.

We can use the method of prime factorisation to determine whether a number is a perfect square.

In this method, if the prime factors can be grouped into two identical groups, then the number is a perfect square.

Example 1

Determine whether each of the following numbers is a perfect square.

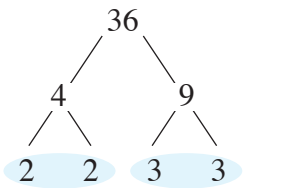
(a) 36 (b) 54

SMART TIPS

Perfect square can be written as a product of two equal factors.
For example,
$225 = 15 \times 15$ or 15^2
225 is a perfect square.

Solution

(a)



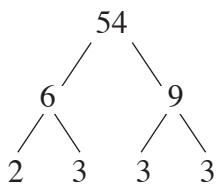
$36 = \underbrace{2 \times 3}_{\text{These prime factors can be grouped into two identical groups.}} \times \underbrace{2 \times 3}_{\text{These prime factors can be grouped into two identical groups.}}$

Thus, 36 is a perfect square.

Let's Discuss

Discuss why the prime factors of a perfect square must be grouped into two identical groups.

(b)



$54 = 2 \times 3 \times 3 \times 3$ ← These prime factors cannot be grouped into two identical groups.

Thus, 54 is not a perfect square.

Self Practice 3.1a

1. Determine whether each of the following numbers is a perfect square.

(a) 45 (b) 100 (c) 214 (d) 324

What is the relationship between squares and square roots?

Exploration Activity 3

Flip Classroom

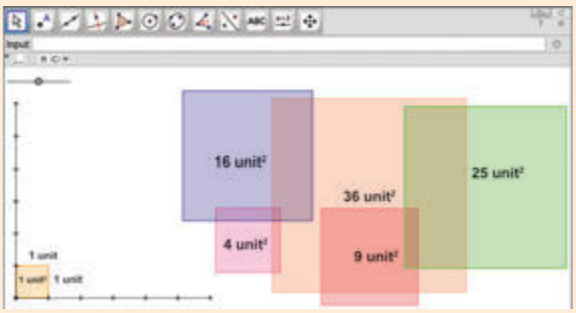
LEARNING STANDARDS

State the relationship between squares and square roots.

Aim: To state the relationship between squares and square roots.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *squares and square roots.ggb* using *GeoGebra*. The screen displayed shows some squares with different areas.
2. Click and drag the squares to the scale shown to determine the length of the sides of the corresponding squares.



3. Copy and complete the table below.

Area (unit²)	1	4	9	16	25	36
Length of the sides (unit)						

4. Based on the results obtained from the table, discuss with your friends the relationship between the area of each square and the length of its sides.

From the results of Exploration Activity 3, it is found that

a square with an area of (unit²)	1,	4,	9,	16,	25,	36
has a side length of (unit)	1,	2,	3,	4,	5,	6

that is, the area of each square is the square of its side length.

SMART TIPS

Finding the square root of the area of a square is equivalent to finding the length of the sides of the square.

For example, for a square with an area of 36 unit², the length of its side is 6 units,

$$\begin{aligned} \text{area (unit}^2) &= 36 \\ &= 6 \times 6 \\ &= 6^2 \end{aligned}$$

We can state that the square of 6 is 36.
Thus, the square root of 36 is 6.
By using the symbol of square root, $\sqrt{\ }$, we write $\sqrt{36} = 6$.

$\sqrt{36}$ is read as 'square root of thirty-six'.

Finding the square and finding the square root are inverse operations.

square

6 ⇄ 36

square root

Example 2

Complete each of the following:

(a) $9 \times 9 = 81$

Thus $\sqrt{81} = \sqrt{\square \times \square}$

$= \square$

(b) $32^2 = 1024$

Thus $\sqrt{1024} = \sqrt{\square^{\square}}$

$= \square$

Solution

(a) $\sqrt{81} = \sqrt{9 \times 9}$

$= 9$

(b) $\sqrt{1024} = \sqrt{32^2}$

$= 32$

Let's Discuss

The square root of a number could be a positive or a negative value.

$(-5) \times (-5) = 25$

Is $\sqrt{25} = -5$ true? Discuss the statement above.

Self Practice 3.1b

1. Copy and complete each of the following:

(a) $5 \times 5 = 25$

Thus,

$\sqrt{25} = \sqrt{\square \times \square}$

$= \square$

(b) $8 \times 8 = 64$

Thus,

$\sqrt{64} = \sqrt{\square \times \square}$

$= \square$

(c) $24^2 = 576$

Thus,

$\sqrt{576} = \sqrt{\square^{\square}}$

$= \square$

How do you determine the square of a number?

LEARNING STANDARDS

Determine the square of a number with and without using technological tools.

We can determine the square of a number by multiplying the number by itself.

Example 3

Find the value of each of the following without using a calculator.

(a) 6^2

(b) $\left(\frac{3}{4}\right)^2$

(c) $(-0.5)^2$

Solution

(a) $6^2 = 6 \times 6$

$= 36$

(b) $\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

(c) $(-0.5)^2 = (-0.5) \times (-0.5)$

$= 0.25$

Example 4

Find the value of each of the following by using a calculator.

(a) 43^2

(b) $\left(-\frac{7}{13}\right)^2$

(c) 2.96^2

Solution

(a) $43^2 = 1849$ Press [4] [3] [x^2] [=]

(b) $\left(-\frac{7}{13}\right)^2 = \frac{49}{169}$ Press [(] [(–)] [7] [$a\frac{b}{c}$] [1] [3] [)] [x^2] [=]

(c) $2.96^2 = 8.7616$ Press [2] [·] [9] [6] [x^2] [=]

Think Smart

What is the largest square number that is less than 200?

Self Practice 3.1c

1. Find the value of each of the following without using a calculator.

(a) 8^2 (b) $\left(-\frac{5}{6}\right)^2$ (c) 1.4^2

2. Find the value of each of the following by using a calculator.

(a) 29^2 (b) $\left(\frac{9}{11}\right)^2$ (c) $(-15.3)^2$

How do you determine the square root of a number?

LEARNING STANDARDS

Determine the square roots of a number without using technological tools.

Example 5

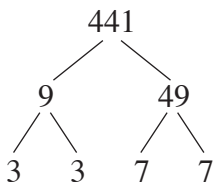
Find the value of each of the following without using a calculator.

(a) $\sqrt{64}$ (b) $\sqrt{441}$

Solution

(a) $\sqrt{64} = \sqrt{8 \times 8}$
$= 8$

(b)



$441 = 3 \times 3 \times 7 \times 7$
$= 3 \times 7 \times 3 \times 7$
$= 21 \times 21$
$\sqrt{441} = \sqrt{21 \times 21}$
$= 21$

Example 6

Find the value of each of the following without using a calculator.

(a) $\sqrt{\frac{4}{25}}$ (b) $\sqrt{2\frac{7}{9}}$ (c) $\sqrt{\frac{27}{48}}$ (d) $\sqrt{0.36}$

SMART TIPS

Prime factorisation is a more systematic method used to find the square root of a larger number.

Solution

(a) $\sqrt{\frac{4}{25}} = \sqrt{\frac{2}{5} \times \frac{2}{5}}$
$= \sqrt{\left(\frac{2}{5}\right)^2}$
$= \frac{2}{5}$

Alternative Method

(a) $\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}}$
$= \frac{\sqrt{2^2}}{\sqrt{5^2}}$
$= \frac{2}{5}$

(b) $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}}$ ← Change to improper fraction first.
$= \sqrt{\left(\frac{5}{3}\right)^2}$
$= \frac{5}{3}$
$= 1\frac{2}{3}$

Think Smart

The square root of a number is the same as the number itself. What is that number?

(c) $\sqrt{\frac{27}{48}} = \sqrt{\frac{\cancel{27}^{9}}{\cancel{48}_{16}}}$ ← Simplify the fraction first.

$= \sqrt{\frac{9}{16}}$

$= \sqrt{\left(\frac{3}{4}\right)^2}$

$= \frac{3}{4}$

(d) $\sqrt{0.36} = \sqrt{0.6^2}$ ← Express as a square in terms of other decimal number.

$= 0.6$

Alternative Method

(d) $\sqrt{0.36} = \sqrt{\frac{36}{100}}$

$= \sqrt{\left(\frac{6}{10}\right)^2}$

$= \frac{6}{10}$

$= 0.6$

Self Practice 3.1d

1. Find the value of each of the following without using a calculator.

(a) $\sqrt{81}$ (b) $\sqrt{49}$ (c) $\sqrt{121}$ (d) $\sqrt{900}$

(e) $\sqrt{\frac{49}{81}}$ (f) $\sqrt{7\frac{1}{9}}$ (g) $\sqrt{\frac{50}{128}}$ (h) $\sqrt{2.25}$

How do you determine the square root of a positive number using technological tools?

Example 7

Calculate the value of each of the following by using a calculator and give your answer correct to two decimal places.

(a) $\sqrt{89}$ (b) $\sqrt{154.7}$ (c) $\sqrt{6\frac{2}{7}}$

LEARNING STANDARDS

Determine the square roots of a positive number using technological tools.

Solution

(a) $\sqrt{89} = 9.43$ (2 d.p.) Press [√] [8] [9] [=]

(b) $\sqrt{154.7} = 12.44$ (2 d.p.) Press [√] [1] [5] [4] [·] [7] [=]

(c) $\sqrt{6\frac{2}{7}} = 2.51$ (2 d.p.) Press [√] [6] [$a^b/_c$] [2] [$a^b/_c$] [7] [=]

Self Practice 3.1e

1. Find the value of each of the following by using a calculator. Give your answer correct to two decimal places.

(a) $\sqrt{43}$ (b) $\sqrt{37.81}$ (c) $\sqrt{\frac{7}{15}}$ (d) $\sqrt{12\frac{5}{6}}$

How do you estimate the square and square root of a number?

Example 8

Estimate the value of
(a) 27.5^2 (b) $\sqrt{54}$

LEARNING STANDARDS

Estimate
(i) the square of a number,
(ii) the square roots of a number.

Solution

(a) 27.5 is between 20 and 30.
27.5^2 is between 20^2 and 30^2.
that is, 27.5^2 is between 400 and 900.
Thus, $27.5^2 \approx 900$

(b) 54 is between perfect squares 49 and 64.
$\sqrt{54}$ is between $\sqrt{49}$ and $\sqrt{64}$,
that is, $\sqrt{54}$ is between 7 and 8.
Thus, $\sqrt{54} \approx 7$

Scan the QR Code or visit **https://goo.gl/UxjF6c** to open the file *Example 8_pdf* to learn about the estimation using number line.

Self Practice 3.1f

1. Estimate the value of each of the following:

(a) 61^2 (b) 22.5^2 (c) 8.7^2 (d) $(-0.188)^2$
(e) $\sqrt{34}$ (f) $\sqrt{17.6}$ (g) $\sqrt{128}$ (h) $\sqrt{0.85}$

What generalisation can be made when two square roots are multiplied?

LEARNING STANDARDS

Make generalisation about multiplication involving:
(i) square roots of the same numbers,
(ii) square roots of different numbers.

Exploration Activity 4

Pair

Aim: To make a generalisation about multiplication involving square roots.

Instruction:
- Perform the activity in pairs.
- Open the folder downloaded from page vii.

1. Open the file *multiplication of square roots.pdf* and print the file.
2. Complete the boxes for Questions 1 and 2.
3. What can you conclude from the products of multiplication obtained in Questions 1 and 2?

From the results of Exploration Activity 4, it is found that

- the product of two equal square root numbers is the number itself, that is, $\sqrt{a} \times \sqrt{a} = a$.
- the product of two different square root numbers is the square root of the product of the two numbers, that is, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.

How do you solve problems?

MATHEMATICS APPLICATION ZONE

Maslina wishes to paste a photo on a piece of cardboard. Both the photo and the cardboard are squares. The length of the cardboard is 12 cm and the area of the photo is 90.25 cm^2. How should Maslina paste the photo such that the photo lies in the middle of the cardboard?

Solution

Length of photo $= \sqrt{90.25}$
$= \sqrt{9.5 \times 9.5}$
$= 9.5$ cm

The remaining length of side of the cardboard after the photo is pasted $= 12 - 9.5$
$= 2.5$ cm

Distance of the photo from the side of the cardboard
$= 2.5 \div 2$
$= 1.25$ cm

Thus, the photo has to be pasted at a distance of 1.25 cm from the sides of the cardboard such that the photo lies in the middle of the cardboard.

LEARNING STANDARDS

Pose and solve problems involving squares and square roots.

Let's Discuss

Given the area of a square cardboard is 156.25 cm^2 whereas the area of a square photo is less than 156.25 cm^2 but greater than 90.25 cm^2. Discuss the possible length of the photo used if the measurement of its length is a whole number.

Scan the QR Code or visit **https://youtu.be/Nsb77HbXcCM** to learn about the application of squares and square roots.

Self Practice 3.1g

1. Ai Ling has a piece of square cloth. The area of the cloth is between 6 400 cm^2 and 12 100 cm^2. She wants to use the cloth to sew a piece of square tablecloth to cover the surface of a square table. The length of each side of the table is 92 cm.

(a) What is the length of the cloth, in cm, that Ai Ling should sew?
[Assume that the length of the cloth is a whole number.]

(b) Ai Ling plans to decorate the surrounding sides of the tablecloth with white lace that is 4.5 m long so that the tablecloth will look nice. Is the length of the white lace enough? Give a reason for your answer.

Mastery Q 3.1

Open the folder downloaded from page vii for extra questions of Mastery Q 3.1.

1. Determine whether each of the following numbers is a perfect square. Use the prime factorisation method to support your answer.
(a) 216 (b) 1 000 (c) 1 024

2. The prime factorisation of 100 is $2 \times 2 \times 5 \times 5$. Explain how you could find the square root of 100 by using the prime factorisation method.

3. Copy and complete each of the following based on the relationship between square and square root.

is equal to	6^2	10^2	14^2	19^2	22^2
	as	as	as	as	
	36	100	196	361	484
is equal to	$\sqrt{36}$	$\sqrt{100}$	$\sqrt{196}$	$\sqrt{361}$	$\sqrt{484}$
	as	as	as	as	
	☐	☐	☐	☐	☐

4. Find the value of each of the following without using a calculator.
(a) $(-6)^2$ (b) $\left(\frac{2}{7}\right)^2$ (c) $\left(-4\frac{1}{3}\right)^2$ (d) $(-8.1)^2$
(e) $\sqrt{361}$ (f) $\sqrt{\frac{9}{49}}$ (g) $\sqrt{2\frac{14}{25}}$ (h) $\sqrt{1.21}$

5. Calculate the value of each of the following by using a calculator. Give your answers correct to two decimal places for (e) to (h).
(a) 127^2 (b) $(-34.6)^2$ (c) 0.097^2 (d) $\left(-2\frac{5}{8}\right)^2$
(e) $\sqrt{76}$ (f) $\sqrt{108.4}$ (g) $\sqrt{\frac{11}{28}}$ (h) $\sqrt{2\frac{3}{5}}$

6. A pyramid has a square base area of 52 900 m^2. Find the length of each side of the base of the pyramid.

7. Estimate the value of each of the following:
(a) 297^2 (b) 51.9^2 (c) $(-0.038)^2$ (d) $(-8.12)^2$
(e) $\sqrt{14}$ (f) $\sqrt{220}$ (g) $\sqrt{8.3}$ (h) $\sqrt{0.5}$

8. A group of 100 members from a Cultural Club participated in the Malaysia Citrawarna Parade. They made various formations throughout the parade.
(a) When the members of the cultural group made a formation in the shape of a square, state the number of members in each row of the square.
(b) At a certain instance, the members of the group formed two squares simultaneously. Determine the number of members in each row of each square.

3.2 Cubes and Cube Roots

What are cubes and perfect cubes?

LEARNING STANDARDS

Explain the meaning of cubes and perfect cubes.

Exploration Activity 5

Pair

Aim: To explore the formation of cubes.
Instruction: Perform the activity in pairs.

1. The diagram shows three cubes which are made up of unit cubes. Observe the three cubes shown.

SMART TIPS

The number of unit cubes in a cube is the volume of the cube.

2. Copy and complete the table below.

Length of the sides of a cube (unit)	Volume of the cube in the form of repeated multiplication (unit^3)	Number of unit cubes (unit^3)
1	$1 \times 1 \times 1$	

3. Discuss with your friends and write the relationship between the number of unit cubes and the length of the sides of the cube.

From the results of Exploration Activity 5, it is found that a cube with

a side length of (unit)	1, 2, 3, …
has a number of unit cubes of (unit^3)	1, 8, 27, …

For example, for a cube with a side length of 2 units,
number of unit cubes $= 2 \times 2 \times 2$
$= 8$ unit^3

We can state that the cube of 2 is 8.
The cube of 2 is written as 2^3.
Thus, we write $2^3 = 8$.

2^3 is read as 'two cubed' or 'cube of two'.

Exploration Activity 6

Group

Aim: To explain the meaning of perfect cubes.
Instruction: Perform the activity in groups of four.

1. Arrange unit blocks beginning with 1 unit block, followed by 2 unit blocks, 3 unit blocks and so on to form a cube (if possible).

2. Copy and complete the table below.

Number of unit blocks	1	2	3	4	5	6	7	8	9	10
Can the arrangement form a cube? (Mark ✓ or ✗)	✓	✗	✗							

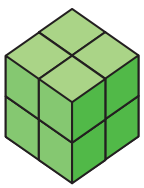
3. Write the numbers that represent the number of unit blocks that can be arranged to form a cube.

4. What is the relationship between the numbers that represent the number of unit blocks and the formation of a cube?

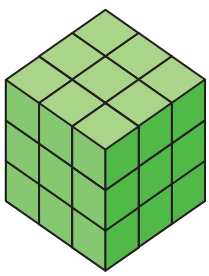
From the results of Exploration Activity 6, it is found that only a certain number of unit blocks can be arranged to form a cube.

For example,

1 unit block



8 unit blocks



27 unit blocks

The numbers of unit blocks that can form a cube are 1, 8 and 27.

The numbers 1, 8 and 27 are known as **perfect cubes**.

Try This *State the subsequent perfect cubes.*

Did You Know

The volume of a table salt crystal is a perfect cube.

Think Smart

Zaiton said,
"$2^3 = 2 \times 3 = 6$."
How do you explain to Zaiton that her statement is false?

How do you determine whether a number is a perfect cube?

LEARNING STANDARDS

Determine whether a number is a perfect cube.

We can also use the method of prime factorisation to determine whether a number is a perfect cube. In this method, if the prime factors can be grouped into three identical groups, then the number is a perfect cube.

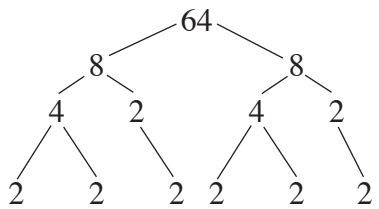
Example 9

Determine whether each of the following numbers is a perfect cube.

(a) 64 (b) 240

Solution

(a)

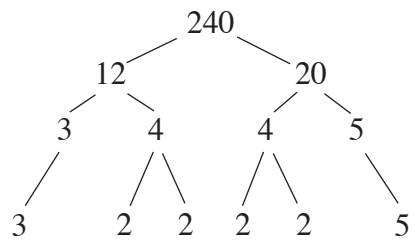


$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

These prime factors can be grouped into three identical groups.

Thus, 64 is a perfect cube.

(b)



$240 = 3 \times 2 \times 2 \times 2 \times 2 \times 5$

These prime factors cannot be grouped into three identical groups.

Thus, 240 is not a perfect cube.

SMART TIPS

Perfect cube can be written as a product of three equal factors.
For example,
$64 = 4 \times 4 \times 4$
64 is a perfect cube.

Think Smart

This number is a perfect square and also a perfect cube. What is this number?

Let's Discuss

Discuss why the prime factors of a perfect cube must be grouped into three identical groups.

Self Practice 3.2a

1. By using the prime factorisation method, determine whether each of the following numbers is a perfect cube.

(a) 27 (b) 45 (c) 215 (d) 343

What is the relationship between cubes and cube roots?

LEARNING STANDARDS

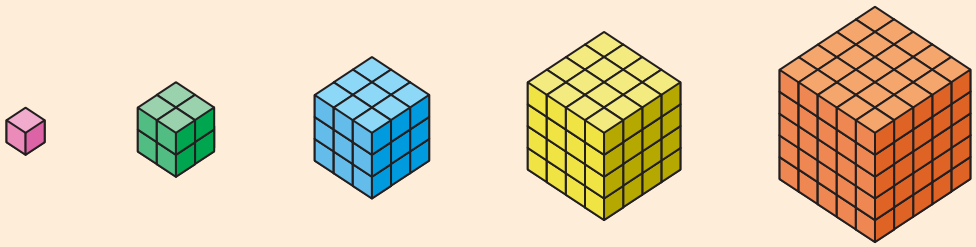
State the relationship between cubes and cube roots.

Exploration Activity 7

Group

Aim: To state the relationship between cubes and cube roots.
Instruction: Perform the activity in groups of four.

1. Observe cubes *A* to *E* in the diagram below.



A *B* *C* *D* *E*

2. Complete the following table for the length of the sides of each cube.

Cube	*A*	*B*	*C*	*D*	*E*
Volume (unit^3)	1	8	27	64	125
Length of the sides (unit)					

3. Based on the results from the table, discuss with your friends the relationship between the volume of each cube and the length of its sides.

From the results of Exploration Activity 7, it is found that

a cube with a volume of (unit^3)	1,	8,	27,	64,	125
has a side length of (unit)	1,	2,	3,	4,	5

that is, the volume of each cube is the cube of its side length.

For example, for a cube with a volume of 8 unit^3, the length of its side is 2 units,

volume (unit^3) = 8
= 2 × 2 × 2
= 2^3

We can state that the cube of 2 is 8.
Thus, the cube root of 8 is 2.

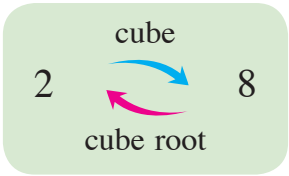
By using the symbol of cube root, $\sqrt[3]{\ }$,
we write $\sqrt[3]{8} = 2$.

SMART TIPS

Finding the cube root of the volume of a cube is equivalent to finding the length of the sides of the cube.

$\sqrt[3]{8}$ is read as 'cube root of eight'.

Finding the cube and finding the cube root are inverse operations.



Think Smart

The cube root of a number is the same as the number itself. What is the number?

Example 10

Complete each of the following:

(a) $4 \times 4 \times 4 = 64$

Thus $\sqrt[3]{64} = \sqrt[3]{\square \times \square \times \square}$

$= \square$

(b) $(-0.5) \times (-0.5) \times (-0.5) = -0.125$

Thus $\sqrt[3]{-0.125} = \sqrt[3]{\square \times \square \times \square}$

$= \square$

(c) $\left(\frac{1}{6}\right)^3 = \frac{1}{216}$

Thus $\sqrt[3]{\frac{1}{216}} = \sqrt[3]{\square^{\square}}$

$= \square$

Solution

(a) $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4}$

$= 4$

(b) $\sqrt[3]{-0.125} = \sqrt[3]{-0.5 \times (-0.5) \times (-0.5)}$

$= -0.5$

(c) $\sqrt[3]{\frac{1}{216}} = \sqrt[3]{\left(\frac{1}{6}\right)^3}$

$= \frac{1}{6}$

Self Practice 3.2b

1. Copy and complete each of the following:

(a) $8 \times 8 \times 8 = 512$

Thus, $\sqrt[3]{512} = \sqrt[3]{\square \times \square \times \square}$

$= \square$

(b) $0.3 \times 0.3 \times 0.3 = 0.027$

Thus, $\sqrt[3]{0.027} = \sqrt[3]{\square \times \square \times \square}$

$= \square$

(c) $\left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$

Thus, $\sqrt[3]{-\frac{1}{8}} = \sqrt[3]{\square^{\square}}$

$= \square$

How do you determine the cube of a number?

LEARNING STANDARDS

Determine the cube of a number with and without using technological tools.

Example 11

Find the value of each of the following without using a calculator.

(a) 4^3 (b) 0.2^3 (c) $\left(-\frac{3}{5}\right)^3$

Solution

(a) $4^3 = 4 \times 4 \times 4$
$= 64$

(b) $0.2^3 = 0.2 \times 0.2 \times 0.2$
$= 0.008$

(c) $\left(-\frac{3}{5}\right)^3 = \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right)$
$= -\frac{27}{125}$

SMART TIPS

The cube of a positive number is always a positive value whereas the cube of a negative number is always a negative value.

Example 12

Find the value of each of the following using a calculator.

(a) 18^3 (b) $\left(-4\frac{1}{2}\right)^3$ (c) $(-6.3)^3$

Solution

(a) $18^3 = 5\,832$ Press [1] [8] [x^3] [=]

(b) $\left(-4\frac{1}{2}\right)^3 = -91\frac{1}{8}$ Press [(] [(–)] [4] [$a^b/_c$] [1] [$a^b/_c$] [2] [)] [x^3] [=]

(c) $(-6.3)^3 = -250.047$ Press [(] [(–)] [6] [·] [3] [)] [x^3] [=]

Self Practice 3.2c

1. Determine the value of each of the following without using a calculator.

(a) 6^3 (b) $(-7)^3$ (c) $\left(-\frac{2}{9}\right)^3$ (d) $(-0.3)^3$ (e) $\left(2\frac{3}{5}\right)^3$

2. Find the value of each of the following using a calculator.

(a) 26^3 (b) $(-5.1)^3$ (c) $\left(\frac{3}{10}\right)^3$ (d) $\left(-1\frac{7}{11}\right)^3$ (e) $\left(4\frac{4}{5}\right)^3$

How do you determine the cube root of a number?

Example 13

Find the value of each of the following without using a calculator.

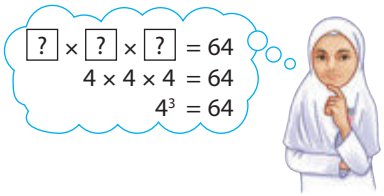
(a) $\sqrt[3]{64}$ (b) $\sqrt[3]{216}$

LEARNING STANDARDS

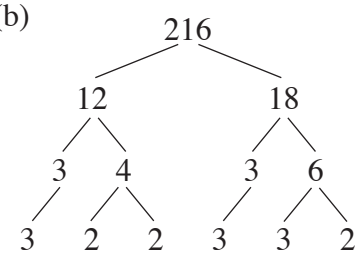
Determine the cube root of a number without using technological tools.

Solution

(a) $\sqrt[3]{64} = \sqrt[3]{4^3}$
$= 4$



(b)



$216 = 3 \times 2 \times 2 \times 3 \times 3 \times 2$
$= (3 \times 2) \times (3 \times 2) \times (3 \times 2)$
$= 6 \times 6 \times 6$
$\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6}$
$= 6$

SMART TIPS

Prime factorisation is a more systematic method used to find the cube root of a larger number.

Example 14

Find the value of each of the following without using a calculator.

(a) $\sqrt[3]{\frac{8}{125}}$ (b) $\sqrt[3]{-\frac{81}{192}}$ (c) $\sqrt[3]{3\frac{3}{8}}$

Solution

(a) $\sqrt[3]{\frac{8}{125}}$
$= \sqrt[3]{\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}}$
$= \sqrt[3]{\left(\frac{2}{5}\right)^3}$
$= \frac{2}{5}$

(b) $\sqrt[3]{-\frac{81}{192}}$
$= \sqrt[3]{-\frac{\cancel{81}^{27}}{\cancel{192}_{64}}}$ ← Simplify the fraction first.
$= \sqrt[3]{-\frac{27}{64}}$
$= \sqrt[3]{\left(-\frac{3}{4}\right)^3}$
$= -\frac{3}{4}$

(c) $\sqrt[3]{3\frac{3}{8}} = \sqrt[3]{\frac{27}{8}}$ ← Convert to improper fraction first.
$= \sqrt[3]{\left(\frac{3}{2}\right)^3}$
$= \frac{3}{2}$
$= 1\frac{1}{2}$

Alternative Method

(a) $\sqrt[3]{\frac{8}{125}} = \frac{\sqrt[3]{8}}{\sqrt[3]{125}}$
$= \frac{\sqrt[3]{2^3}}{\sqrt[3]{5^3}}$
$= \frac{2}{5}$

SMART TIPS

The cube root of a positive number is always a positive value whereas the cube root of a negative number is always a negative value.

Example 15

Find the value of each of the following without using a calculator.

(a) $\sqrt[3]{0.027}$ (b) $\sqrt[3]{-0.008}$

Solution

(a) $\sqrt[3]{0.027}$
$= \sqrt[3]{0.3^3}$
$= 0.3$

Express as a cube in terms of other decimal number.

(b) $\sqrt[3]{-0.008}$
$= \sqrt[3]{(-0.2)^3}$
$= -0.2$

Alternative Method

(b) $\sqrt[3]{-0.008}$
$= \sqrt[3]{-\frac{8}{1\,000}}$
$= \sqrt[3]{\left(-\frac{2}{10}\right)^3}$
$= -\frac{2}{10}$
$= -0.2$

Self Practice 3.2d

1. Given $9\,261 = 3^3 \times 7^3$, find $\sqrt[3]{9\,261}$ without using a calculator.

2. Find $\sqrt[3]{2\,744}$ by using the prime factorisation method.

3. Find the value of each of the following without using a calculator.

(a) $\sqrt[3]{27}$ (b) $\sqrt[3]{-125}$ (c) $\sqrt[3]{343}$ (d) $\sqrt[3]{-1\,000}$

4. Find the value of each of the following without using a calculator.

(a) $\sqrt[3]{\frac{8}{125}}$ (b) $\sqrt[3]{-\frac{1}{27}}$ (c) $\sqrt[3]{\frac{24}{81}}$ (d) $\sqrt[3]{1\frac{61}{64}}$

(e) $\sqrt[3]{0.001}$ (f) $\sqrt[3]{-0.064}$ (g) $\sqrt[3]{-0.216}$ (h) $\sqrt[3]{0.000343}$

How do you determine the cube root of a number using technological tools?

LEARNING STANDARDS

Determine the cube root of a number using technological tools.

Example 16

Calculate the value of each of the following by using a calculator. Give your answer correct to two decimal places.

(a) $\sqrt[3]{24}$ (b) $\sqrt[3]{-104.8}$ (c) $\sqrt[3]{-1\frac{2}{9}}$

Solution

(a) $\sqrt[3]{24} = 2.88$ (2 d.p.) Press $\sqrt[3]{}$ 2 4 =

(b) $\sqrt[3]{-104.8} = -4.71$ (2 d.p.) Press $\sqrt[3]{}$ (–) 1 0 4 · 8 =

(c) $\sqrt[3]{-1\frac{2}{9}} = -1.07$ (2 d.p.) Press $\sqrt[3]{}$ (–) 1 $a\frac{b}{c}$ 2 $a\frac{b}{c}$ 9 =

Self Practice 3.2e

1. Find the value of each of the following using a calculator and give your answer correct to two decimal places.

(a) $\sqrt[3]{15}$ (b) $\sqrt[3]{-74}$ (c) $\sqrt[3]{164.2}$ (d) $\sqrt[3]{\frac{7}{9}}$ (e) $\sqrt[3]{-1\frac{2}{5}}$

How do you estimate the cube and cube root of a number?

LEARNING STANDARDS

Estimate
(i) the cube of a number,
(ii) the cube root of a number.

Example 17

Estimate the value of
(a) 4.2^3 (b) $\sqrt[3]{180}$

Solution

(a) 4.2 is between 4 and 5.
4.2^3 is between 4^3 and 5^3,
that is, 4.2^3 is between 64 and 125.
Thus, $4.2^3 \approx 64$

(b) 180 is between perfect cubes 125 and 216.
$\sqrt[3]{180}$ is between $\sqrt[3]{125}$ and $\sqrt[3]{216}$,
that is, $\sqrt[3]{180}$ is between 5 and 6.
Thus, $\sqrt[3]{180} \approx 6$

Scan the QR Code or visit **https://goo.gl/UxjF6c** to open the file *Example 17_pdf* to learn about the estimation using number line.

Self Practice 3.2f

1. Estimate the value of each of the following:
(a) 2.1^3 (b) $(-9.6)^3$ (c) 19.7^3 (d) $(-43.2)^3$

2. Estimate the value of each of the following:
(a) $\sqrt[3]{7}$ (b) $\sqrt[3]{69}$ (c) $\sqrt[3]{-118}$ (d) $\sqrt[3]{-26.8}$

How do you solve problems?

MATHEMATICS APPLICATION ZONE

LEARNING STANDARDS

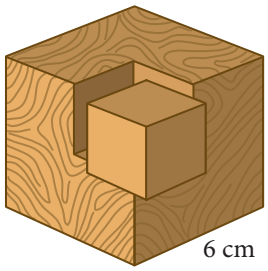
Solve problems involving cubes and cube roots.

A sculptor carves a small cube from a wooden cube block that has a side length of 6 cm. If the volume of the remaining wooden block is 189 cm^3, find the length of the small cube that is being removed.

Solution

Understanding the problem

- Length of the wooden block = 6 cm
- Volume of the remaining wooden block after a small cube is removed = 189 cm^3
- Find the length of the small cube.



Devising a plan

- Volume of the wooden block = Cube of the length of its side
- Volume of the small cube
  $= \left(\begin{matrix}\text{Volume of the}\\ \text{wooden block}\end{matrix}\right) - \left(\begin{matrix}\text{Volume of the remaining}\\ \text{wooden block}\end{matrix}\right)$
- Length of the small cube
  = Cube root of the volume of the small cube

Implementing the strategy

Volume of the wooden block $= 6^3$
$= 216\ cm^3$

Volume of the small cube
$= \left(\begin{matrix}\text{Volume of the}\\ \text{wooden block}\end{matrix}\right) - \left(\begin{matrix}\text{Volume of the remaining}\\ \text{wooden block}\end{matrix}\right)$
$= 216 - 189$
$= 27\ cm^3$

Length of the small cube $= \sqrt[3]{27}$
$= 3$ cm

Doing reflection

Volume of the small cube $= 3^3$
$= 27\ cm^3$

Volume of the wooden block $= 27 + 189$
$= 216\ cm^3$

Thus, the length of the wooden block $= \sqrt[3]{216}$
$= 6$ cm

Creative & Innovative

Material:
Manila card

Task:
Design a closed box to fill eight ping pong balls by using the least material.

Reflection:
What is the shape of the box that saves the most amount of material?

Think Smart

Amira shapes a cuboid measuring 5 cm, 2 cm and 5 cm from plasticine. How many identical cuboids are needed to form a big cube?

Self Practice 3.2g

1. Malik wants to make a cubic frame using a piece of wire with a length of 150 cm. If the volume of the cube is 2 197 cm^3, is the length of the wire sufficient? Give a reason for your answer.

How do you perform computations involving different operations on squares, square roots, cubes and cube roots?

LEARNING STANDARDS

Perform computations involving addition, subtraction, multiplication, division and the combination of those operations on squares, square roots, cubes and cube roots.

Example 18

Find the value of each of the following:

(a) $0.5^2 + \sqrt[3]{1000}$

(b) $(-3)^3 - \sqrt{64}$

(c) $\sqrt{25} + (-0.2)^2 \div \sqrt[3]{0.008}$

(d) $\sqrt[3]{-3\frac{3}{8}} \times (\sqrt{36} - 2^3)^2$

Solution

(a) $0.5^2 + \sqrt[3]{1000}$
$= 0.25 + 10$
$= 10.25$

(b) $(-3)^3 - \sqrt{64}$
$= -27 - 8$
$= -35$

(c) $\sqrt{25} + (-0.2)^2 \div \sqrt[3]{0.008}$
$= 5 + 0.04 \div 0.2$
$= 5 + 0.2$
$= 5.2$

Solve the ÷ operation first.

(d) $\sqrt[3]{-3\frac{3}{8}} \times (\sqrt{36} - 2^3)^2$
$= \sqrt[3]{-\frac{27}{8}} \times (6 - 8)^2$
$= -\frac{3}{2} \times (-2)^2$
$= -\frac{3}{2_1} \times \overset{2}{\cancel{4}}$
$= -6$

SMART TIPS

Find the value of squares, square roots, cubes or cube roots.
↓
Solve the operation in the brackets.
↓
Solve the operations × and ÷ from left to right.
↓
Solve the operations + and – from left to right.

Self Practice 3.2h

1. Calculate the value of each of the following:

(a) $\sqrt{49} + 3^2$

(b) $\sqrt[3]{27} - 1.5^2$

(c) $\sqrt[3]{-64} \times 0.2^3$

(d) $(-2)^2 \div \sqrt{100}$

(e) $\sqrt{2\frac{1}{4}} - \sqrt[3]{15\frac{5}{8}}$

(f) $\sqrt[3]{\frac{1}{125}} \times 0.3^2$

(g) $\sqrt{\frac{12}{27}} \div \left(-\frac{2}{3}\right)^3$

(h) $(-5)^2 + \sqrt{2\frac{7}{9}} - 2^3$

(i) $(\sqrt{16} - 6)^2 \times \sqrt[3]{-\frac{54}{128}}$

Mastery Q 3.2

Open the folder downloaded from page vii for extra questions of Mastery Q 3.2.

1. Determine whether each of the following numbers is a perfect cube. Explain how you could support your answer by using the prime factorisation method.

(a) 128 (b) 343 (c) 1 000

2. The prime factorisation of 3 375 is $3 \times 3 \times 3 \times 5 \times 5 \times 5$.

Explain how you could find the cube root of 3 375 by using the prime factorisation method.

3. Find the value of each of the following without using a calculator.

(a) $(-5)^3$ (b) $\left(\frac{4}{5}\right)^3$ (c) $\left(-1\frac{1}{6}\right)^3$ (d) $(-3.2)^3$

(e) $\sqrt[3]{125}$ (f) $\sqrt[3]{-512}$ (g) $\sqrt[3]{729}$ (h) $\sqrt[3]{-27\,000}$

(i) $\sqrt[3]{\frac{8}{125}}$ (j) $\sqrt[3]{-\frac{64}{343}}$ (k) $\sqrt[3]{-0.512}$ (l) $\sqrt[3]{1.331}$

4. Calculate the value of each of the following using a calculator. Give your answers correct to two decimal places for (e) to (h).

(a) 202^3 (b) $(-17.6)^3$ (c) 0.041^3 (d) $\left(-2\frac{3}{7}\right)^3$

(e) $\sqrt[3]{34.8}$ (f) $\sqrt[3]{215.7}$ (g) $\sqrt[3]{-0.94}$ (h) $\sqrt[3]{-\frac{7}{11}}$

5. Estimate the value of each of the following:

(a) 2.9^3 (b) $(-10.12)^3$ (c) 14.87^3 (d) $(-0.88)^3$

(e) $\sqrt[3]{65}$ (f) $\sqrt[3]{344}$ (g) $\sqrt[3]{-728.9}$ (h) $\sqrt[3]{8\frac{1}{8}}$

6. The photo shows a decorative cubic box. The area of each face of the box is 2 500 mm^2.

(a) Find the length, in mm, of the decorative box.

(b) Write the volume of the decorative box in cubic notation.

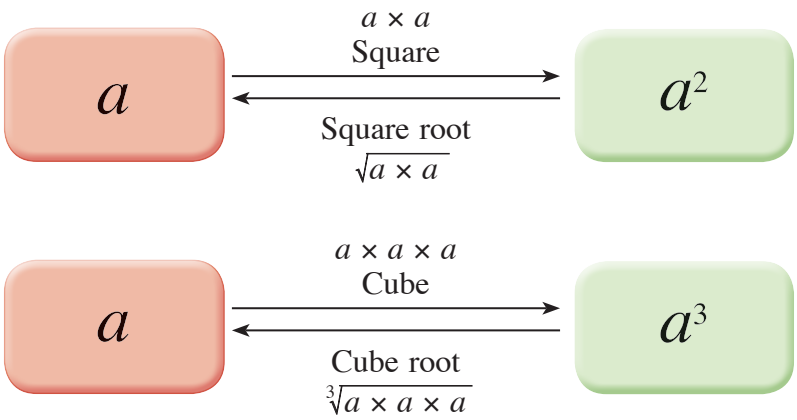
7. Find the value of each of the following:

(a) $\sqrt[3]{8} + (-0.3)^2$ (b) $4^2 \times \sqrt[3]{-125}$

(c) $\sqrt{36} \div \left(2\frac{1}{2}\right)^2$ (d) $3^2 - \sqrt[3]{27} \div (-1)^3$

(e) $5^2 \times \sqrt[3]{-216} \div \sqrt{\frac{4}{9}}$ (f) $\sqrt[3]{-\frac{1}{343}} \times \left(2^3 - \sqrt{2\frac{7}{9}}\right)$

SUMMARY



At the end of this chapter, I can...	Very good	Work harder
explain the meaning of squares, perfect squares, cubes and perfect cubes.		
determine whether a number is a – perfect square. – perfect cube.		
state the relationship between squares, square roots, cubes and cube roots.		
determine the square and cube of a number with and without using technological tools.		
determine the square root and cube root of a number without using technological tools.		
determine the square root of a positive number and the cube root of a number using technological tools.		
estimate the square, square root, cube and cube root of a number.		
make generalisation about multiplication involving – square roots of the same numbers. – square roots of different numbers.		
pose and solve problems involving squares, square roots, cubes and cube roots.		
perform computations involving addition, subtraction, multiplication, division and the combination of those operations on squares, square roots, cubes and cube roots.		

Let's PRACTISE

Test Yourself

1. Mark (✓) for the number which is a perfect square.

27	( )	32	( )	18	( )	4	( )
81	( )	8	( )	125	( )	49	( )

2. Copy and complete the operation steps below by filling up the empty boxes with suitable numbers.

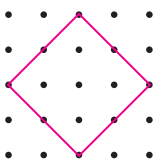
$$\sqrt{1\frac{11}{25}} - (-0.1)^3 = \sqrt{\frac{\square}{25}} - (-0.1)^3$$

$$= \frac{\square}{5} - (\square)$$

$$= \square$$

Self Mastery

3. The diagram shows a square drawn by Siti. She claims that the length of the side of the square is $\sqrt{8}$ units. Show how you would validate Siti's answer.



4. Application Aviation

A helicopter landing pad is a square and has an area of 400 m^2. Use the prime factorisation method to find the length of the side of the landing pad.

5.

$$512 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$

(a) Based on the mathematical statement above, Fong Yee states that 512 is a perfect cube. Explain how you would support Fong Yee's answer.

(b) Fong Yee also states that 512 is not a perfect square. Explain the reason Fong Yee said so.

6. Mohan used a can of paint to paint a square backdrop. A can of paint could cover the entire backdrop of 38 m^2. Estimate the length of the side of the backdrop.

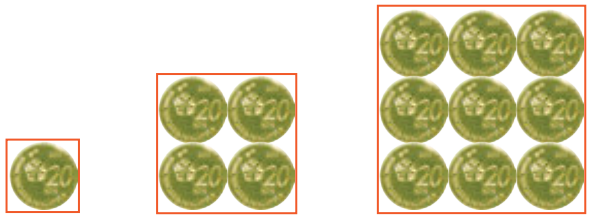
7. A big cube with a side length of 30 cm is cut into 27 small cubes of the same size. Find

(a) the length of the side of each small cube,

(b) the area of the top surface of each small cube.

Challenge Yourself

8. Amirul arranges 20 sen coins in the shapes of squares as shown in the diagram.

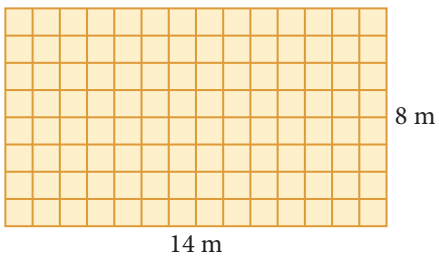


(a) Calculate the total value, in RM, of the
(i) fourth square,
(ii) 10th square.

(b) If Amirul has 20 sen coins with an amount of RM60, determine the arrangement of the 20 sen coins to form the largest square.

9. Application Construction

Stella wants to beautify the patio of her house by placing square patio stones on it. Each patio stone has an area of 1 m^2. She makes a sketch of the layout plan as shown in the diagram.



(a) How many patio stones are needed for the construction?

(b) If Stella suggests using the same number of patio stones as in (a) but changes the layout plan into the shape of a square, is it still possible for Stella to work? Explain your answer.

10. Application History

Hypatia was an Egyptian mathematician who was born in the year 370 A.D. In a finding, Hypatia posed the following problems:

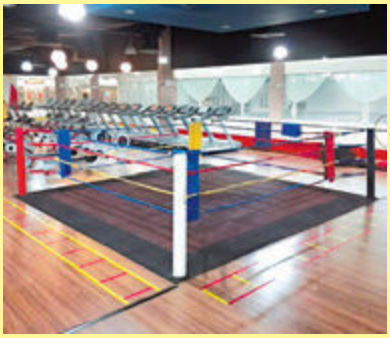
- This number is the sum of two square numbers.
- The square of this number is also the sum of two square numbers.

Square numbers:
1, 4, 9, 16, ...
$5 = 1 + 4$
$5^2 = 9 + 16$

One of the numbers that satisfies the constraints posed by Hypatia is 5. Find three other numbers.

ASSIGNMENT

The photo shows a square boxing ring. Other than boxing, there are other sports which use square floors as surfaces.



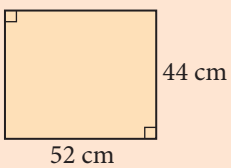
By conducting research via Internet, reference books or visiting a library, find out what other sports also use square floors as surfaces. Find out the length of the side and the area of this square surface. Relate the rules of this sports in describing the purpose of having a square floor as its surface.

Exploring MATHEMATICS

Chess is played on a square board. The board is made up of 32 light squares and 32 dark squares.

You have a piece of rectangular board with measurements of 44 cm × 52 cm. You decide to use the board to make your own chessboard such that

- each square grid on the board has a side length which is a whole number.
- each chess piece is placed on a square grid with an area of not less than 9 cm^2.



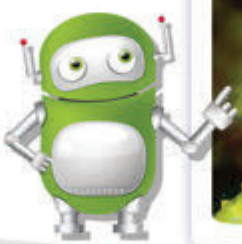
Determine all the possible measurements of the chessboard that you can create.

CHAPTER 4

Ratios, Rates and Proportions

What will you learn?

- Ratios
- Rates
- Proportions
- Ratios, Rates and Proportions
- Relationship between Ratios, Rates and Proportions, with Percentages, Fractions and Decimals



Why study this chapter?

Ratios, rates and proportions are involved in the fields that require the concept of comparisons. An astronomer measures the distance in the solar system by comparing each distance in the solar system with the distance from the Earth to the Sun. A nurse uses the concept of rates when taking a patient's pulse. Discuss other fields which involve the concept of comparisons.

The *Hippocampus kuda* is a type of seahorse found in river estuaries. These seahorses are facing extinction and must be conserved and appreciated. To estimate the seahorse population at a certain habitat, marine biologists will tag the first sample of the seahorses and then release them back to the original sites where they were collected from.

After a certain period of time, the second sample is collected. The marine biologists will record the number of tagged seahorses. How do the marine biologists estimate the population of the seahorses in the river estuaries by using this method?

Walking through Time

Vitruvian Man

The Vitruvian Man, is a drawing by Leonardo da Vinci around the year 1490 which depicts a human body inscribed in a circle and a square. In this drawing, the human body is said to be drawn based on certain ratios and proportions which were suggested by a Roman architect called Vitruvius.

For more information:

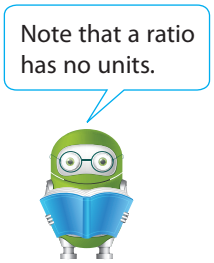
https://goo.gl/VNOYjx

Word Link

• rate	• *kadar*
• proportion	• *kadaran*
• ratio	• *nisbah*
• equivalent ratio	• *nisbah setara*
• percentage	• *peratusan*

Open the folder downloaded from page vii for the audio of Word Link.

4.1 Ratios

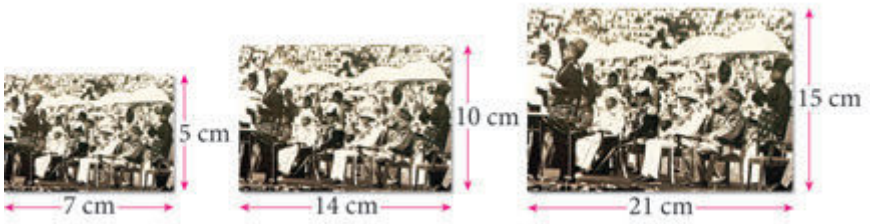


Ratio is used to compare two or more quantities of the same kind that are measured in the same unit. For example, the ratio of 5000 g to 9 kg can be represented as

$$5000\text{ g} : 9\text{ kg} = 5\text{ kg} : 9\text{ kg}$$
$$= 5 : 9$$

The ratio of a to b is written as $a : b$.

How do you represent the relationship of three quantities?



LEARNING STANDARDS

Represent the relation between three quantities in the form of $a : b : c$.

The picture above shows three photos of different sizes.
How do you represent the relationship of the sizes of these three photos in a ratio?

Example 1

Represent the ratio of 0.02 m to 3 cm to 4.6 cm in the form of $a : b : c$.

Solution

$$0.02\text{ m} : 3\text{ cm} : 4.6\text{ cm} = 2\text{ cm} : 3\text{ cm} : 4.6\text{ cm}$$
$$= 2 : 3 : 4.6$$
$$= 20 : 30 : 46$$
$$= 10 : 15 : 23$$

Self Practice 4.1a

1. Represent the relationship of the following three quantities in the form of $a : b : c$.

(a) 2 weeks to 16 days to 1 week

(b) 0.1 kg to 50 g to 0.25 kg

(c) 4 minutes to 120 seconds to 1.6 hours

(d) $3\frac{1}{5}$ m to 480 cm to 6400 mm

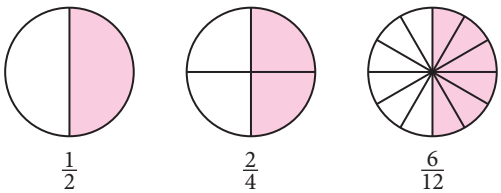
2. Tahir pays RM5.60 for a plate of *nasi beriani*, RM1.20 for a glass of tea and 30 sen for a piece of *kuih*. Represent the relationship of the prices of *nasi beriani*, tea and *kuih* in the form of $a : b : c$.

What are equivalent ratios?

LEARNING STANDARDS

Identify and determine the equivalent ratios in numerical, geometrical or daily situation contexts.

Observe the equivalent fractions that represent the shaded parts in the diagram below.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{6}{12}$

The areas of the shaded parts are always of the same size.

$\frac{1}{2} \overset{\times 2}{=} \frac{2}{4} \overset{\times 3}{=} \frac{6}{12}$

SMART TIPS

Equivalent ratios can be found by writing the ratios as equivalent fractions.

These fractions can be expressed in ratios as follows.

$1 : 2 \overset{\times 2}{=} 2 : 4 \overset{\times 3}{=} 6 : 12$

1 : 2, 2 : 4 and 6 : 12 are known as **equivalent ratios**. To find an equivalent ratio, multiply or divide each part of the ratio by the same whole number.

Think Smart

Are 23 : 46 and $\frac{1}{6} : \frac{1}{3}$ equivalent to 1 : 2?

Example 2

Which of the following ratios are equivalent to 27 : 45?

9 : 15 5 : 3 54 : 90 $\frac{1}{2} : \frac{5}{6}$

Solution

$27 : 45 = 27 \times 2 : 45 \times 2$
$= 54 : 90$

$27 : 45 = 27 \div 9 : 45 \div 9$
$= 3 : 5$

$27 : 45 = 27 \div 3 : 45 \div 3$
$= 9 : 15$

$27 : 45 = 27 \times \frac{1}{54} : 45 \times \frac{1}{54}$
$= \frac{1}{2} : \frac{5}{6}$

SMART TIPS

Ratio 3 : 5 ≠ 5 : 3.

Thus, ratios which are equivalent to 27 : 45 are 9 : 15, 54 : 90 and $\frac{1}{2} : \frac{5}{6}$.

Alternative Method

$\frac{1}{2} : \frac{5}{6} = \frac{1}{2} \times 54 : \frac{5}{6} \times 54$
$= 27 : 45$

Example 3

Puan Habibah mixed 4 cups of vinegar with 8 cups of water to obtain a natural cleaning solution to clean the glass window of her house. Identify and determine two possible equivalent ratios for this cleaning solution.

Solution

$4 : 8 = 4 \times 2 : 8 \times 2$ $\qquad$ $4 : 8 = 4 \div 2 : 8 \div 2$

$= 8 : 16$ $\qquad$ $= 2 : 4$

The two possible equivalent ratios of 4 : 8 are 8 : 16 and 2 : 4.

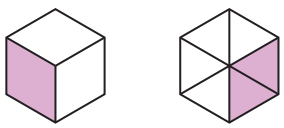
Self Practice 4.1b

1. Which of the following ratios are equivalent to 18 : 24 : 45?

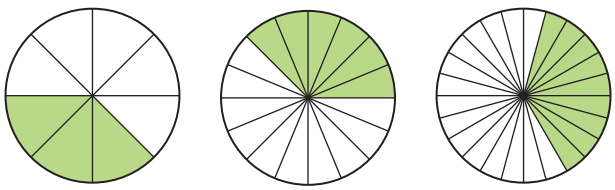
3 : 4 : 9 $\qquad$ 36 : 48 : 90 $\qquad$ 0.6 : 0.8 : 1.5 $\qquad$ $\frac{2}{5} : \frac{8}{15} : 1$

2. Identify and determine the equivalent ratios of the shaded parts in each of the following diagrams.

(a)



(b)



3. At a construction site, a worker prepares a concrete mixture which acts as the basic solid foundation of a building. The concrete mixture is obtained by adding 10 parts of cement to 20 parts of sand and 30 parts of gravel. Identify and determine two possible equivalent ratios for this mixture.

How do you express ratios in their simplest form?

To express a ratio in its simplest form, divide the quantities by the Highest Common Factor (HCF) or multiply the quantities by the Lowest Common Multiple (LCM).

LEARNING STANDARDS

Express ratios of two and three quantities in simplest form.

Example 4

Express each of the following ratios in its simplest form.

(a) 800 g : 1.8 kg $\qquad$ (b) 32 : 24 : 20

(c) $\frac{3}{5} : \frac{7}{10}$ $\qquad$ (d) 0.04 : 0.12 : 0.56

Solution

(a) $800 \text{ g} : 1.8 \text{ kg} = 800 \text{ g} : 1\,800 \text{ g}$ ← Convert to the same unit.

$= \frac{800}{200} : \frac{1\,800}{200}$ ← Divide both parts by 200.

$= 4 : 9$

SMART TIPS

A ratio of $a : b$ is said to be in its simplest form if a and b are integers with no common factors other than 1.

(b) $32 : 24 : 20 = \frac{32}{4} : \frac{24}{4} : \frac{20}{4}$ ← Divide the three parts by 4, that is, the HCF of 32, 24 and 20.

$= 8 : 6 : 5$

(c) $\frac{3}{5} : \frac{7}{10} = \frac{3}{5} \times 10 : \frac{7}{10} \times 10$ ← Multiply both parts by 10, that is, the LCM of 5 and 10.

$= 6 : 7$

(d) $0.04 : 0.12 : 0.56 = 0.04 \times 100 : 0.12 \times 100 : 0.56 \times 100$ ← Multiply the three parts by 100.

$= 4 : 12 : 56$

$= \frac{4}{4} : \frac{12}{4} : \frac{56}{4}$ ← Divide the three parts by 4, that is, the HCF of 4, 12 and 56.

$= 1 : 3 : 14$

Self Practice 4.1c

1. Express each of the following ratios in its simplest form.

(a) 240 g : 1.6 kg

(b) 30 : 42 : 48

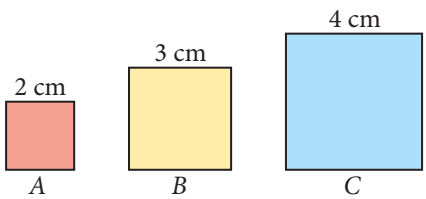
(c) $\frac{2}{5} : \frac{8}{9}$

(d) 0.09 : 0.12 : 0.24

Mastery Q 4.1

Open the folder downloaded from page vii for extra questions of Mastery Q 4.1.

1. The diagram shows three squares, A, B and C.



(a) Represent each of the following in the form of $a : b : c$.

(i) The length of square A to the length of square B to the length of square C.

(ii) The perimeter of square A to the perimeter of square B to the perimeter of square C.

(iii) The area of square A to the area of square B to the area of square C.

(b) Write the equivalent ratios based on your answers in (a).

(c) What is the relationship between the numbers in the ratio of areas and the numbers in the ratio of length?

2. In a workshop, a mechanic uses a jack with a force of 120 pounds to lift a car with a mass of 1 350 kg. State the ratio of the mass of the car to the force required to lift the car. Express the ratio in its simplest form. (1 pound = 0.45 kg)

3. The table shows the number of boys and girls in three Form 1 classes.

Students	Class 1 Amanah	Class 1 Bestari	Class 1 Cekap
Boys	12	9	9
Girls	16	20	12

(a) Which classes have equivalent ratios of the number of girls to the number of boys?

(b) Write the ratio of the number of girls in class 1 Amanah to the number of girls in class 1 Bestari to the number of girls in class 1 Cekap in the form of $a : b : c$.

(c) Class 1 Amanah and class 1 Bestari are combined during the Physical Education and Health Education lesson. Find the ratio of the number of girls to the number of boys in the combined class. Express your answer in its simplest form.

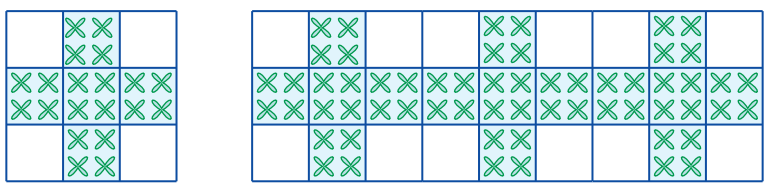
4. Identify and determine the possible equivalent ratios for each of the following.

is equivalent to

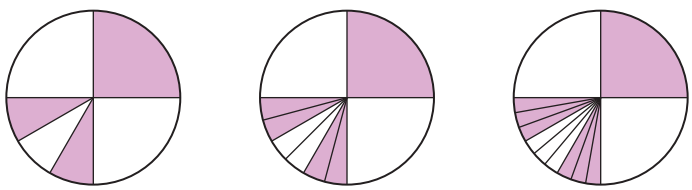
1 : 5 as ☐ 4 : 9 : 2 as ☐ 37 : 74 as ☐ 6 : 1 as ☐ 0.3 : 1.2 : 0.5

5. Identify and determine the equivalent ratios of the shaded parts in each of the following diagrams.

(a)



(b)



6. A farmer prepares a soil medium by mixing 3 parts of loamy soil, 2 parts of peat and 1 part of river sand. Identify and determine two possible equivalent ratios for this soil medium.

4.2 Rates

What is the relationship between ratios and rates?

LEARNING STANDARDS

Determine the relationship between ratios and rates.

Exploration Activity 1

Individual

Aim: To determine the relationship between ratios and rates.

Instruction: Open the folder downloaded from page vii.

1. Open the file *ratio and rates.pdf* and print the file.
2. State the ratio of two quantities for the measurements involved in each of the situations given.
3. State the quantities involved and also their units of measurement.

Situation	Ratio in the form $\frac{a}{b}$	Quantities involved	Units of measurement
A car travels 285 km in 3 hours.	$\frac{285\text{ km}}{3\text{ hours}}$	Distance and time	km and hour
A plant grows 24 cm in 4 months.			
A baby's mass increases by 1.3 kg in 60 days.			
Karim's pulse rate is 75 beats per minute.			
The force acting on the surface area of 1 square metre is 2 Newtons.			

In Exploration Activity 1, we compare two quantities measured in different units. For example, in the ratio $\frac{285\text{ km}}{3\text{ hours}}$, we compare the distance travelled in km with the time taken in hours. The ratio $\frac{285\text{ km}}{3\text{ hours}}$ is known as a **rate**.

Rates show how two quantities with different units are related to each other.

SMART TIPS

Rate is a special ratio that compares two quantities with different units of measurement.

Let's Discuss

Whether $\frac{1.5\text{ kg}}{3\text{ kg}}$ or $\frac{1.5\text{ kg}}{3\text{ g}}$ is a rate? Discuss.

Example 5

State the rate and the two quantities (including the units) involved in each of the following situations.

(a) Fatin buys 2 kg of mangoes at the total price of RM10.

(b) A car consumes 1 litre of petrol for a distance of 12 km.

Solution

(a) $\text{Rate} = \frac{\text{RM10}}{\text{2 kg}}$

The two quantities involved are mass (kg) and total amount of money (RM).

(b) $\text{Rate} = \frac{\text{12 km}}{\text{1 litre}}$

The two quantities involved are distance (km) and volume (litre).

Conversion of units of rates

Two stalls sell cherry tomatoes from Cameron Highlands. Which stall sells cherry tomatoes at a cheaper price?

To compare the price rates, convert the units first.

Stall *A*

Stall *B*

The price of cherry tomatoes at stall $B = \frac{\text{RM8}}{\text{500 g}}$

$= \frac{8 \times 2}{500 \times 2}$

$= \frac{16}{1\,000}$ ← 1 000 g = 1 kg

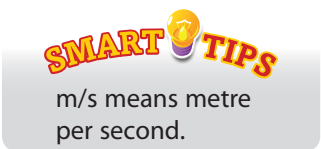
$= \text{RM16 per kg}$

Thus, the price of cherry tomatoes at stall *A* is cheaper than the price of cherry tomatoes at stall *B*.

Example 6

(a) Rajan is riding his bicycle at a speed of 5 m/s. Convert 5 m/s to km/h.

(b) The density of a type of metal is 2 700 kg per m^3.
State the density of this metal in g per cm^3.



Solution

(a) 5 m/s

$= \frac{5 \text{ m}}{1 \text{ s}}$

$= 5 \text{ m} \div 1 \text{ s}$

$= \frac{5}{1\,000} \text{ km} \div \frac{1}{60 \times 60} \text{ h}$

$= \frac{5}{1\,000} \times \frac{60 \times 60}{1}$

$= 18 \text{ km/h}$

(b) Density

$= \frac{2\,700 \text{ kg}}{1 \text{ m}^3}$

$= \frac{2\,700 \times 1\,000}{100 \times 100 \times 100}$ ← 1 kg = 1 000 g

$= 2.7 \text{ g/cm}^3$

$= 2.7 \text{ g per cm}^3$

1 m^3
= 1 m × 1 m × 1 m
= 100 cm × 100 cm × 100 cm

Self Practice 4.2a

1. State the rate and the two quantities (including the units) involved in each of the following situations.

(a) The train fare for two adult passengers travelling from Johor Bahru to Kuala Lumpur is RM154.

(b) 20 litres of water flow out of a water tank each time the water is pumped.

(c) The tuition fee paid by Haruiri is RM240 for 4 subjects.

(d) The price of the fertiliser used on a 10 hectare farm is RM500.

(e) The shaft of an engine makes 600 revolutions in 3 seconds.

2. The table shows the speeds of two objects, *A* and *B*. Convert the units of measurement to determine which object moves faster.

Object	Speed
A	25 m per second
B	8 km per hour

3. The mass per unit area of a type of metal sheet is 3 kg per m^2. State the rate in g per 100 cm^2.

4. An oil palm plantation uses fertiliser at a rate of 350 kg per hectare. State the rate of fertiliser consumption in g per m^2. [1 hectare = 10 000 m^2]

Mastery Q 4.2

Open the folder downloaded from page vii for extra questions of Mastery Q 4.2.

1. The mass of a piece of 5 cm^3 aluminium is 13.5 g. Find the density of the aluminium in g per cm^3.

2.

Orang-utan	Mass at birth (kg)	Mass after 60 days (kg)
Borneo	0.3	6.3
Sumatera	0.7	7.7

(a) Based on the information in the table above, state

(i) the ratios of the mass at birth to the mass after 60 days for the two species of orang-utan,

(ii) the rate of growth in kg per 60 days for the two species of orang-utan.

(b) State the two quantities involved for the rate obtained in (a)(ii).

3. Halim wants to buy milk. The cartons of milk are sold in three different sizes as shown in the diagram.

(a) Write the price rate for each carton of milk.

(b) Determine the price per litre for the 500 m*l* carton of milk.

(c) Which carton of milk offers the most affordable price? Justify your answer.

RM2.25 RM4.00 RM7.50

4.3 Proportions

What is the relationship between ratios and proportions?

LEARNING STANDARDS

Determine the relationship between ratios and proportions.

Exploration Activity 2

Pair

Aim: To determine the relationship between ratios and proportions.
Instruction: Perform the activity in pairs.

1. Select a page from a novel to copy. The time allocated for copying is 5 minutes.
2. Your friend acts as a timekeeper to tell you the time to begin and the time to stop copying.
3. Copy at a comfortable rate to minimise spelling mistakes.
4. Stop copying when the time is up. Mark the place you have stopped copying.
5. Count the total number of words copied in 5 minutes.
6. Exchange roles and repeat Steps **1** to **5** to obtain data for your friend. Use the same page when copying.
7. Copy and record your findings in the following table.

Name of student		
Number of words		
Time (minutes)	5	5

8. Based on the results in the table, answer the following questions:
   (a) What is the rate of copying in 5 minutes for you and your friend?
   (b) Convert the rate of copying in 5 minutes to the number of words per minute.
   (c) If both of you continued copying at the same rate, how many words could each person copy in 1 hour?

In Exploration Activity 2, you have used proportion to find the number of words that you and your friend copied in 1 hour.
For example,

$$\frac{45 \text{ words}}{5 \text{ minutes}} \overset{\times 12}{=} \frac{540 \text{ words}}{60 \text{ minutes}}$$

Let's Discuss

Based on Exploration Activity 2, discuss the similarities and differences among ratios, rates and proportions. Give an example for each one.

Proportion is a relationship that states that the two ratios or two rates are equal. Proportion can be expressed in the form of fraction.

Example 7

Write a proportion for each of the following situations:

(a) If 10 beans have a mass of 17 g, then 30 beans have a mass of 51 g.

(b) The Le Tour De Langkawi cycling race covers a distance of 1 180 km which starts from Kedah and ends in Melaka. On the map, this distance is 23.6 cm where 1 cm represents 50 km.

Solution

(a) $\frac{17\text{ g}}{10\text{ beans}} = \frac{51\text{ g}}{30\text{ beans}}$ (b) $\frac{50\text{ km}}{1\text{ cm}} = \frac{1\,180\text{ km}}{23.6\text{ cm}}$

Self Practice 4.3a

1. Write a proportion for each of the following situations:

(a) If the price of 3 balls is RM5, then the price of 12 balls is RM20.

(b) The height of four wooden blocks stacked vertically on top of one another is 24 cm. When Raju stacks 13 wooden blocks on top of one another, the height of the stacked blocks is 78 cm.

(c) There are 13 boys and 15 girls in each classroom at Ria Kindergarten. If there are 65 boys in the kindergarten, then there are 75 girls.

How do you determine the unknown value in a proportion?

LEARNING STANDARDS

Determine an unknown value in a proportion.

Example 8

Electricity costs 43.6 sen for 2 kilowatt-hour (kWh). How much does 30 kWh cost?

Solution

Unitary method

The cost of electricity for 2 kWh
= 43.6 sen

The cost of electricity for 1 kWh
$= \frac{43.6\text{ sen}}{2}$
= 21.8 sen

The cost of electricity for 30 kWh
= 30 × 21.8
= 654 sen

Proportion method

Let the cost of electricity for 30 kWh be x sen.

Then,

$$\frac{43.6\text{ sen}}{2\text{ kWh}} \overset{\times 15}{=} \frac{x\text{ sen}}{30\text{ kWh}}$$

$x = 43.6 \times 15$
$= 654$

Cross multiplication method

Let the cost of electricity for 30 kWh be x sen.

Then,

$$\frac{43.6}{2} \quad \frac{x}{30}$$

$$2 \times x = 43.6 \times 30$$

$$x = \frac{43.6 \times 30}{2}$$

$$= 654$$

Thus, the cost of electricity consumption for 30 kWh is RM6.54.

SMART TIPS

If $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$.

Self Practice 4.3b

1. In a sports carnival, there are 200 players in 8 rugby teams. Determine the number of players in 2 teams if each team has the same number of players.
2. During the *SEGAK* test in the Physical Education lesson, Amir is able to do 60 push-ups in 3 minutes. How many push-ups can Amir do in 5 minutes? (Assume that Amir does the push-ups at the same rate.)
3. A farmer plants three chilli plants per 0.5 m^2. How many chilli plants can he plant in an area of 85 m^2?

Mastery Q 4.3

Open the folder downloaded from page vii for extra questions of Mastery Q 4.3.

1. Write a proportion for each of the following situations. Use a suitable variable to represent the required information.
   (a) Puan Jamilah mixes 175 m*l* of olive oil and 50 m*l* of vinegar to make a marinade. What is the volume of vinegar required if Puan Jamilah uses 300 m*l* of olive oil to make the marinade?
   (b) A meteorologist used a rain gauge to measure the amount of rain in a city. He found out that the total rainfall in the city was 7.8 mm within 3 hours. If the rainfall continues at the same rate, how long will it take for the rainfall to reach 11.7 mm?

2. A gear is a simple machine consisting of wheels with teeth. When a large gear rotates 4 times, a small gear will rotate 18 times at the same time. How many times will the large gear rotate if the small gear rotates 54 times?
3. A gardener spends half an hour to mow and weed the lawn which has a measurement of 20 m × 15 m. He is paid RM30 per hour. How much is he paid for a lawn which has a measurement of 40 m × 30 m?

4.4 Ratios, Rates and Proportions

How do you determine the ratio of three quantities when two or more ratios of two quantities are given?

LEARNING STANDARDS

Determine the ratio of three quantities, given two or more ratios of two quantities.

Example 9

If $p : q = 7 : 3$ and $q : r = 3 : 5$, find the ratio of $p : q : r$.

Solution

$p : q = 7 : \textcircled{3}$ $\quad$ $q : r = \textcircled{3} : 5$

same

Thus, $p : q : r = 7 : 3 : 5$.

Example 10

At the reading corner of Class 1 Jujur, the ratio of the number of storybooks to the number of reference books is 2 : 5. The ratio of the number of reference books to the number of magazines is 3 : 2. Find the ratio of the number of storybooks to the number of reference books to the number of magazines.

Solution

Let x = number of storybooks
y = number of reference books
z = number of magazines

$x : y = 2 : 5$ $\quad$ $y : z = 3 : 2$

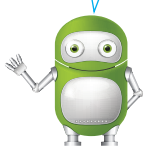
$= 2 \times 3 : 5 \times 3$ $\quad$ $= 3 \times 5 : 2 \times 5$

$= 6 : \textcircled{15}$ $\quad$ $= \textcircled{15} : 10$

same

Thus, $x : y : z = 6 : 15 : 10$, that is, the ratio of the number of storybooks to the number of reference books to the number of magazines is 6 : 15 : 10.

Change the value of y in both ratios to the same number by determining the LCM of 5 and 3.



Self Practice 4.4a

1. If $p : q = 2 : 9$ and $q : r = 9 : 7$, find the ratio of $p : q : r$.

2. A certain amount of money was donated to three charity organisations P, Q and R. The ratio of the amount of donation received by P to Q is 2 : 3. The ratio of the amount of donation received by Q to R is 4 : 1. Find the ratio of the amount of donation received by organisation P to organisation Q to organisation R.

How do you determine the ratio or the related value?

LEARNING STANDARDS

Determine the ratio or the related value given
(i) the ratio of two quantities and the value of one quantity.
(ii) the ratio of three quantities and the value of one quantity.

Example 11

Nurin's mother tries a bread recipe by mixing flour with water. The ratio of the flour to the water is 5 : 3. If Nurin's mother has 480 g of flour, what is the mass of water, in g, that is needed?

Solution

Unitary method

Flour	:	Water
5	:	3
480 g	:	

5 parts of flour = 480 g

$$1 \text{ part of flour} = \frac{480}{5}$$
$$= 96 \text{ g}$$

Mass of water needed = 3 × 96 g
= 288 g

Cross multiplication method

Let m = mass of water needed

Flour	:	Water
5	:	3
480 g	:	m g

Thus, $\frac{m}{3} = \frac{480}{5}$

$$5 \times m = 3 \times 480$$
$$m = \frac{3 \times 480}{5}$$
$$= 288$$

Example 12

In funding a bicycle shelter project, the ratio of the donations contributed by the school canteen operator, Jaya Bookstore and PIBG is 2 : 6 : 5. If the PIBG had donated RM900, find the amounts donated by the school canteen operator and Jaya Bookstore respectively.

Solution

Unitary method

School canteen operator	:	Jaya Bookstore	:	PIBG
2	:	6	:	5
	:		:	RM900

5 parts = RM900

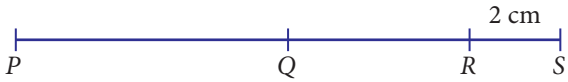
$$1 \text{ part} = \frac{900}{5}$$
= RM180

School canteen operator donated 2 parts.
Amount donated = 2 × RM180
= RM360

Jaya Bookstore donated 6 parts.
Amount donated = 6 × RM180
= RM1 080

Try This *Use cross multiplication method to solve Example 12.*

Example 13



A piece of wire of length 12 cm is divided into three parts. Given that $PQ : QR = 3 : 2$ and $RS = 2$ cm, determine $PQ : QR : RS$.

Solution

$PR = 12 - 2$
$= 10$ cm

The total ratio of PQ and $QR = 3 + 2$
$= 5$

5 parts $= 10$ cm

1 part $= \frac{10}{5}$
$= 2$ cm

$PQ = 3 \times 2$ cm
$= 6$ cm

$QR = 2 \times 2$ cm
$= 4$ cm

Thus, $PQ : QR : RS = 6 : 4 : 2$
$= 3 : 2 : 1$

Self Practice 4.4b

1. The ratio of the price of a *baju kebaya* to the price of a *baju kurung* is 7 : 4. If the price of the *baju kebaya* is RM84, find the price of the *baju kurung*.

2. The ratio of Encik Arif's mass to his son's mass is 3 : 2. If his son's mass is 42 kg, find Encik Arif's mass.

3. Zanariah, Rusita and Hanifah shared the cost of a birthday present for their mother in the ratio 5 : 3 : 2. Hanifah paid RM50 for the present. Calculate the total amount of money Zanariah and Rusita paid.

4. Kadir, Chandran and Ping Wei participated in a Science quiz. The number of questions they had answered was in the ratio 4 : 6 : 3. Chandran had answered 30 questions. Calculate the total number of questions Kadir and Ping Wei had answered.

5. 42 students in Class 1 Dedikasi have registered as members of the Computer Club, Robotics Club and Taekwondo Club. Each student is allowed to register for one club only. It is given that the number of students who had registered as members of the Robotics Club is one third of the number of students who had registered as members of the Computer Club and the number of students who had registered as members of the Taekwondo Club is 14. Determine the ratio of the number of members of the Computer Club to the number of members of the Robotics Club to the number of members of the Taekwondo Club.

How do you determine the value related to a rate?

LEARNING STANDARDS

Determine the value related to a rate.

Example 14

Mr Tan jogs at a steady rate on a treadmill and his heart beats 420 times in 4 minutes. Find the number of times his heart will beat if he jogs on the treadmill at the same rate for 12 minutes.

Solution

$$\text{Heart rate} = \frac{420 \text{ times}}{4 \text{ minutes}}$$

Let the number of heartbeats be x times in 12 minutes.

Proportion method

$$\frac{420 \text{ times}}{4 \text{ minutes}} = \frac{x \text{ times}}{12 \text{ minutes}}$$

$$\frac{420 \text{ times}}{4 \text{ minutes}} \overset{\times 3}{=} \frac{x \text{ times}}{12 \text{ minutes}}$$

$$x = 1\,260$$

Cross multiplication method

$$\frac{420}{4} \quad \frac{x}{12}$$

$$4 \times x = 420 \times 12$$

$$x = \frac{420 \times 12}{4}$$

$$x = 1\,260$$

Mr Tan's heart will beat 1 260 times in 12 minutes.

Try This *Use unitary method to solve Example 14.*

Did You Know

In real-life situations, the heart rate per minute is not constant. This is because the rate varies according to an individual's condition at different times.

Self Practice 4.4c

1. A person who walks for exercise burns 2.9 calories of energy per minute. How many calories does that person burn during a 20-minute walk?
2. Six cartons of lime juice are sold at RM12.25. Determine the price for 24 similar cartons of lime juice.
3. On Monday, Anis used 8 cups of flour to make 60 pieces of buttermilk biscuits. On Tuesday, Anis plans to make 15 pieces of buttermilk biscuits. If the rate of the flour used to make the biscuits remains the same, how many cups of flour should she use?
4. Suppose the exchange rate of USD1 is equivalent to RM3.90. Ahmad goes to the money changer to change RM200 to US Dollars at this rate. The money changer gives Ahmad back a change of RM5 together with the US Dollars. How much US Dollars does Ahmad receive?

How do you solve problems?

LEARNING STANDARDS

Solve problems involving ratios, rates and proportions, including making estimations.

Example 15

A farmer uses the capture, mark, release and recapture method to estimate the population of snails in his vegetable farm. He captures 24 snails and marks on the shell of each of them. Then, he releases them back into the farm. Two weeks later, he captures randomly 30 snails and finds that 5 of them are marked. He uses the following proportion to estimate the population of snails in the vegetable farm.

$$\frac{\text{Number of marked snails}}{\text{Number of recaptured snails}} = \frac{\text{Number of snails which are captured and marked}}{\text{Population of snails in the vegetable farm}}$$

Estimate the population of snails in the vegetable farm.

Solution

Let the population of snails in the farm = x

$$\frac{\text{Number of marked snails}}{\text{Number of recaptured snails}} = \frac{\text{Number of snails which are captured and marked}}{\text{Population of snails in the vegetable farm}}$$

$$\frac{5}{30} = \frac{24}{x}$$

$$\frac{{}^{1}\cancel{5}}{{}_{6}\cancel{30}} = \frac{24}{x}$$

$\times 24$

$$\frac{1}{6} = \frac{24}{x}$$

$\times 24$

$$x = 6 \times 24$$
$$= 144$$

Let's Discuss

How would you find the value of x if you do not change $\frac{5}{30}$ in its simplest form?

The farmer estimates that there are 144 snails in the vegetable farm.

Did You Know

By knowing the pest population in a certain habitat, scientists can estimate the optimum use of pesticides so that the ecological balance can be maintained without affecting the survival of other living organisms.

Self Practice 4.4d

1. The table shows the results of a study to estimate the population of a type of freshwater fish that lives in a lake.

Capture	Number of fish caught	
	Marked	Unmarked
First	60	
Second (After a week)	5	20

Estimate the population of the freshwater fish that live in the lake.

2. A football team played 28 games and won 4 out of every 7 games contested. There were no tie games.
(a) How many games did this football team lose?
(b) Calculate the team's win-loss ratio.
(c) If this trend continues, estimate the number of losses of the team once they have won 20 games.

Mastery Q 4.4

Open the folder downloaded from page vii for extra questions of Mastery Q 4.4.

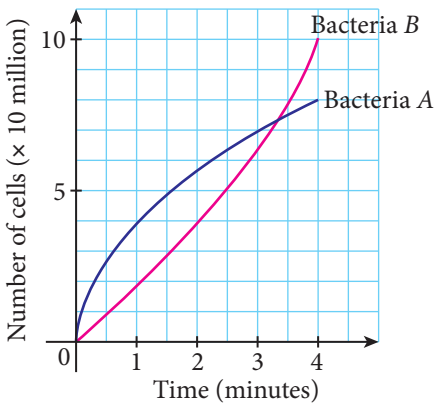
1. The scouts are having an annual camp. Every 12 scouts are under the supervision of a teacher. How many teachers will be involved if there are 90 scouts attending the annual camp?

2. Akmal, Bakri and Cadin share their mother's medical expenses which cost RM4 200. Cadin pays RM2 100 while Akmal pays three quarters of Bakri's amount. Find the ratio of the expenses shared by Akmal to Bakri to Cadin.

3. The table shows the prices of four types of fish. Siti wants to buy 1 kg each of two types of fish and the total amount she is willing to spend is less than RM20. What are the possible combinations of fish that Siti can buy? Show your calculations.

Type of fish	Price
Pomfret (*Bawal*)	RM1.50 per 100 g
Hardtail scad (*Cencaru*)	RM3.20 per 500 g
One finlet scad (*Selar*)	RM2.70 per 300 g
Indian mackerel (*Kembung*)	RM5.40 per 400 g

4. The graph shows the growth rate of bacteria *A* and bacteria *B* in a culture.
(a) Which bacteria shows the fastest growth rate in the first minute? State this growth rate.
(b) What is the difference in the number of cells between bacteria *A* and bacteria *B* in the 4th minute?
(c) Estimate the time when there are equal numbers of the two types of bacteria.



5. A 30 kg bag of fertiliser is labelled 15-20-10. These numbers mean that the contents in this bag of fertiliser contain 15% nitrogen, 20% phosphorus and 10% potassium by mass. The remaining 55% is made up of micronutrients and other fillers.

(a) Find the ratio of nitrogen to phosphorus to potassium.

(b) Calculate the mass, in kg, of the contents of nitrogen, phosphorus and potassium in the bag.

4.5 Relationship between Ratios, Rates and Proportions, with Percentages, Fractions and Decimals

What is the relationship between percentages and ratios?

LEARNING STANDARDS

Determine the relationship between percentages and ratios.

Flashback

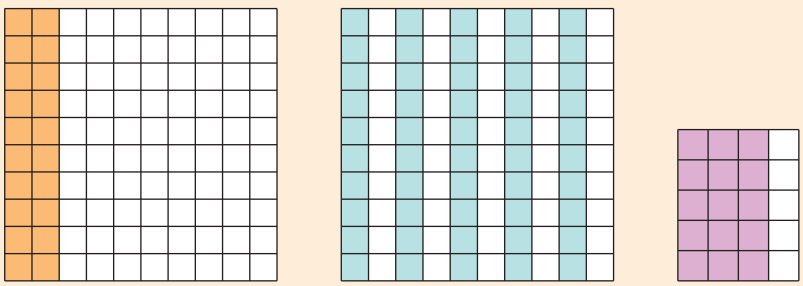
A percentage is a fraction with a denominator of 100.

Exploration Activity 3

Individual

Aim: To determine the relationship between percentages and ratios.

1. Copy and complete the table below.



A *B* *C*

Diagram	The ratio of the number of shaded squares to the number of unshaded squares	The percentage of the shaded squares	The percentage of the unshaded squares
A			
B			
C			

2. Based on the results in the table above, discuss the relationship between percentages and ratios.

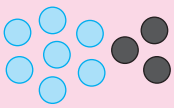
From Exploration Activity 3, it is found that a percentage is a ratio that describes a part of 100.

For example, $20\% = \frac{20}{100}$

'Percentage' means 'per 100'.

Let's Discuss

The diagram shows 10 marbles.
What does the ratio 3 : 7 represent in the diagram?
What is the percentage of black marbles in the diagram?



Did You Know

The ratio that compares the number of parts to 100 parts can be expressed in the form of fraction, decimal and percentage.

Example 16

In a class, the ratio of the number of girls to the number of boys is 3 : 2. Find the percentage of the girls in the class.

Solution

The ratio of the number of girls to the total number of students = 3 : 5

$$= \frac{3}{5}$$

$$\frac{3}{5} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100}$$

Change to a fraction with a denominator of 100.

Alternative Method

The percentage of the girls
$= \frac{3}{5} \times 100\%$
$= 60\%$

Thus, the percentage of the girls in the class is 60%.

Example 17

Zakaria is thrifty. He saves 30% of his daily pocket money. Find the ratio of his daily savings to his total daily pocket money received.

Solution

The percentage of Zakaria's daily savings = 30%

$$= \frac{30}{100}$$

The ratio of his daily savings to his total daily pocket money received = 30 : 100

$$= 3 : 10$$

Self Practice 4.5a

1. In a class, the ratio of the number of students who wear spectacles to the number of students who do not wear spectacles is 1 : 3. Find the percentage of the students who do not wear spectacles in the class.

2. It is known that 35% of the hard disk of a computer has been filled with data. Find the ratio of the capacity that has been filled with data to the capacity that has not been filled with data.

How do you determine the percentage of a quantity by applying the concept of proportions?

Flashback

A proportion is a relationship that states that two ratios are equal.

LEARNING STANDARDS

Determine the percentage of a quantity by applying the concept of proportions.

Example 18

Determine the percentage of each of the following quantities by applying the concept of proportions.

(a) A box contains 8 ribbons. Two of the ribbons are blue. What is the percentage of blue ribbons in the box?

(b) Puan Jorana has RM300. She spends RM15 for her transport fee. What is the percentage of the amount of money she spends?

Solution

(a) Let the percentage of blue ribbons in the box be x.

$$\frac{x}{100} = \frac{\text{Number of blue ribbons}}{\text{Total number of ribbons}}$$ ← Write a proportion

$$\frac{x}{100} = \frac{2}{8}$$

$$8 \times x = 2 \times 100$$

$$x = \frac{2 \times 100}{8}$$

$$= 25$$

Thus, 25% of the ribbons in the box are blue.

(b) Let the percentage of the amount of money spent be y.

$$\frac{\text{Amount of money spent}}{\text{Total amount of money}} = \frac{y}{100}$$

$$\frac{15}{300} \overset{\div 3}{=} \frac{y}{100}$$

$$y = 5$$

Puan Jorana spends 5% of the total amount of money.

Example 19

At a sale carnival, Encik Rosli chooses a shirt from a rack which displays '45% price reduction'. The original price of the shirt is RM85.00. When Encik Rosli scans the price tag of the shirt, the scanner shows that the price is RM57.80. By applying the concept of proportions, determine whether this percentage discount corresponds to the percentage reduction displayed. Give a reason for your answer.

Solution

Let p be the percentage discount of the original price.

Thus, $$\frac{p}{100} = \frac{\text{Selling price}}{\text{Original price}}$$

$$\frac{p}{100} = \frac{57.80}{85.00}$$

$$85p = 5780$$

$$p = \frac{5780}{85}$$

$$= 68$$

The selling price is 68% of the original price.

Percentage discount obtained = 100% – 68%
= 32%

The percentage discount obtained by Encik Rosli is less than the percentage reduction displayed.

Self Practice 4.5b

1. Determine the percentage of each of the following quantities by applying the concept of proportions.

(a) 14 out of 56 students completed their homework in 1 hour. What is the percentage of the students who had completed their homework in 1 hour?

(b) A class has 45 students. 18 of them are girls. What is the percentage of girls in the class?

(c) The bee hummingbird is the smallest bird in the world. It can weigh as little as 2 grams. The ostrich is the largest bird in the world and it can weigh as much as 150 kilograms. What percent of the mass of a bee hummingbird is the mass of an ostrich?

2. A shoe factory outlet is having a sales promotion in conjunction with the school holidays. A pair of shoes with an original price of RM45 is sold at a 25% discount. By applying the concept of proportions, determine the amount a consumer could save when he buys this pair of shoes during the sales promotion.

How do you solve problems?

MATHEMATICS APPLICATION ZONE

LEARNING STANDARDS

Solve problems involving relationship between ratios, rates and proportions with percentages, fractions and decimals.

The table shows the monthly rental rates and the deposits required when renting apartments in Taman Bukit Damai.

Type of apartment	Two-room	Three-room
Rental rate (RM per month)	450	550
Deposit	1 month rent	2 months rent

(a) Ben and Farid rent the two-room apartment and the three-room apartment respectively. Find the ratio of the initial payment made by Ben to the initial payment made by Farid.

(b) The monthly salaries of Ben and Farid are RM3 750 and RM5 000 respectively. By applying the concept of proportions, determine the percentage of the monthly rental spent to the monthly salary of each of them.

Solution

(a) Initial payment made by Ben $= 450 + 450$
$= \text{RM}900$

Initial payment made by Farid $= 550 + (2 \times 550)$
$= \text{RM}1\,650$

Ratio of initial payment made by Ben to Farid $= 900 : 1\,650$
$= 6 : 11$

(b) Let x be the percentage of the monthly rental spent to the monthly salary of Ben.

$$\frac{x}{100} = \frac{450}{3\,750}$$

$$x = \frac{450}{3\,750} \times 100$$

$$= 12$$

Ben spends 12% of his monthly salary on rental.

Let y be the percentage of the monthly rental spent to the monthly salary of Farid.

$$\frac{y}{100} = \frac{550}{5\,000}$$

$$y = \frac{550}{5\,000} \times 100$$

$$= 11$$

Farid spends 11% of his monthly salary on rental.

Career in Mathematics

Chemists use proportions to calculate the amount of each pigment required to produce the appropriate colour of paint.

Self Practice 4.5c

1. There are 40 passengers on a bus. At the next bus stop, 8 passengers get off and 18 passengers get on the bus.

(a) By applying the concept of proportions, determine the percentage of the passengers who get off the bus compared to the number of passengers originally on the bus.

(b) What is the ratio of the passengers who get on the bus at the bus stop compared to the new total number of passengers on the bus?

Mastery Q 4.5

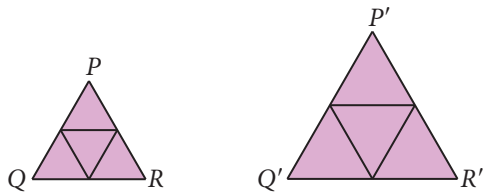
Open the folder downloaded from page vii for extra questions of Mastery Q 4.5.

1. In a basket, the ratio of the number of red apples to the number of green apples is 3 : 5.

(a) Write the fraction of the number of red apples over the total number of apples. Hence, express this fraction in decimal.

(b) What is the percentage of red apples in the basket?

2.



Original After enlargement

Karim uses a photocopy machine to enlarge the diagram of PQR to 150% of its original size.

(a) Write the ratio of the length of $P'Q'$ to the length of PQ.

(b) Is the ratio of the length of $P'R'$ to the length of PR same as the ratio of the length of $P'Q'$ to the length of PQ?

(c) Use your knowledge on ratios and percentages to explain the meaning of 'enlarge diagram PQR to 150% of its original size'.

3. A book with an original price of RM25 is sold at a 30% discount in a bookshop. If the same book is sold online at the original price of RM20, the customers are given a 15% discount. By applying the concept of proportions, determine which one is the better choice for buying the book. Justify your choice.

SUMMARY

Ratio	Rate	Proportion
Compares two or three quantities measured in the same unit.	Compares two quantities measured in different units.	A relationship that states that two ratios or two rates are equal.
Written in the form of $a : b$ or $a : b : c$.	Can be expressed as a fraction involving different units. For example: Speed is measured in km/h.	Can be expressed as a fraction. For example: $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ Thus, a is proportional to b.
Can be expressed as a percentage, a fraction and a decimal.	Cannot be expressed as a percentage.	Can be solved using the unitary method, the proportion method or the cross multiplication method.

At the end of this chapter, I can...	Very good	Work harder
represent the relation between three quantities in the form of $a : b : c$.		
identify and determine the equivalent ratios in numerical, geometrical or daily situation contexts.		
express the ratios of two and three quantities in simplest form.		
determine the relationship between ratios and rates, ratios and proportions.		
determine an unknown value in a proportion.		
determine the ratio of three quantities, given two or more ratios of two quantities.		
determine the ratio or the related value given (i) the ratio of two quantities and the value of one quantity. (ii) the ratio of three quantities and the value of one quantity.		
determine the value related to a rate.		
solve problems involving ratios, rates and proportions, including making estimations.		
determine the relationship between percentages and ratios.		
determine the percentage of a quantity by applying the concept of proportions.		
solve problems involving relationship between ratios, rates and proportions with percentages, fractions and decimals.		

Test Yourself

1. Copy and match the following pairs of equivalent ratios.

6 : 9 •	• 1 : 15
6 : 90 •	• 2 : 3
9 : 60 •	• 3 : 20

2.

Aquarium *A*

Aquarium *B*

Aquarium *A* contains 16 goldfish and aquarium *B* contains 20 goldfish.

(a) State the ratio of the number of goldfish in aquarium *A* to the number of goldfish in aquarium *B*.

(b) A few goldfish are subsequently added into each aquarium such that the ratio in (a) remains the same. Find the minimum number of goldfish added into each of the aquariums.

3. The information below shows the ratios of the scores obtained by Group *P* and Group *Q* in a quiz.

Ratio of the scores obtained by Group *P*				
Chong	:	Rahim	:	Hassan
5	:	4	:	7

Ratio of the scores obtained by Group *Q*				
Nurin	:	Bala	:	Shanthy
3	:	2	:	1

The total score obtained by Group *P* is 144 and the total score obtained by Group *Q* is 168.

(a) Find Rahim's score.

(b) State the ratio of Hassan's score to Bala's score.

(c) Who obtained the highest score? What is the score?

Self Mastery

4.

Anis goes to a grocery store to buy her favourite brand of orange juice. She finds that the orange juice is sold in different volumes and prices. Which bottle of orange juice offers the most affordable price?

5. Puan Kavitha has a water tank which measures 120 cm in length, 60 cm in width and 50 cm in height. Water is filled into the tank at a rate of 2.4 litres per minute. If Puan Kavitha starts to fill the empty tank with water at 0630 hours, at what time will the water tank be full? ($1\,l = 1\,000\ cm^3$)

6. Atong uses the following proportion to estimate the height of a tree.

$$\frac{\text{Height of the tree}}{\text{Length of the tree's shadow}} = \frac{\text{Height of the student}}{\text{Length of the student's shadow}}$$

Atong's height is 1.55 m and one afternoon, he finds that his shadow is 0.93 m on the ground. Estimate the height of the tree that casts a 6-m shadow on the ground.

Challenge Yourself

7. The table shows the prices of RON 95 petrol on two different days.

Date	29 February 2016	6 March 2016
Price of RON 95 petrol	RM1.75 per litre	RM1.60 per litre

(a) If Lai Huat pays RM15 every time he fills up his motorcycle fuel tank with RON 95 petrol, what is the difference in volume, in litres, of the petrol filled on 29 February 2016 compared to 6 March 2016?

(b) Lai Huat suggests moving to his new house on 8 March 2016.

	Monthly rental (RM)	**Commuting distance (km)**
Current house	300	24
New house	340	18

Lai Huat fills up his motorcycle fuel tank with RON 95 petrol to travel from his house to the workplace. If the fuel consumption rate for his motorcycle is 20 km per litre, should Lai Huat move to his new house? Give a reason and show your calculations. (Assume that Lai Huat works an average of 20 days per month.)

8. The table shows the parking rates at a car park.

Parking rate	**Monday – Friday (8:00 a.m. – 5:00 p.m.)**
First 2 hours or a part thereof	RM1.60
Every subsequent hour or a part thereof	RM1.00
After 5:00 p.m. to 10:00 p.m. (per entry)	RM2.00
Lost ticket	RM20.00

(a) On Wednesday, Puan Zaiton parks her car from 1030 hours to 1400 hours. How much does Puan Zaiton need to pay?

(b) On Thursday, Mr Ong parks his car at 0800 hours. Upon his return to the car park at 2000 hours, he finds that he has left his parking ticket in the office on the 16th floor. Should Mr Ong go back to his office to take the parking ticket or pay the penalty by reporting the lost ticket? Justify your answer.

ASSIGNMENT

Black glutinous rice porridge is a popular dessert in Malaysia. Look at the black glutinous rice porridge recipe below. If this recipe serves 6 people, calculate the quantity of each ingredient that is required to serve all the students in your class or all your family members at home. Use your knowledge on ratios and rates to prepare this recipe. Try this recipe in school or at home with the help of your teacher or family members.

Black Glutinous Rice Porridge

Ingredients:

- 200 g of black glutinous rice (soaked overnight in water)
- 125 g of palm sugar (*gula melaka*)
- 270 m*l* of coconut milk (add more if you prefer a richer taste)
- 6 cups of water
- 3 pieces of *pandan* leaves, halved and knotted.

Method:

1. Wash and drain the black glutinous rice thoroughly.
2. Place the glutinous rice in a pot filled with 5 cups of water and the knotted *pandan* leaves.
3. Bring the glutinous rice to a boil and then lower the heat to a simmer until the rice is cooked and soft. Stir the glutinous rice occasionally.
4. Meanwhile, add the palm sugar (*gula melaka*) and a cup of water to another pot. Heat up the pot and stir the mixture on low heat until a thick syrup is formed.
5. Add the palm sugar (*gula melaka*) mixture together with coconut milk to the cooked glutinous rice. Stir until the mixture is well blended.
6. Turn off the heat. The black glutinous rice porridge is ready to be served warm or chilled.

Exploring MATHEMATICS

Leonardo da Vinci (famous for the painting of Mona Lisa), Michelangelo (an artist, architect and poet of the Italian Renaissance) and a few other artists studied the relationship of mathematical ratios on the human body. One of the ratios they used was $\frac{\text{height of head}}{\text{height of the whole body}} = \frac{2}{5}$.

Find pictures or photos of people in magazines or newspapers. Measure their entire body to determine the ratio. Is the ratio always $\frac{2}{5}$?

CHAPTER 5

Algebraic Expressions

What will you learn?

- Variables and Algebraic Expression
- Algebraic Expressions Involving Basic Arithmetic Operations

Why study this chapter?

In the field of algebra, you will learn the method to represent an unknown value with a letter. Thus, daily life problems can be represented in simple mathematical models by writing the relationship between the quantities involved in algebraic language. Discuss daily life problems which involve unknown values.

During the school holidays, a tour company offers a promotional package of three days two nights holiday in Pulau Pangkor. How can you determine the total cost for the different number of adults and children?

Walking through Time

Abu Abdullah Muhammad Ibn Musa Al-Khwarizmi

The term 'algebra' was derived from the Arabic word 'al-jabr' based on the book entitled 'al-jabr wa'l Muqabalah' written by an Arabic mathematician, Muhammad Ibn Musa Al-Khwarizmi. He is also known as the 'Father of Algebra' for his contributions in the field of algebra.

For more information:

http://goo.gl/qAuzp9

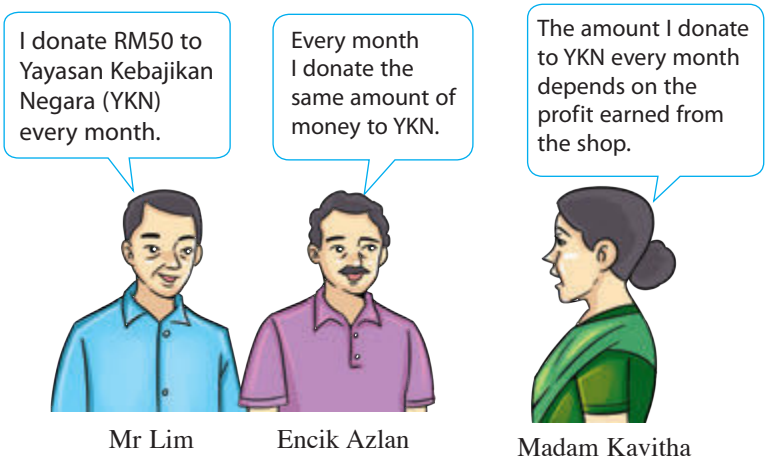
Word Link

• letter	• *huruf*
• coefficient	• *pekali*
• variable	• *pemboleh ubah*
• term	• *sebutan*
• algebraic term	• *sebutan algebra*
• like terms	• *sebutan serupa*
• unlike terms	• *sebutan tidak serupa*
• algebraic expression	• *ungkapan algebra*

Open the folder downloaded from page vii for the audio of Word Link.

5.1 Variables and Algebraic Expression

How do you use letters to represent variables?



LEARNING STANDARDS

Use letters to represent quantities with unknown values. Hence, state whether the value of the variable varies or fixed, with justification.

Based on the situation above, we know the total amount of money donated by Mr Lim every month. However, we do not know the amount of money donated by Encik Azlan and Madam Kavitha. The total amount of money donated by Encik Azlan and Madam Kavitha is a quantity with an unknown value. The quantity is known as a **variable**.

We can use letters to represent variables. For example:

Every month, Encik Azlan donates RMx and Madam Kavitha donates RMy to Yayasan Kebajikan Negara.

Between the variables x and y, which one has a fixed value and which one has a varied value?

SMART TIPS

- A variable has a fixed value if the represented quantity is always constant at any time.
- A variable has a varied value if the represented quantity changes over time.

Example 1

Represent each of the following variables with an appropriate letter. Hence, determine whether the variable has a fixed value or a varied value. Justify your answers.

(a) The annual interest rate for a fixed deposit offered by a bank.

(b) The travelling time taken by Faizal from his house to the school every day.

Solution

(a) k represents the annual interest rate for a fixed deposit.
k has a fixed value because the interest rate for the fixed deposit does not change throughout the year.

(b) t represents the travelling time taken by Faizal from his house to the school every day.
t has a varied value because the travelling time of Faizal changes every day.

Let's Discuss

The market price for 1 gram of gold is RMy per day. Is y a variable with a fixed value or a varied value? Discuss this situation.

Self Practice 5.1a

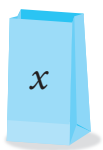
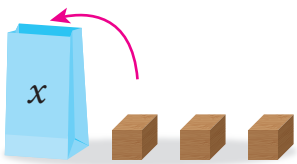
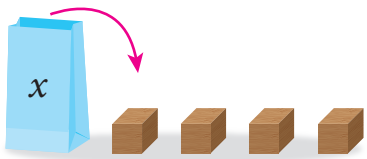
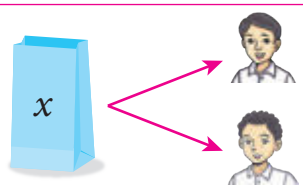
1. Represent each of the following variables with an appropriate letter. Hence, determine whether the variable has a fixed value or a varied value. Justify your answers.
(a) The mass of each student in your class.
(b) The mark obtained by Zaini in a Mathematics test.
(c) The distance between Arman's house and his school.
(d) The temperature at the peak of Gunung Kinabalu in a day.

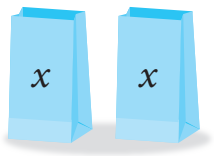
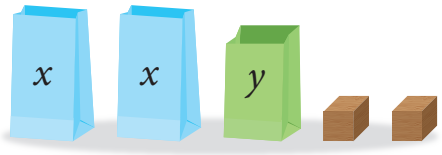
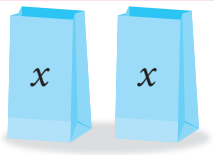
LEARNING STANDARDS

Derive algebraic expressions based on arithmetic expressions that represent a situation.

How do you derive an algebraic expression from a situation?

Study the situations below where the number of blocks in each bag is not known.

Situation	Total number of blocks
A bag contains x blocks. 	x
3 blocks are added into the bag. 	$x + 3$
4 blocks are taken out from the bag. 	$x - 4$
The blocks in the bag are divided equally between 2 students. 	$\frac{x}{2}$

Two bags each contains x blocks respectively.		$2 \times x = 2x$
Two bags each contains x blocks respectively while another bag contains y blocks, and 2 extra blocks are added.		$2x + y + 2$
Two bags each contains x blocks respectively where the mass of each block is p gram.		Total mass of the blocks in both bags $= 2 \times x \times p$ $= 2xp$

The total number or the mass of blocks which are written in **numbers** and **variables**, for example, x, $x + 3$, $x - 4$, $\frac{x}{2}$, $2x$, $2x + y + 2$, $2xp$ are called **algebraic expressions**.

Example 2

Yusri buys 5 apples which cost x sen each and 8 oranges which cost y sen each. Write an algebraic expression for the total amount of money that he paid.

Solution

The total price of the apples $= 5 \times x$
$= 5x$

The total price of the oranges $= 8 \times y$
$= 8y$

The total payment $= 5x + 8y$

Self Practice 5.1b

1. Write an algebraic expression for each of the following situations:
(a) Subtract 7 from a number, x.
(b) The sum of y and z is divided by 9.
(c) The total number of people for x tents if each tent can accommodate 4 people.
(d) Madam Neo bought m kg of bream which costs RMp per kilogram and n kg of hardtail scad which costs RMq per kilogram. What is the total payment?
(e) The age of Nazmi is h years old and Jagjit is k years old whereas the age of Izhar is twice that of Jagjit. What is the difference in age between Nazmi and Izhar?

How do you determine the values of algebraic expressions?

LEARNING STANDARDS

Determine the values of algebraic expressions given the values of variables and make connection with appropriate situations.

The value of an algebraic expression can be determined by substituting the variables with given values.

Example 3

Given that $x = 3$ and $y = 2$, find the value of $8x - 5y + 7$.

Solution

$$\begin{aligned} 8x - 5y + 7 &= 8(3) - 5(2) + 7 \\ &= 24 - 10 + 7 \\ &= 21 \end{aligned}$$

Smart

A scientific calculator can be used to determine the value of an expression. For instance, in Example 3:

1. Press
   8 | ALPHA | X | – | 5 | ALPHA | Y | + | 7 | CALC
   Screen displayed
   X?
2. Enter the given value of x.
   Press 3 | =
   Screen displayed
   Y?
3. Enter the given value of y.
   Press 2 | =
   Screen displayed
   8X – 5Y + 7
   21

Example 4

In a class, $\frac{1}{3}$ of the boys are members of *Kadet Remaja Sekolah* whereas 9 girls are not members of *Kadet Remaja Sekolah*.

(a) Write an expression for the total number of members of *Kadet Remaja Sekolah* in the class.

(b) If there are 12 boys and 16 girls in the class, calculate the total number of members of *Kadet Remaja Sekolah* in the class.

Solution

(a) Let the number of boys = x
and the number of girls = y
Thus, the total number of members of *Kadet Remaja Sekolah* = $\frac{1}{3}x + y - 9$

(b) When $x = 12$ and $y = 16$,
the total number of members of *Kadet Remaja Sekolah* $= \frac{1}{3}(12) + 16 - 9$
$= 4 + 16 - 9$
$= 11$

Self Practice 5.1c

1. Given that $p = 5$, $q = 2$ and $r = -4$, find the value for each of the following expressions:
(a) $2p + q$ (b) $3q - 4r + 8$ (c) $5(p - r)$ (d) $\frac{r}{2} + 7q - 3$

2. Encik Adnan and Mr Tan donate rice to the flood victims. Encik Adnan donates 8 bags of rice and each bag has a mass of x kg. Mr Tan donates 4 bags of rice and each bag has a mass of y kg.
(a) Write an expression for the total mass of rice donated by them.
(b) If $x = 5$ and $y = 10$, calculate the total mass of rice donated by them.

3. Jane and Kamalesh buy m and n *Hari Raya* greeting cards respectively at a price of RMp for each card. These cards will be given to their Muslim friends.
(a) Write an expression for the difference in the amount of money that they had paid.
(b) If $m = 8$, $n = 6$ and $p = 1.5$, calculate the difference in the amount of money they had paid.

What are the terms and the coefficients in an expression?

In an expression, for example, $2x + 3xy$,
$2x$ is the product of the number 2 and the variable x.
$3xy$ is the product of the number 3 and the variables x and y.

$2x$ dan $3xy$ are known as **algebraic terms**.

Example 5

Identify the algebraic terms for each of the following algebraic expressions.

(a) $x + 5x$ (b) $\frac{x}{8} - 7y$

(c) $pq - 2q + 13$ (d) $m^2 - 2m + n - 6$

Solution

(a) $x + 5x$
Algebraic terms are x and $5x$.

(b) $\frac{x}{8} - 7y$
Algebraic terms are $\frac{x}{8}$ and $7y$.

(c) $pq - 2q + 13$
Algebraic terms are pq, $2q$ and 13.

(d) $m^2 - 2m + n - 6$
Algebraic terms are m^2, $2m$, n and 6.

An algebraic term can be written as the product of the variable and its factor. For example,

$3xy = 3x \times y$ — $3x$ is a factor of the variable y. So, $3x$ is the coefficient of the variable y.
$= 3y \times x$ — $3y$ is a factor of the variable x. So, $3y$ is the coefficient of the variable x.
$= 3 \times xy$ — xy is the variable.

What is the coefficient of the variable xy?

LEARNING STANDARDS

Identify the terms in an algebraic expression. Hence, state the possible coefficients for the algebraic terms.

Communication Corner

Is x an algebraic term? Explain.

Did You Know

The algebraic term that consists of one variable with the power 1 is called a linear algebraic term.

SMART TIPS

In an algebraic expression, a number is also considered as a term.

Think Smart

Is term xy an algebraic expression? Give your reasons.

Example 6

In the term $-3k^2mn$, state the coefficient of

(a) k^2mn (b) $-mn$ (c) $3k^2$

Solution

(a) $-3k^2mn = -3 \times k^2mn$
The coefficient of k^2mn is -3.

(b) $-3k^2mn = 3k^2 \times (-mn)$
The coefficient of $-mn$ is $3k^2$.

(c) $-3k^2mn = -mn \times 3k^2$
The coefficient of $3k^2$ is $-mn$.

Self Practice 5.1d

1. Identify all the terms for each of the following algebraic expressions:

(a) $6k + 2k$ (b) $x^2 - 9xy$

(c) $\frac{ab}{3} + 2a - 5b$ (d) $4pq - \frac{7x}{2} + 8p^2q - 1$

2. In the term $-8xy^2$, state the coefficient of

(a) xy^2 (b) $8x$ (c) y^2 (d) $-x$

What are like terms and unlike terms?

LEARNING STANDARDS
Identify like and unlike terms.

$-2m$ and $\frac{3}{7}m$	$\frac{pq}{3}$ and $6.9pq$	$6ab$ and $7ba$	$5k^2$ and $-k^2$

Diagram (a)

Each pair of terms in Diagram (a) has the same variable with the same power. The pair of terms is known as **like terms**.

$2v$ and $2w$	$5xy$ and $-yz$	$9x^2$ and $8x$	$3m^2n$ and $\frac{1}{4}mn^2$

Diagram (b)

Each pair of terms in Diagram (b) does **not** have the same variable with the same power. The pair of terms is known as **unlike terms**.

Let's Discuss

Discuss whether $\frac{xy}{2}$ and $\frac{5x}{y}$ are like terms or unlike terms.

Example 7

Identify whether each of the following pairs of terms is like terms or unlike terms:

(a) $4xy, \frac{xy}{2}$ (b) $12pq, 12pr$ (c) $3abc, 0.5bca$ (d) $-7h, 6h^2$

Solution

(a) Like terms ← Same variable xy.

(b) Unlike terms ← Variables pq and pr are different.

(c) Like terms ← Variable abc is equal to bca.

(d) Unlike terms ← The powers of the variable h are different.

Self Practice 5.1e

1. Identify whether each of the following pairs of terms is like terms or unlike terms:

(a) $5k, -0.1k$ (b) $4y, y^2$ (c) $4srt, 11rts$ (d) $\frac{3ab}{2}, -8bc$

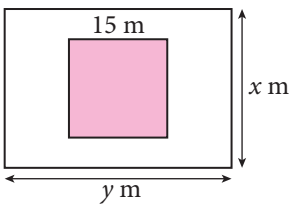
Mastery Q 5.1

Open the folder downloaded from page vii for extra questions of Mastery Q 5.1.

1. Mr Gan invests a certain amount of money in Amanah Saham. A dividend is paid based on a specific rate every year. Represent each variable in this situation with a suitable letter. Hence, explain whether each variable has a fixed value or a varied value.

2. A watermelon with a mass of 5 kg and two durians with the same mass are placed on an electronic balance. If the reading shown on the balance is m kg, write an expression for the mass of a durian.

3. The diagram shows a rectangular park. The shaded square is planted with flowers. The rest of the area is covered with bricks. Write an expression for the area covered with bricks.



4. (a) Given that $x = 5$ and $y = -2$, find the value of $xy + \frac{x}{2} - 6y$.

(b) Given that $a = 7$, $b = 3$ and $c = -4$, find the value of $3(b - a) - 5ac + 14$.

5. Azlan has n coins, consisting of x 10-sen coins, $3x$ 20-sen coins and the rest are 50-sen coins, in a coin box.

(a) State an expression for the number of 50-sen coins in the coin box.

(b) Find the total amount of money in the coin box if $x = 6$ and the number of 50-sen coins is twice the number of 20-sen coins.

6. A ribbon with a length of p cm is cut into three parts. The length of the first part and the second part are x cm and $2x$ cm respectively.

(a) Write an expression for the length of the third part.

(b) If $x = 10$ and the length of the second part is four times the length of the third part, calculate the value of p.

7. Copy and complete the table by stating the possible coefficients for the following algebraic term.

Algebraic term	Coefficient	Variable
$-10abc$		

5.2 Algebraic Expressions Involving Basic Arithmetic Operations

How do you add and subtract two or more algebraic expressions?

When adding and subtracting two or more algebraic expressions, gather the like terms first. Then, add or subtract the like terms.

LEARNING STANDARDS

Add and subtract two or more algebraic expressions.

Example 8

Simplify each of the following.

(a) $(3x + 5y) + (8x - y - 9)$

(b) $(12mn - 4p) + (6 + 7p) - (10mn + p - 2)$

Solution

(a) $(3x + 5y) + (8x - y - 9)$

$= 3x + 5y + 8x - y - 9$

$= 3x + 8x + 5y - y - 9$ ← Gather the like terms.

$= 11x + 4y - 9$ ← Simplify the like terms.

(b) $(12mn - 4p) + (6 + 7p) - (10mn + p - 2)$

$= 12mn - 4p + 6 + 7p - 10mn - p + 2$

$= 12mn - 10mn - 4p + 7p - p + 6 + 2$

$= 2mn + 2p + 8$

When arranging like terms, the operation sign which lies before the term must be transferred together.

SMART TIPS

- When the '+' sign which lies before the brackets is removed, the sign for each term in the brackets remains unchanged.
- When the '–' sign which lies before the brackets is removed, the sign for each term in the brackets changes from: '+ to –'; '– to +'.

$-(a + b) = -a - b$
$-(a - b) = -a + b$
$-(-a + b) = +a - b$
$-(-a - b) = +a + b$

Self Practice 5.2a

1. Simplify each of the following.

(a) $(3x - 2y) + (5x + 9y)$

(b) $(6ab + 2bc + 10) - (ab + 3bc - 2)$

(c) $(4xy + 5k) - (-3k + 7) + (13xy - k)$

(d) $(7p - 8q + 6pq) + (q - 2p + pq) - (10pq - p - 4q)$

(e) $\frac{2}{3}fg - (9mn - \frac{1}{2}fg) + (3mn - \frac{1}{6}fg)$

What is the product of the repeated multiplication of algebraic expressions?

LEARNING STANDARDS

Make generalisation about repeated multiplication of algebraic expressions.

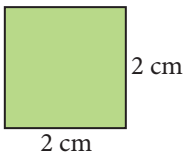
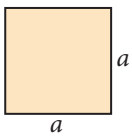
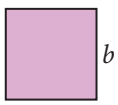
Exploration Activity 1

Group

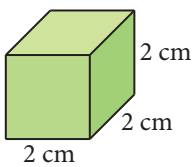
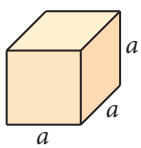
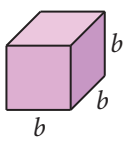
Aim: **To make a generalisation about the repeated multiplication of algebraic expressions.**

Instruction: Perform the activity in groups of four.

1. Find the area for each of the following squares in the form of repeated multiplication.

		
Area = ☐ cm × ☐ cm = ☐^{2} cm^2	Area = ☐ × ☐ = ☐☐	Area = ☐ × ☐ = ☐☐

2. Find the volume for each of the following cubes in the form of repeated multiplication.

		
Volume = ☐ cm × ☐ cm × ☐ cm = ☐^{3} cm^3	Volume = ☐ × ☐ × ☐ = ☐☐	Volume = ☐ × ☐ × ☐ = ☐☐

3. Based on the results above, what generalisations can be made about

(a) $a \times a \times a \times a$?

(b) $a \times a \times a \times a \times \ldots \times a$, where the multiplication of a is repeated n times?

(c) (i) $(a + b) \times (a + b)$?

(ii) $(a + b) \times (a + b) \times (a + b)$?

(iii) $(a + b) \times (a + b) \times (a + b) \times (a + b)$?

(iv) $(a + b) \times (a + b) \times (a + b) \times (a + b) \times \ldots \times (a + b)$, where the multiplication of $(a + b)$ is repeated n times?

From the results of Exploration Activity 1, it is found that

$\underbrace{a \times a}_{\text{Repeated multiplication of } a \text{ by 2 times}} = a^2$ $\underbrace{a \times a \times a}_{\text{Repeated multiplication of } a \text{ by 3 times}} = a^3$ $\underbrace{a \times a \times a \times \ldots \times a}_{\text{Repeated multiplication of } a \text{ by } n \text{ times}} = a^n$ ← Power of n

Hence, generalisations regarding the repeated multiplication of algebraic expressions are recognised as follows.

$(a + b) \times (a + b) = (a + b)^2$
$(a + b) \times (a + b) \times (a + b) = (a + b)^3$
$(a + b) \times (a + b) \times (a + b) \times (a + b) = (a + b)^4$

In general,

$\underbrace{(a + b) \times (a + b) \times (a + b) \times \ldots \times (a + b)}_{\text{Repeated multiplication of the algebraic expression } (a + b) \text{ by } n \text{ times}} = (a + b)^n$ ← Power of n

Example 9

Simplify each of the following:

(a) $m \times m \times m \times m$ (b) $(x + 7) \times (x + 7)$

(c) $(p - 3q) \times (p - 3q) \times (p - 3q)$

Solution

(a) $\underbrace{m \times m \times m \times m}_{\text{Multiplication is repeated 4 times}} = m^4$

(b) $\underbrace{(x + 7) \times (x + 7)}_{\text{Multiplication is repeated 2 times}} = (x + 7)^2$

(c) $\underbrace{(p - 3q) \times (p - 3q) \times (p - 3q)}_{\text{Multiplication is repeated 3 times}} = (p - 3q)^3$

Example 10

Write each of the following in the form of repeated multiplication:

(a) $(x + 4y)^2$ (b) $(9p - q)^3$

Solution

(a) $(x + 4y)^2 = (x + 4y)(x + 4y)$ (b) $(9p - q)^3 = (9p - q)(9p - q)(9p - q)$

Self Practice 5.2b

1. Simplify each of the following:

(a) $pq \times pq \times pq$ (b) $(6a - 1) \times (6a - 1)$

(c) $(8x + 3y) \times (8x + 3y) \times (8x + 3y)$

2. Write each of the following in the form of repeated multiplication:

(a) $(2 + 7x)^2$ (b) $(h - 4k)^3$ (c) $(5p + q)^4$

How do you multiply and divide algebraic expressions?

To find the product of algebraic expressions with one term, gather all the same variables, and then multiply the number with number and the variable with variable.

LEARNING STANDARDS

Multiply and divide algebraic expressions with one term.

Example 11

Simplify $3ab^2 \times 4a^3b$.

Solution

$$3ab^2 \times 4a^3b = 3 \times a \times b \times b \times 4 \times a \times a \times a \times b$$ ← Write as the product of factors.

$$= 3 \times 4 \times a \times a \times a \times a \times b \times b \times b$$ ← Gather the numbers and the same variables.

$$= 12a^4b^3$$

The quotient of algebraic expressions with one term can be obtained by eliminating the common factors.

Example 12

Simplify $20m^4n^2 \div 5m^2n^3$.

Solution

$$20m^4n^2 \div 5m^2n^3 = \frac{20m^4n^2}{5m^2n^3}$$ ← Write in fraction form.

$$= \frac{\overset{4}{\cancel{20}} \times \cancel{m} \times \cancel{m} \times m \times m \times \cancel{n} \times \cancel{n}}{\underset{1}{\cancel{5}} \times \cancel{m} \times \cancel{m} \times \cancel{n} \times \cancel{n} \times n}$$ ← Simplify.

$$= \frac{4m^2}{n}$$

Let's Discuss

By using a cube with sides of x cm, discuss how you would show the quotient for $x^3 \div x$ and $x^3 \div x^2$.

Example 13

Simplify $21xy \times 6x \div 14y^3z$.

Solution

$$21xy \times 6x \div 14y^3z = \frac{\overset{3}{\cancel{21}} \times x \times \cancel{y} \times \overset{3}{\cancel{6}} \times x}{\underset{z}{\cancel{14}} \times \cancel{y} \times y \times y \times z}$$

$$= \frac{9x^2}{y^2z}$$

CHAPTER 5

Self Practice 5.2c

1. Find the product for each of the following:

(a) $3x \times 5x^3$ (b) $-4mn \times 7m^2$ (c) $\frac{2}{3}p^4q \times 6pr$

2. Find the quotient for each of the following:

(a) $8x^6y^4 \div 2xy^3$ (b) $4ab^3 \div 6a^2b$ (c) $12p^5r \div (-10pq)$

3. Simplify each of the following:

(a) $2mn \times 5m^2 \div 3n^3$ (b) $6xy \div 20px^2 \times (-5p^6y)$

Mastery Q 5.2

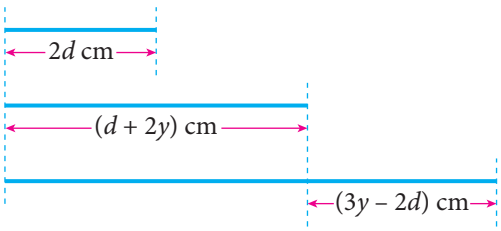
Open the folder downloaded from page vii for extra questions of Mastery Q 5.2.

1. Simplify each of the following:

(a) $(x + pq) - (3y - \frac{pq}{2} - 4) + (\frac{1}{3}x - 5y + 7)$

(b) $\frac{6ab - 9mn}{3} - 2(4mn - 3ab)$

2. In the diagram, a rope is cut into three parts. Write the expression for the length of the rope in terms of d and y.



3. The age of Azhar's mother was four times the age of Azhar last year. If Azhar is n years old now, state the age of Azhar's mother seven years later in terms of n.

4. It is given that $(ax + b)(ax + b)(ax + b) = (9x - 2)^n$, where a, b and n are integers. Determine the values of a, b and n.

5. Aina makes a cubic model from a manila card. If the volume of the cube is $(2 + 3p)^3$ cm^3, find the total surface area of the cube in terms of p.

6. Simplify each of the following:

(a) $\frac{18xy \times 10y^3z}{15xz^2}$ (b) $-\frac{8pq}{12p^2q} \times (-3p^2q^3)$

7. Copy and fill in the boxes with the correct algebraic terms:

(a) $\square \times 3pqr = 15p^2qr^3$ (b) $\square \div 2xy^2z = 7x^2yz$

8. The area of the rectangle is $12a^3b^2$ cm^2. Express the length of the rectangle in terms of ab.

SUMMARY

ALGEBRAIC EXPRESSIONS

- **Variables**
  - **Fixed value**
    The value of the variable is fixed at any time.
  - **Varied value**
    The value of the variable varies over time.
- **Algebraic terms**
  - **Like terms**
    Contain the same variables with the same power.
    Example: $2x$ and $-\frac{3}{5}x$
  - **Unlike terms**
    Contain different variables or the same variables with different powers.
    Example: • $2x$ and $6y$ • $2x$ and $-7x^2$
- **Addition and subtraction**
  Example: • $5a + 6a = 11a$ • $12bc - 3bc = 9bc$
- **Multiplication and division**
  Example: • $5mn \times 2n$
  $= 5 \times 2 \times m \times n \times n$
  $= 10mn^2$
  • $\frac{36pq}{4p} = \frac{{}^{9}\cancel{36} \times \cancel{p} \times q}{{}_{1}\cancel{4} \times \cancel{p}}$
  $= 9q$

At the end of this chapter, I can...	Very good	Work harder
use letters to represent quantities with unknown values. Hence state whether the value of the variable varies or is fixed with justification.		
derive algebraic expressions for a situation.		
determine the values of algebraic expressions.		
identify the terms in an algebraic expression and hence state the possible coefficients.		
identify like and unlike terms.		
add and subtract two or more algebraic expressions.		
make generalisation about repeated multiplication of algebraic expressions.		
multiply and divide algebraic expressions with one term.		

Let's PRACTISE

Test Yourself

1. It is given that $(3x^2 + 7y - 1) - (x^2 + 2y - 5) + (6x^2 - y) = ax^2 + by + c$, where a, b and c are integers. Determine the values of a, b and c.

2. Pavathy buys x m of batik cloth which costs RM12 per metre and y m of curtain which costs RM7 per metre. If she pays RM120 to the cashier, express the balance in terms of x and y.

3. If $p^3 + 2q = -5$ and $4px = 6$, find the value of $p^3 - (4px - 2q)$.

Self Mastery

4. Kumar buys four pineapples at a price of RMx each. He pays RM20 and receives 80 sen in change. What is the price of a pineapple?

5. In a Mathematics test, Su Lin obtains double the marks of Daud and their total marks are $3k$. If Hafiz obtains 10 marks more than Su Lin, state Hafiz's marks in terms of k.

6. Zuriana's mother gives a certain amount of money to Zuriana to buy satay and *otak-otak*. Zuriana buys m sticks of satay which costs RMx for 5 sticks and receives a balance of 80 sen. Then, she buys $2m$ pieces of *otak-otak* which costs RMy per piece and receives a balance of 60 sen.

(a) Write an algebraic expression for the total payment of the satay and *otak-otak*.

(b) If $m = 10$, $x = 4$ and $y = 1.2$, find the total amount of money that Zuriana received from her mother.

Challenge Yourself

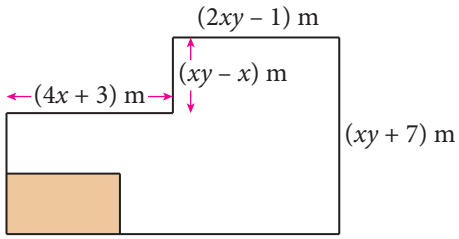
7. A number is added to 7, and its result is x. If the number is divided by 2, its result is y. Explain how you would determine the value of $x + y$ if the value of the number is known.

8. Application Business

An electrical shop buys 120 filament lamps at a cost of RMp each and 180 LED lamps at a cost of RMq each. The shop then sells the lamps with 2 filament lamps and 3 LED lamps at a price of RM$(3h + 4k)$ during their sales promotion. If the shop is able to sell all the lamps, express the profits earned in terms of p, q, h and k.

9. Application Construction

The diagram shows a farm owned by Norhaimi. The rectangular shaded area has not been fertilised. The rest of the area is planted with vegetables. Norhaimi intends to fence up the area planted with vegetables. Express the length of fence required in terms of x and y.



10. Application Science

The temperature in the unit of degree Celsius (°C) can be converted into degree Fahrenheit (°F) by using the expression $\frac{9}{5}T + 32$, where T is the temperature in the unit of degree Celsius. When a type of liquid is heated from 18°C to 33°C, what is the change in temperature of the liquid in the unit of degree Fahrenheit?

ASSIGNMENT

Scan the QR Code or visit the website to get information regarding the uses of algebraic expression in our daily lives.

Hence, write a report about the importance and the application of algebraic expression in our daily lives. Present your report in class.

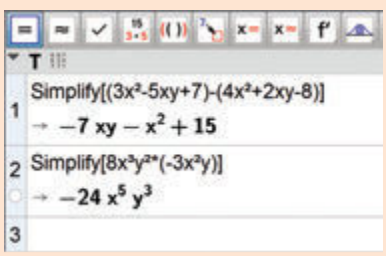
https://goo.gl/ev3Gzs

Exploring MATHEMATICS

A The *GeoGebra* software can be used to simplify the algebraic expression involving basic arithmetic operations.

1. Select on the menu *View* → *CAS*.

2. Use the instruction *Simplify* and type the expression involving basic arithmetic operations and then press Enter in the *CAS* space. For example,
(i) Simplify $[(3x^2 - 5xy + 7) - (4x^2 + 2xy - 8)]$
(ii) Simplify $[8x^3y^2(-3x^2y)]$



3. Enhance your exploration with other expressions involving basic arithmetic operations.

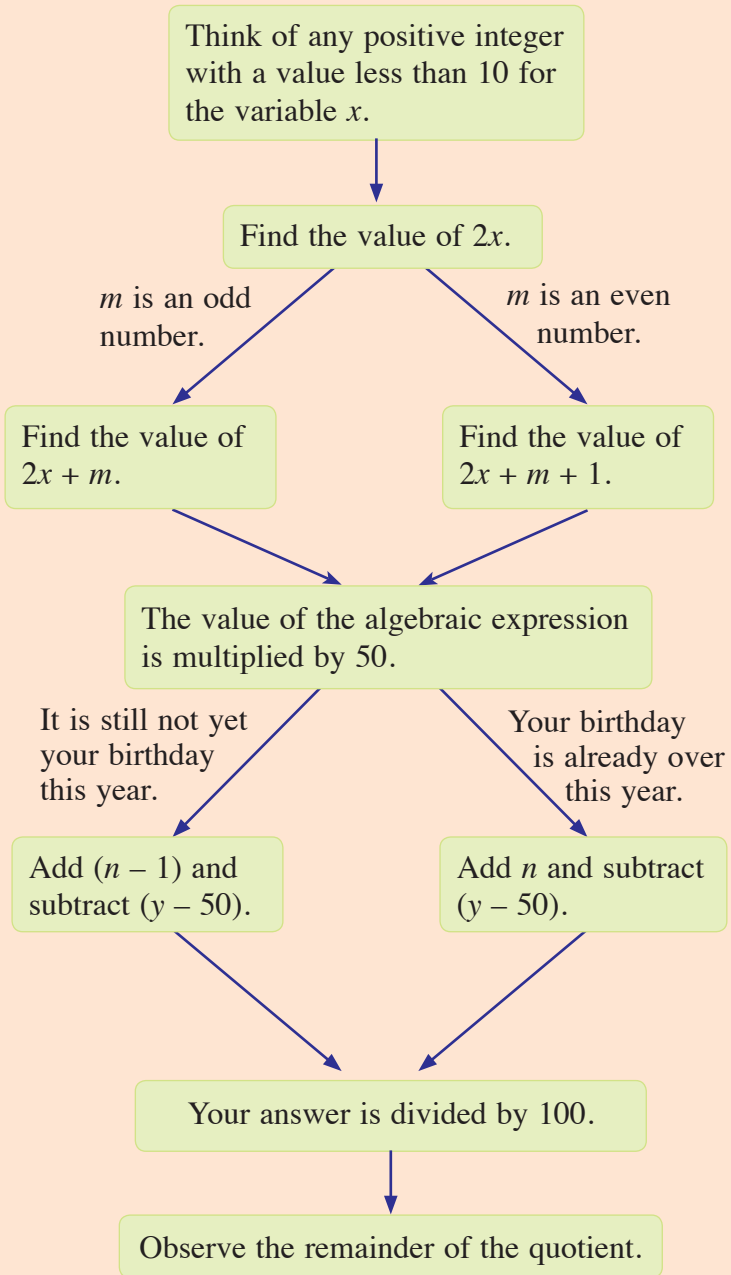
B By following the given instructions and the keys, carry out the following activity.

Key:

m = your birth month
For example,
January = 1
February = 2
March = 3
and so on.

Key:

n = this year
y = your birth year



What is represented by the remainder of the final answer? State your comment.

CHAPTER 6

Linear Equations

What will you learn?

- Linear Equations in One Variable
- Linear Equations in Two Variables
- Simultaneous Linear Equations in Two Variables

Why study this chapter?

Various problems which involve daily situations can be solved by interpreting the information in a mathematical sentence expressed in the form of linear equations. Discuss with your teacher the situations or other fields that involve solution of linear equations.

21st century learning is one of the implementations in the Malaysia Education Blueprint (PPPM) 2013 - 2025. In the 21st century classroom, the students are arranged in groups to enable them to hold discussions and carry out various activities.

For example, a class consisting 30 students is needed to be arranged in groups to achieve the goals.

It is given that the number of boys in the class is 6 more than the number of girls. How do you determine the number of groups that can be formed such that each group has 2 girls?

Walking through Time

Diophantus of Alexandria

Diophantus of Alexandria was a Greek mathematician who contributed significantly to the knowledge of solving algebraic equations. He was known as the Father of Algebra.

For more information:

http://goo.gl/9AoB9

Word Link

• trial and improvement	• *cuba jaya*
• equality	• *kesamaan*
• numerical value	• *nilai berangka*
• working backwards / backtracking	• *pematahbalikan*
• variable	• *pemboleh ubah*
• substitution	• *penggantian*
• elimination	• *penghapusan*
• solution	• *penyelesaian*
• linear equation	• *persamaan linear*
• simultaneous linear equations	• *persamaan linear serentak*
• root of an equation	• *punca persamaan*

Open the folder downloaded from page vii for the audio of Word Link.

6.1 Linear Equations in One Variable

LEARNING STANDARDS

Identify linear equations in one variable and describe the characteristics of the equations.

What are linear equations in one variable?

Diagram (a)

Diagram (b)

Note that the amount of money in Diagram (a) is equal to the amount of money in Diagram (b). This situation can be written as:

$$\text{Amount of money in Diagram (a)} = \text{Amount of money in Diagram (b)}$$

The symbol '=' is used to show the relationship between two quantities that have the same value. The mathematical sentence that involves equality is known as **equation**.

For example, $x + 2 = 5$ and $y - 7 = 11$

Did You Know

The symbol '≠' is used for relationship that consists of different values.
For example, $2 \neq 5$

Exploration Activity 1

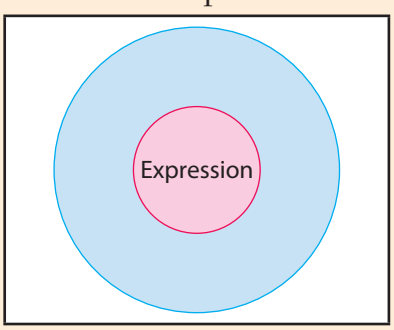
Individual

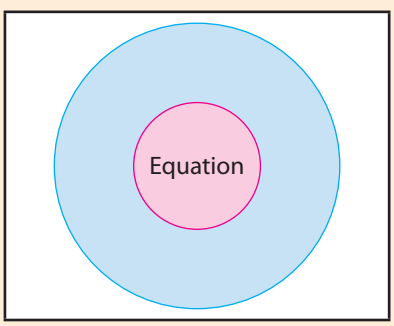
Aim: To identify algebraic expressions and equations.

1. Observe the mathematical sentences written on the cards in the diagram below.

$7h + 2$	$x + 5 = 8$	$\frac{y}{2} - 10 = 1$	$k^2 + 1 = 6$	$6p + 4q = 9$
$x - \frac{2y}{3} = 7$	$3n + 1 = 6n$	$5a^3 - 4$	$3h - 1 = 5k$	$x + 2x$

2. Classify the mathematical sentences into expressions and equations. Copy and complete the circle map below.





3. Compare your answers with those of your friends.

Look at the following equations obtained from the results of Exploration Activity 1.

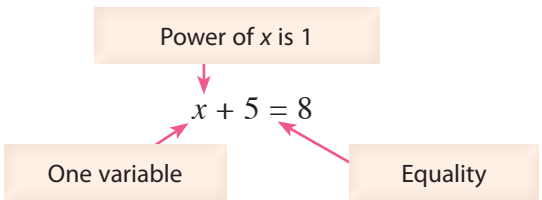
$x + 5 = 8$	$\frac{y}{2} - 10 = 1$	$6p + 4q = 9$
$x - \frac{2y}{3} = 7$	$3n + 1 = 6n$	$3h - 1 = 5k$

These equations are known as **linear equations** since the power of the variables is one.

Among the linear equations, it is found that the following equations have only one variable and the power of the variable is one.

$x + 5 = 8$	$\frac{y}{2} - 10 = 1$	$3n + 1 = 6n$

These equations are known as **linear equations in one variable**.



Communication Corner

Discuss why $k^2 + 1 = 6$ is a non-linear equation.

Did You Know

Expressions such as xy, $\frac{1}{x}$ and $\frac{x}{y}$ are non-linear expressions.

SMART TIPS

The characteristics of a linear equation in one variable:
- Has only one variable
- The power of the variable is one

Let's Discuss

Discuss whether each of the following equations is a linear equation in one variable.
(a) $x = 0$
(b) $\frac{1}{x} + 2 = 6$
(c) $x(x - 1) = 9$

Example 1

Explain whether each of the following equations is a linear equation in one variable:
(a) $3x + 2 = 5$ (b) $p - 4q = 6$
(c) $2(k - 7) = \frac{k}{3}$ (d) $y^2 + 3y = 1$

Solution

(a) Yes, because the equation has one variable, x and the power of x is 1.
(b) No, because the equation has two variables, p and q.
(c) Yes, because the equation has one variable, k and the power of k is 1.
(d) No, because the highest power of the variable y is 2.

Self Practice 6.1a

1. Explain whether each of the following equations is a linear equation in one variable:
(a) $m + 5 = 12$ (b) $3(p - 2) = -7$ (c) $9x + 8y = 10$ (d) $k^2 - 5k = 4$

How do you form linear equations based on a given situation and vice-versa?

LEARNING STANDARDS

Form linear equations in one variable based on a statement or a situation, and vice-versa.

Example 2

Form a linear equation in one variable for each of the following:

(a) Subtract 8 from a number, its remainder is 2.

(b) Peck Chin bought five pens which cost y sen each and an exercise book which costs RM3. The total amount that she paid was RM7.

Solution

(a) Let the number be x.

$x - 8 = 2$

(b) The price of five pens $= 5 \times y$

$= 5y$

$5y + 300 = 700$

RM3 = 300 sen

RM7 = 700 sen

SMART TIPS

- Each term in a linear equation must have the same unit.
- It is unnecessary to write the unit while forming linear equations.

Example 3

Write a statement or situation for each of the following equations.

(a) $2x + 5 = 19$, where x is a number.

(b) $y - 2 = 8$, where y is the age of Rajes now.

Solution

(a) Twice a number added to 5, the result is 19.

(b) Rajes was 8 years old 2 years ago.

Self Practice 6.1b

1. Form a linear equation in one variable for each of the following:

(a) The quotient of a number and 6 is 12.

(b) The price of chicken is RMy per kilogram. Rozita bought 5 kg of chicken and made a total payment of RM40.

(c) The perimeter of a rectangle with length $2x$ cm and width 5 m is 14 m.

2. Write a statement or situation for each of the following equations:

(a) $p - 1 = 6$, where p is a number.

(b) $x + 10 = 78$, where x is Edri's score in a test.

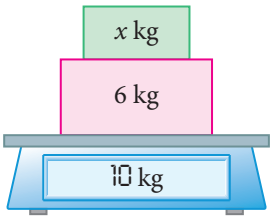
(c) $4m = 50$, where m is the mass, in kg, of a pack of rice.

How do you solve linear equations in one variable?

LEARNING STANDARDS

Solve linear equations in one variable.

Two blocks with mass of 6 kg and x kg respectively are placed on the scales as shown in the diagram.



The reading shown by the scales is equal to the total mass of both the blocks. Therefore, the linear equation formed is $x + 6 = 10$.

If we replace the x kg block with a block of a mass of 4 kg, the reading on the scales is equal to 10 kg. This shows that 4 is a numerical value that satisfies the equation $x + 6 = 10$. This numerical value is known as the solution of the equation $x + 6 = 10$.

The **solution of linear equation** is the numerical value that satisfies the equation.

The linear equation in one variable can be solved by using three methods as follows:
(a) Trial and improvement method
(b) Application of equality concept
(c) Backtracking method

Did You Know

Solution of linear equation is also known as the **root** of the equation.

Example 4

Solve the linear equation $2x + 1 = 7$ by using
(a) trial and improvement method (b) application of equality concept
(c) backtracking method

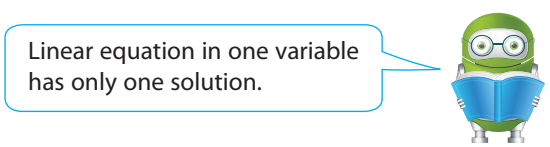
Solution

(a) **Trial and improvement method**

$2x + 1 = 7$

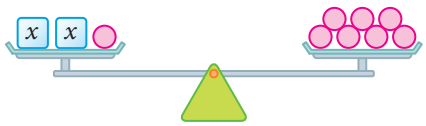
Value of x	Left hand side $= 2x + 1$	Review	Reflection
1	$2(1) + 1 = 3$	$3 \neq 7$, the value 3 is less than 7.	Substitute x with a value greater than 1.
2	$2(2) + 1 = 5$	$5 \neq 7$, the value 5 is close to 7.	Substitute x with a value greater than 2.
3	$2(3) + 1 = 7$	Left hand side = 7	$x = 3$ is the solution.

Thus, $x = 3$



(b) **Application of equality concept**

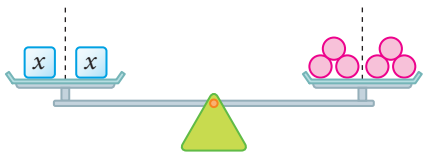
$2x + 1 = 7$



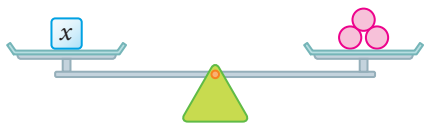
$2x + 1 - 1 = 7 - 1$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$



Thus, $x = 3$



SMART TIPS

In a linear equation, the value on the left hand side is always equal to the value on the right hand side. Therefore, the mathematical operations performed on both sides of the linear equation must be the same in order to obey the equality concept.

(c) **Backtracking method**

$2x + 1 = 7$

Let the initial value = x and the final value = 7.
List the mathematical operations starting from the value of x to become 7.

$x \longrightarrow \times 2 \longrightarrow + 1 \longrightarrow = 7$

By using backtracking method, we have to consider the reverse operation so that 7 can become the value of x.

$x \longleftarrow \div 2 \longleftarrow - 1 \longleftarrow = 7$

So the value of x can be determined by writing

$7 - 1 = 6 \longrightarrow 6 \div 2 = 3$

Thus, $x = 3$

SMART TIPS

The backtracking method uses working backwards strategy and usually is used to solve problems that have known final value but the initial value is unknown.

Let's Discuss

Is backtracking method appropriate for solving all the linear equations? Discuss.

Example 5

Solve the following equations:

(a) $\frac{2x + 13}{5} = 7$ (b) $5(x - 4) = x + 16$

Solution

(a) $\frac{2x + 13}{5} = 7$

$\frac{2x + 13}{5} \times 5 = 7 \times 5$ ← Multiply both sides by 5.

$2x + 13 = 35$

$2x + 13 - 13 = 35 - 13$ ← Subtract 13 from both sides of the equation.

$2x = 22$

$\frac{2x}{2} = \frac{22}{2}$ ← Divide both sides by 2.

$x = 11$

(b) $5(x - 4) = x + 16$

$5x - 20 = x + 16$ ← Expand the equation.

$5x - 20 + 20 = x + 16 + 20$ ← Add 20 to both sides.

$5x = x + 36$

$5x - x = x + 36 - x$ ← Subtract x from both sides of the equation.

$4x = 36$

$\frac{4x}{4} = \frac{36}{4}$ ← Divide both sides by 4.

$x = 9$

Think Smart

Can you solve the equation $3(2x + 3) = 6x + 9$? Explain.

SMART TIPS

Check your answer by substituting the value of the variable obtained into the equation and then test whether the value on the left hand side is equal to the value on the right hand side.

Visit http://goo.gl/nB5Sq for the game about solving linear equations in one variable.

Self Practice 6.1c

1. Solve the following linear equations:

(a) $x - 10 = 3$ (b) $4x - 1 = 7$ (c) $\frac{x}{6} + 3 = 5$

(d) $\frac{x + 8}{2} = 9$ (e) $3(x + 2) = 5x$ (f) $\frac{2}{3}x - 4 = x + 1$

How do you solve problems?

LEARNING STANDARDS

Solve problems involving linear equations in one variable.

MATHEMATICS APPLICATION ZONE

After 10 years, Jalil's age will be thrice his age this year. What is Jalil's age?

Solution

Understanding the problem

After 10 years, Jalil's age is thrice his age this year.

Devising a plan

Let Jalil's current age = x years old

After 10 years, Jalil's age = $3 \times x$

$= 3x$

Linear equation in one variable that can be formed is $x + 10 = 3x$.

Implementing the strategy

$$x + 10 = 3x$$
$$x + 10 - x = 3x - x$$
$$10 = 2x$$
$$\frac{10}{2} = \frac{2x}{2}$$
$$5 = x$$

Jalil is 5 years old.

Doing reflection

When $x = 5$, $5 + 10 = 3(5)$

$= 15$

SMART TIPS

The steps to solve problems involving linear equations in one variable:

- Identify the variable in the problem and represent the variable with a letter.
- ↓
- Form a linear equation based on the information given.
- ↓
- Solve the equation to find the value of the variable.

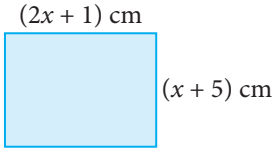
Think Smart

The sum of three consecutive odd numbers is 63. What are the numbers?

Self Practice 6.1d

1. In a mathematics test, Azmah scored 17 marks more than Yazid while Suzana's score is twice of Yazid's score. If their total score is 161, what is Azmah's score?

2. The diagram shows a rectangle. If the perimeter of the rectangle is 66 cm, what is the area of the rectangle?



Mastery Q 6.1

Open the folder downloaded from page vii for extra questions of Mastery Q 6.1.

1. Write the linear equation in one variable for each of the given situations:

(a) The difference between x and 8 is 15, where x is greater than 8.

(b) The price of an exercise book is thrice the price of a pen. Yahya buys 2 exercise books and 8 pens and makes a total payment of RM42.

(c) Harjit's age is p years old. He was born when his mother was 34 years old. Now, his mother's age is thrice his age.

2. Solve the following equations using the trial and improvement method:

(a) $x - 6 = 10$ (b) $2x + 3 = 11$ (c) $14 - 3x = 8$

3. Solve the following equations by applying the equality concept:

(a) $5x + 1 = 3x$ (b) $2y - 4 = 5y + 8$ (c) $\frac{2}{3}x - 9 = 7$

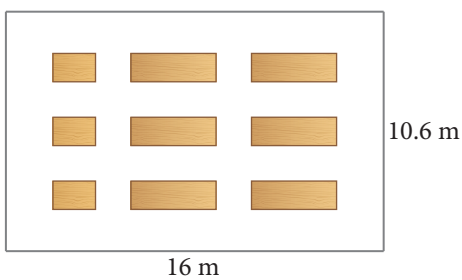
4. Solve the following equations using the backtracking method:

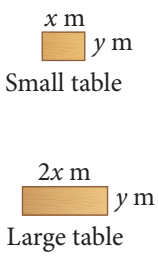
(a) $3x + 2 = 12$ (b) $2(x + 4) = 22$ (c) $\frac{7y - 6}{9} = 4$

5. There are 35 students participating in a community activity to clean up their school. The number of girls who participated is 5 less than the number of boys. How many boys participated in the activity?

6. The time spent by Nadia to answer the geography quiz is 30 minutes more than the time spent to answer the history quiz. The time spent to answer the history quiz is half the time spent to answer the geography quiz. What is the total time that Nadia took to answer both quizzes?

7.





The diagram shows the plan for the arrangement of tables in a room. The distance between the tables is 1.5 m and the distance between the tables and the wall is 2 m. Determine the surface areas of a small table and a large table.

6.2 Linear Equations in Two Variables

What are linear equations in two variables?

LEARNING STANDARDS

Identify linear equations in two variables and describe the characteristics of the equations.

Look at the following linear equations obtained from the results in Exploration Activity 1.

$6p + 4q = 9$ $\quad x - \frac{2y}{3} = 7$ $\quad 3h - 1 = 5k$

These linear equations have two variables and the power of each variable is 1. These equations are known as **linear equations in two variables**.

Linear equation in two variables is a linear equation that has two variables and the power of each variable is one.

Linear equation in two variables can be written in the general form of

$$ax + by = c \quad \text{where } a \text{ and } b \text{ are non-zero.}$$

Which of the following linear equations in two variables is written in the general form?

$6p + 4q = 9$ $\quad x - \frac{2y}{3} = 7$ $\quad 3h - 1 = 5k$

Example 6

Explain whether the following equations are linear equations in two variables:

(a) $3p - q = 6$ (b) $7x^2 + 5y = 9$

(c) $2(k + 8) = 5k$ (d) $\frac{y}{6} + 4 = -2x$

Let's Discuss

Discuss why $xy + 2y = 3$ and $\frac{1}{x} + \frac{1}{y} = 5$ are not linear equations in two variables.

Solution

(a) Yes, because the equation has two variables, p and q, and the power of each variable is 1.

(b) No, because the power of variable x is 2.

(c) No, because the equation has only one variable.

(d) Yes, because the equation has two variables, x and y, and the power of each variable is 1.

Self Practice 6.2a

1. Explain whether the following equations are linear equations in two variables:

(a) $h - 7k = 6$ (b) $3m + 5 = 9m - 1$ (c) $\frac{2x}{3} - 8 = 4y$ (d) $p(p + 3) = 2q$

How do you form linear equations in two variables?

Example 7

LEARNING STANDARDS

Form linear equations in two variables based on a statement or a situation, and vice-versa.

Form a linear equation in two variables for each of the following:

(a) The difference between two numbers is 18.

(b) The amount of pocket money saved by Ahmad is RMx and the amount of pocket money saved by Norita is RMy. Their total savings is RM600.

(c) A bus and a van can accommodate m and n passengers respectively. The total number of passengers for two buses and five vans is 100.

Solution

(a) Let the two numbers be p and q respectively.

$p - q = 18$

(b) Total savings = 600

$x + y = 600$

(c) Total number of passengers = 100

$2m + 5n = 100$

Example 8

Write the situation for each of the following linear equations in two variables.

(a) $p - q = 6$, where p and q represent the number of boys and the number of girls respectively who participated in an expedition held during the last school holidays.

(b) $4x + 5y = 35$, where x and y represent the price, in RM, of a plate of *nasi lemak* and a plate of fried noodles respectively.

Solution

(a) In an expedition held during the last school holidays, the number of boys is 6 more than the number of girls.

(b) The total price for 4 plates of *nasi lemak* and 5 plates of fried noodles is RM35.

Self Practice 6.2b

1. Form a linear equation in two variables for each of the following:

(a) In a long jump event, the total points obtained by Satria team and Perdana team were 258.

(b) The difference between the largest angle and the smallest angle in a triangle is 15°.

(c) The total price of tickets for a water theme park for eight adults and five children is RM265.

(d) The total marks for a test paper which consists of Section A and Section B is 40. Each correct answer in Section A and Section B contributes 1 mark and 2 marks respectively.

2. Write the situation for each of the following linear equations:

(a) $x + y = 465$, where x and y represent the number of aluminium cans and the number of glass bottles collected respectively in a recycling campaign.

(b) $p - q = 3$, where p and q represent the length and width respectively, in cm, of a rectangle.

How do you determine the possible solutions of linear equations in two variables?

LEARNING STANDARDS

Determine and explain possible solutions of linear equations in two variables.

I have read a total of 7 novels and storybooks this week for the NILAM Programme. Can you guess the number of novels and the number of storybooks I have read?

The equation that can be formed from the situation above is $x + y = 7$ and it has a few pairs of different values for x and y. All the pairs of values of x and y are possible solutions for the equation.

A **linear equation in two variables** has **many possible pairs of values of solutions**.

Example 9

Write three possible pairs of solutions for $2x + y = 6$.

Solution

$2x + y = 6$

When $x = 0$,
$2(0) + y = 6$
$y = 6$

When $x = 1$,
$2(1) + y = 6$
$y = 6 - 2$
$y = 4$

When $x = 2$,
$2(2) + y = 6$
$y = 6 - 4$
$y = 2$

Thus, three possible pairs of solutions are $x = 0, y = 6$; $x = 1, y = 4$ and $x = 2, y = 2$.

SMART TIPS

Each pair of solutions can be writen in the ordered pair (x, y). For example, (0, 6), (1, 4) and (2, 2).

Smart

A scientific calculator can be used to determine the values of solutions for linear equations in two variables.

1. Write equation in the form $y = 6 - 2x$.
Press
ALPHA | Y | ALPHA | = | 6 | – | 2 | ALPHA | X | CALC
Screen displayed
X?

2. Enter the value x. For example 0.
Press 0 | =
Screen displayed
6 – 2X
6
Thus, $y = 6$.

3. Press CALC and enter the different values of x for the other pairs of solutions.

CHAPTER 6

Example 10

Osman has bought a total of five jerseys for Harimau Team and Kancil Team. What are the possible numbers of jerseys that can be bought for each team?

Solution

Let the number of jerseys for Harimau Team = x
and the number of jerseys for Kancil Team = y

Thus, $x + y = 5$

Number of jerseys for Harimau Team	Number of jerseys for Kancil Team
$x = 1$	$y = 4$
$x = 2$	$y = 3$
$x = 3$	$y = 2$
$x = 4$	$y = 1$

SMART TIPS

The steps to determine the possible solutions of linear equations in two variables:

Choose a value for one of the variables.

↓

Substitute the value into the linear equation.

↓

Solve the equation to find the value of the other variable.

Let's Discuss

Besides the pairs of solutions listed above, is there any other pair of solutions for the number of jerseys that Osman bought? Discuss.

Self Practice 6.2c

1. Write three possible pairs of solutions for the following equations:

(a) $x - y = 7$ (b) $y = 1 - 2x$ (c) $3x - \frac{y}{2} = 6$

2. On sports day, Sani won 4 medals all of which were gold and bronze medals. How many gold and bronze medals did Sani possibly win?

How do you represent graphically linear equations in two variables?

LEARNING STANDARDS

Represent graphically linear equations in two variables.

Exploration Activity 2

Flip Classroom

Aim: To represent graphically linear equations in two variables.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *linear equation graphically.pdf* and print the file.
2. Complete the table for each given equation.

(a) $x - y = -2$

x	0	1	2	3	4	5
y	2					
(x, y)	(0, 2)					

(b) $x - 2y = 8$

x	0	2	4	6	8	10
y						
(x, y)						

(c) $x + y = -5$

x						
y	0	−1	−2	−3	−4	−5
(x, y)						

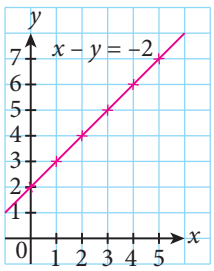
(d) $2x + y = 2$

x						
y	10	8	6	4	2	0
(x, y)						

3. Based on the ordered pairs obtained from the tables above, plot all the points on the printed grid paper.
4. Draw a graph by connecting all the points.
5. Study the shape of your graphs. What is your observation?
6. Compare the shape of the graphs obtained from the given linear equations.
7. Discuss your findings.

From the results of Exploration Activity 2, it is found that straight line graphs are obtained when all the ordered pairs of the linear equations in two variables are plotted and connected.

All the points on the straight line are the solutions of the linear equation.



Self Practice 6.2d

1. Which of the following diagrams represent graphically a linear equation in two variables?

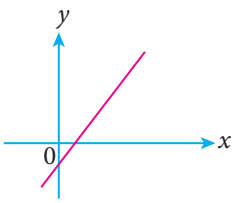


Diagram (a)

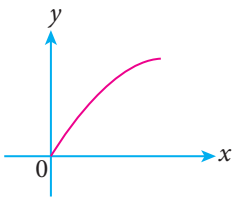


Diagram (b)

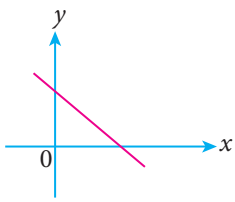


Diagram (c)

Mastery Q 6.2

Open the folder downloaded from page vii for extra questions of Mastery Q 6.2.

1. Form a linear equation in two variables for each of the following:
(a) The total price of x headscarves at RM30 each is RM8 more than the total price of y shawls at RM20 each.
(b) The total age of a father and his twin children is 130.
(c) In a school, the number of female teachers is twice the number of male teachers.

2. Write two possible pairs of solutions for each of the following equations:
(a) $x + y = 7$ (b) $y - 2 = 5x$

3. Draw graphs to represent each of the following linear equations based on the given values of x:
(a) $x - y = 2$; $x = 0, 1, 2, 3, 4, 5$ (b) $2x + y = 4$; $x = -2, -1, 0, 1, 2$
(c) $y - \frac{x}{2} = 3$; $x = -6, -4, -2, 0, 2$

4. A shirt costs RM20 and a pair of pants costs RM10. Find the possible number of shirts and pants that Sheimah can buy with a total payment of RM80. What is the maximum number of shirts that Sheimah can buy?

5. Pei San saves only 10 sen and 20 sen coins in her coin box. Her total savings is RM5. Draw a graph to represent the situation.

6.3 Simultaneous Linear Equations in Two Variables

How do you form simultaneous linear equations and represent them graphically?

LEARNING STANDARDS

Form simultaneous linear equations based on daily situations. Hence, represent graphically the simultaneous linear equations in two variables and explain the meaning of simultaneous linear equations.

Faizah rears a total of 7 chickens and ducks in a coop. The cost of rearing a chicken is RM2 per week whereas the cost of rearing a duck is RM1 per week. The total cost for rearing the chickens and ducks is RM12 per week. How many chickens and ducks can Faizah rear?

Based on the situation above, let x and y be the number of chickens and ducks being reared respectively,

thus $x + y = 7$ ← Total number of chickens and ducks is 7.

and $2x + y = 12$ ← Total rearing cost of chickens and ducks is RM12 per week.

Both the equations formed are linear equations in two variables.

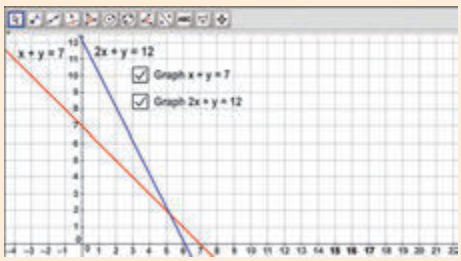
To determine the number of chickens and the number of ducks, we need to find the values of x and y that satisfy both the linear equations.

Exploration Activity 3a

Aim: To explore simultaneous linear equations.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *intersecting.ggb* using *GeoGebra*.
2. Click on both the boxes to display the straight line graph for the equations $x + y = 7$ and $2x + y = 12$.
3. Observe the two straight lines displayed and state the point of intersection (x, y) of both the straight lines.



4. Determine whether the point of intersection (x, y) is the solution for both the linear equations.

From the results of Exploration Activity 3a, it is found that the point of intersection of the two straight lines is the common pair of solutions for both the linear equations.

Linear equations $x + y = 7$ and $2x + y = 12$ are **simultaneous linear equations** in two variables because both the linear equations have two similar variables.

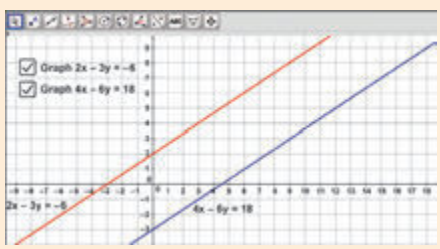
The solution of simultaneous linear equations that has one point of intersection is known as a **unique solution**.

Exploration Activity 3b

Aim: To explore simultaneous linear equations.

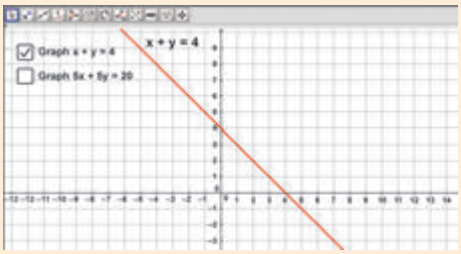
Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *parallel.ggb* using *GeoGebra*.
2. Click on both the boxes to display the straight line graph for the equations $2x - 3y = -6$ and $4x - 6y = 18$.
3. Observe the two straight lines displayed.



4. Open the file *overlapping.ggb* using *GeoGebra*.

5. Click on both the boxes to display the straight line graph for the equations $x + y = 4$ and $5x + 5y = 20$.



6. Observe the two straight lines displayed.

7. Copy and record all your results in the following table.

Linear equations displayed	Condition of both straight lines	Point of intersection
$2x - 3y = -6$ $4x - 6y = 18$		
$x + y = 4$ $5x + 5y = 20$		

8. Discuss with your friends your findings.

Exploration Activities 3a and 3b show that there are three cases involving solution of simultaneous linear equations as shown in the table below.

Condition of both straight lines	Type of solution
Intersecting	Unique solution
Parallel	No solution
Overlapping	Infinite solutions

Self Practice 6.3a

1. Form simultaneous linear equations for each given daily situation. Hence, represent graphically the simultaneous linear equations in two variables.
   (a) Puan Siti intends to reward her students with 6 Malay dictionaries and English dictionaries. The prices of a Malay dictionary and an English dictionary are RM20 and RM40 respectively. Puan Siti buys x Malay dictionaries and y English dictionaries at a total cost of RM160.
   (b) A total of 12 students comprising boys and girls are divided equally into two groups. The number of boys in each group is 2 more than the number of girls.

How do you solve simultaneous linear equations in two variables?

LEARNING STANDARDS

Solve simultaneous linear equations in two variables using various methods.

The simultaneous linear equations in two variables can be solved by using

(i) graphical method
(ii) substitution method
(iii) elimination method

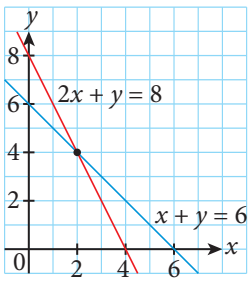
Example 11

Solve the following simultaneous linear equations by using graphical method.

$$x + y = 6 \quad \text{and} \quad 2x + y = 8$$

Solution

From the graph drawn, the point of intersection is (2, 4). Thus, the solution is $x = 2$ and $y = 4$.



Example 12

Solve the following simultaneous linear equations by using substitution method.

$$x - 3y = 7 \quad \text{and} \quad 5x + 2y = 1$$

Solution

$$x - 3y = 7 \quad \ldots\ldots\ldots\ldots \text{①}$$
$$5x + 2y = 1 \quad \ldots\ldots\ldots\ldots \text{②}$$

From ①, $x = 7 + 3y \quad \ldots\ldots \text{③}$ ← Express x in terms of y.

Substitute ③ into ②.

$$5(7 + 3y) + 2y = 1$$
$$35 + 15y + 2y = 1$$
$$35 + 17y = 1$$
$$17y = 1 - 35$$
$$17y = -34$$
$$y = \frac{-34}{17}$$
$$y = -2$$

Substitute $y = -2$ into ③.

$x = 7 + 3(-2)$
$= 1$

Thus, $x = 1$ and $y = -2$.

SMART TIPS

The steps for solving simultaneous linear equations in two variables using substitution method:

- Express one of the variables in terms of the other variable.
- Substitute the expression into the other linear equation.
- Solve the linear equation in one variable.
- Substitute the value obtained into the expressed equation to find the value of the other variable.

Example 13

Solve the following simultaneous linear equations by using elimination method.

(a) $x + 2y = 9$ and $3x - 2y = 15$

(b) $2x + 5y = 14$ and $3x + 4y = 7$

Solution

(a) $x + 2y = 9$ ..... ①

$3x - 2y = 15$ ..... ②

Identify the variable with the same coefficient.

① + ②: $4x + 0 = 24$

Eliminate the variable y by adding ① and ②.

$4x = 24$

$x = 6$

Substitute $x = 6$ into ①.

$6 + 2y = 9$

$2y = 9 - 6$

$2y = 3$

$y = \frac{3}{2}$

Thus, $x = 6$ and $y = \frac{3}{2}$.

(b) $2x + 5y = 14$ ..... ①

$3x + 4y = 7$ ..... ②

- Multiply ① and ② to equate the coefficient of x.
- The LCM for 2 and 3 is 6.

① × 3: $6x + 15y = 42$ ..... ③

② × 2: $6x + 8y = 14$ ..... ④

③ − ④: $0 + 7y = 28$

Eliminate the variable x by subtracting ④ from ③.

$7y = 28$

$y = 4$

Substitute $y = 4$ into ①.

$2x + 5(4) = 14$

$2x + 20 = 14$

$2x = 14 - 20$

$2x = -6$

$x = -3$

Thus, $x = -3$ and $y = 4$.

SMART TIPS

The steps for solving simultaneous linear equations in two variables using elimination method:

Multiply one or both equations with a number so that the coefficient of one of the variables is equal.

↓

Add or subtract both the equations to eliminate one of the variables.

↓

Solve the linear equation in one variable.

↓

Substitute the value obtained into the original equation to find the value of the other variable.

Scan the QR Code or visit https://youtu.be/L0DJskZw3y0 to learn about the solution of Example 13(b) by using scientific calculator.

Self Practice 6.3b

1. Solve the following simultaneous linear equations:

(a) $y = 3x + 1$
$x + 2y = 16$

(b) $x + y = 5$
$2x - y = 22$

(c) $4x + 3y = 8$
$x - 3y = 2$

(d) $8x + 3y = -4$
$5x + 2y = 6$

How do you solve problems?

MATHEMATICS APPLICATION ZONE

LEARNING STANDARDS

Solve problems involving simultaneous linear equations in two variables.

Yesterday I brought my wife and three children who are under 12 years old to the Zoo Negara, and the total cost of tickets was RM97.

Last week, I brought my wife and a 5-year-old child together with two of my friends to the Zoo Negara. The total cost of tickets was RM139.

Based on the conversation above, how much is the ticket for an adult and a child?

Solution

Understanding the problem

- Total cost of tickets for 2 adults and 3 children is RM97.
- Total cost of tickets for 4 adults and one child is RM139.

Devising a plan

Let the cost of the ticket for an adult = RMx
and the cost of the ticket for a child = RMy
Form two simultaneous linear equations and solve them.

Implementing the strategy

$2x + 3y = 97$ ..... ①
$4x + y = 139$ ..... ②
① × 2: $4x + 6y = 194$ ..... ③
$4x + y = 139$ ..... ②

③ − ②: $0 + 5y = 55$
$5y = 55$
$y = 11$

Substitute $y = 11$ into the equation ①.

$2x + 3(11) = 97$
$2x + 33 = 97$
$2x = 64$
$x = 32$

Thus, the cost of the ticket for an adult is RM32 and the cost of the ticket for a child is RM11.

Doing reflection

Total cost of tickets for 2 adults and 3 children = 2(32) + 3(11)
= RM97

Total cost of tickets for 4 adults and 1 child = 4(32) + 11
= RM139

Self Practice 6.3c

1. During the *Hari Keusahawanan* in a school, 800 booklets of coupons were sold. The price of each booklet of coupons was RM30 and RM50 respectively. The total amount collected was RM30 000. How many booklets of RM30 and RM50 coupons were sold?

2. The length of a rectangular swimming pool is p m and its width is q m. It is given that the length of the swimming pool is twice its width. If the perimeter of the swimming pool is 150 m, find the values of p and q.

Mastery Q 6.3

Open the folder downloaded from page vii for extra questions of Mastery Q 6.3.

1. Form simultaneous linear equations based on the following statement.

The difference between two numbers is 5. When the larger number is multiplied by 2, the sum of both the numbers is 7.

Hence, represent graphically the simultaneous linear equations in two variables and state the type of the solution.

2. Solve the following simultaneous linear equations:

(a) $x + 4y = 14$
$3x + 2y = 12$

(b) $3m - 2n = 19$
$5m + 7n = 11$

(c) $\frac{1}{3}p + q = 4$
$\frac{p - q}{4} = 2$

(d) $\frac{f}{2} + \frac{g}{5} = 3$
$2g - f = 10$

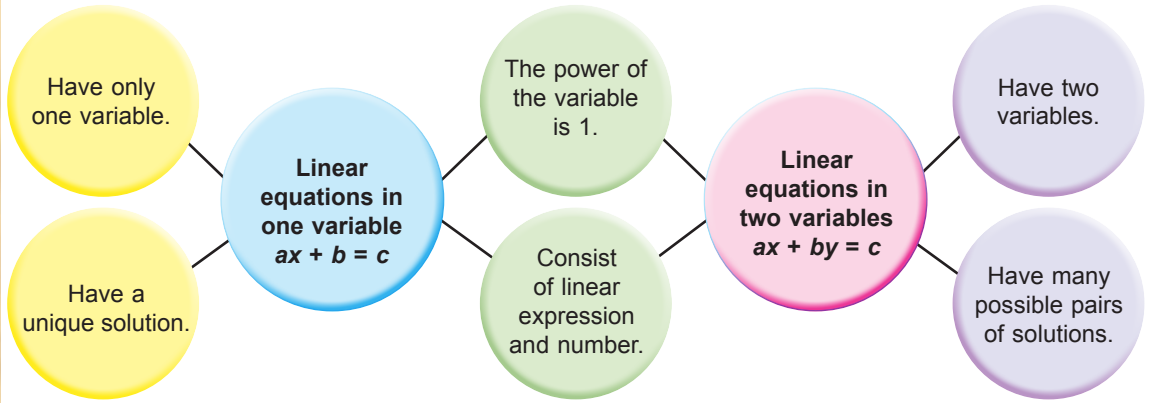
3. A wire with a length of 100 cm is cut into three parts. The length of wire for the first and second parts are the same. The length of the third part of the wire exceeds the sum of the first two parts by 4 cm. Calculate the length of each part.

4. Lai Yee and Khadijah have 60 stamps. When Lai Yee gives 5 stamps to Khadijah, the number of Lai Yee's stamps is twice the number of Khadijah's stamps. How many stamps does each of them have at first?

5. After six years, the age of Devaki's father is thrice the age of Devaki. If the age of Devaki's father was seven times the age of Devaki two years ago, what are their ages this year?

6. Sarah has more money than Hui Chin. If Sarah gives RM10 to Hui Chin, both of them will have the same amount of money. If Hui Chin gives RM5 to Sarah, Sarah will have four times the amount of money that Hui Chin has. How much does each of them have?

SUMMARY



At the end of this chapter, I can...	Very good	Work harder
identify linear equations in one variable and describe the characteristics of the equations.		
form linear equations in one variable based on a statement or a situation, and vice-versa.		
solve linear equations in one variable.		
solve problems involving linear equations in one variable.		
identify linear equations in two variables and describe the characteristics of the equations.		
form linear equations in two variables based on a statement or a situation, and vice-versa.		
determine and explain possible solutions of linear equations in two variables.		
represent graphically linear equations in two variables.		
form simultaneous linear equations based on daily situations. Hence, represent graphically the simultaneous linear equations in two variables and explain the meaning of simultaneous linear equations.		
solve simultaneous linear equations in two variables using various methods.		
solve problems involving simultaneous linear equations in two variables.		

Let's PRACTISE

Test Yourself

1. Meena has a packet of sweets. She gives half of the sweets to her brother. After eating 3 sweets, there are 5 sweets left in the packet. How many sweets were there in the packet initially?

2. In a cross-country run organized by a school, the students who completed the run within one hour would obtain 2 points for their team. A total of 280 students managed to obtain the points. The number of boys who obtained the points was 60 more than the number of girls. How many points did the girls obtain during the cross-country run?

Self Mastery

3. The total savings of Ella and Zahida was RM2 000. Ella and Zahida each donated $\frac{1}{4}$ and $\frac{1}{5}$ of their savings to an old folks home. The total amount donated by them was RM440. What are the balances of Ella's and Zahida's savings now?

4. An online dealer sells two types of clothes, *baju kurung* and *baju kebaya*. The profit obtained from a *baju kurung* is RM10 less than the profit obtained from a *baju kebaya*. He earns a profit of RM275 from the sale of 5 *baju kurung* and 8 *baju kebaya* during the first week. In the second week, he sells 9 *baju kurung* and 7 *baju kebaya*. What is his profit in the second week?

5. The perimeter of a rectangle is 56 cm. When its length is reduced by 2 cm and its width is increased by 4 cm, a square is formed. What is the area of the rectangle?

6. The incomplete receipt shows the expenses of Liza and Kei Ling at Sedap Restaurant. The price of a cup of coffee is RM1 more than the price of a piece of curry puff. What are the prices of a cup of coffee and a piece of curry puff?

Sedap Restaurant	
2 cups of coffee	RM
4 curry puffs	RM
Total	RM6.80

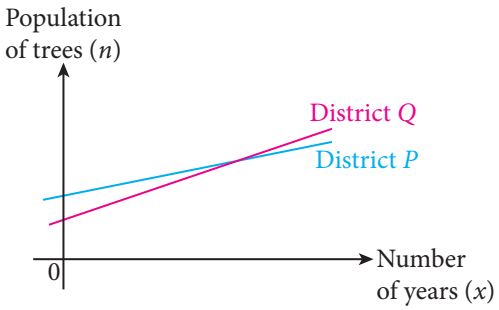
7. Encik Rizal and his wife brought their children below 12 years old along with them for a holiday in Pulau Redang. They spent a total amount of RM1 150 for the package tour. The fees for an adult and a child are RM350 and RM150 respectively. How many children did he bring with him?

Challenge Yourself

8. Asnita : My age is x years more than your age.
Reslynna : I was 13 years old x years ago.
Asnita : My age will be 31 years old after x years.

Based on their conversation, what are the ages of Asnita and Reslynna?

9. The straight line graph represents the population of trees planted in the districts P and Q since the year 2010. The linear equations represented by the straight line graphs of districts P and Q are $n - 3x = 16$ and $n - 5x = 10$ respectively, where n is the population of trees planted and x is the number of years after 2010. In which year would the population of trees planted in both the districts be the same? State the population of the trees in that year.



10. Cikgu Latif recorded the time for a 100 m sprint by three students Amir, Ben and Ravi. The recorded time of Ben and Ravi is the same. The average time for the three students is 13.3 s. However, Cikgu Latif noticed that Amir's time was recorded wrongly as 15.3 s instead of 13.5 s. Calculate the actual average time of the three students.

11. A wholesaler supplies two kinds of fruits, pineapples and watermelons, to stalls A and B. The mass of the fruits supplied are as shown in the table.

Stall	Mass (kg)	
	Pineapple	Watermelon
A	15	40
B	25	60

The total payments received by the wholesaler from stalls A and B are RM90 and RM140 respectively. Determine the price per kilogram for each kind of fruit.

12. Application Commerce

A company produces two types of pen drive, P and Q. In the year 2015, the profit earned from both the pen drives was RM350 000. In the year 2016, the profit earned from type P pen drive increased by 25% while the profit earned from type Q pen drive decreased by 10%. If the total profit for the year 2016 was RM395 500, find the profit earned from each type of the pen drive in the year 2016.

ASSIGNMENT

Linear equations are often used to solve various daily problems. Besides that, linear equations are also used in other fields such as management, finance, computer knowledge, science, engineering, construction and health.

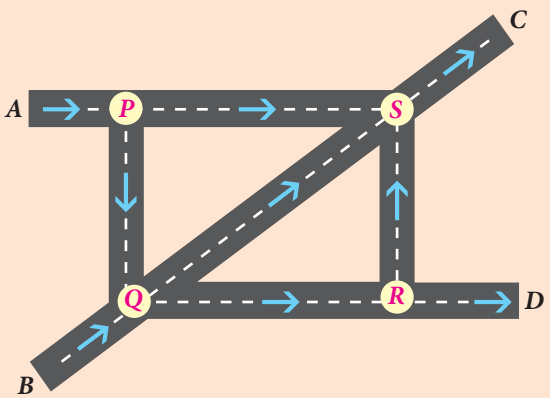
Carry out a study and write a report about the importance of linear equations in the stated fields.

Scan the QR Code or visit http://goo.gl/nDPH2m to explore the usage of linear equations in various fields.

Exploring MATHEMATICS

Linear equations can be used to study and solve the traffic problems.

The diagram shows a road map with four junctions, P, Q, R and S, where each road is a one-way road.



The flow rate of vehicles during the day is given in the table.

Traffic	Average flow rate of vehicles (Number of vehicles per hour)
Vehicles from A	110
Vehicles from B	75
Vehicles that move along PQ	35
Vehicles that move along QS	80
Vehicles that move along RS	20

By using the equality concept, form some linear equations from the given information. Hence, determine the average flow rate of vehicles that travel to C and D.

CHAPTER 7

Linear Inequalities

What will you learn?

- Inequalities
- Linear Inequalities in One Variable

Why study this chapter?

As a basic knowledge in fields that need to apply the concept of limits. An architect needs to consider the height limit of a structure constructed, an engineer needs to determine the speed and weight limit of a vehicle designed, a financial planner uses the idea of limits to calculate the optimum profit of a company based on its operation costs. Discuss other fields that involve the concept of limits.

Body Mass Index (BMI) is used to determine ideal body weight. A survey carried out by the World Health Organisation in the year 2011 revealed that 44.2% of the population in Malaysia have BMI more than 25.

Based on the table given, what is their body weight classification? What is your body weight classification?

Body Weight Classification	BMI
Underweight	BMI < 18.5
Normal weight	$18.5 \leqslant$ BMI < 25.0
Overweight	$25.0 \leqslant$ BMI < 30.0
Obese	BMI $\geqslant 30.0$

Scan the QR Code to watch the video about healthy lifestyle practices.

http://goo.gl/fPfbq6

Walking through Time

Thomas Harriot

The symbols '>' and '<' were introduced by an English surveyor Thomas Harriot in his book published in the year 1631. The inequality symbol is believed to have originated from the equality symbol '='.

For more information:

http://goo.gl/JooUWU

Word Link

• inequality	• *ketaksamaan*
• linear inequality in one variable	• *ketaksamaan linear dalam satu pemboleh ubah*
• simultaneous linear inequalities	• *ketaksamaan linear serentak*
• converse property	• *sifat akas*
• transitive property	• *sifat transitif*
• additive inverse	• *songsangan terhadap penambahan*
• multiplicative inverse	• *songsangan terhadap pendaraban*

Open the folder downloaded from page vii for the audio of Word Link.

7.1 Inequalities

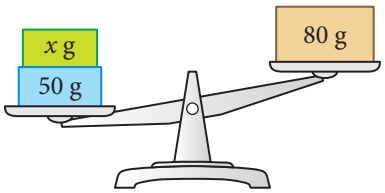
LEARNING STANDARDS

Compare the values of numbers, describe inequality and hence, form algebraic inequality.

What are inequalities?

In our daily lives, we often make comparisons between two quantities with different values. We compare the quantities in terms of number, price, temperature, size, height, mass and so on.

Which smart phone brand is cheaper?



Which has a larger mass?

By comparison, we found that

- the price of smart phone brand *A* is cheaper than brand *B*.
- the mass $(x + 50)$ g is greater than 80 g.

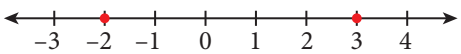
The relationship between two quantities that do not have the same value is known as an **inequality**.

Let's Discuss

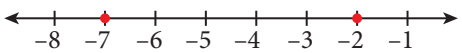
Discuss the relationship between two quantities in real-life situations which involve the usage of 'more than' or 'less than'.

Compare the values of numbers

In chapter 1, we have learned how to compare the value of an integer with another integer based on its position on a number line. In this chapter, we shall write this comparison by using mathematical symbols.
For example:



Observe the position of the pair of numbers –2 and 3 on the number line.
–2 lies to the left of 3,
therefore –2 is less than 3.
The symbol '$<$' is used to represent 'less than'.
So, '–2 is less than 3' is written as '$-2 < 3$'.



Observe the position of the pair of numbers –7 and –2 on the number line.
–2 lies to the right of –7,
therefore –2 is more than –7.
The symbol '$>$' is used to represent 'greater than'.
So, '–2 is greater than –7' is written as '$-2 > -7$'.

$-2 < 3$ and $-2 > -7$ are inequalities.

Example 1

Fill in the boxes with the symbol '>' or '<' so that it becomes a true statement. Hence, write an inequality for each statement by using 'is greater than' or 'is less than'.

(a) $-\frac{5}{6}$ ☐ 4 (b) 4^2 ☐ 9

SMART TIPS

Symbol	Meaning
>	greater than
<	less than

Solution

(a) $-\frac{5}{6}$ < 4 (b) 4^2 > 9

$-\frac{5}{6}$ is less than 4. 4^2 is greater than 9.

Self Practice 7.1a

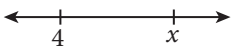
1. Fill in the boxes with the symbol '>' or '<' so that it becomes a true statement. Hence, write an inequality for each statement by using 'is greater than' or 'is less than'.

(a) -6 ☐ 0 (b) $\frac{1}{7}$ ☐ $\frac{1}{4}$ (c) 0.42 ☐ 0.072

(d) 4.5 ☐ $\sqrt{4.5}$ (e) 10 cm ☐ 50 mm (f) 1 200 g ☐ 1.6 kg

Describe inequality and form algebraic inequality

Observe the number line below.



The number line compares 4 with another unknown number, x. We can describe the relationship between 4 and x in an inequality as 'x is greater than 4' and the inequality is written as '$x > 4$'.

Example 2

In the number line below, describe the relationship between x and 8 as an inequality by using 'is less than'.

Hence, form an algebraic inequality for the relationship.

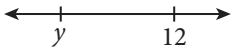
Solution

x is less than 8.

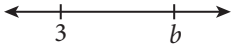
$x < 8$

Self Practice 7.1b

1. Based on the number line, describe the relationship between y and 12 as an inequality by using 'is greater than'. Hence, form an algebraic inequality for the relationship.

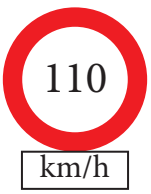


2. Based on the number line, describe the relationship between 3 and b as an inequality by using 'is less than'. Hence, form an algebraic inequality for the relationship.



Identify relationship

(a) is greater than or equal to
(b) is less than or equal to



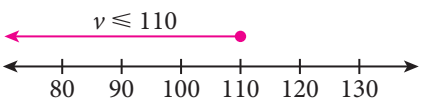
The road sign in the diagram shown can be found along the highway. This road sign reminds drivers that the speed limit of vehicles along the highway must not exceed 110 km/h.

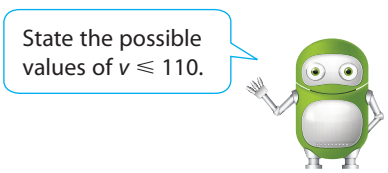
If v represents the speed in km/h, then $v = 110$ and $v < 110$. Both these equality and inequality can be combined by using the symbol '$\leqslant$' and are written as $v \leqslant 110$.

SMART TIPS

The possible values of the variable v can be an integer, decimal or fraction.

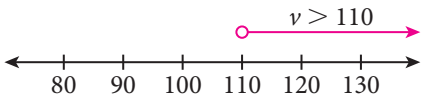
We can use a number line to represent the inequality relationship $v \leqslant 110$ as shown in the diagram below.





How are we going to use the number line to represent the speed of the vehicles that do not follow the speed limit along the highway? What are the risks that would be faced by these irresponsible drivers?

The vehicles that do not follow the road sign have speeds greater than 110 km/h and the speeds can be represented on the number line as shown in the diagram below.



State the possible values of $v > 110$.

Example 3

Fatimah earns overtime pay when she works at least 9 hours a day.

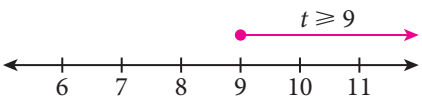
(a) If t is the number of working hours per day, describe an inequality based on the situation above by using 'is greater than or equal to' or 'is less than or equal to'.

(b) Represent the inequality on a number line and hence form an algebraic inequality for the relationship.

Solution

(a) t is greater than or equal to 9.

(b)



The algebraic inequality is $t \geqslant 9$.

SMART TIPS

Symbol	Meaning
$\geqslant$	greater than or equal to
$\leqslant$	less than or equal to

Scan the QR Code or visit https://youtu.be/xTA_VbuMPME to learn about representing linear inequality on a number line.

Let's Discuss

Fatimah did not earn any overtime pay.

(a) Describe an inequality based on the situation above by using 'is greater than' or 'is less than'.
(b) Represent the inequality on a number line and hence form an algebraic inequality for the relationship.

Self Practice 7.1c

1. Describe an inequality based on each of the following situations by using 'is greater than or equal to' or 'is less than or equal to':

(a) The road sign shows the load of a lorry, m, that is allowed to cross the bridge.

(b) The age, t, to vote is 21 years old.

Represent the inequality on a number line and hence form an algebraic inequality for the relationship.

What are the properties of inequalities?

LEARNING STANDARDS

Make generalisation about inequality related to
(i) the converse and transitive properties, additive and multiplicative inverse,
(ii) basic arithmetic operations.

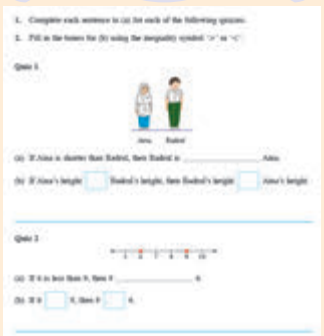
Exploration Activity 1

Pair

Aim: To explore the converse property and the transitive property of inequalities.

Instruction:
- Perform the activity in pairs.
- Open the folder downloaded from page vii.

1. Open the file *converse and transitive properties.pdf* and print the file.
2. Complete the sentence in (a) for each quiz.
3. Fill in the blanks in (b) using the inequality symbol '>' or '<'.



From the results of Exploration Activity 1, it is found that

- if a is less than b, then b is greater than a.
  Converse property of inequality: If $a < b$, then $b > a$.
- if a is less than b and b is less than c, then a is less than c.
  Transitive property of inequality: If $a < b < c$, then $a < c$.

Example 4

(a) Write the converse property of inequality
(i) $10 < 24$, (ii) $-4 > -13$.

(b) Write the transitive property of inequality $-5 < -1 < 9$.

Solution

(a) (i) $24 > 10$ (ii) $-13 < -4$
(b) $-5 < 9$

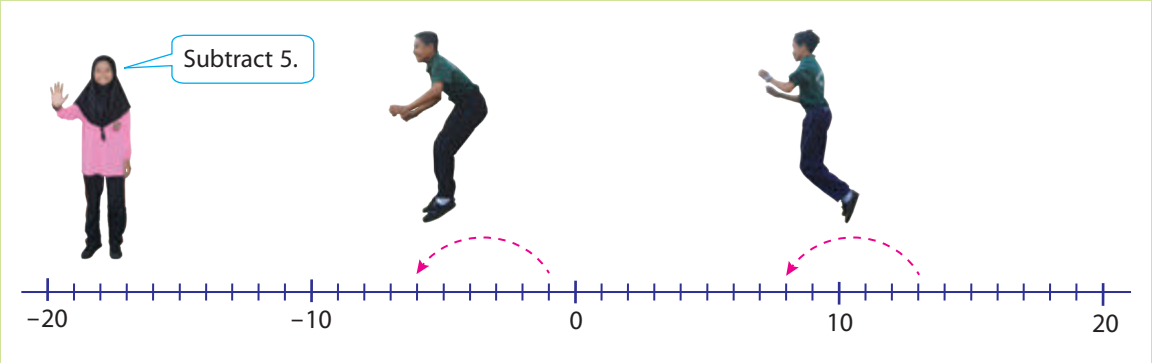
Self Practice 7.1d

1. Write the converse property of each of the following inequalities.
   (a) $5 < 14$ (b) $8 > -8$ (c) $-23 > -32$
   (d) $6.7 > 1.5$ (e) $\frac{1}{13} > \frac{1}{14}$ (f) $-2 > -11.8$
2. Write the transitive property of each of the following inequalities.
   (a) $-2 < 5 < 10$ (b) $-15 < -8 < 0$ (c) $-4.56 < -1.52 < 2.01$
   (d) $-17\frac{2}{9} < 11\frac{1}{5} < 20\frac{1}{3}$ (e) $\frac{1}{20} < \frac{1}{14} < \frac{1}{7}$ (f) $-8 < -4.3 < \frac{1}{9}$

Learning Outside the Classroom

Aim: **To make a generalisation about the inequality related to the basic arithmetic operations.**

Instruction: Perform the activity in groups of four.



1. Use a rope to represent a number line that shows the integers from –20 to 20.
2. Get two students to participate. Each of them will choose a different positive even number that is less than 20.
3. Then, the students have to stand at the position of each chosen number on the number line.
4. Get another student to write down the inequality to compare both the numbers. Observe the direction of the inequality symbol and determine who has the greater number.
5. The fourth student will give instructions related to basic arithmetic operations (+, −, ×, ÷) for the numbers chosen by both the students.

   Examples of instructions: add 3, subtract −4, multiply by −1, divide by 2
6. Both the students will shift their positions to show the results after performing each of the basic arithmetic operations. Extend the number line on the rope if necessary.
7. Determine whose number is greater after performing each basic operation. Write an inequality to compare the two new numbers.
8. Repeat Steps 3 to 7 with the same original numbers chosen but using a different basic operation.
9. Predict the operation that will retain the direction of the inequality symbol and the operation that will reverse the direction of the inequality symbol.
10. Make a generalisation about the inequality related to the operations.
11. Repeat the steps with two different negative even numbers.

From the results of the activity, we can conclude that:

- The inequality symbol **remains unchanged** when adding or subtracting a positive or negative number to or from both sides of the inequality.

If $a < b$,
then $a + c < b + c$.

If $a < b$,
then $a - c < b - c$.

If $a < b$,
then $a + (-c) < b + (-c)$.

If $a < b$,
then $a - (-c) < b - (-c)$.

SMART TIPS

If $2 < 6$
then $2 + 3 < 6 + 3$
$5 < 9$

If $2 < 6$
then $2 - 3 < 6 - 3$
$-1 < 3$

If $2 < 6$
then $2 + (-3) < 6 + (-3)$
$-1 < 3$

If $2 < 6$
then $2 - (-3) < 6 - (-3)$
$5 < 9$

- (i) The inequality symbol **remains unchanged** when multiplying or dividing both sides of the inequality by a positive number.

If $a < b$,
then $a \times c < b \times c$.

If $a < b$,
then $\frac{a}{c} < \frac{b}{c}$.

SMART TIPS

If $2 < 6$
then $2 \times 3 < 6 \times 3$
$6 < 18$

If $2 < 6$
then $\frac{2}{2} < \frac{6}{2}$
$1 < 3$

(ii) The direction of the inequality symbol is reversed when multiplying or dividing both sides of the inequality by a negative number.

If $a < b$,
then $a \times (-c) > b \times (-c)$.

If $a < b$,
then $\frac{a}{-c} > \frac{b}{-c}$.

SMART TIPS

If $2 < 6$
then $2 \times (-3) > 6 \times (-3)$
$-6 > -18$

If $2 < 6$
then $\frac{2}{-2} > \frac{6}{-2}$
$-1 > -3$

- When both sides of the inequality are multiplied by -1, the direction of the inequality symbol is reversed.

If $a < b$,
then $-a > -b$.

It is known as **additive inverse**.

SMART TIPS

If $2 < 6$
then $2 \times (-1) > 6 \times (-1)$
$-2 > -6$

- When performing reciprocal of both numbers on both sides of the inequality, the direction of the inequality symbol is reversed.

If $a < b$,
then $\frac{1}{a} > \frac{1}{b}$.

It is known as **multiplicative inverse**.

SMART TIPS

If $2 < 6$
then $\frac{1}{2} > \frac{1}{6}$

Self Practice 7.1e

Fill in each box with the symbol '<' or '>'.

1. (a) (i) $-8 \square 16$ (ii) $-8 + 2 \square 16 + 2$ (iii) $-8 - 2 \square 16 - 2$

(b) (i) $8 \square -16$ (ii) $8 + 5 \square -16 + 5$ (iii) $8 - 5 \square -16 - 5$

(c) (i) $-8 \square -16$ (ii) $-8 + 1 \square -16 + 1$ (iii) $-8 - 1 \square -16 - 1$

2. (a) (i) $8 \square 16$ (ii) $8 \times 2 \square 16 \times 2$ (iii) $8 \div 2 \square 16 \div 2$

(b) If $8 < 16$ and $c > 0$, then $8c \square 16c$ and $\frac{8}{c} \square \frac{16}{c}$.

(c) If $16 > 8$ and $c > 0$, then $16c \square 8c$ and $\frac{16}{c} \square \frac{8}{c}$.

3. (a) (i) $6 \square 12$ (ii) $6 \times (-3) \square 12 \times (-3)$ (iii) $6 \div (-3) \square 12 \div (-3)$

(iv) $-6 \square -12$ (v) $\frac{1}{6} \square \frac{1}{12}$

(b) If $6 < 12$ and $d < 0$, then $6d \square 12d$ and $\frac{6}{d} \square \frac{12}{d}$.

(c) If $12 > 6$ and $d < 0$, then $12d \square 6d$ and $\frac{12}{d} \square \frac{6}{d}$.

Mastery Q 7.1

Open the folder downloaded from page vii for extra questions of Mastery Q 7.1.

1. Fill in the boxes with the symbol '>' or '<' so that each of the following statements becomes true.

(a) $(-5)^2 \square (-6)^2$ (b) $0.1 \square \sqrt[3]{0.008}$ (c) $6 + x \square 8 + x$

(d) $m + 3 \square m$ (e) $10 - k \square 8 - k$ (f) $2x + 5 \square 2x - 5$

2.

(a) Based on the number line above, describe the relationship between x and y as an inequality by using 'is greater than'.

(b) Hence, form an algebraic inequality for the relationship.

3. A minimum deposit of RM100 is required to open a bank account.
(a) Describe an inequality for the minimum deposit required to open a bank account by using 'is greater than or equal to' or 'is less than or equal to'.
(b) If a is the minimum deposit required to open a bank account, represent the inequality on a number line and form an algebraic inequality for the relationship.

4. Represent the following inequalities on number lines.
(a) $x > 3$ (b) $x < 15$ (c) $x \geqslant -19$ (d) $-5 \geqslant x$
(e) $y \leqslant 8.3$ (f) $p \geqslant -5.7$ (g) $x < -\frac{3}{5}$ (h) $7.8 > q$

5. Fill in the boxes with the symbol '>' or '<' so that each of the following statements becomes true.
(a) If $x < y$, then y ☐ x. (b) If $p < q$ and $q < 0$, then p ☐ 0.
(c) If $-2 > x$ and $x > y$, then -2 ☐ y. (d) If $x > y$, then $\frac{x}{10}$ ☐ $\frac{y}{10}$.
(e) If $x > y$, then $(-5)x$ ☐ $(-5)y$. (f) If $u > 0$, then $(-3)u$ ☐ 0.

CHAPTER 7

7.2 Linear Inequalities in One Variable

How do you form linear inequalities based on the daily life situations and vice-versa?

LEARNING STANDARDS
Form linear inequalities based on daily life situations, and vice-versa.

Example 5

Construct a linear inequality based on each situation below.
(a) Pak Samad is a *gasing uri* maker in Kelantan. The time, t days, Pak Samad spends in making a *gasing uri* is less than 42 days.
(b) In a fishing competition, the participants can win a prize if the length, l cm, of the fish caught is at least 32 cm.
(c) Madam Chen bakes a cake that has a mass of not more than 2 kg. The mass of the cake, x kg, is received by each neighbour, if Madam Chen cuts the cake into 10 equal slices for her neighbours.
(d) Mr Mohan is a businessman. He intends to donate 3% of the profit earned from his business to a local charity every month. The profit, p, in RM, Mr Mohan has to earn each month if his donation is to exceed RM240 per month.

SMART TIPS

Symbol ⩾
• At least • Not less than • Minimum
Symbol ⩽
• At most • Not more than • Maximum

Solution
(a) $t < 42$ (b) $l \geqslant 32$ (c) $x \leqslant \frac{2}{10}$ (d) $\frac{3}{100}p > 240$

Example 6

Write a situation based on each of the linear inequalities given:

(a) $h \geq 110$ where h is the height of the passenger, in cm, that is allowed to ride the roller coaster.

(b) $T < -5$ where T is the temperature, in °C, of the freezer compartment of a refrigerator.

(c) $m > 4\,600$ where m is Mr Siva's monthly salary, in RM.

Solution

(a) The height of the passenger that is allowed to ride the roller coaster must be at least 110 cm.

(b) The temperature of the freezer compartment of a refrigerator is less than –5°C.

(c) Mr Siva's monthly salary is more than RM4 600.

Self Practice 7.2a

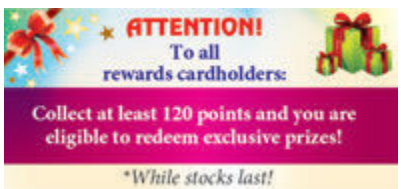
1. Construct a linear inequality based on each of the following situations:

(a) The price, RMx, of a double-storey terrace house is RM450 000 and above.

(b) The passing mark of a Mathematics test is 50. Hajar obtained y marks and she failed the test.

(c) The total number of participants, k, of 5 debating teams if each team cannot have more than 6 people for the competition.

(d) Puan Kalsom has a reward points card that shows one point earned for every RM5 spent. Puan Kalsom spends RMq and is eligible to redeem exclusive prizes.



2. Write a situation based on each of the linear inequalities given:

(a) $n \leq 4$ where n is the number of passengers in a taxi.

(b) $A > 1\,000$ where A is the area of an apartment in m^2.

(c) $4y \geq 60$, where y is the expenditure, in RM, of a customer who patronizes the restaurant.

How do you solve problems involving linear inequalities?

Linear inequality in one variable is an unequal relationship between a number and a variable with power of one.

LEARNING STANDARDS

Solve problems involving linear inequalities in one variable.

For example, the algebraic inequalities such as

$3x < 7$ (The power of the variable x is 1)

and $y - 4 > 5 + 2y$ (The power of the variable y is 1)

are known as **linear inequalities in one variable**.

Solving a linear inequality in x is to find the values of x that satisfy the inequality. The process of solving linear inequalities is similar to the process of solving linear equations. However, we need to consider the direction of the inequality symbol when solving linear inequalities.

Example 7

Solve each of the following inequalities:

(a) $x - 2 \leq 6$ (b) $7x \geq 28$

(c) $-\frac{x}{3} < 9$ (d) $7 - 4x > 15$

Solution

(a) $x - 2 \leq 6$

$x - 2 + 2 \leq 6 + 2$ ← Add 2 to both sides of the inequality.

$x \leq 8$

(b) $7x \geq 28$

$\frac{7x}{7} \geq \frac{28}{7}$ ← Divide both sides of the inequality by 7.

$x \geq 4$

(c) $-\frac{x}{3} < 9$

$-\frac{x}{3} \times (-3) > 9 \times (-3)$ ← Multiply both sides of the inequality by –3 and reverse the inequality symbol.

$x > -27$

(d) $7 - 4x > 15$

$7 - 4x - 7 > 15 - 7$ ← Subtract 7 from both sides of the inequality.

$-4x > 8$

$\frac{-4x}{-4} < \frac{8}{-4}$ ← Divide both sides of the inequality by –4 and reverse the inequality symbol.

$x < -2$

SMART TIPS

To solve linear inequalities that involve multiplication or division, we need to multiply or divide both sides of the inequality with an appropriate number so that the coefficient of the variable becomes 1.

Let's Discuss

What are the possible solutions for each of the following inequalities if x is an integer?

(a) $x \geq 3$

(b) $x \leq -5$

SMART TIPS

Linear inequality in one variable has more than one possible solution.

MATHEMATICS APPLICATION ZONE

In conjunction with a reading campaign, a stall is having a book sales promotion by selling every book at a price of RM12.50. Ghani spends not more than RM80 to buy some books from the stall. Calculate the maximum number of books that Ghani can buy.

Scan the QR Code or visit https://youtu.be/BoCI6rCvoDQ to learn about solving inequality on a number line.

Solution

Understanding the problem

- The price of a book is RM12.50.
- Ghani spends not more than RM80 to buy books.
- Calculate the maximum number of books that can be bought.

Devising a plan

- Write a linear inequality in one variable to represent the situation given.
- Solve the inequality and interpret the solutions.

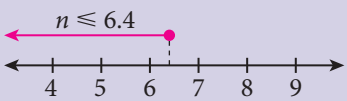
Implementing the strategy

Let n be the number of books,
then the total expenditure is $12.5n$.

So, $12.5n \leqslant 80$

$$\frac{12.5n}{12.5} \leqslant \frac{80}{12.5}$$

$$n \leqslant 6.4$$



Since n is the number of books, then n must be a whole number.

Therefore, the maximum number of books that Ghani can buy is 6.

Doing reflection

When $n = 6$, $12.5n = 12.5 \times 6$
$= 75\ (< 80)$

When $n = 7$, $12.5n = 12.5 \times 7$
$= 87.5\ (> 80)$

Thus, the maximum number of books that can be bought is 6, which is correct.

Self Practice 7.2b

1. Solve each of the following inequalities:

(a) $x + 3 \geqslant 10$ (b) $-2x < 18$ (c) $-7 > \frac{x}{3}$ (d) $16 - 5x \leqslant -4$

2. Fatimah is working part-time as a canned drink seller. She is paid 10 sen for every canned drink sold. She wants to earn at least RM20 per hour. Calculate the number of canned drinks that she needs to sell in an hour in order to achieve her target.

3. Sadiah has RM120 in her savings account and she saves RM40 per month. What is the minimum number of months that Sadiah has to save her money so that her savings can exceed RM500? (Give your answer to the nearest whole number.)

4. A car rental company offers two types of rental packages:

Package *A*	The basic rental payment is RM40 and an extra payment of RM8 for every rental hour.
Package *B*	No basic rental payment but RM15 for every rental hour.

What is the maximum time, in hours, of the car rental such that package *B* will be cheaper? (Give your answer to the nearest whole number.)

How do you solve simultaneous linear inequalities?

LEARNING STANDARDS

Solve simultaneous linear inequalities in one variable.

Based on the World Health Report, the daily consumption of sugar is between 25 g and 37.5 g.

If m gram represents the quantity of daily sugar consumption, then we can write

$$m > 25 \text{ and } m < 37.5$$

The two inequalities are simultaneous linear inequalities in one variable. Therefore, the amount of sugar, in grams, an individual consumes, can be any values between 25 and 37.5, such as 27, 32 and 34.8.

These values are the common values of the simultaneous linear inequalities. The solutions of simultaneous linear inequalities in one variable are the common values of the simultaneous linear inequalities.

Example 8

Solve the following simultaneous linear inequalities:

(a) $2x + 5 < 11$ and $3x - 10 < 5$ (b) $8x + 5 \geqslant 5x - 13$ and $3x - 4 > 9x + 20$

Solution

(a) $2x + 5 < 11$

$2x < 11 - 5$

$2x < 6$

$x < 3$

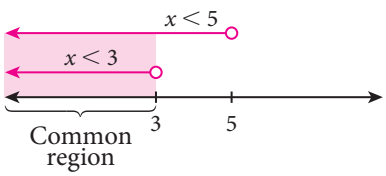
$3x - 10 < 5$

$3x < 5 + 10$

$3x < 15$

$x < 5$

Simplify each linear inequality to the simplest form.

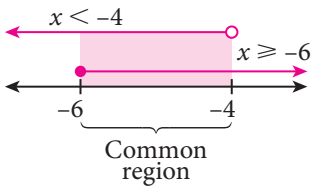


Determine the common values of both the inequalities by using a number line.

Since x needs to satisfy $x < 3$ and $x < 5$, we find the common region of both solutions. The solution is $x < 3$.

(b) $8x + 5 \geqslant 5x - 13$
$8x \geqslant 5x - 18$
$3x \geqslant -18$
$x \geqslant -6$

$3x - 4 > 9x + 20$
$3x > 9x + 24$
$-6x > 24$
$x < -4$



The solution is $-6 \leqslant x < -4$.

Career in Mathematics

Actuaries use inequalities to determine the amount of premium that a client needs to pay in an insurance policy. They also use inequalities to predict the total amount of money that needs to be paid when clients make an insurance claim.

Self Practice 7.2c

1. Solve the following simultaneous linear inequalities:

(a) $x + 2 > 4$ and $4x > 20$
(b) $x - 3 \leqslant -6$ and $3x - 4 \leqslant 5$
(c) $2x + 3 > 0$ and $9x - 2 \leqslant 16$
(d) $5x - 3 < 2$ and $4x + 6 \geqslant x + 3$
(e) $5x - 7 < 13$ and $7x + 4 < 16$
(f) $6x + 5 > 3x + 14$ and $13x - 4 \geqslant 9x$

Mastery Q 7.2

Open the folder downloaded from page vii for extra questions of Mastery Q 7.2.

1. The manager of a clothing store has set some goals for his sales staff. Construct a linear inequality based on each of the following situations:
(a) The minimum total monthly sales in RM, x, is 18 000.
(b) At the end of the month, the total time, t, spent in counting the inventory of the store is at most 8 hours.
(c) The total daily sales, h, must be more than RM700.

2. Write a situation based on each of the linear inequalities given:
(a) $x \leqslant 30$ where x is the speed of the vehicle, in km/h, when approaching the school area.
(b) $m > 1\,100$ where m is the mass of a car in kg.
(c) $y < 900$ where y is the parents' salary, in RM, of a student who qualifies to apply for a scholarship.

3. If Mr Tan keeps a daily balance of at least RM1 200 in his current account, the bank will not impose any service charge to his current account:
(a) Represent the situation on a number line.
(b) If x represents Mr Tan's daily balance, write an inequality that represents the possible values of x when the bank does not impose any service charge to his current account.

4. The salt content in a packet of instant noodles is not more than 800 mg. It is recommended that the maximum daily intake of salt in food is 2 300 mg. Find the maximum packets of instant noodles that can be consumed such that the total salt intake is less than 2 300 mg.

5. Ghanesh has received a restaurant coupon.

(a) Ghanesh buys a set meal which costs RM10.50. If m is the price of the second set meal, construct a linear inequality to represent the values of m such that Ghanesh can use the coupon.

(b) Represent the inequality by using a number line.

6. The maximum capacity of a lift is 960 kg. Assuming that the mass of each boy is 45 kg, find the possible maximum number of boys that could take the lift at any time.

7. Madam Chong's mass is 72 kg. After participating in a fitness programme, her mass decreases at a rate of 3 kg per month. Find the minimum number of months that Madam Chong has to participate in the programme so that her mass becomes less than 52 kg. (Give your answer to the nearest whole number.)

8. Solve the following simultaneous linear inequalities:

(a) $10 - 3x > 8 - 2x$ and $14 - 2x < 9 - 8x$

(b) $\frac{x}{2} - 1 < 3$ and $\frac{3x}{5} - 2 \leqslant x$

(c) $\frac{x}{9} < \frac{2}{3}$ and $\frac{5 - 2x}{7} \geqslant 1$

SUMMARY

LINEAR INEQUALITIES

Inequality involving addition

If $a < b$, then $a + c < b + c$.

Inequality involving subtraction

If $a < b$, then $a - c < b - c$.

Inequality involving multiplication

- If $a < b$, then $a \times c < b \times c$.
- If $a < b$, then $a \times (-c) > b \times (-c)$.

Inequality involving division

- If $a < b$, then $\frac{a}{c} < \frac{b}{c}$.
- If $a < b$, then $\frac{a}{-c} > \frac{b}{-c}$.

At the end of this chapter, I can...	Very good	Work harder
compare the values of numbers, describe inequality and hence, form algebraic inequality.		
make generalisation about inequality related to the converse and transitive properties, additive and multiplicative inverse, and basic arithmetic operations.		
form linear inequalities based on daily life situations, and vice-versa.		
solve problems involving linear inequalities in one variable.		
solve simultaneous linear inequalities in one variable.		

Test Yourself

1. Given that $p < q$. Compare the values of the following pairs of numbers by using the symbol '$<$' or '$>$'.

(a) $p + 5$ ☐ $q + 5$ (b) $\frac{1}{3}p$ ☐ $\frac{1}{3}q$ (c) $-p$ ☐ $-q$

2. Stella is x years old whereas her son is 18 years old. Construct an inequality that shows the relationship between their ages

(a) in the current year, (b) after 3 years, (c) 5 years ago.

3. Zain bought x pieces of *Hari Raya* greeting cards at a price of RM1.20 each. He paid RM20 and received a balance of more than RM5. Construct an inequality based on the information given.

4. Mr Koh has three pieces of RM50 notes, two pieces of RM10 notes and n pieces of RM1 notes in his wallet.

(a) Express, in terms of n, the total amount of money in Mr Koh's wallet.

(b) If the total amount of money is less than RM178, find the possible values of n such that $n > 0$.

Self Mastery

5. Solve the following inequalities:

(a) $3x + 7 < 19$ (b) $15x - 6 < 8x + 8$

6. Yoke Ling has four metal balls. Each ball has the same mass. Diagram (a) and Diagram (b) show the positions of the balances when Yoke Ling weighs one metal ball and four metal balls respectively.

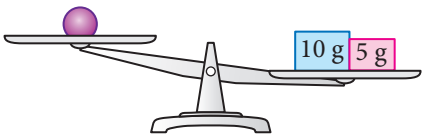


Diagram (a)

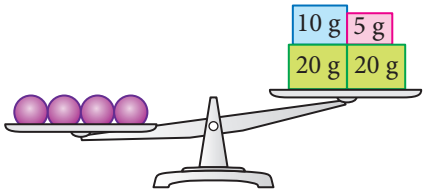


Diagram (b)

Yoke Ling's friends said that the mass of one metal ball could be 12 g, 13 g, 14 g or 15 g. Which of the given mass could be the mass of one metal ball?

Challenge Yourself

7. The Youth and Sports Complex in a district provides badminton court facilities. The management of the complex charges an annual membership fee of RM50. The rental rate per hour of the badminton court for a member and a non-member is shown in the table. When a person enrols as a new member, what is the minimum number of hours for the court rental in a year so that the total cost paid by the member would be more affordable compared to a non-member?

Rental rate per hour	
Non-member	RM15
Member	RM12

8. The mass of a metal sphere is 15 g and the mass of a box is 200 g. Chan puts n metal spheres into the box. If the total mass of the box and the metal spheres is more than 290 g,

(a) form a linear inequality based on the situation above.

(b) find the smallest value of n.

9. Umang is offered a job as a mobile phone sales agent by two companies.

Satria Company offers wages with a fixed rate of RM50 per day and an extra commission of 3% from her total sales.

Perdana Company offers wages with a fixed rate of RM35 per day and an extra commission of 5% from her total sales.

Calculate the minimum total sales, to the nearest RM, that Umang needs to obtain such that Perdana company is a better choice.

10. Solve the following simultaneous linear inequalities:

(a) $4 - 3x \geqslant -5$ and $3x + 1 \geqslant -11$

(b) $\frac{3x}{2} - 1 > 3$ and $3 - x \leqslant 7$

(c) $\frac{2x - 5}{3} \leqslant 3$ and $\frac{5 - x}{2} \leqslant 1$

(d) $\frac{x - 4}{3} \geqslant 2 - x$ and $\frac{3x - 1}{4} < 2$

ASSIGNMENT

Haze is a phenomenon that happens when small particles exist in large quantities in the air. These could block the sunlight from reaching the Earth, hence impairing visibility.

Write a report in the form of an essay about the haze phenomenon that includes the following.

- The causes of haze.
- The effects of haze.
- How does the Air Pollution Index (API) act as a guide to identify the quality of the air?
- The level of haze that occurred in your area and the preventive measures taken following that incident.

Scan the QR Code and visit http://apims.doe.gov.my/v2/ to help you in preparing the report.

Exploring MATHEMATICS

In an amusement park, fixed revenue is generated from advertising sponsors whereas fixed expenses incurred is the maintenance cost of the machines. Besides that, variable revenue is the number of visitors who visit the amusement park whereas variable expenses is the employees' wages. An operations manager needs to estimate the operating expenses and revenues to ensure that the amusement park obtains a profit.

The table below shows the daily expenses and revenues estimated by the operations manager of an amusement park which offers 12 rides. Copy and complete the information in the table below.

Daily expenses	
Total variable operating cost per visitor	RM30
Total fixed cost (RM10 000 + RM2 500 per ride)	
Daily revenues	
Admission ticket per visitor	RM76
Food per visitor	RM50
Souvenirs per visitor	RM30
Parking per visitor	RM5
Total variable revenues per visitor	
Fixed revenue from sponsors	RM8 000

Construct and solve the inequalities to determine the minimum number of visitors needed to visit the amusement park per day in order to obtain a profit.
Interpret your solutions.

CHAPTER 8

Lines and Angles

What will you learn?

- Lines and Angles
- Angles related to Intersecting Lines
- Angles related to Parallel Lines and Transversals

Why study this chapter?

Lines and angles constitute the basic knowledge in geometry and construction. Discuss the objects around you that involve the knowledge of lines and angles.

Lines and angles are used in various fields. In engineering, lines and angles provide the visual beauty and stability to the structure of a building. In visual arts, lines and angles produce stunning patterns. How are lines and angles used in photography?

Walking through Time

Euclid of Alexandria (325 B.C. – 265 B.C.) was a mathematician who had contributed much to geometry. He wrote a set of books entitled 'The Element'. In the books, Euclid defined points and lines as the fundamentals in geometrical constructions.

Euclid of Alexandria

For more information:

http://goo.gl/mhn3oT

Word Link

• intersecting lines	• *garis bersilang*
• transversal	• *garis rentas lintang*
• parallel lines	• *garis selari*
• perpendicular line	• *garis serenjang*
• congruency	• *kekongruenan*
• perpendicular bisector	• *pembahagi dua sama serenjang*
• angle bisector	• *pembahagi dua sama sudut*
• adjacent angles	• *sudut bersebelahan*
• vertically opposite angles	• *sudut bertentang bucu*
• angle of elevation	• *sudut dongak*
• conjugate angles	• *sudut konjugat*
• interior angles	• *sudut pedalaman*
• complementary angles	• *sudut pelengkap*
• supplementary angles	• *sudut penggenap*
• reflex angle	• *sudut refleks*
• alternate angles	• *sudut selang-seli*
• corresponding angles	• *sudut sepadan*
• angle of depression	• *sudut tunduk*
• line segment	• *tembereng garis*

Open the folder downloaded from page vii for the audio of Word Link.

8.1 Lines and Angles

Lines and angles exist in our surroundings. For instance, coconut leaves which resemble lines at certain angles exhibit the beauty of art in nature.

What are congruency of line segments and congruency of angles?

LEARNING STANDARDS

Determine and explain the congruency of line segments and angles.

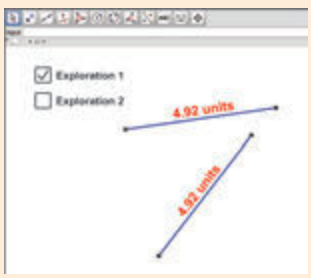
Exploration Activity 1

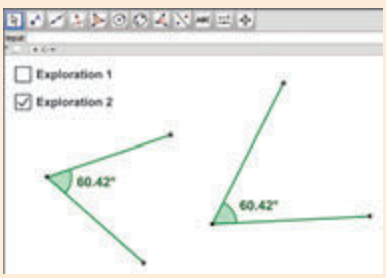
Flip Classroom

Aim: To determine congruency of line segments and congruency of angles.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Congruent segments angles.ggb* using *GeoGebra*.





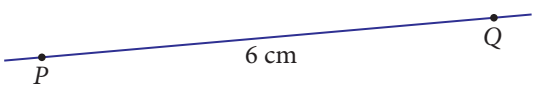
2. Click and drag the points in both explorations displayed.
3. Discuss with your friends what you have observed.

From the results of Exploration Activity 1, it is found that

(i) in Exploration 1, the two line segments displayed always have the same length. Line segments having the same length are known as **congruent line segments**.

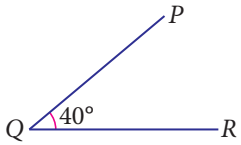
(ii) in Exploration 2, the two angles displayed always have the same size. Angles having the same size are known as **congruent angles**.

A line segment is denoted using capital letters at both ends of the segment. For instance,



Thus, $PQ = 6$ cm.

An angle is denoted using the symbol '∠' and capital letters at the vertex and at the ends of the two arms of the angle. For instance,

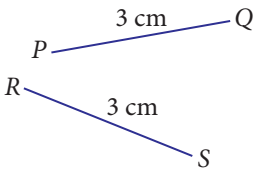


Thus, $\angle PQR = 40°$ or $\angle RQP = 40°$
or $P\hat{Q}R = 40°$ or $R\hat{Q}P = 40°$
or $\angle Q = 40°$

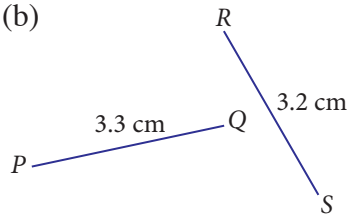
Self Practice 8.1a

1. Explain whether line segments PQ and RS in each of the following diagrams are congruent:

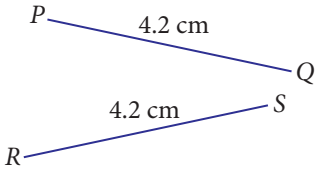
(a)



(b)

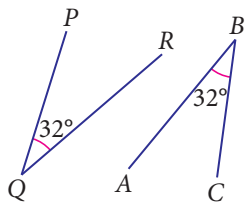


(c)

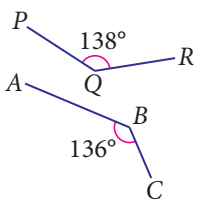


2. Explain whether $\angle PQR$ and $\angle ABC$ in each of the following diagrams are congruent:

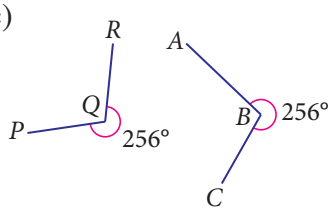
(a)



(b)



(c)



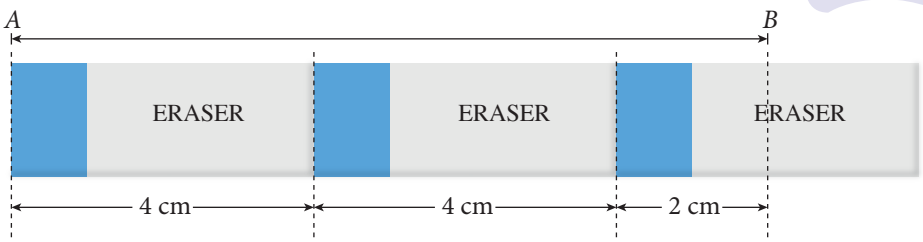
CHAPTER 8

How do you estimate and measure the length of a line segment and the size of an angle?

LEARNING STANDARDS

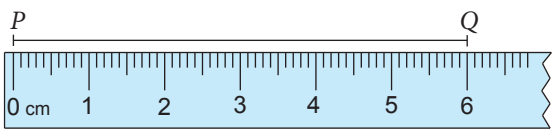
Estimate and measure the size of line segments and angles, and explain how the estimation is obtained.

The length of a line segment can be estimated by comparing the length with objects of a known length. For instance, if an eraser is known to have a length of 4 cm, then the length of line AB below can be estimated as approximately 10 cm.



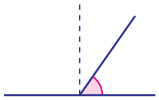
The length of a line segment can be measured more accurately by using a ruler.

The length of line segment PQ is exactly 6 cm.

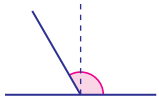


The size of an angle can be estimated by recognising the right angle first. An angle that appears more than a right angle has an angle greater than 90°. An angle that appears less than a right angle has an angle less than 90°. For instance,

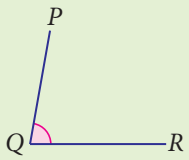
Right angle = 90°



An angle less than 90°



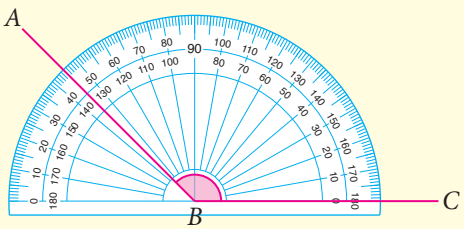
An angle greater than 90°



$\angle PQR$ appears slightly less than a right angle.
Thus, $\angle PQR$ can be estimated at about 80°.

From what you have learnt, the size of an angle can be measured more accurately using a protractor.

Size of $\angle ABC$ is exactly 135°.



Self Practice 8.1b

1. Estimate the length of each of the following line segments. Explain how you would do the estimation.

(a)

(b)

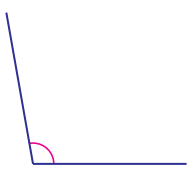
2. Measure each of the following line segments.

(a)

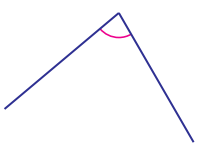
(b)

3. Estimate the size of each of the following angles. Explain how you would do the estimation.

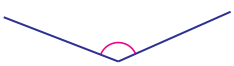
(a)



(b)

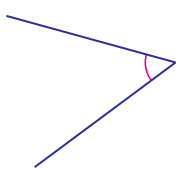


(c)

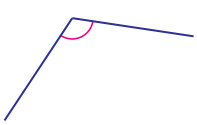


4. Using a protractor, measure each of the following angles accurately:

(a)



(b)



(c)

What are the properties of the angle on a straight line, a reflex angle and the angle of one whole turn?

Flashback

Explain the meanings of an acute angle and an obtuse angle.

LEARNING STANDARDS

Recognise, compare and explain the properties of angles on a straight line, reflex angles, and one whole turn angles.

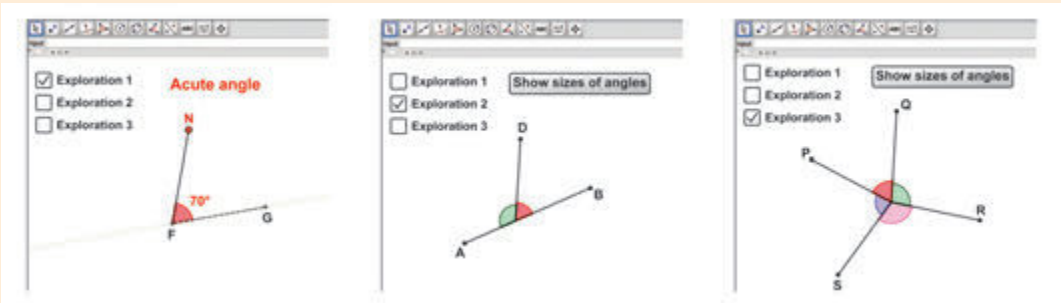
Exploration Activity 2

Flip Classroom

Aim: **To explain the properties of the angle on a straight line, a reflex angle and the angle of one whole turn.**

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Angles straight line.ggb* using *GeoGebra*.



2. Select 'Exploration 1'. Click and drag the red point and observe the change of the angle displayed.
   (i) Click and drag the red point so that the angle is on a straight line. What is the size of the angle displayed?
   (ii) Click and drag the red point so that the angle goes through one whole turn. What is the size of the angle displayed?
3. Click and drag the black points to change the position of the angle for further exploration.
4. Discuss with your friends, then compare and explain the meanings of the angle on a straight line, a reflex angle and the angle of one whole turn.
5. Select 'Exploration 2'.
6. Explain how you would determine the sum of the angles displayed.
7. Click and drag the black points and click on the button displayed on the screen for further exploration.
8. Select 'Exploration 3' and repeat Steps 6 and 7.
9. Discuss with your friends and state all the conclusions that can be made regarding your explorations.

From the results of Exploration Activity 2, it is found that

(i) the angle on a straight line is 180°. Thus, the sum of angles on a straight line is 180°.

(ii) a reflex angle is an angle with a size more than 180° and less than 360°.

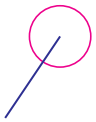
(iii) the angle of one whole turn is 360°. Thus, the sum of angles at a point is 360°.

Self Practice 8.1c

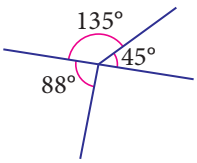
1. For each of the following angles, state whether the angle is an angle on a straight line, a reflex angle or an angle of one whole turn.

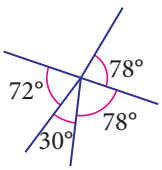
(a) 

(b) 

(c) 

2. Copy each of the following diagrams. Identify and mark (i) the angle on a straight line, (ii) the reflex angle and (iii) the angle of one whole turn. Explain how you obtained your answers.

(a) 

(b) 

What are the properties of complementary angles, supplementary angles and conjugate angles?

LEARNING STANDARDS

Describe the properties of complementary angles, supplementary angles and conjugate angles.

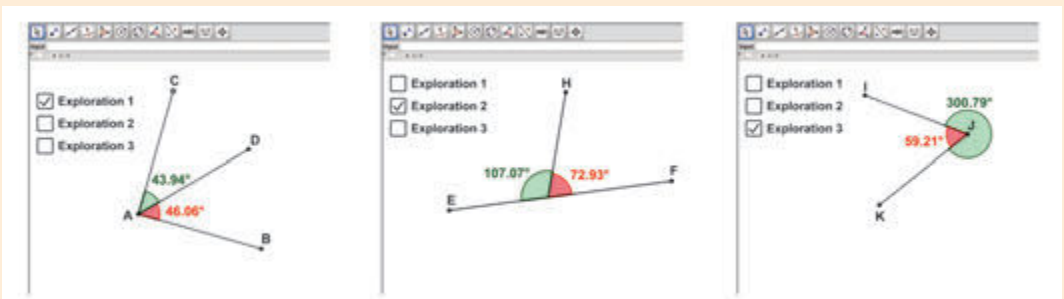
Exploration Activity 3

Flip Classroom

Aim: To describe the properties of complementary angles, supplementary angles and conjugate angles.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Complementary angles.ggb* using *GeoGebra*.



2. Select 'Exploration 1'. Determine the sum of the two angles displayed. What do you observe?
3. Click and drag the black points and repeat Step **2**. Discuss with your friends and state a conclusion.
4. Repeat Steps **2** and **3** for 'Exploration 2' and 'Exploration 3'.
5. State the conclusions that you can make.

From the results of Exploration Activity 3, it is found that

(i) in Exploration 1, the sum of the two angles is always 90°. These angles are known as **complementary angles**.

(ii) in Exploration 2, the sum of the two angles is always 180°. These angles are known as **supplementary angles**.

(iii) in Exploration 3, the sum of the two angles is always 360°. These angles are known as **conjugate angles**.

Self Practice 8.1d

1. Explain whether each of the following statements is TRUE or FALSE.
(a) 60° is the complement of 30°.
(b) 125° is the supplement of 45°.
(c) 300° and 60° are conjugate angles.
(d) 142° and 38° are supplementary angles.
(e) If angle A is the conjugate of angle B, then $A = 360° - B$.

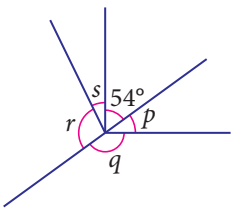
How do you solve problems?

LEARNING STANDARDS

Solve problems involving complementary angles, supplementary angles and conjugate angles.

Example 1

In the diagram, p and 54° are complementary angles. It is given that p and q are supplementary angles and the angle conjugate to r is 260°. Calculate the values of p, q, r and s.



Solution

$p + 54° = 90°$ ← Complementary angles
Thus, $p = 90° - 54°$
$= 36°$

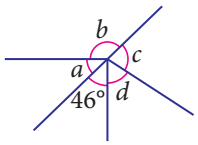
$p + q = 180°$ ← Supplementary angles
$36° + q = 180°$
Thus, $q = 180° - 36°$
$= 144°$

$r + 260° = 360°$ ← Conjugate angles
Thus, $r = 360° - 260°$
$= 100°$

$s + 100° + 144° + 36° + 54° = 360°$ ← The angle of one whole turn
$s + 334° = 360°$
$s = 360° - 334°$
$= 26°$

Self Practice 8.1e

1. In the diagram, a and 46° are complementary angles. It is given that a and b are supplementary angles and the angle conjugate to c is 283°. Calculate the values of a, b, c and d.



2. It is given that p and q are supplementary angles such that $p > q$. If the difference between p and q is 52°, calculate the values of p and q.

How do you perform a geometrical construction?

LEARNING STANDARDS

Construct
(i) line segments,
(ii) perpendicular bisectors of line segments,
(iii) perpendicular line to a straight line,
(iv) parallel lines
and explain the rationale of construction steps.

Logos, house plans or technical designs are drawings that require accurate measurements. These drawings can be done with the help of compasses and straight edge tools only, any geometrical tool such as set squares, rulers or by using geometry software.

The work of using geometrical tools or geometry software to do drawings of accurate measurements is known as a **geometrical construction**.

SMART TIPS

Use a sharp pencil while performing geometrical construction and do not erase all construction lines.

(i) Line segments

A section of a straight line with a fixed length is known as a **line segment**.

Example 2

Construct the line segment AB with a length of 8 cm using only a pair of compasses and a ruler.

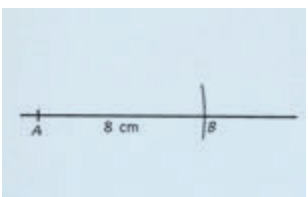
Let's Discuss

Why is the construction of a line segment using only a ruler less accurate? Explain.

Solution

1 Draw a line and mark point A on the line.

2 Measure a distance of 8 cm on the compasses.

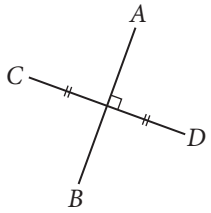


3 From point A, with the distance of 8 cm on the compasses, mark point B on the line.

Scan the QR Code or visit https://youtu.be/ElKif-Hy2mA to watch the video of the construction of a line segment. Then, explain the rationale of the construction steps.

(ii) Perpendicular bisectors

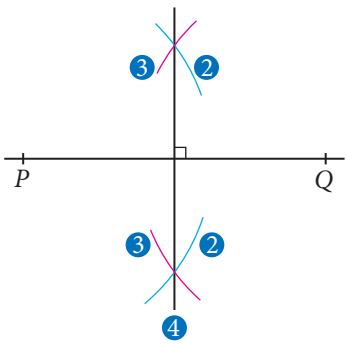
If line AB is perpendicular to line segment CD and divide CD into two parts of equal length, then line AB is known as the **perpendicular bisector** of CD.

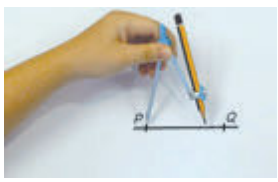


Example 3

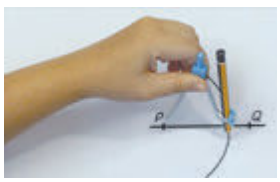
Construct the perpendicular bisector of line segment PQ using only a pair of compasses and a ruler.

Solution



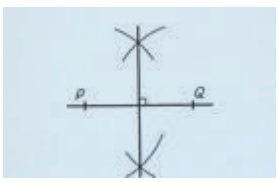


1. Adjust and set the compasses to a distance slightly more than half the length of line PQ.



2. Construct two arcs from P, one above and one below PQ.

3. Without altering the distance on the compasses, construct two arcs from Q, one above and one below PQ.



4. Draw a line joining the points of intersection of the arcs constructed in Steps **2** and **3**.

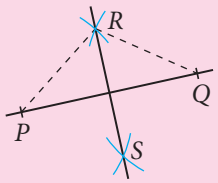
Scan the QR Code or visit https://youtu.be/P6M50r8Mblk to watch the video of the construction of a perpendicular bisector. Then, explain the rationale of the construction steps.

Let's Discuss

In the construction of perpendicular bisector,

(a) if the distance measured on the compasses is less than half the length of PQ, what will happen? Discuss.

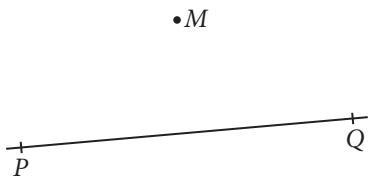
(b) is the perpendicular bisector of PQ, which is RS, the axis of symmetry of triangle PQR? Discuss and explain.



(iii) Perpendicular line to a straight line

Example 4

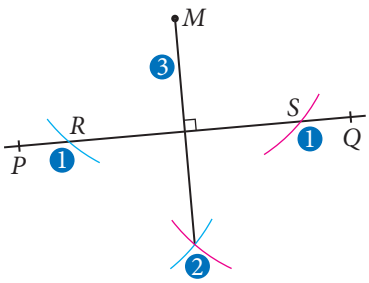
Using only a pair of compasses and a ruler, construct the perpendicular line from point M to the straight line PQ.



Flashback

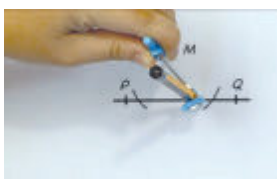
If a line is perpendicular to line PQ, then the line is known as **perpendicular line** to line PQ.

Solution

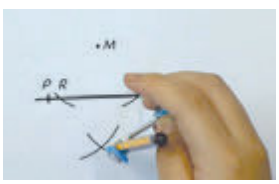


Let's Discuss

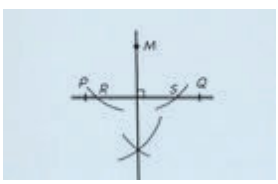
Referring to the construction of a perpendicular line in Example 4, what is the relationship between the construction methods of a perpendicular line and a perpendicular bisector?



1. With the same distance on the compasses from M, construct two arcs from M intersecting PQ. Label the two points of intersection as R and S.



2. With the same distance on the compasses from R and from S, construct an arc from R and from S respectively so that the arcs intersect.

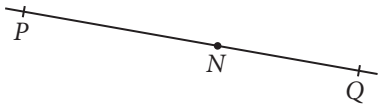


3. Draw a line joining M and the point of intersection of the arcs.

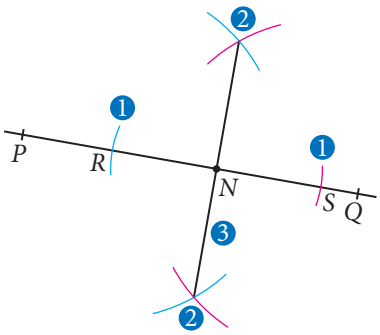
Scan the QR Code or visit https://youtu.be/Raz8EGk4x0Y to watch the video of the construction of a perpendicular line to a straight line passing through a point. Then, explain the rationale of the construction steps.

Example 5

Using only a pair of compasses and a ruler, construct the perpendicular line to PQ passing through point N.

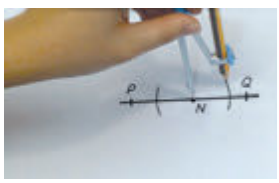


Solution



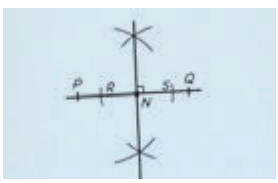
Let's Discuss

Referring to the construction of a perpendicular line in Example 5, what is the relationship between the construction methods of a perpendicular line and a perpendicular bisector? Discuss and explain.



1. With the same distance on the compasses, construct two arcs from N intersecting PQ. Label the two points of intersection as R and S.

2. Widen the compasses to a distance more than NR or NS, and with the same distance from R and from S on the compasses, construct two arcs above and below PQ respectively so that the arcs intersect.



3. Draw a line joining N and the points of intersection of the arcs constructed in Step **2**.

Scan the QR Code or visit https://youtu.be/FoxcZU8P7JA to watch the video of the construction of a perpendicular line to a straight line passing through a point on the straight line. Then, explain the rationale of the construction steps.

Scan the QR Code or visit https://youtu.be/Kow8PWTwIRM to watch the video of the construction of a perpendicular line to a straight line using a set square. Hence, construct the perpendicular line in Example 4 and Example 5 by using only a set square and a ruler.

(iv) Parallel lines

Example 6

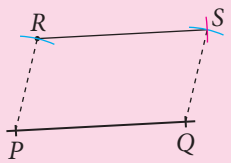
Using only a pair of compasses and a ruler, construct the line that is parallel to PQ passing through point R.

Flashback

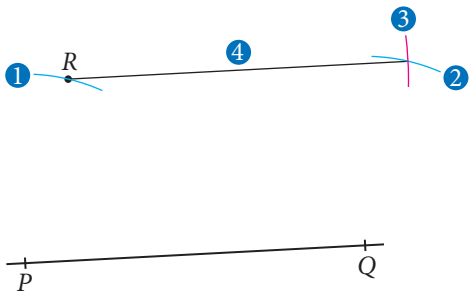
Lines that will never meet even when they are extended are known as **parallel lines**.

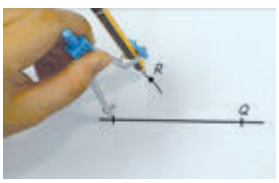
Let's Discuss

Explain how the construction of parallel lines relates to the construction of parallelogram $PQSR$.

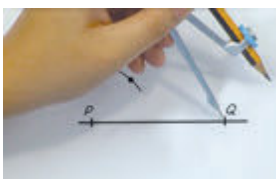


Solution

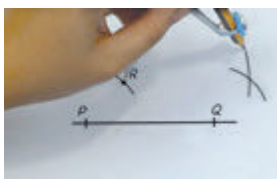




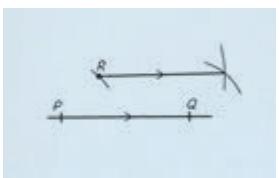
1 With the distance on the compasses equals to the length of PR, construct an arc from P passing through R.



2 Keeping the same distance on the compasses, construct an arc from Q.



3 With the distance on the compasses equals to the length of PQ, construct an arc from R intersecting the arc constructed in Step **2**.



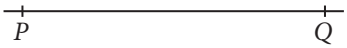
4 Draw a line joining R and the point of intersection constructed in Step **3**.

Scan the QR Code or visit https://youtu.be/Xwa44YIHcCg to watch the video of the construction of parallel lines. Then, explain the rationale of the construction steps.

Scan the QR Code or visit https://youtu.be/J8Im4Rz5poc to watch the video of the construction of parallel lines using a set square. Hence, construct the parallel lines in Example 6 by using only a set square and a ruler.

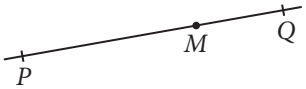
Self Practice 8.1f

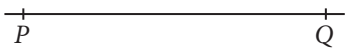
1. Construct each of the following line segments:
(a) $AB = 6$ cm (b) $PQ = 5.4$ cm (c) $RS = 7.3$ cm

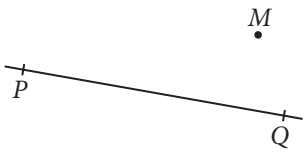
2. Copy and construct the perpendicular bisector of each of the following line segments:
(a) 

(b) 

3. For each of the following, copy and construct the perpendicular line to line segment PQ passing through point M.
(a) 

(b) 

4. For each of the following, copy and construct a line parallel to line PQ passing through point M.
(a) 

(b) 

CHAPTER 8

How do you construct angles and angle bisectors?

LEARNING STANDARDS
Construct angles and angle bisectors, and explain the rationale of construction steps.

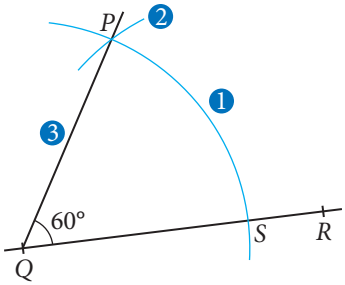
(i) Constructing an angle of 60°

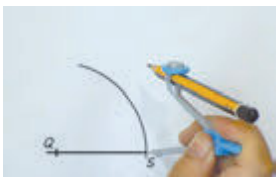
Other than using a protractor to construct an angle, an **angle of 60°** can be constructed using only a pair of compasses and a ruler based on the **concept of interior angles of an equilateral triangle**. The construction of three line segments of equal length forms a triangle with **interior angles of 60°**.

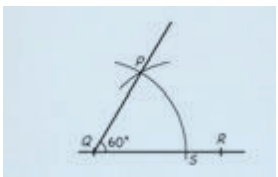
Example 7

Using only a pair of compasses and a ruler, construct line PQ so that $\angle PQR = 60°$.

Solution







1. Construct a long arc from Q intersecting QR. Mark the point of intersection as S.

2. Keeping the same distance on the compasses, construct an arc from S intersecting the arc constructed in Step **1**. Mark the point of intersection as P.

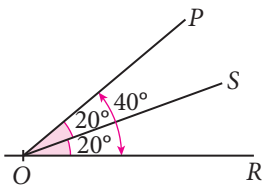
3. Draw a line joining point P and point Q. Thus, $\angle PQR = 60°$.

Scan the QR Code or visit https://youtu.be/urXMY5G2-6Y to watch the video of the construction of an angle of 60°. Then, explain the rationale of the construction steps.

(ii) Angle bisectors

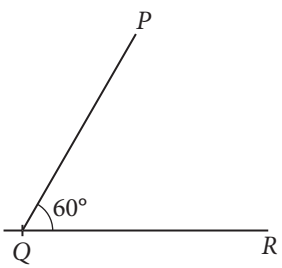
If a line divides an angle into two equal angles, the line is known as an **angle bisector**.

For instance, line OS is the angle bisector of $\angle POR$.

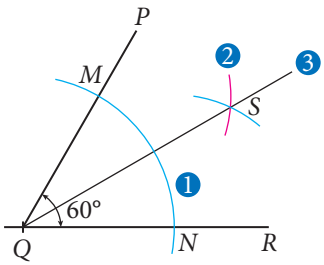


Example 8

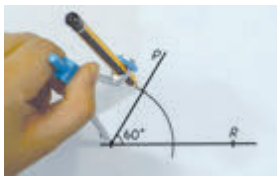
Using only a pair of compasses and a ruler, construct the angle bisector of $\angle PQR$.

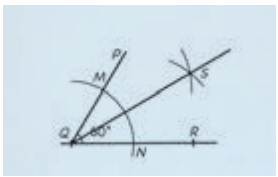


Solution



Scan the QR Code or visit https://youtu.be/9-X-WDvdfFI to watch the video of the construction of an angle bisector. Then, explain the rationale of the construction steps.



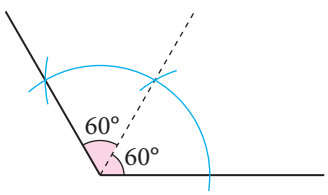


1. Construct a long arc from Q intersecting both arms QP and QR. Mark the two points of intersection as M and N.

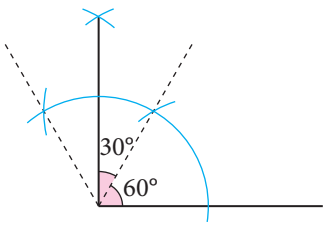
2. With the same distance on the compasses, construct an arc from M and from N respectively so that the arcs intersect. Mark the point of intersection as S.

3. Draw a line joining point Q and point S. QS is the angle bisector of $\angle PQR$.

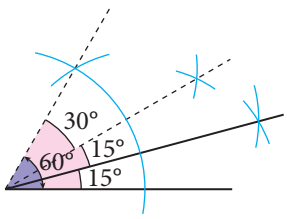
It is found that $\angle PQS = \angle SQR = 30°$. We have actually constructed an angle of 30°. Beginning with the construction of an angle of 60° and the knowledge of an angle bisector, we can construct other angles accurately using only a pair of compasses and a ruler. For instance,



120°



90°



15°

Let's Discuss

(a) What is the rationale for the construction of angles of 120°, 90° and 15° as shown above?
(b) Discuss and state other angles that can be constructed by using only a pair of compasses and a ruler.

Self Practice 8.1g

1. For each of the following angles, explain briefly how you would construct the angle using only a pair of compasses and a ruler. Then, construct the angle.
(a) 90° (b) 45° (c) 75° (d) 105°

Mastery Q 8.1

Open the folder downloaded from page vii for extra questions of Mastery Q 8.1.

1. Determine whether each of the following statements is TRUE or FALSE.
(a) A reflex angle is greater than the angle on a straight line.
(b) The sum of an obtuse angle and an acute angle yields a reflex angle.
(c) Two adjacent angles on a straight line are supplementary angles.
(d) The sum of two conjugate angles is the angle of one whole turn.
(e) If m and n are conjugate angles, then either m or n is a reflex angle but not both of them.

2. It is given that p and q are congruent angles and also complementary angles. State the values of p and q.

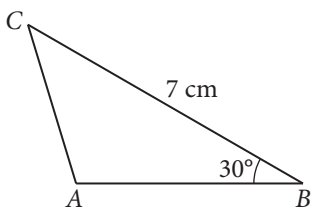
3. It is given that $PQ = 7$ cm. If PQ and RS are congruent line segments, state the length of line segment RS.

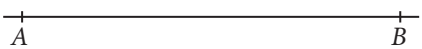
4. If p and q are supplementary angles and the difference between p and q is 30°, find the values of p and q such that $p > q$.

5. If p and q are conjugate angles and the size of q is four times that of p, find the values of p and q.

6. (a) Using only a ruler and a pair of compasses,
(i) copy the given line segment PQ and construct a triangle PQR beginning from the line PQ such that $QR = 6$ cm and $\angle RPQ = 60°$,
(ii) then, construct the perpendicular line to PR passing through point Q.
(b) Based on your construction in (a), measure the perpendicular distance from Q to line PR.

7. The diagram shows a triangle ABC. Using only a ruler and a pair of compasses,
(a) copy the given line segment AB and construct triangle ABC according to the given measurements beginning from straight line AB.





(b) then, construct the perpendicular bisector of line AC. The perpendicular bisector intersects line AC at point P and intersects line BC at point Q. Measure $\angle PQC$.

8.2 Angles related to Intersecting Lines

What are vertically opposite angles and adjacent angles at intersecting lines?

LEARNING STANDARDS

Identify, explain and draw vertically opposite angles and adjacent angles at intersecting lines, including perpendicular lines.

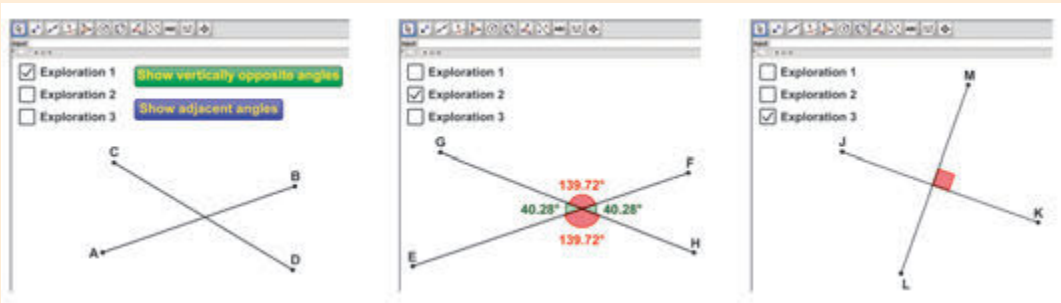
Exploration Activity 4

Flip Classroom

Aim: To explore angles at intersecting lines.

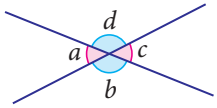
Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Angles intersecting lines.ggb* using *GeoGebra*.



2. Select 'Exploration 1'. The screen displayed shows two intersecting lines.
3. Click at 'Show vertically opposite angles' and 'Show adjacent angles'.
4. Discuss with your friends and explain the meanings of vertically opposite angles and adjacent angles at intersecting lines. Then, state another pair of vertically opposite angles and another pair of adjacent angles.
5. Select 'Exploration 2'. Click and drag the black points and observe the displayed angles.
6. Discuss with your friends and explain the properties of vertically opposite angles and adjacent angles at intersecting lines.
7. Select 'Exploration 3'. The screen displayed shows two lines perpendicular to each other. What conclusion can you make about the angles at the intersecting lines?
8. Discuss with your friends and make an overall conclusion about angles at intersecting lines.

From the results of Exploration Activity 4, it is found that



(i) **vertically opposite angles** at intersecting lines are equal.

(ii) the sum of **adjacent angles** at intersecting lines is 180°.

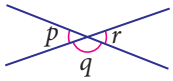
$\angle a = \angle c$ $\quad \angle a + \angle d = 180°$ $\quad \angle c + \angle b = 180°$

$\angle b = \angle d$ $\quad \angle d + \angle c = 180°$ $\quad \angle b + \angle a = 180°$

(iii) if two intersecting lines are perpendicular to each other, then all angles at the intersecting lines have the same size of 90°.

Example 9

The diagram shows two intersecting lines. Identify and state



(a) the vertically opposite angles.

(b) the adjacent angles at the intersecting lines.

Solution

(a) Angles p and r are the vertically opposite angles.

(b) Angles p and q, q and r are the adjacent angles at the intersecting lines.

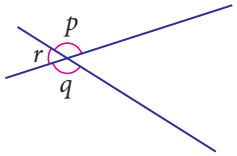
Self Practice 8.2a

1. For each of the following diagrams, identify and state

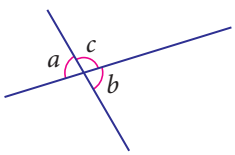
(i) the vertically opposite angles.

(ii) the adjacent angles at the intersecting lines.

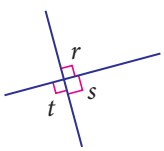
(a)



(b)



(c)

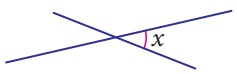


2. Copy each of the following diagrams, mark and label

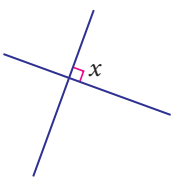
(i) the angle y if angles x and y are vertically opposite angles.

(ii) the angle z if angles x and z are adjacent angles at the intersecting lines.

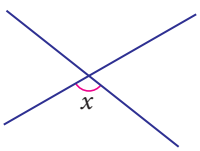
(a)



(b)



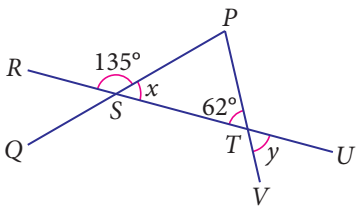
(c)



How do you determine the values of the angles at intersecting lines?

Example 10

In the diagram, PSQ, $RSTU$ and PTV are straight lines. Find the values of x and y.



LEARNING STANDARDS

Determine the values of angles related to intersecting lines, given the values of other angles.

Solution

$x + 135° = 180°$ ← Adjacent angles at intersecting lines

$x = 180° - 135°$

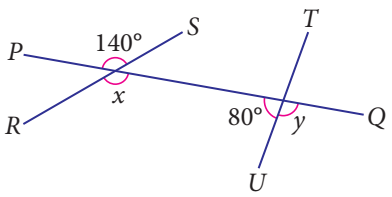
$= 45°$

$y = 62°$ ← Vertically opposite angles

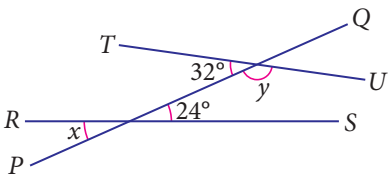
Self Practice 8.2b

1. In the following diagrams, PQ, RS and TU are straight lines. Find the values of x and y.

(a)



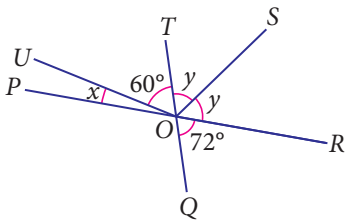
(b)



How do you solve problems?

Example 11

In the diagram, POR and TOQ are straight lines. Find the values of x and y.



LEARNING STANDARDS

Solve problems involving angles related to intersecting lines.

Solution

$\angle POT = 72°$

$x + 60° = 72°$ ← $\angle POT$ and $\angle QOR$ are vertically opposite angles.

$x = 72° - 60°$

$= 12°$

$\angle TOR + 72° = 180°$ ← $\angle QOR$ and $\angle TOR$ are adjacent angles at intersecting lines.

$\angle TOR = 180° - 72°$

$= 108°$

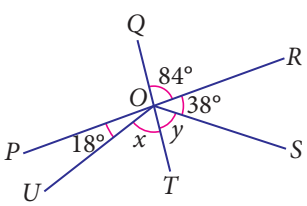
$y + y = 108°$

$2y = 108°$

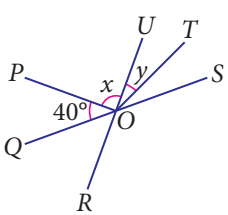
$y = 54°$

Self Practice 8.2c

1. In the diagram, POR and QOT are straight lines. Find the values of x and y.



2. In the diagram, QOS and ROU are straight lines. OT is the bisector of $\angle UOS$. $\angle POQ$ and $\angle QOR$ are complementary angles. Find the values of x and y.

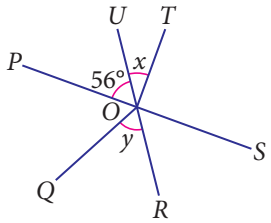


CHAPTER 8

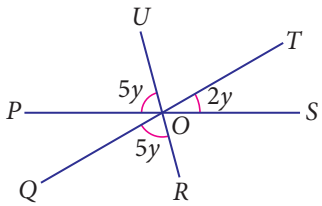
Mastery Q 8.2

Open the folder downloaded from page vii for extra questions of Mastery Q 8.2.

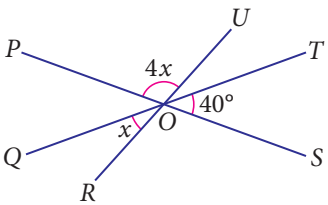
1. In the diagram, POS and UOR are straight lines. OQ is the bisector of $\angle POR$. $\angle POU$ and $\angle UOT$ are complementary angles. Find the values of x and y.



2. In the diagram, POS, QOT and UOR are straight lines. Find the value of y.



3. In the diagram, POS, QOT and ROU are straight lines. Find the value of x.

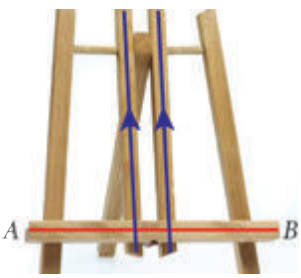


8.3 Angles related to Parallel Lines and Transversals

What are parallel lines and transversals?

LEARNING STANDARDS

Recognise, explain and draw parallel lines and transversals.

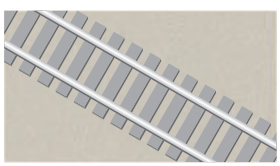


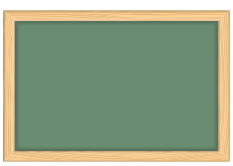
Parallel lines can be denoted using arrow heads '$\rightarrow\!\!\!\!-$' as shown in the photograph.

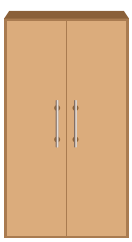
A straight line that intersects two or more straight lines is known as a **transversal**. For instance, straight line AB shown in the photograph is a transversal.

Self Practice 8.3a

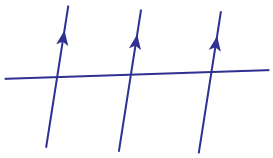
1. Copy the following objects. Draw and mark parallel lines on the copied objects.

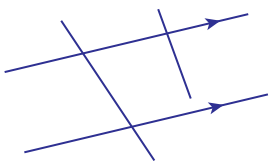
(a) 

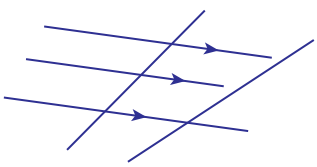
(b) 

(c) 

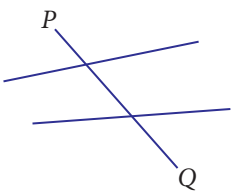
2. Copy each of the following diagrams and label the transversal as AB. Then draw another transversal and label the line as PQ.

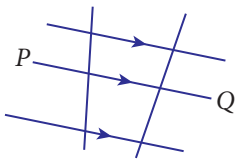
(a) 

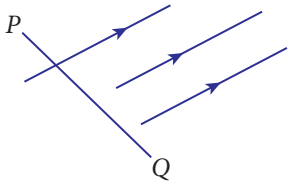
(b) 

(c) 

3. For each of the following diagrams, determine if the line labelled as PQ is a transversal or not, and make justification.

(a) 

(b) 

(c) 

What are corresponding angles, alternate angles and interior angles?

LEARNING STANDARDS

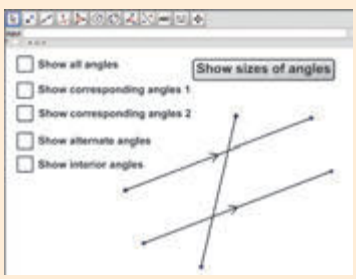
Recognise, explain and draw corresponding angles, alternate angles and interior angles.

Exploration Activity 5

Flip Classroom

Aim: To explore angles related to parallel lines.

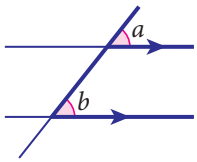
Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.



1. Open the file *Angles parallel lines.ggb* using *GeoGebra*.
2. Click and drag the points on the screen displayed and identify the parallel lines and the transversal.
3. Select 'Show corresponding angles 1' and 'Show corresponding angles 2' to explore the properties of corresponding angles.
4. Click at 'Show sizes of angles' and drag the points on the screen displayed if necessary.
5. State another pair of corresponding angles.
6. Discuss with your friends and explain the properties of corresponding angles.
7. Select 'Show alternate angles' and 'Show interior angles' and continue exploring alternate angles and interior angles.
8. State another pair of alternate angles and another pair of interior angles.
9. Discuss with your friends and explain the properties of alternate angles and interior angles.
10. State the conclusions that can be made from your explorations.

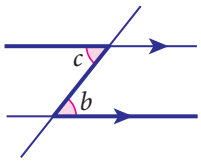
From the results of Exploration Activity 5, it is found that when a transversal intersects two or more parallel lines,

(i) the corresponding angles are equal.



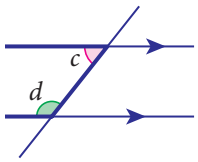
Thus, $\angle a = \angle b$

(ii) the alternate angles are equal.



Thus, $\angle b = \angle c$

(iii) the sum of the interior angles is 180°.

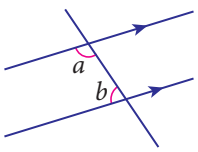


Thus, $\angle c + \angle d = 180°$

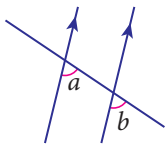
Self Practice 8.3b

1. Copy each of the following diagrams. Explain and state whether the two angles labelled as a and b are corresponding angles, alternate angles or interior angles. Then label another pair of angles of the same type with c and d.

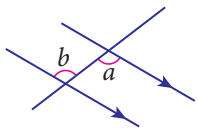
(a)



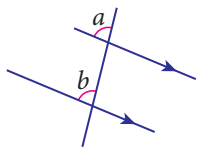
(b)



(c)



(d)



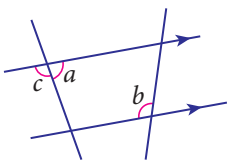
2. Copy each of the following diagrams and label

(i) angle p if a and p are corresponding angles.

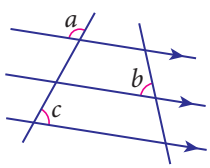
(ii) angle q if b and q are alternate angles.

(iii) angle r if c and r are interior angles.

(a)



(b)



How do you determine whether two straight lines are parallel?

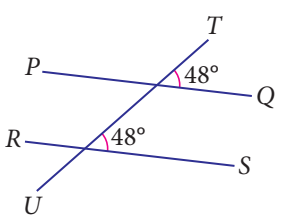
LEARNING STANDARDS

Determine whether two straight lines are parallel based on the properties of angles related to transversals.

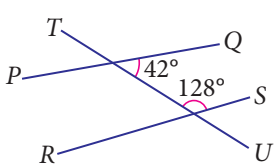
Example 12

In each of the following diagrams, determine whether straight line PQ and straight line RS are parallel.

(a)



(b)



CHAPTER 8

Solution

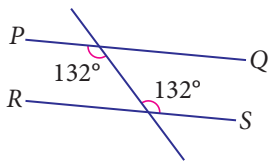
(a) PQ and RS are parallel. ← The corresponding angles, 48°, are equal.

(b) $42° + 128° = 170°$
$\neq 180°$ ← The sum of the interior angles should be 180° if PQ and RS are parallel.
Thus, PQ and RS are not parallel.

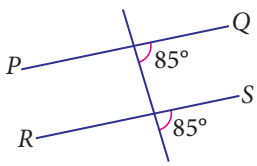
Self Practice 8.3c

1. For each of the following diagrams, determine whether line PQ and line RS are parallel.

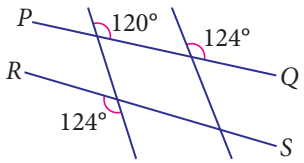
(a)



(b)



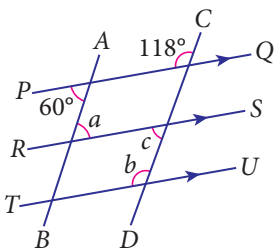
(c)



How do you determine the values of angles related to parallel lines and transversals?

Example 13

In the diagram, PQ, RS and TU are parallel lines. Find the values of a, b and c.



LEARNING STANDARDS

Determine the values of angles related to parallel lines and transversals, given the values of other angles.

Solution

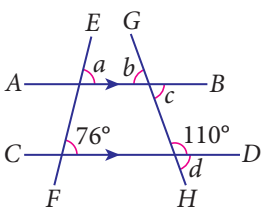
$a = 60°$ ← Alternate angles

$b = 118°$ ← Corresponding angles

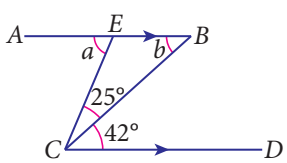
$b + c = 180°$ ← Interior angles
$118° + c = 180°$
$c = 180° - 118°$
$= 62°$

Self Practice 8.3d

1. In the diagram, EF and GH are straight lines. Find the values of a, b, c and d.



2. In the diagram, find the values of a and b.



What are angles of elevation and depression?

The concept of angles is very important in land surveying. A land surveyor uses angles to determine the distance. The two types of angles used are the angle of elevation and the angle of depression.

LEARNING STANDARDS

Recognise and represent angles of elevation and angles of depression in real-life situations.

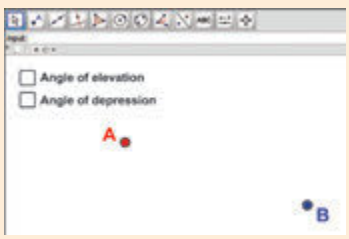
Exploration Activity 6

Flip Classroom

Aim: To recognise the angle of elevation and the angle of depression.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

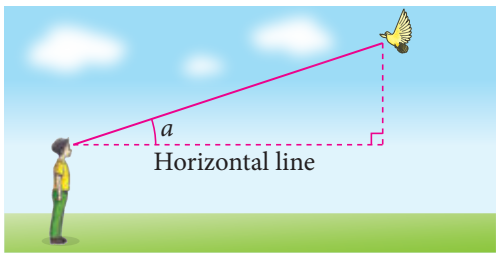
1. Open the file *Angles of elevation and depression.ggb* using *GeoGebra*.
2. Select 'Angle of elevation'. The screen displayed shows point *A* and point *B* at different levels.
3. Click at the button on the screen to explore the angle of elevation.
4. Observe the animation on the screen and explain in general the meaning of the angle of elevation.



5. Click and drag point *A* or point *B* to different positions and click the button on the screen for further exploration about the angle of elevation.
6. Select 'Angle of depression' and repeat Steps 3 to 5 to explore the angle of depression.
7. Discuss with your friends the angle of elevation and the angle of depression.
   (a) How are the positions of *A* and *B* related to the angle of elevation and the angle of depression?
   (b) If *A* is at a higher level than *B* or vice versa, explain the angle of elevation or the angle of depression between *A* and *B*.
8. Discuss with your friends and state the conclusions that can be made about the angle of elevation and the angle of depression.

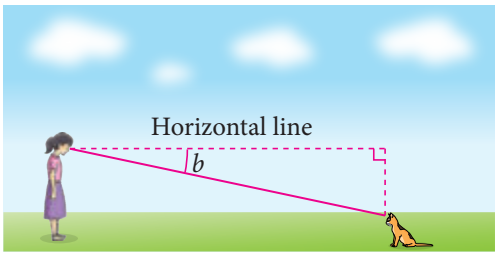
From the results of Exploration Activity 6, it is found that

(i) When a person sees an object which is at a higher level, the angle between the line of sight and the horizontal line is known as the **angle of elevation**.



Angle a is the angle of elevation of the bird from the sight of Jefri.

(ii) When a person sees an object which is at a lower level, the angle between the line of sight and the horizontal line is known as the **angle of depression**.



Angle b is the angle of depression of the cat from the sight of Kim.

Let's Discuss

When we figure out the angle of elevation and the angle of depression between two objects at different levels,

(a) is the angle of elevation always the same as the angle of depression?

(b) what is the relation between the angle of elevation and the angle of depression?

Let's Discuss

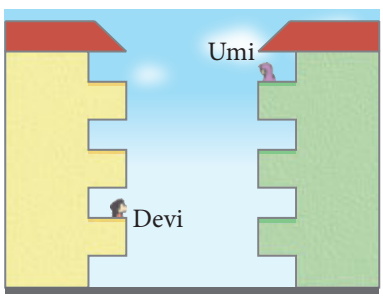
"The angle of elevation and the angle of depression are always acute angles."
Is this statement true? Discuss.

Example 14

The diagram shows the positions of Devi and Umi at two buildings on a horizontal ground. Draw and label

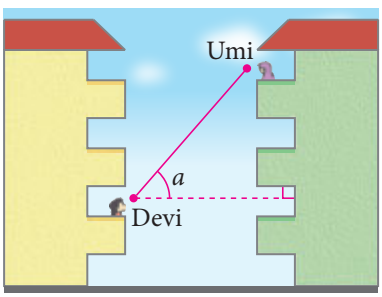
(a) angle a to denote the angle of elevation of Umi from Devi.

(b) angle b to denote the angle of depression of Devi from Umi.

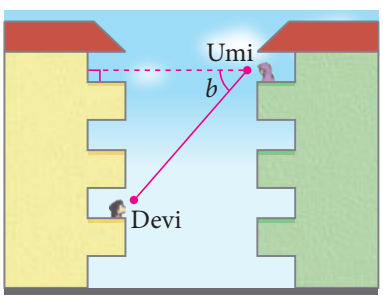


Solution

(a)



(b)

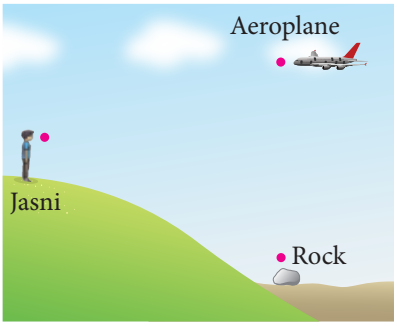


Self Practice 8.3e

1. Jasni is standing on top of a hill observing an aeroplane and a rock. Represent the situation by drawing and labelling

(a) angle a which represents the angle of elevation of the aeroplane from Jasni.

(b) angle b which represents the angle of depression of the rock from Jasni.



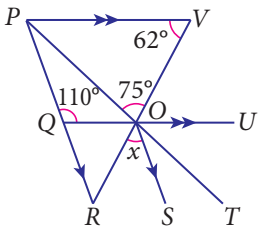
How do you solve problems?

Example 15

In the diagram, POT, QOU, PQR and VOR are straight lines.

(a) Find the value of x.

(b) Assuming PV is a horizontal line and $\angle PVO$ is the angle of depression of O from V, find the angle of elevation of P from O.



LEARNING STANDARDS

Solve problems involving angles related to parallel lines and transversals.

Solution

(a) $\angle QOS = \angle PQO$ ← Alternate angles
$= 110°$

$\angle QOR = \angle PVR$ ← Corresponding angles
$= 62°$

Thus, $x = 110° - 62°$
$= 48°$

(b) $\angle QOV + 62° = 180°$ ← The sum of the interior angles
$\angle QOV = 180° - 62°$
$= 118°$
$\angle POQ = 118° - 75°$
$= 43°$

Thus, the angle of elevation of P from O is 43°.

Career in Mathematics

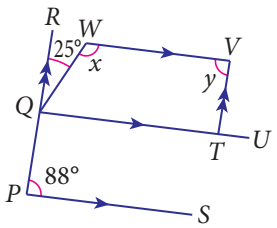
Photographers and videographers use the knowledge of lines and angles to help them capture photographs and record videos skilfully.

Self Practice 8.3f

1. In the diagram, PQR and QTU are straight lines.

(a) Find the values of x and y.

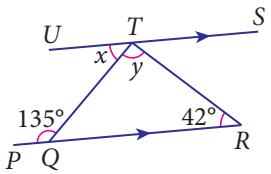
(b) Assuming WV is a horizontal line, find the angle of elevation of V from T.



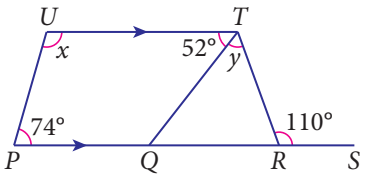
Mastery Q 8.3

Open the folder downloaded from page vii for extra questions of Mastery Q 8.3.

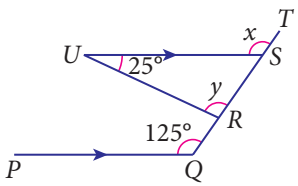
1. In the diagram, PQR and UTS are straight lines. Find the values of x and y.



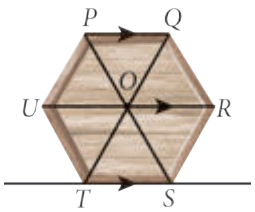
2. In the diagram, $PQRS$ is a straight line. Find the values of x and y.



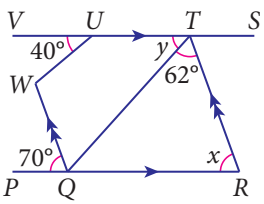
3. In the diagram, $QRST$ is a straight line. Find the values of x and y.



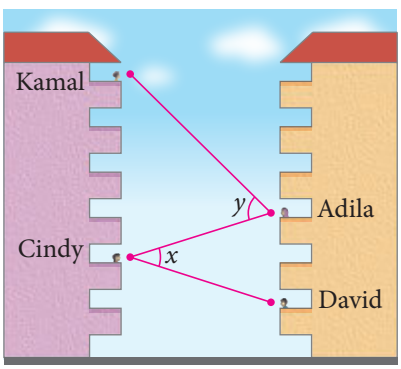
4. The diagram shows a hexagonal wooden frame $PQRSTU$ resting on a horizontal ground.
(a) State the angle of elevation of Q from O.
(b) State the angle of depression of T from O.



5. In the diagram, $VUTS$ and PQR are straight lines.
(a) Find the values of x and y.
(b) Find the reflex angle QWU.
(c) Assuming $VUTS$ is a horizontal line, find the angle of depression of R from T.



6. The diagram shows the positions of Kamal, Cindy, Adila and David in two condominiums. The angle of elevation of Cindy from David is 15°, the angle of elevation of Adila from Cindy is 18° and the angle of depression of Adila from Kamal is 40°. Find the values of x and y.



SUMMARY

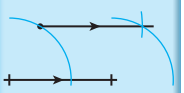
LINES AND ANGLES

Line segment

Perpendicular bisector

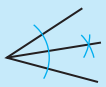
Perpendicular line

Parallel lines



Construction of 60°

Angle bisector



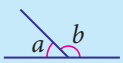
The angle on a straight line

Reflex angle

The angle of one whole turn

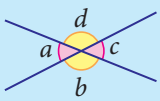
Complementary angles

Supplementary angles



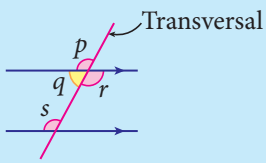
Conjugate angles

Angles related to intersecting lines



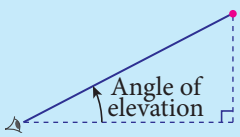
- Vertically opposite angles
  $\angle a = \angle c$ and $\angle b = \angle d$
- Adjacent angles at intersecting lines
  $\angle a + \angle d = 180°$
  $\angle d + \angle c = 180°$
  $\angle c + \angle b = 180°$
  $\angle b + \angle a = 180°$

Angles related to parallel lines and transversals

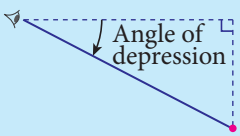


- Corresponding angles
  $\angle p = \angle s$
- Alternate angles
  $\angle r = \angle s$
- Interior angles
  $\angle q + \angle s = 180°$

- Angle of elevation



- Angle of depression

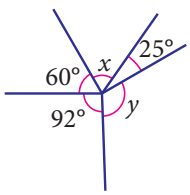


At the end of this chapter, I can...	Very good	Work harder
determine and explain the congruency of line segments and angles.		
estimate and measure the size of line segments and angles.		
recognise, compare and explain the properties of angles on a straight line, reflex angles, and one whole turn angles.		
describe the properties and solve problems involving complementary angles, supplementary angles and conjugate angles.		
construct line segments, perpendicular bisectors of line segments, perpendicular line to a straight line, parallel lines and explain the rationale of construction steps.		
construct angles and angle bisectors and explain the rationale of construction steps.		
identify, explain and draw vertically opposite angles and adjacent angles at intersecting lines, including perpendicular lines.		
determine the values of angles and solve problems involving angles related to intersecting lines.		
recognise, explain and draw parallel lines, transversals, corresponding angles, alternate angles and interior angles.		
determine whether two straight lines are parallel.		
determine the values of angles and solve problems involving angles related to parallel lines and transversals.		
recognise and represent angles of elevation and angles of depression in real-life situations.		

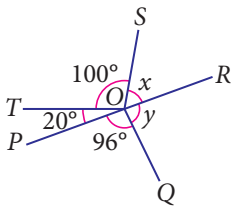
Let's PRACTISE

Test Yourself

1. In the diagram, x and 25° are complementary angles. Find the values of x and y.



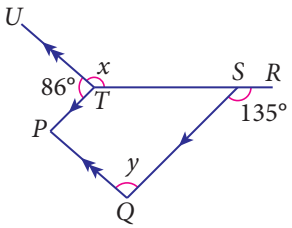
2. The diagram shows a straight line POR. Find the values of x and y.



3. Using only a pair of compasses and a ruler, construct a line segment AB with a length of 8 cm. Then construct point C such that $\angle ABC = 60°$ and $BC = 5$ cm. Then, construct the perpendicular line from C to AB.

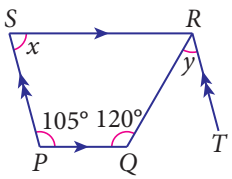
Self Mastery

4. TSR is a straight line as shown in the diagram. Find the values of x and y.



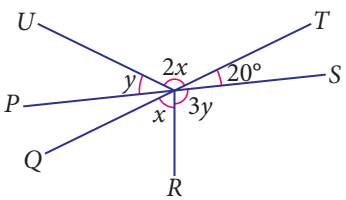
5. If x and y are supplementary angles and $x : y = 2 : 3$, find the values of x and y.

6. (a) In the diagram, find the values of x and y.
(b) Hence, using only a pair of compasses and a ruler, construct the trapezium $PQRS$ such that $PQ = 4$ cm and $PS = 6$ cm.

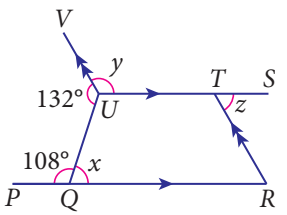


Challenge Yourself

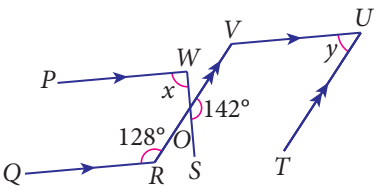
7. PS and QT are straight lines as shown in the diagram. Find the values of x and y.



8. In the diagram, UTS and PQR are straight lines. Find the values of x, y and z.



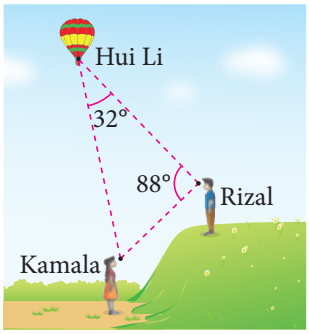
9. The diagram shows two straight lines, WOS and ROV. Find the values of x and y.



10. The diagram shows the positions of three friends. Hui Li is on a hot-air balloon, Kamala is on a horizontal ground and Rizal is on top of a hill. The angle of depression of Kamala from Hui Li is 78°. Based on the information given in the diagram, find

(a) the angle of depression of Rizal from Hui Li.

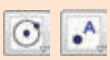
(b) the angle of elevation of Rizal from Kamala.



ASSIGNMENT

Hafeeza is assigned to take photographs during her elder sister's wedding. Explain how the knowledge of lines like parallel lines and perpendicular lines as well as the knowledge of angles like angle of elevation and angle of depression can help Hafeeza solve problems like positioning the cameras, choice of lenses, perspective issue and composition issue of subjects in the photographs and others. Write a report and present your finding during the lesson.

Exploring MATHEMATICS

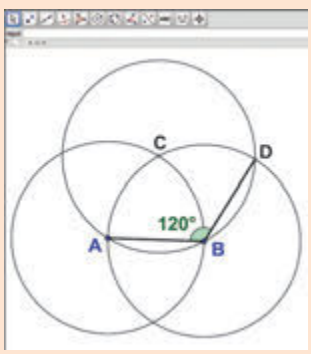
Open the folder downloaded from page vii. Open the file *Angle of 120.ggb* using *GeoGebra*. The screen displayed shows the construction of 120° using only  in *GeoGebra*. Explain the rationale behind the construction.

Using only  in *GeoGebra*, and beginning with a given line segment *AB*, construct each of the following:

(a) The perpendicular bisector of *AB*.

(b) The line perpendicular to *AB* passing through a given point.

(c) An angle of 30° such that *AB* is one of the arms of the angle.



Present your work in class during the lesson by explaining the rationale behind the constructions that you have done.

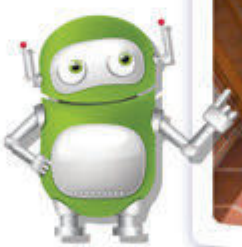
CHAPTER 9

Basic Polygons

What will you learn?

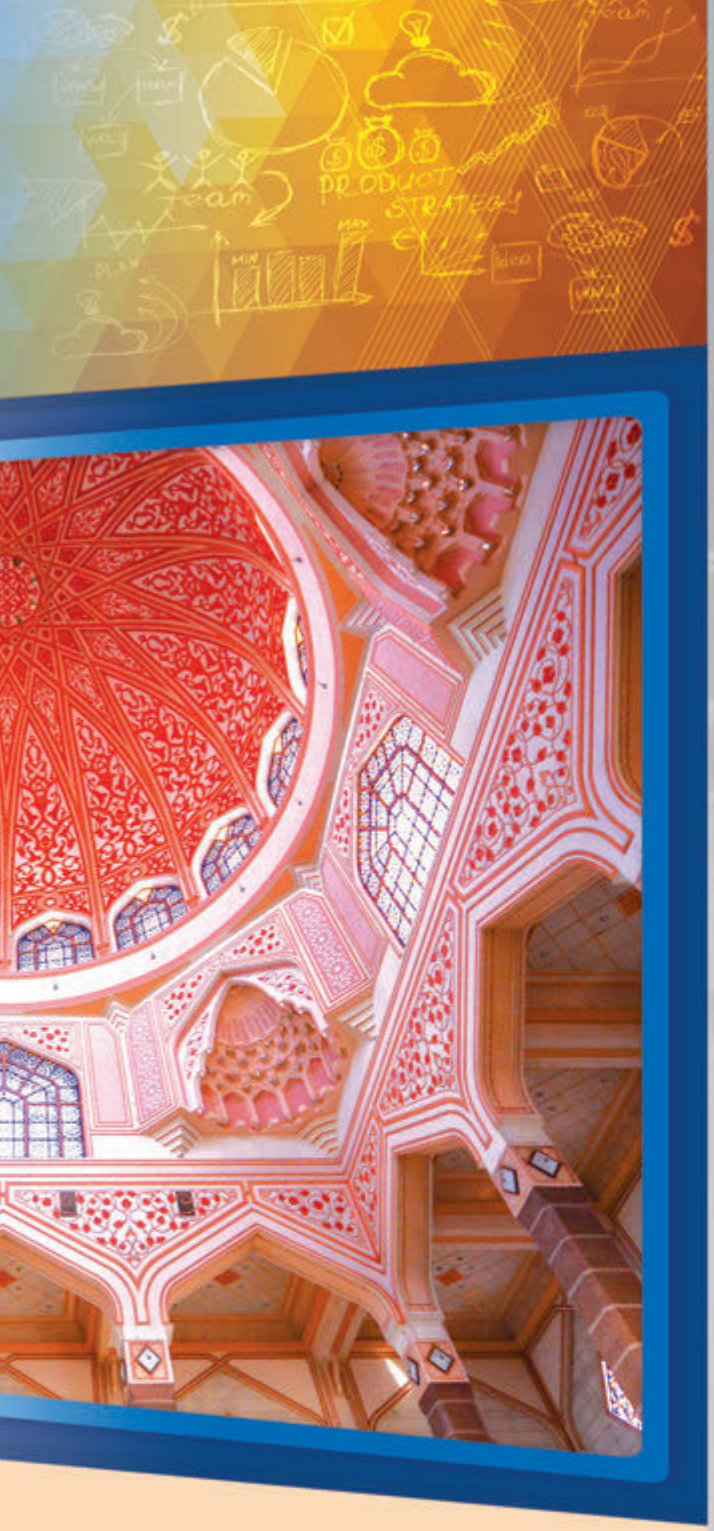
- Polygons
- Properties of Triangles and the Interior and Exterior Angles of Triangles
- Properties of Quadrilaterals and the Interior and Exterior Angles of Quadrilaterals

Why study this chapter?



As the basic knowledge about the uses of polygons in the field of building construction. Discuss other fields that involve the use of polygons.

Putra Mosque is one of the main landmarks in Putrajaya and a tourist attraction because of the beautiful and magnificent designs. The combination of the various polygons exhibits the uniqueness in the artwork.



How are polygons used in the artwork of buildings? What types of polygons are used?

Walking through Time

Carl Friedrich Gauss (1777 – 1855) was a German mathematician who contributed much in the field of mathematics. One of his major discoveries was introducing the techniques to construct a 17-sided polygon using only a pair of compasses and a straight edge.

Carl Friedrich Gauss

For more information:

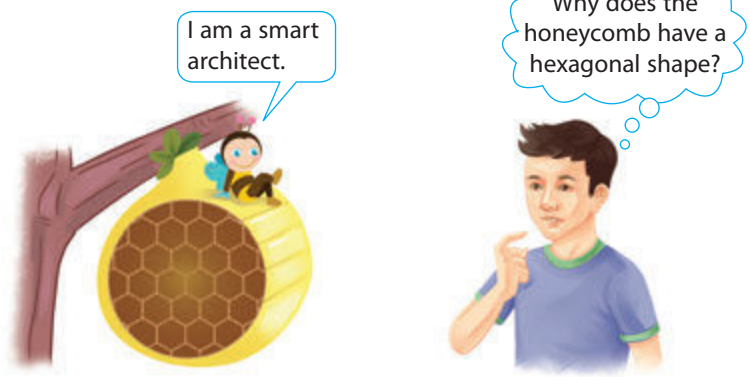
https://goo.gl/Tvk2d3

Word Link

• vertex	• *bucu*
• conjecture	• *konjektur*
• kite	• *lelayang*
• diagonal	• *pepenjuru*
• rhombus	• *rombus*
• square	• *segi empat sama*
• parallelogram	• *segi empat selari*
• rectangle	• *segi empat tepat*
• triangle	• *segi tiga*
• side	• *sisi*
• quadrilateral	• *sisi empat*
• obtuse angle	• *sudut cakah*
• interior angle	• *sudut pedalaman*
• exterior angle	• *sudut peluaran*
• right angle	• *sudut tegak*
• acute angle	• *sudut tirus*
• trapezium	• *trapezium*

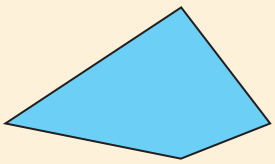
Open the folder downloaded from page vii for the audio of Word Link.

9.1 Polygons



Flashback

A polygon is an enclosed figure on a plane bounded by 3 or more straight sides.



What is the relationship between the number of sides, vertices and diagonals of a polygon?

LEARNING STANDARDS

State the relationship between the number of sides, vertices and diagonals of polygons.

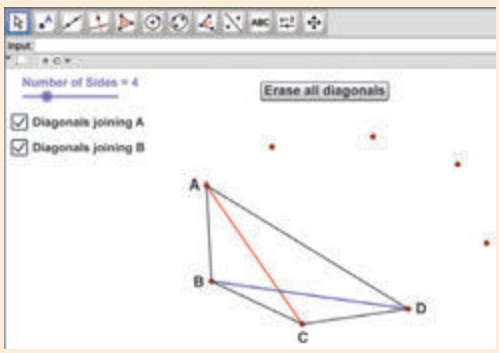
Exploration Activity 1

Flip Classroom

Aim: To explore the number of sides, vertices and diagonals of a polygon.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Diagonals of polygon.ggb* using *GeoGebra*.
2. Click and drag the slider 'Number of Sides' to change the number of sides of the polygon displayed.
3. Click at the checkboxes to display the diagonals of polygon.
4. Record the number of sides, the number of vertices and the number of diagonals displayed.



5. Open the spreadsheet file *Table of diagonals of polygon.xls* and type all the values obtained in Step **4** in the cells provided.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Polygon with vertices	Number of sides	Number of vertices	Number of diagonals joining								Total number of diagonals
2				A (red)	B (blue)	C (green)	D (purple)	E (orange)	F (grey)			
3												
4	A, B, C	3										0
5	A, B, C, D	4										
6	A, B, C, D, E	5										
7	A, B, C, D, E, F	6										
8	A, B, C, D, E, F, G	7										
9	A, B, C, D, E, F, G, H	8										
10		9										
11		10										

6. Based on the table in the spreadsheet, explain the relationship between the number of sides and the number of vertices of a polygon.
7. Discuss with your friends the relationship between the number of sides and the number of diagonals of a polygon.
8. Complete the table in the spreadsheet for a polygon of 9 sides and of 10 sides.

From the results of Exploration Activity 1, it is found that for a polygon
- number of vertices = number of sides
- number of diagonals can be determined according to the following steps.

Identify the number of sides of the polygon. → Minus 3 from the number of sides. Let the result be m. → The value m is multiplied by 2. Thus we get $2m$. → The value $2m$ is added to all the integers with values from less than m to 1.

Example 1

Find the number of vertices and the number of diagonals of a polygon with 10 sides.

Solution

Number of vertices = Number of sides
= 10

$10 - 3 = 7$

Thus, the number of diagonals $= 2(7) + 6 + 5 + 4 + 3 + 2 + 1$
$= 35$

SMART TIPS

The number of diagonals of a polygon having n sides can also be calculated by using the following formula.

Number of diagonals $= \frac{n(n-3)}{2}$

https://youtu.be/3Zkh0_aCebU

Self Practice 9.1a

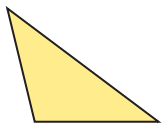
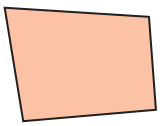
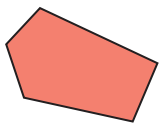
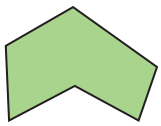
1. Find the number of vertices and the number of diagonals of a polygon with
(a) 6 sides, (b) 9 sides, (c) 12 sides, (d) 20 sides.

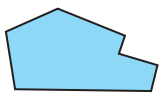
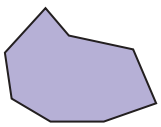
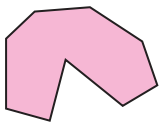
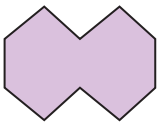
How do you draw, label and name a polygon?

LEARNING STANDARDS

Draw polygons, label vertices of polygons and name the polygons based on the labelled vertices.

A polygon is named according to the number of its sides.

Name of polygon	 Triangle	 Quadrilateral	 Pentagon	 Hexagon
Number of sides	3	4	5	6

Name of polygon	 Heptagon	 Octagon	 Nonagon	 Decagon
Number of sides	7	8	9	10

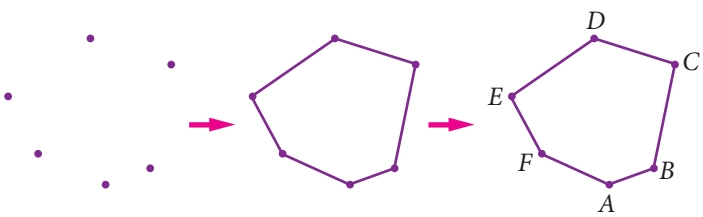
A polygon can be drawn according to the following steps.

Identify the number of sides of the polygon. → Mark points equal in number to the number of sides. → Join all the points with straight lines to form a closed figure. → Label the vertices and name the polygon.

Example 2

Draw a polygon with six sides. Then label and name the polygon.

Solution

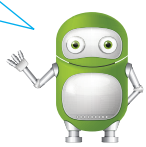


Thus, the polygon is a hexagon *ABCDEF*.

SMART TIPS

The vertices of a polygon are usually labelled in alphabetical order and the polygon is named either clockwise or anticlockwise of the vertices.

Make sure that when drawing a polygon, do not mark two or more points that can form a straight line.



Self Practice 9.1b

1. Draw each of the following polygons according to the given number of sides, then label and name the polygon.
   (a) 5 sides (b) 8 sides (c) 10 sides

Mastery Q 9.1

Open the folder downloaded from page vii for extra questions of Mastery Q 9.1.

1. For each of the following, state whether the statement is TRUE or FALSE.
   (a) A polygon with 11 sides has 11 vertices.
   (b) A polygon with 12 sides has 54 diagonals.

2. Draw a polygon with 8 sides. Then label and name the polygon. Finally, based on the results from Exploration Activity 1, draw all the diagonals systematically.

9.2 Properties of Triangles and the Interior and Exterior Angles of Triangles

What are the properties of a triangle?

LEARNING STANDARDS

Recognise and list geometric properties of various types of triangles. Hence classify triangles based on geometric properties.

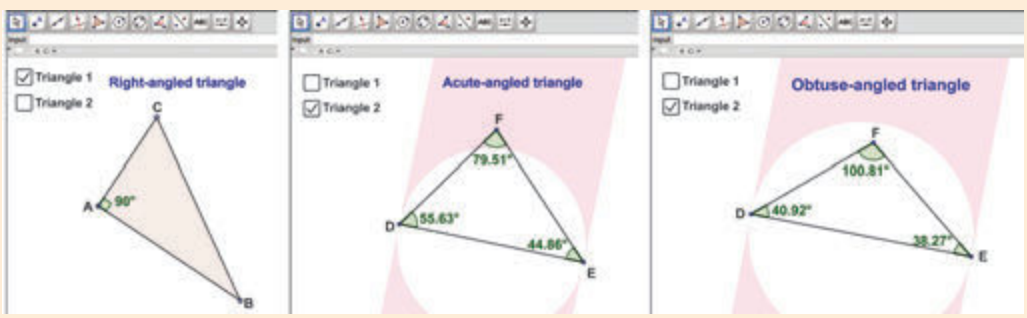
Exploration Activity 2

Flip Classroom

Aim: To explore the geometric properties of a triangle.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from vii.

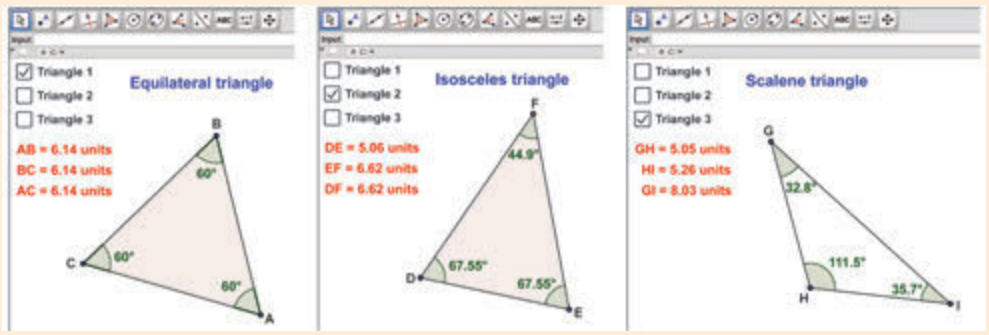
1. Open the file *Triangles properties of angles.ggb* using *GeoGebra*.



2. Click at Triangle 1. Click and drag points A, B and C to change the triangle displayed. State the properties of the right-angled triangle.

3. Click at Triangle 2. Click and drag points D, E and F to change the triangle displayed. Observe the change of types of triangle when point F lies in the pink region or in the white region. State the properties of the acute-angled triangle and the obtuse-angled triangle.

4. Open the file *Triangles properties of sides.ggb* using *GeoGebra*.



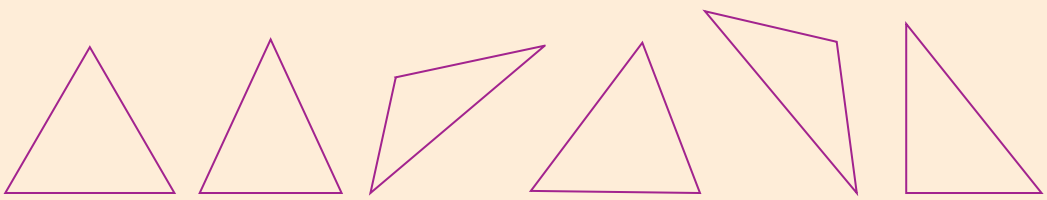
5. Click at Triangle 1. Click and drag points A and B to change the triangle displayed. Observe the change in the length of the sides and the interior angles of the triangle.

6. Repeat the exploration in Step 5 for Triangle 2 and Triangle 3.

7. State the properties of an equilateral triangle, an isosceles triangle and a scalene triangle.

8. Discuss with your friends the properties of the various types of triangles.

9. Open and print the file *Triangles axes of symmetry.pdf*. Cut out the triangles in the printout.

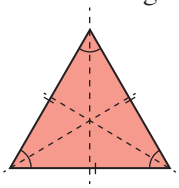
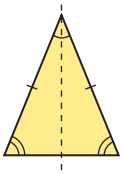
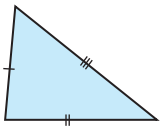


10. By folding each of the triangles, or otherwise, explain how you can find the number of axes of symmetry for each type of triangle.

From the results of Exploration Activity 2, it is found that triangles can be classified based on the geometric properties of their interior angles or the length of their sides.

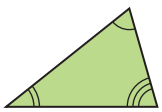
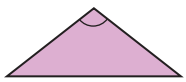
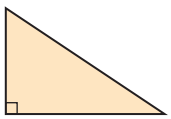
Triangles in Table (a) are classified according to the geometric properties of their sides.

Table (a)

Type of triangle	Equilateral triangle 	Isosceles triangle 	Scalene triangle 
Number of axes of symmetry	3	1	None
Geometric properties	• All the sides are of the same length. • Every interior angle is 60°.	• Two of the sides have the same length. • The two base angles are of the same size.	• All the sides are of different lengths. • All the interior angles are of different sizes.

Triangles in Table (b) are classified according to the geometric properties of their angles.

Table (b)

Type of triangle	Acute-angled triangle 	Obtuse-angled triangle 	Right-angled triangle 
Geometric properties	• Every interior angle is an acute angle.	• One of the interior angles is an obtuse angle.	• One of the interior angles is a right angle (90°).

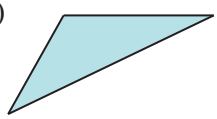
Let's Discuss

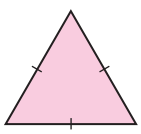
Discuss and explain whether each of the following statements is TRUE or FALSE.

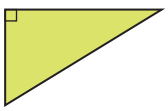
(a) An equilateral triangle is also an isosceles triangle.
(b) An isosceles triangle could be an obtuse-angled triangle.
(c) A right-angled triangle could be an isosceles triangle.
(d) A right-angled triangle may have axes of symmetry.

Self Practice 9.2a

1. State the type of triangles for each of the following.

(a) 

(b) 

(c) 

(d) 

How do you determine the interior angles and exterior angles of a triangle?

LEARNING STANDARDS

Make and verify conjectures about
(i) the sum of interior angles,
(ii) the sum of interior angle and adjacent exterior angle,
(iii) the relation between exterior angle and the sum of the opposite interior angles
of a triangle.

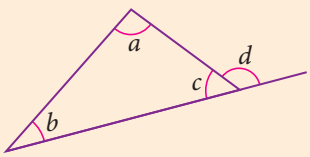
Exploration Activity 3

Flip Classroom

Aim: **To explore the interior angles and exterior angles of a triangle.**

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

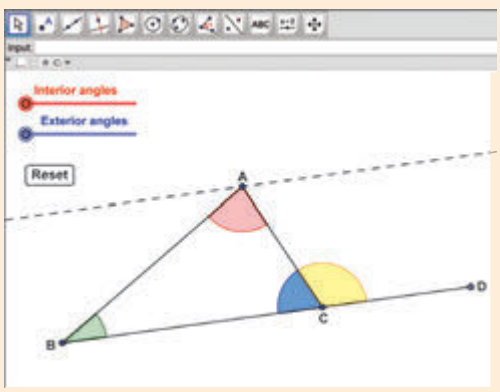
1. Copy and complete the following table to make conjectures about the sum of angles shown in the diagram. Then continue the exploration to verify the conjectures.



Sum of angles	Conjecture
$a + b + c$	
$c + d$	
$a + b$	

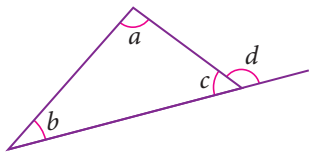
(*Notes: To make a conjecture means to make a guess based on observations.*)

2. Open the file *Interior and exterior angles of triangle.ggb* using *GeoGebra*.
3. Click and drag the slider 'Interior angles' towards the right. Explain what you observe.
4. Click at 'Reset' or click and drag the slider back to its original position.
5. Click and drag points *A*, *B* or *C* to change the shape of the triangle and repeat Step 3.
6. Discuss with your friends your findings.



7. Repeat Steps 3 to 6 for slider 'Exterior angles'.
8. Consider the angle on a straight line, explain the relationship between the interior angle of a triangle (the angle in blue colour) and its adjacent exterior angle (the angle in yellow colour). Explain all the conclusions arrived at.

From the results of Exploration Activity 3, it is found that for a triangle,
(i) the sum of all the interior angles is 180°.
(ii) the sum of an interior angle and its adjacent exterior angle is 180°.
(iii) an exterior angle is the sum of two opposite interior angles.



$$a + b + c = 180°$$
$$c + d = 180°$$
$$d = a + b$$

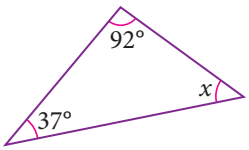
Flashback

The angle on a straight line is 180°.

Example 3

Find the value of x in each of the following diagrams.

(a)



(b)

Solution

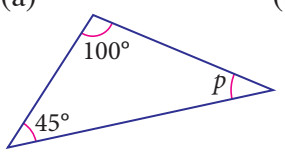
(a) $92° + 37° + x = 180°$ ← The sum of all the interior angles is 180°.
$129° + x = 180°$
$x = 180° - 129°$
$= 51°$

(b) $x = 79° + 28°$ ← The exterior angle is the sum of two opposite interior angles.
$= 107°$

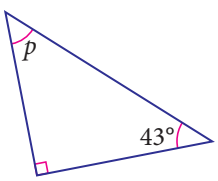
Self Practice 9.2b

1. Find the value of p in each of the following triangles.

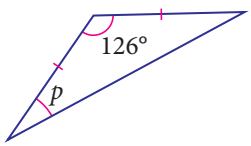
(a)



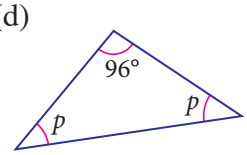
(b)



(c)

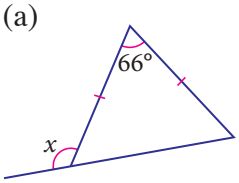


(d)

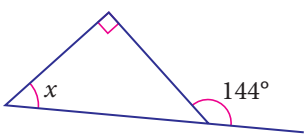


2. Find the value of x in each of the following diagrams.

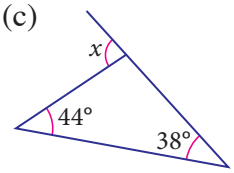
(a)



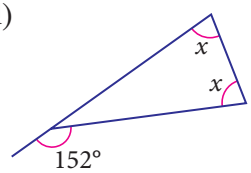
(b)



(c)



(d)



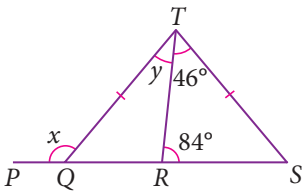
How do you solve problems?

LEARNING STANDARDS

Solve problems involving triangles.

Example 4

In the diagram, $PQRS$ is a straight line. Calculate the values of x and y.



Solution

$\angle RST + 84° + 46° = 180°$ ← Sum of the interior angles of ΔRST.

$\angle RST = 180° - 84° - 46°$

$= 50°$

$\angle TQS = 50°$ ← Angles at the base of an isosceles triangle.

$x + 50° = 180°$ ← Sum of the interior angle and its adjacent exterior angle.

$x = 180° - 50°$

$= 130°$

$\angle QTS + 50° + 50° = 180°$ ← Sum of the interior angles of ΔQST.

$\angle QTS = 180° - 50° - 50°$

$= 80°$

$y + 46° = 80°$

$y = 80° - 46°$

$= 34°$

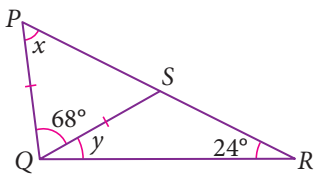
Did You Know

The symbol Δ is used to denote a triangle.

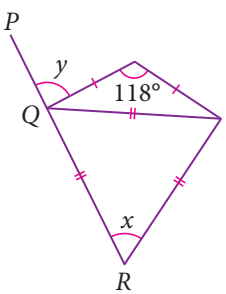
CHAPTER 9

Self Practice 9.2c

1. In the diagram, PSR is a straight line. Calculate the values of x and y.



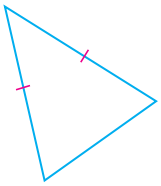
2. In the diagram, PQR is a straight line. Calculate the values of x and y.

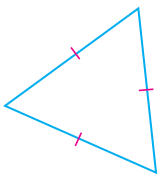


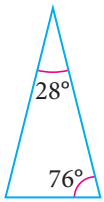
Mastery Q 9.2

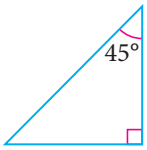
Open the folder downloaded from page vii for extra questions of Mastery Q 9.2.

1. For each of the following triangles, state the number of axes of symmetry.

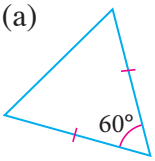
(a) 

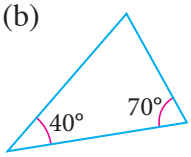
(b) 

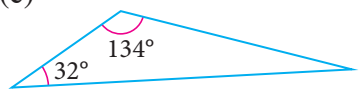
(c) 

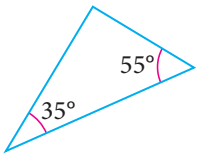
(d) 

2. Identify the type of triangle for each of the following triangles.

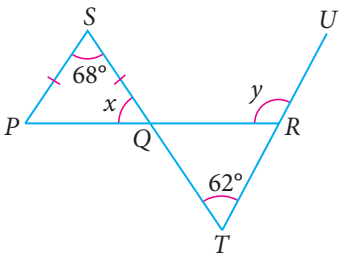
(a) 

(b) 

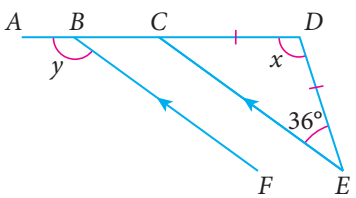
(c) 

(d) 

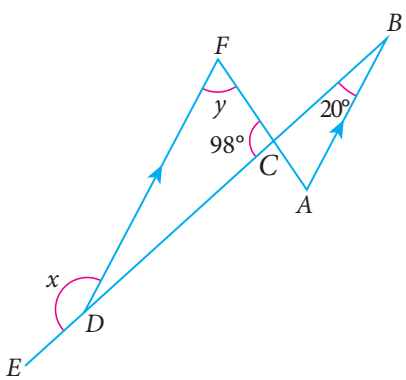
3. In the diagram, *PQR*, *SQT* and *TRU* are straight lines. Calculate the values of x and y.



4. In the diagram, *ABCD* is a straight line. Calculate the values of x and y.



5. In the diagram, *BCDE* and *ACF* are straight lines. Calculate the values of x and y.



9.3 Properties of Quadrilaterals and the Interior and Exterior Angles of Quadrilaterals

What are the properties of a quadrilateral?

LEARNING STANDARDS

Describe the geometric properties of various types of quadrilaterals. Hence classify quadrilaterals based on the geometric properties.

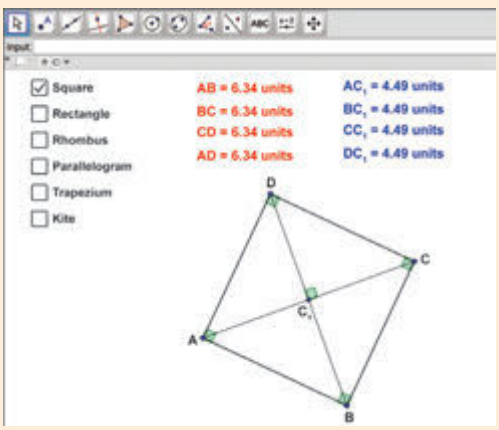
Exploration Activity 4

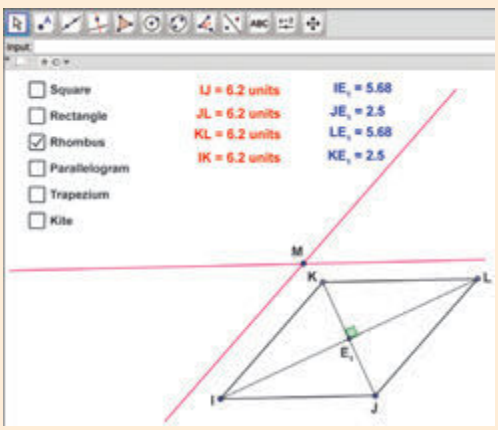
Flip Classroom

Aim: To explore the geometric properties of a quadrilateral.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Quadrilaterals geometric properties.ggb* using *GeoGebra*.

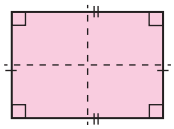
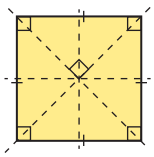
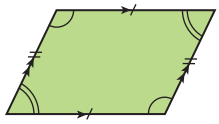
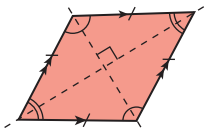
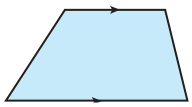
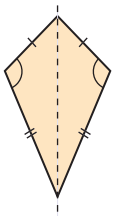




2. Click the checkbox for the first type of quadrilateral. Click and drag the vertices of the quadrilateral to change the dimensions of the quadrilateral. Explain the properties of the quadrilateral.
3. Repeat the exploration in Step 2 for all the other types of quadrilaterals.
4. Discuss with your friends the geometric properties of the various types of quadrilaterals.
5. Open and print the file *Quadrilaterals axes of symmetry.pdf*. Cut out the quadrilaterals in the printout.

6. By folding each of the quadrilaterals, or otherwise, explain how you can find the number of axes of symmetry for each type of quadrilateral.

The following table shows the types of quadrilaterals and their geometric properties.

Type of quadrilateral	Number of axes of symmetry	Geometric properties
Rectangle 	2	• The opposite sides are parallel and of equal length. • All of its interior angles are 90°. • The diagonals are of equal length and are bisectors of each other.
Square 	4	• All the sides are of equal length. • The opposite sides are parallel. • All of its interior angles are 90°. • The diagonals are of equal length and are perpendicular bisectors of each other.
Parallelogram 	None	• The opposite sides are parallel and of equal length. • The opposite angles are equal. • The diagonals are bisectors of each other.
Rhombus 	2	• All the sides are of equal length. • The opposite sides are parallel. • The opposite angles are equal. • The diagonals are perpendicular bisectors of each other.
Trapezium 	None	• Only one pair of opposite sides is parallel.
Kite 	1	• Two pairs of adjacent sides are of equal length. • One pair of opposite angles is equal. • One of the diagonals is the perpendicular bisector of the other. • One of the diagonals is the angle bisector of the angles at the vertices.

Self Practice 9.3a

1. Explain the common properties of a rhombus and a square.
2. Explain the geometric properties of a rectangle compared to a parallelogram.

Let's Discuss

Discuss and explain whether each of the following statements is TRUE or FALSE.

(a) A square is also a rhombus.

(b) A trapezium may have axes of symmetry.

How do you determine the interior angles and the exterior angles of a quadrilateral?

LEARNING STANDARDS

Make and verify the conjectures about
(i) the sum of interior angles of a quadrilateral,
(ii) the sum of interior angle and adjacent exterior angle of a quadrilateral, and
(iii) the relationship between the opposite angles in a parallelogram.

Exploration Activity 5

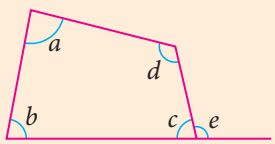
Flip Classroom

Aim: To explore the interior angles and the exterior angles of a quadrilateral.

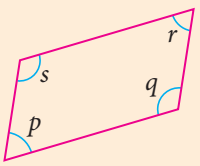
Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Copy and complete the following table to make conjectures about quadrilaterals and parallelograms. Then continue the exploration to verify the conjectures.

(a)



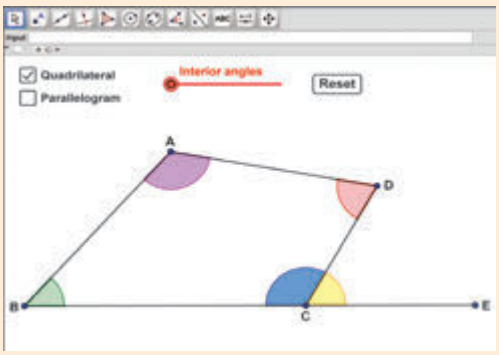
(b)

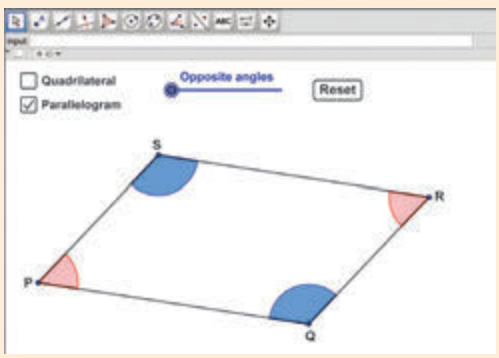


Sum of angles	Conjecture
$a + b + c + d$	
$c + e$	

Angles	Conjecture
p and r	
q and s	

2. Open the file *Interior angles of quadrilateral.ggb* using *GeoGebra*.

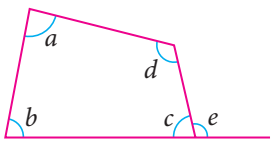




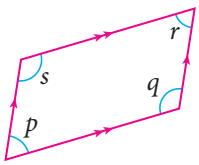
3. Then, select 'Quadrilateral' to display a quadrilateral with its interior and exterior angles.
4. Click and drag the slider 'Interior angles' towards the right. Hence, explain what you observe.
5. Click at 'Reset' or drag the slider back to its original position.
6. Click and drag points A, B, C or D to change the shape of the quadrilateral and repeat Step 4.
7. Discuss with your friends your findings.

8. Consider the angle on a straight line, explain the relationship between the interior angle of a quadrilateral (the angle in blue colour) and its adjacent exterior angle (the angle in yellow colour).
9. Select 'Parallelogram' to display the interior angles of a parallelogram.
10. Click and drag the slider 'Opposite angles' towards the right. Explain what you observe.
11. Click at 'Reset' or drag the slider back to its original position.
12. Click and drag points P, Q or S to change the shape of the parallelogram and repeat Step 10.
13. Explain all the conclusions arrived at.

From the results of Exploration Activity 5, it is found that
(a) the sum of the interior angles of a quadrilateral is 360°.
(b) the sum of an interior angle of a quadrilateral and its adjacent exterior angle is 180°.
(c) the opposite angles in a parallelogram (or rhombus) are equal.



$a + b + c + d = 360°$
$c + e = 180°$



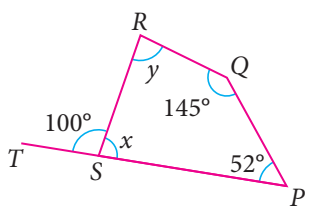
$p = r$
$q = s$

Scan the QR Code or visit https://youtu.be/p7lPCvwE0vY to watch a video about the sum of the interior angles of a quadrilateral.

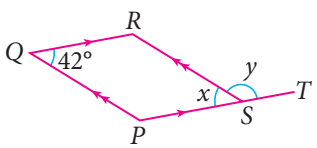
Example 5

In each of the following diagrams, PST is a straight line. Calculate the values of x and y.

(a)



(b)



Solution

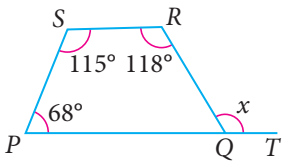
(a) $x + 100° = 180°$ ← Sum of the interior angle and its adjacent exterior angle is 180°.
$x = 180° - 100°$
$= 80°$

$y + 80° + 52° + 145° = 360°$ ← Sum of the interior angles of a quadrilateral is 360°.
$y + 277° = 360°$
$y = 360° - 277°$
$= 83°$

(b) $x = 42°$ ← Opposite angles in a parallelogram are equal.
$y + 42° = 180°$
$y = 180° - 42°$
$= 138°$

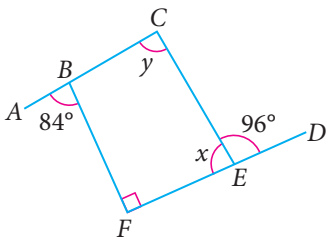
Self Practice 9.3b

1.



In the diagram, PQT is a straight line. Calculate the value of x.

2.



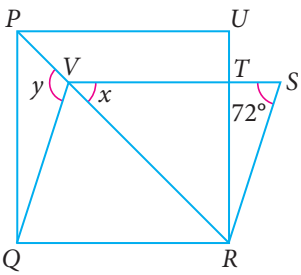
In the diagram, ABC and DEF are straight lines. Calculate the values of x and y.

How do you solve problems involving quadrilaterals?

LEARNING STANDARDS

Solve problems involving quadrilaterals.

Example 6



In the diagram, $PQRU$ is a square and $QRSV$ is a parallelogram. PVR is a straight line. Find the values of x and y.

Career in Mathematics

An architect uses the knowledge of polygons to design a building.

Solution

$\angle PQR = 90°$ dan $PQ = QR$

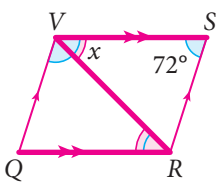
Thus, $\angle QRV = \frac{90°}{2}$

$x = 45°$ ← $x = \angle QRV$

$\angle QVS + 72° = 180°$

$\angle QVS = 180° - 72°$

$= 108°$



Therefore, $\angle QVR + x = 108°$

$\angle QVR = 108° - 45°$

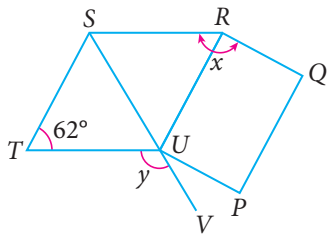
$= 63°$

Hence, $y = 180° - 63°$

$= 117°$

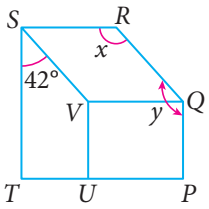
Self Practice 9.3c

1.



In the diagram, $PQRU$ is a rectangle and $RSTU$ is a rhombus. SUV is a straight line. Calculate the values of x and y.

2.



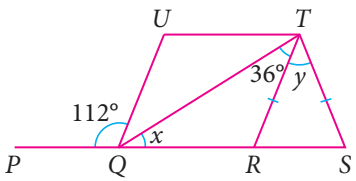
In the diagram, $PQVU$ is a rectangle, $QRSV$ is a parallelogram and $STUV$ is a trapezium. Calculate the values of x and y.

How do you solve problems involving the combinations of triangles and quadrilaterals?

LEARNING STANDARDS

Solve problems involving the combinations of triangles and quadrilaterals.

Example 7



In the diagram, $QRTU$ is a parallelogram and $PQRS$ is a straight line. Find the values of x and y.

Solution

$\angle QRT = 112°$ ← $\angle QRT$ and 112° are corresponding angles.

$x + 112° + 36° = 180°$ ← Sum of the interior angles of ΔQRT.

$x = 180° - 112° - 36°$

$= 32°$

$\angle TRS + 112° = 180°$ ← Sum of the interior angle and its adjacent exterior angle.

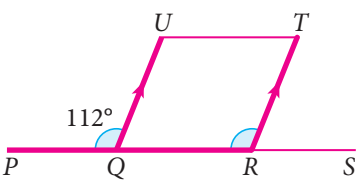
$\angle TRS = 180° - 112°$

$= 68°$

$y + 68° + 68° = 180°$ ← Sum of the interior angles of ΔRST.

$y = 180° - 68° - 68°$

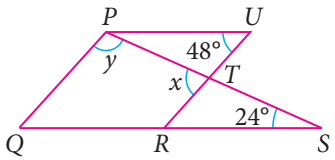
$= 44°$



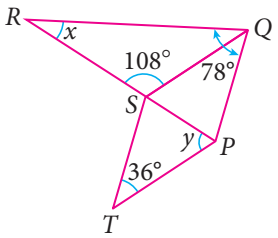
CHAPTER 9

Self Practice 9.3d

1. In the diagram, *PQRU* is a parallelogram. *PTS*, *QRS* and *RTU* are straight lines. Find the values of x and y.



2. In the diagram, *PQST* is a parallelogram. *PSR* is a straight line. Find the values of x and y.



Mastery Q 9.3

Open the folder downloaded from page vii for extra questions of Mastery Q 9.3.

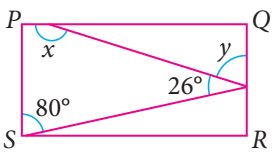
1. State the similarities and differences between the geometric properties of a parallelogram and a rhombus.

2. Using suitable mathematical notation, copy and denote all the geometric properties of the following quadrilaterals.

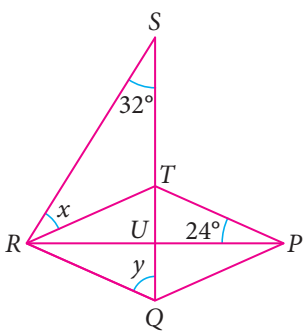
(a)

(b)

3. In the diagram, *PQRS* is a rectangle. Find the values of x and y.

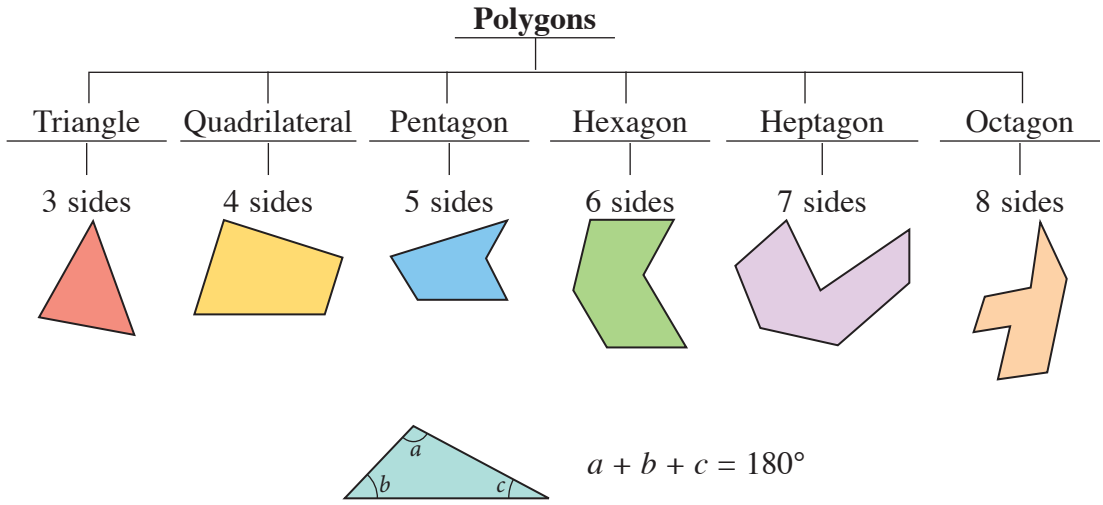


4. In the diagram, *PQRT* is a rhombus. *STUQ* and *PUR* are straight lines. Find the values of x and y.



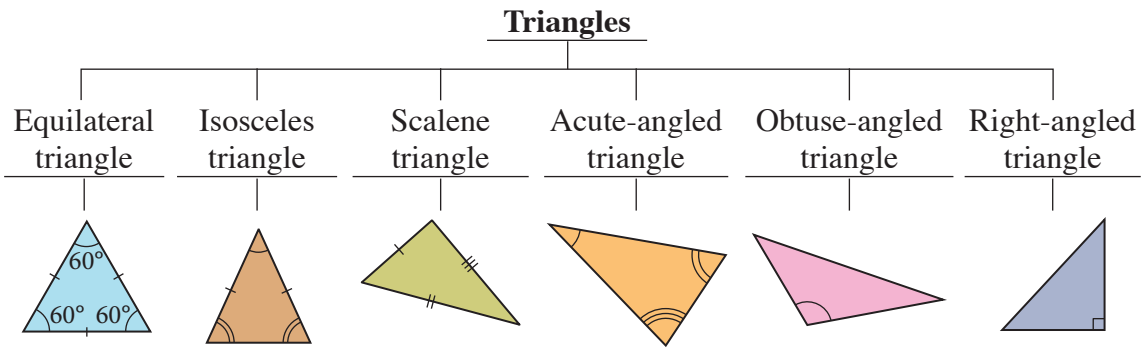
SUMMARY

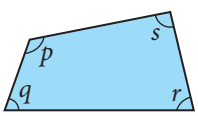
Polygons



$a + b + c = 180°$

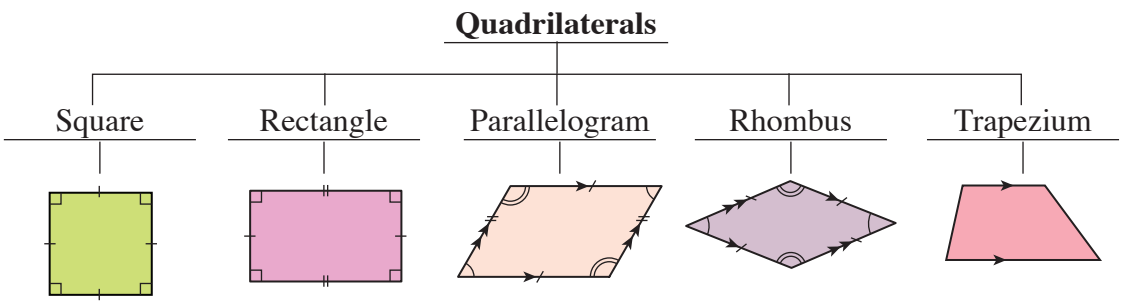
Triangles





$p + q + r + s = 360°$

Quadrilaterals



At the end of this chapter, I can...	Very good	Work harder
state the relationship between the number of sides, vertices and diagonals of polygons.		
draw polygons, label vertices of polygons and name the polygons based on the labelled vertices.		
recognise and list geometric properties of various types of triangles. Hence, classify triangles based on geometric properties.		
make and verify conjectures about (i) the sum of interior angles, (ii) the sum of interior angle and adjacent exterior angle, (iii) the relation between exterior angle and the sum of the opposite interior angles. of a triangle.		
solve problems involving triangles.		
describe the geometric properties of various types of quadrilaterals. Hence classify quadrilaterals based on geometric properties.		
make and verify the conjectures about (i) the sum of interior angles of a quadrilateral, (ii) the sum of interior angle and adjacent exterior angle of a quadrilateral, and (iii) the relationship between the opposite angles in a parallelogram.		
solve problems involving quadrilaterals.		
solve problems involving the combinations of triangles and quadrilaterals.		

Test Yourself

1. Mark ✓ for a TRUE statement and ✗ for a FALSE statement.

(a) A right-angled triangle has an axis of symmetry if one of the interior angles is 45°. ☐

(b) If the axis of symmetry of an equilateral triangle PQR passes through vertex P, then the axis of symmetry is the angle bisector of the angle at P. ☐

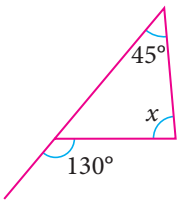
(c) A diagonal of a rectangle is the perpendicular bisector of the other diagonal. ☐

(d) A square and a rhombus are quadrilaterals and their diagonals intersect at right angles. ☐

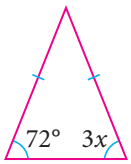
2. Determine the types of
(a) quadrilaterals which have two axes of symmetry.
(b) triangles which do not have an axis of symmetry.
(c) quadrilaterals with all the sides having the same length.
(d) quadrilaterals with all the interior angles being 90°.

3. Find the value of x in each of the following diagrams.

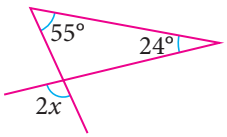
(a)



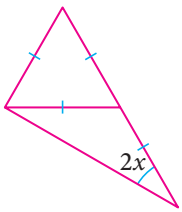
(b)



(c)

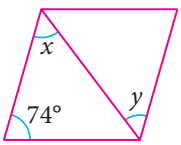


(d)

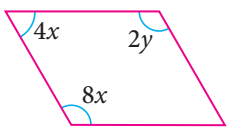


Self Mastery

4. The diagram shows a rhombus. Find the values of x and y.



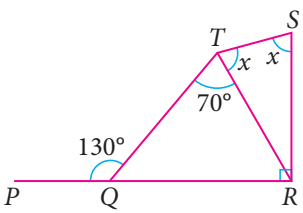
5. A parallelogram is shown in the diagram. Find the values of x and y.



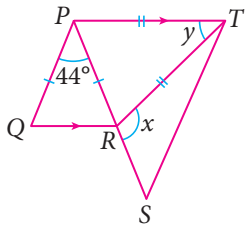
6. Four angles of a quadrilateral are in the ratio of 3 : 4 : 5 : 6. Explain how you would calculate the biggest angle of the quadrilateral.

Challenge Yourself

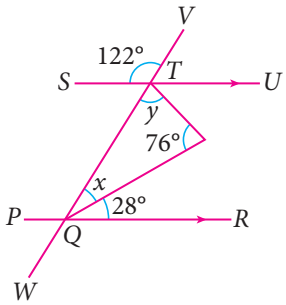
7. In the diagram, PQR is a straight line. Find the value of x.



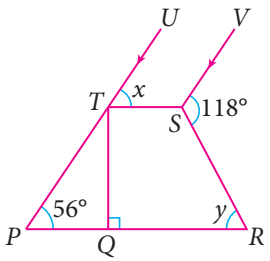
8. *PRS* is a straight line as shown in the diagram. Find the values of x and y.



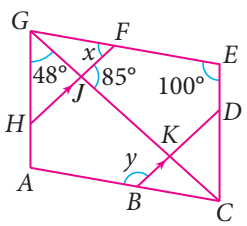
9. *PQR*, *STU* and *VTQW* are straight lines. Find the values of x and y.



10. *PRST* is a trapezium. *PQR* and *PTU* are straight lines. Find the values of x and y.



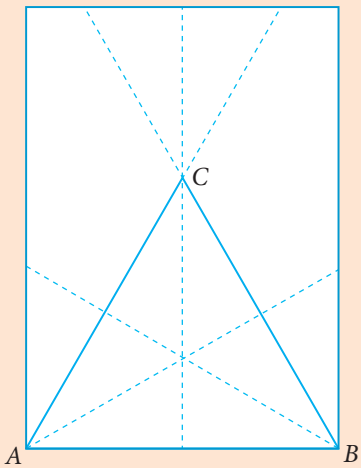
11. *ACEG* is a parallelogram. *BKD*, *HJF* and *CKJG* are straight lines. Find the values of x and y.



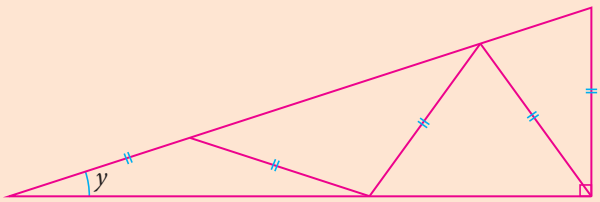
ASSIGNMENT

You have been asked to do a study regarding the use of polygons in your school buildings. Write a report to explain how the use of polygons have enhanced the visual beauty of your school.

A The diagram shows a piece of A4-size paper. Without using any geometrical tools but only by folding the paper, explain the rationale behind the folding to obtain an equilateral triangle *ABC*. (*Hint: The folding lines have been drawn in the diagram as a guide.*)

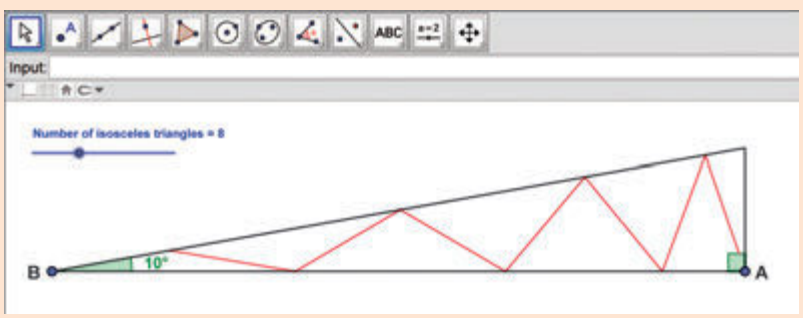


B



In the diagram, four isosceles triangles are inscribed in a right-angled triangle. Explain how you would find the value of *y*.

Next, open the file *Isosceles triangles inscribed.ggb* from the folder downloaded from page vii using *GeoGebra*.



Click and drag the slider 'Number of isosceles triangles', point *A* and point *B* to change the information displayed. With the help of the information, explain the relationship between the value of *y* and the number of isosceles triangles that can be drawn.

CHAPTER 10

Perimeter and Area

What will you learn?

- Perimeter
- Area of Triangles, Parallelograms, Kites and Trapeziums
- Relationship between Perimeter and Area

Why study this chapter?

Perimeter and area are essentially basic knowledge of geometry and its related fields, hence they are widely used in agriculture, landscape and graphic designs. Discuss other fields that use perimeter and area.

Kedah is well known as *Jelapang Padi Malaysia* because almost the entire western part of the state is covered with paddy fields. Besides being the main rice producer in Malaysia, the scenery of the vast paddy fields is also one of the main attractions in Kedah.

Do you know the total area of all the paddy fields in Kedah? What is the perimeter of each paddy field?

Walking through Time

Rhind Papyrus

The word 'perimeter' is derived from the Greek word 'peri' which means 'around' and 'meter' which means 'measure'. Notes regarding the concept of area were found in a mathematical document named Rhind Papyrus around 1650 B.C. in Egypt. In this document, a mistake in the formula for finding the area of quadrilaterals was noted. However, the formula is still valid for finding the areas of certain geometric shapes.

For more information:

https://goo.gl/9QTPb0

Word Link

• conjecture	• *konjektur*
• width	• *lebar*
• kite	• *lelayang*
• area	• *luas*
• length	• *panjang*
• perimeter	• *perimeter*
• formula	• *rumus*
• square	• *segi empat sama*
• parallelogram	• *segi empat selari*
• rectangle	• *segi empat tepat*
• triangle	• *segi tiga*
• height	• *tinggi*
• trapezium	• *trapezium*
• square unit	• *unit persegi*

Open the folder downloaded from page vii for the audio of Word Link.

10.1 Perimeter

How do you determine the perimeter?

Perimeter is the total length around an enclosed area. For example, we can determine the perimeter of various shapes of swimming pools by finding the total length of all the sides of the swimming pool.

LEARNING STANDARDS

Determine the perimeter of various shapes when the side lengths are given or need to be measured.

Photo (a)

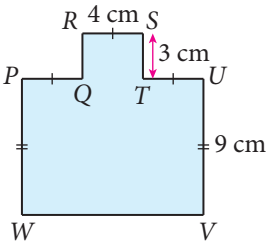
Photo (b)

How do you measure the length of a curved swimming pool in Photo (b)?

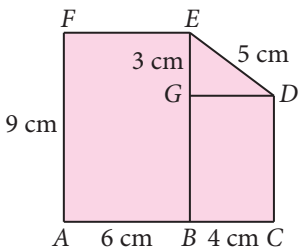
Example 1

Determine the perimeter of each of the following shapes.

(a)



(b)



Solution

(a) Perimeter $= PQ + QR + RS + ST + TU + UV + VW + WP$
$= 4 + 3 + 4 + 3 + 4 + 9 + 12 + 9$
$= 48$ cm

$VW = PQ + RS + TU$
$= 4 + 4 + 4$
$= 12$ cm

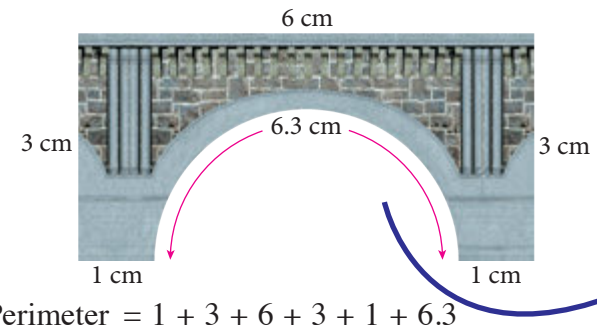
(b) Perimeter $= AB + BC + CD + DE + EF + FA$
$= 6 + 4 + 6 + 5 + 6 + 9$
$= 36$ cm

$CD = AF - GE$
$= 9 - 3$
$= 6$ cm

Example 2

Measure the perimeter of the object in the diagram below.

Solution



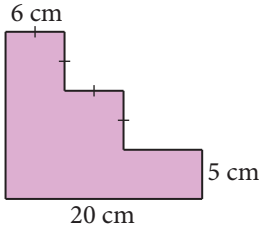
Measure the length of the curve by using a thread.

Perimeter $= 1 + 3 + 6 + 3 + 1 + 6.3$
$= 20.3$ cm

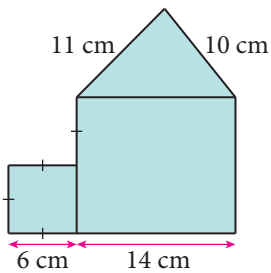
Self Practice 10.1a

1. Determine the perimeter of each of the following shapes.

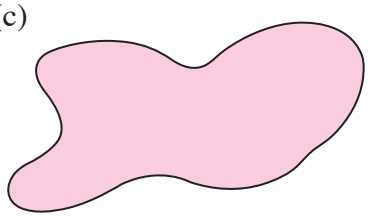
(a)



(b)



(c)



How do you estimate the perimeter accurately?

We can use a square grid paper or a graph paper to estimate the perimeter of a shape.

LEARNING STANDARDS

Estimate the perimeter of various shapes, and then evaluate the accuracy of estimation by comparing with the measured value.

Exploration Activity 1

Pair

Aim: To estimate the perimeter of various shapes.
Instruction: Perform the activity in pairs.

1. Trace the shape of the object on a grid of equal squares with sides of 1 cm.
2. Estimate the perimeter of each shape based on the grid.
3. Measure the perimeter of each shape using a ruler or a thread.
4. Record your estimation and measurement.
5. Compare the estimated value with the measured value to evaluate the accuracy of your estimation.

The accuracy of estimated perimeter can be evaluated by comparing the estimated value with the measured value. The smaller the difference between the values of estimated perimeter and measured perimeter, the more precise the estimated value.

Let's Discuss

Besides using grid paper or graph paper, discuss other methods which can be used to estimate the perimeter of a shape.

Did You Know

Percentage error

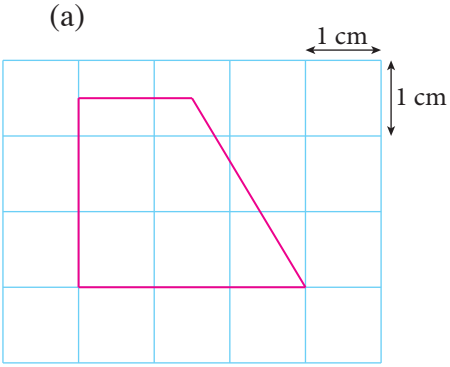
$$= \frac{\text{Difference between estimated value and actual value}}{\text{Actual value}} \times 100\%$$

The smaller the percentage error, the more precise the estimated value.

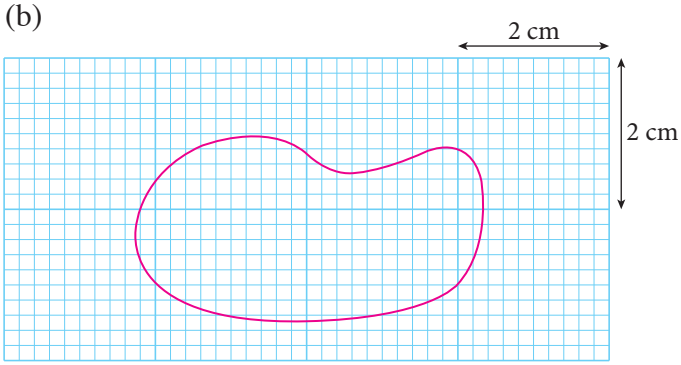
Self Practice 10.1b

1. Estimate the perimeter of each of the following shapes. Hence, measure the perimeter of the shape by using a ruler or a thread to evaluate the accuracy of estimation.

(a)



(b)



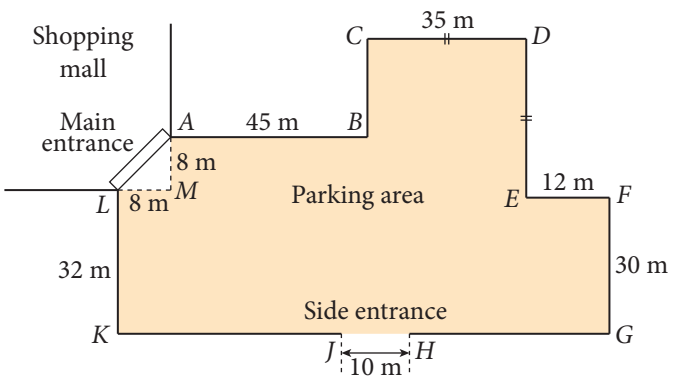
How do you solve problems?

MATHEMATICS APPLICATION ZONE

LEARNING STANDARDS

Solve problems involving perimeter.

The diagram shows the parking area in front of a shopping mall. The shopping mall management intends to fence up the entire parking area, except the main entrance and another side entrance. If the cost of fencing is RM80 per metre, what will be the cost incurred by the shopping mall?



Solution

Total length of the parking area to be fenced up
= (*AB* + *BC* + *CD* + *DE* + *EF* + *FG* + *GK* + *KL*) – *HJ*
= (45 + 25 + 35 + 35 + 12 + 30 + 100 + 32) – 10
= 314 – 10
= 304 m

Total cost to fence up the parking area
= 304 × RM80 ← Total length of fence × RM80
= RM24 320

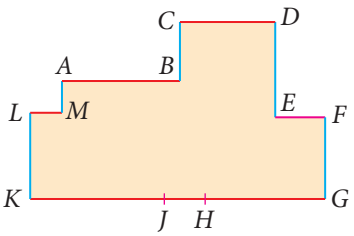
BC = *DE* + *FG* – (*AM* + *LK*)
= (35 + 30) – (8 + 32)
= 25 m

GK = *LM* + *AB* + *CD* + *EF*
= 8 + 45 + 35 + 12
= 100 m

Alternative Method

Perimeter of the enclosed parking area
= 2 × [(*LM* + *AB* + *CD* + *EF*) + (*DE* + *FG*)]
= 2 × [(8 + 45 + 35 + 12) + (35 + 30)]
= 2 × (100 + 65)
= 2 × 165
= 330 m

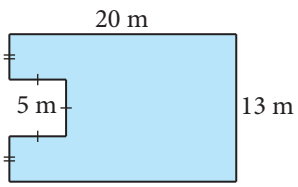
Total length of the parking area to be fenced up
= 330 – *HJ* – *LM* – *MA*
= 330 – 10 – 8 – 8
= 304 m



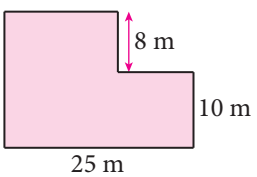
KG = *LM* + *AB* + *CD* + *EF*
CB + *AM* + *LK* = *DE* + *FG*

Self Practice 10.1c

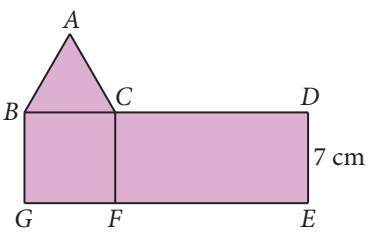
1. The diagram shows the plan of a swimming pool. What is the perimeter of the swimming pool?



2. The diagram shows the plan of Encik Yahya's house. Encik Yahya wants to install colorful LED lights around his house as decorations to celebrate *Hari Raya*. Calculate the installation cost if the cost to install LED lights is RM20 per metre.



3. In the diagram, *ABC* is an equilateral triangle, *BCFG* is a square and *CDEF* is a rectangle. The perimeter of the whole diagram is 65 cm, find the length of *GE*.

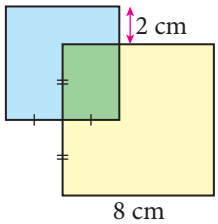


Mastery Q 10.1

Open the folder downloaded from page vii for extra questions of Mastery Q 10.1.

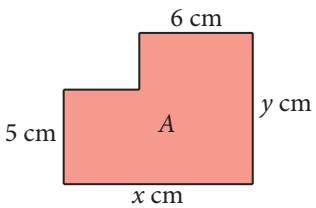
1. The perimeter of a rectangular laboratory is 64 m. If the length of the laboratory is 23 m, find the width of the laboratory.

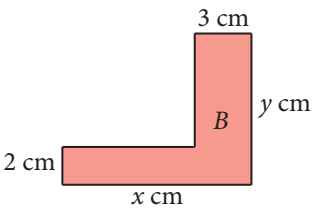
2.



The diagram shows two squares overlapping each other. Calculate the perimeter of the whole diagram.

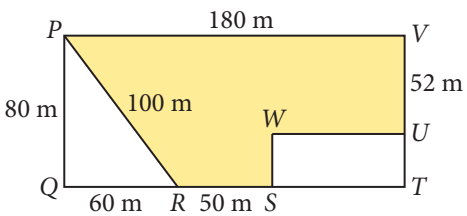
3.





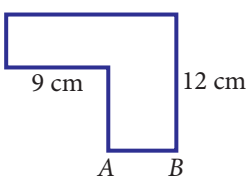
The diagram shows two shapes, *A* and *B*. Prove that both of the shapes have equal perimeter.

4.



The diagram shows a piece of rectangular land *PQTV* owned by Mr Rhuben. Parts of the land in the shapes of triangle *PQR* and rectangle *STUW* have been given to his brother. Mr Rhuben plans to fence up his land. What is the cost of fencing if each metre of fence costs RM50?

5. A wire with a length of 54 cm is bent to form a shape as shown in the diagram. What is the length of side *AB*?



10.2 Area of Triangles, Parallelograms, Kites and Trapeziums

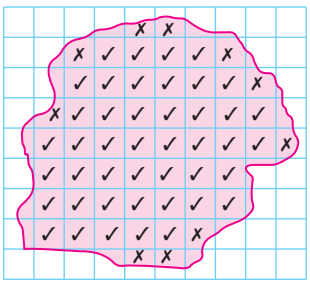
How do you estimate the area of various shapes?

LEARNING STANDARDS

Estimate the area of various shapes using various methods.

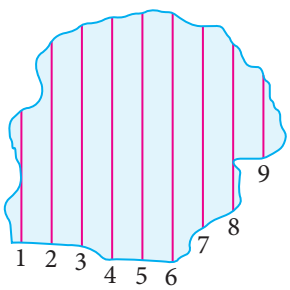
Various methods can be used to estimate the area of an irregular shape.

(i) Using a grid with equal squares of sides 1 unit



Number of full 1 unit^2 squares (✓) = 44
Number of half full or more than half full squares (✗) = 10
Hence, the area of the shape is about 44 + 10 = 54 unit^2.

(ii) Drawing lines 1 unit apart



The lines are 1 unit apart from each other. Hence, we can assume one line as a rectangle with a width of 1 unit.

Line	1	2	3	4	5	6	7	8	9
Length (unit)	4.4	6.6	7.2	7.8	8.0	8.0	6.4	5.4	2.6
Area (unit^2) = Length × 1 unit	4.4	6.6	7.2	7.8	8.0	8.0	6.4	5.4	2.6

Total area = 4.4 + 6.6 + 7.2 + 7.8 + 8.0 + 8.0 + 6.4 + 5.4 + 2.6
= 56.4 unit^2

Hence, the area of the shape is approximately 56.4 unit^2.

Self Practice 10.2a

1. Collect some leaves from your school compound. Sketch the shape of each leaf on a piece of paper. Then estimate the area of each leaf in cm^2, using suitable methods.

How do you derive the formulae for the area of various shapes?

LEARNING STANDARDS

Derive the formulae of the area of triangles, parallelograms, kites and trapeziums based on the area of rectangles.

Exploration Activity 2

Group

Aim: To derive the formula for the area of triangles.
Instruction: Perform the activity in groups of four.

1. Referring to Diagram (a) as shown,

(a) is the area of ΔABD the same as the area of ΔBCD?

(b) state the area of rectangle $ABCD$ in terms of a and b.

(c) hence, state the area of ΔABD in terms of a and b.

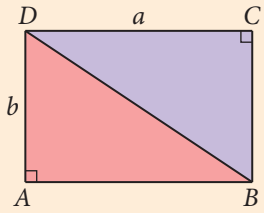


Diagram (a)

2. Referring to Diagram (b) as shown,

(a) state the area of ΔBFE in terms of b and c based on rectangle $BFEC$.

(b) state the area of ΔAFE in terms of b and d based on rectangle $AFED$.

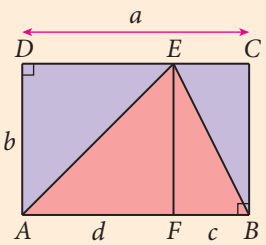


Diagram (b)

(c) the area of ΔABE = ☐ + ☐

$= \frac{1}{2}b($ ☐ $)$

$=$ ☐

Distributive law

3. Referring to Diagram (c) as shown,

(a) what is the length of DE in terms of a and c?

(b) state the area of ΔAFE in terms of a, b and c based on rectangle $AFED$.

(c) state the area of ΔBFE in terms of b and c based on rectangle $BFEC$.

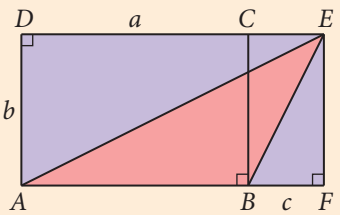


Diagram (c)

(d) hence the area of ΔABE = area of ΔAFE − area of ΔBFE

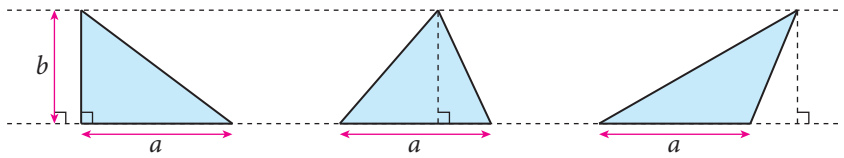
$= \frac{1}{2}b($ ☐ $)$ − ☐

$= \frac{1}{2}ba$ + ☐ − ☐

Distributive law

4. Discuss with your friends your findings.
   (a) Compare the results of the area obtained in Steps **1**, **2** and **3**. What can you conclude?
   (b) What is the general formula to calculate the area of a triangle?
5. State all the conclusions that can be made.

From the results of Exploration Activity 2, it is found that



the area of a triangle with the length of base a and height b is calculated by

$$\text{area of triangle} = \frac{1}{2}ab$$

In general, **the area of a triangle** $= \frac{1}{2}$ **× length of base × height**

Exploration Activity 3

Group

Aim: To derive the formula for the area of parallelograms.
Instruction: Perform the activity in groups of four.

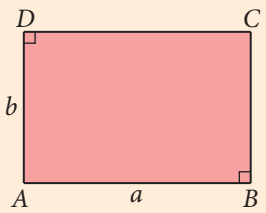


Diagram (a)

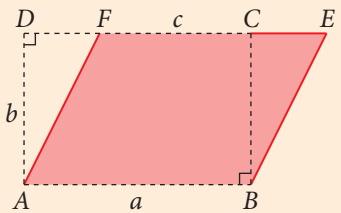
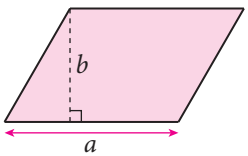


Diagram (b)

1. Diagram (a) shows a rectangle $ABCD$ with length of base a and height b. Referring to Diagram (a), state the area of $ABCD$ in terms of a and b.
2. Diagram (b) shows a parallelogram $ABEF$ with the length of base a and height b. Referring to Diagram (b),
   (a) state the length of CE in terms of a and c based on the properties of parallelogram.
   (b) state the length of DF in terms of a and c.
   (c) explain your findings.

3. (a) What can you conclude about the area of triangle BCE and triangle ADF?
   (b) Is the area of $ABCD$ the same as the area of $ABEF$?
   (c) Hence, state the area of the parallelogram $ABEF$ in terms of a and b.
4. Discuss with your friends and state the conclusions that can be made.

From the results of Exploration Activity 3, it is found that the area of a parallelogram with the length of base a and height b is calculated by



$$\text{area of parallelogram} = ab$$

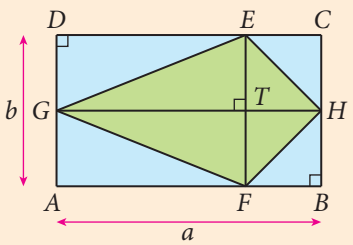
In general, **the area of a parallelogram = length of base × height**

Exploration Activity 4

Group

Aim: To derive the formula for the area of kites.
Instruction: Perform the activity in groups of four.

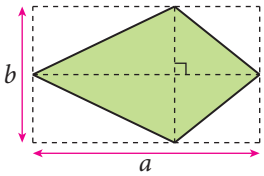
1. The diagram shows a kite inscribed in a rectangle with measurements a and b. Referring to the diagram, discuss with your friends and complete the following table.



Area of ΔEHT	$= \frac{1}{2} \times$ area of rectangle $ECHT$
Area of ΔETG	$= \frac{1}{2} \times$ area of rectangle ☐
Area of ΔEHG	$=$ ☐ × area of rectangle ☐
Area of ΔFHG	$=$ ☐ × ☐
Area of kite $EHFG$	$=$ ☐ × ☐

2. Discuss with your friends and state the conclusions that can be made.

From the results of Exploration Activity 4, it is found that the area of a kite with diagonals of lengths a and b is calculated by



$$\text{area of kite} = \frac{1}{2}ab$$

In general,

the area of a kite = $\frac{1}{2}$ × product of the lengths of the two diagonals

Let's Discuss

The formula for the area of a parallelogram and the formula for the area of a kite can be used to find the area of a rhombus. Discuss.

Exploration Activity 5

Group

Aim: To derive the formula for the area of trapeziums.
Instruction: Perform the activity in groups of four.

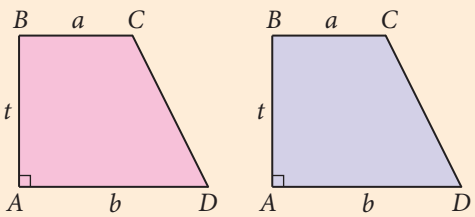


Diagram (a)

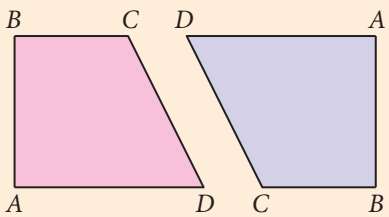
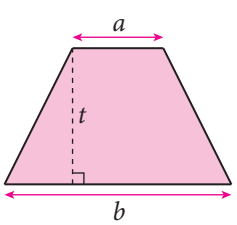


Diagram (b)

1. Draw two identical trapeziums on a piece of manila card and cut out both the trapeziums. Then, record the measurements of the trapeziums as shown in Diagram (a).
2. Rotate one of the trapeziums to a suitable position so that it can be joined to the other trapezium, as shown in Diagram (b).
3. (a) What is the shape obtained when two trapeziums are joined together?
   (b) What is the length of the base of the combined shape?
   (c) State the area of the combined shape in terms of a, b and t.
   (d) Hence, state the area of one trapezium.
4. Discuss with your friends and state the conclusions that can be made.

From the results of Exploration Activity 5, it is found that the area of a trapezium with lengths of two parallel sides a and b respectively and height t is calculated by



$$\text{area of trapezium} = \frac{1}{2}(a + b)t$$

In general,

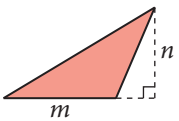
the area of a trapezium = $\frac{1}{2}$ × (sum of the lengths of the two parallel sides) × height

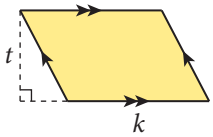
Let's Discuss

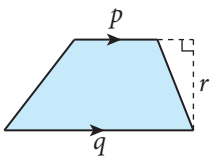
Open the file *Area of trapezium.ggb* from the folder downloaded from page vii using *GeoGebra*. Click and drag the slider 'Dissect', the points *A*, *B*, *C* and *D* to do an exploration. Discuss with your friends and explain how you would derive the formula for the area of a trapezium based on your exploration of the *GeoGebra* file. Present your findings during the lesson.

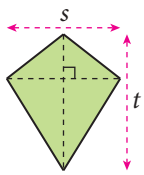
Self Practice 10.2b

1. Write an expression to represent the area of each of the following shapes.

(a) 

(b) 

(c) 

(d) 

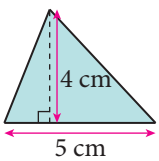
How do you solve problems involving various shapes?

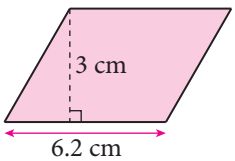
LEARNING STANDARDS

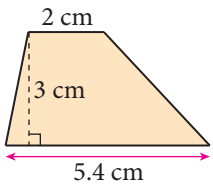
Solve problems involving areas of triangles, parallelograms, kites, trapeziums and the combinations of these shapes.

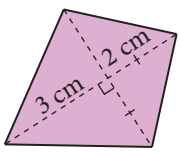
Example 3

Calculate the area of each of the following shapes.

(a) 

(b) 

(c) 

(d) 

Solution

(a) Area $= \frac{1}{2} \times$ length of base $\times$ height
$= \frac{1}{2} \times 5 \times 4$
$= 10 \text{ cm}^2$

(b) Area = length of base × height
= 6.2 × 3
= 18.6 cm²

(c) Area $= \frac{1}{2} \times$ sum of lengths of two parallel sides $\times$ height
$= \frac{1}{2} \times (2 + 5.4) \times 3$
$= \frac{1}{2} \times 7.4 \times 3$
$= 11.1 \text{ cm}^2$

Career in Mathematics

A housing developer uses the knowledge of perimeter and area to plan a housing project.

(d) Area $= \frac{1}{2} \times$ product of the lengths of the two diagonals

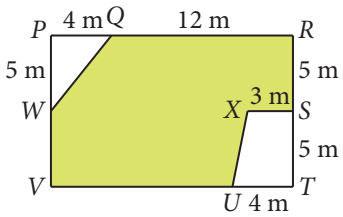
$= \frac{1}{2} \times (3 + 2) \times (2 + 2)$ ← $\frac{1}{2} \times$ length of long diagonal $\times$ length of short diagonal

$= \frac{1}{2} \times 5 \times 4$

$= 10 \text{ cm}^2$

MATHEMATICS APPLICATION ZONE

The diagram shows a rectangular plot of land *PRTV*. The triangular region *PQW* and the trapezium-shaped region *UTSX* are used for planting banana trees and rambutan trees respectively. The remaining shaded region is used to plant papaya trees. Calculate the area of the region used to plant papaya trees.



Solution

Area of *PRTV*
= Length × width
= (12 + 4) × (5 + 5)
= 16 × 10
= 160 m²

Area of *PQW*
$= \frac{1}{2} \times$ length of base × height
$= \frac{1}{2} \times 4 \times 5$
$= 10 \text{ m}^2$

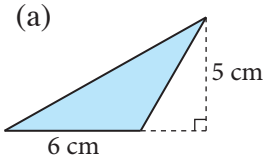
Area of *UTSX*
$= \frac{1}{2} \times$ (sum of the lengths of the two parallel sides) × height
$= \frac{1}{2} \times (3 + 4) \times 5$
$= \frac{1}{2} \times 7 \times 5$
$= 17.5 \text{ m}^2$

Thus, the area of the shaded region = 160 − 10 − 17.5 ← Area of *PRTV* − Area of *PQW* − Area of *UTSX*
= 132.5 m²

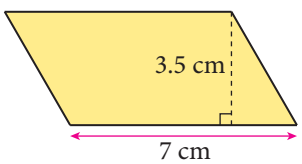
Self Practice 10.2c

1. Calculate the area of each of the following shapes.

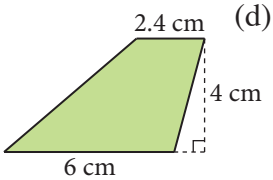
(a)



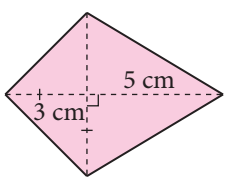
(b)



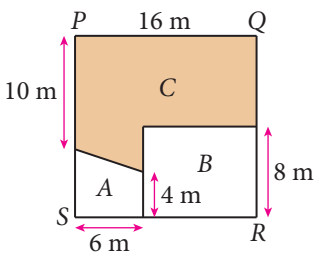
(c)



(d)



2. A square-shaped exhibition site *PQRS* is divided into three regions *A*, *B* and *C* with *A* being trapezium-shaped and *B* being rectangular. Find the area of the shaded region *C*.

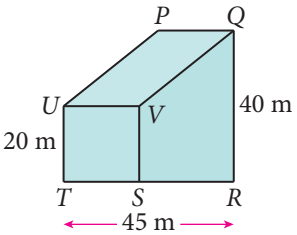


Mastery Q 10.2

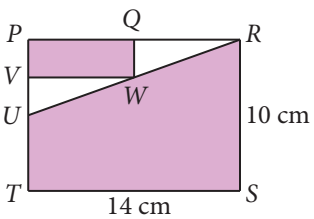
Open the folder downloaded from page vii for extra questions of Mastery Q 10.2.

1. By using a suitable computer program or a pencil, draw an irregular shape. Then, estimate the area of the shape in cm^2, using a suitable method.

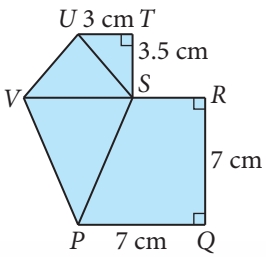
2. The diagram shows a playground which is a combination of three different shapes. *TSVU* is a square, *SRQV* is a trapezium and *PQVU* is a parallelogram. Calculate the total area of the playground.



3. In the diagram, *PRST* and *PQWV* are rectangles. *Q*, *V* and *U* are midpoints of *PR*, *PU* and *PT* respectively. Find the area of the shaded region.



4. In the diagram, *VSR* is a straight line and *PSUV* is a kite. Calculate the area of the whole diagram.



CHAPTER 10

10.3 Relationship between Perimeter and Area

What is the relationship between perimeter and area?

LEARNING STANDARDS

Make and verify the conjecture about the relationship between perimeter and area.

Kassim has a plot of empty land. He wants to fence up a region of the land to plant some vegetables. He has a 20 m length of fence. How should he construct the fencing such that he can have a region with the largest area?

State your conjectures for each of the following.

- How does the area of a rectangle change when its perimeter is fixed?
- How does the perimeter of a rectangle change when its area is fixed?

Then, perform the following activity to verify your conjectures.

Exploration Activity 6

Aim: To explore the relationship between the perimeter and the area of a rectangle.
Instruction: Explore by yourself before the lesson begins and discuss in groups of four during the lesson.

A **1.** Copy and complete Table (a) by stating the values for the length and width of a rectangle which has a perimeter of 36 cm.

Table (a)

Length (cm)	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
Width (cm)	1.0	2.0															
Area (cm^2)	17.0	32.0															
Difference between length and width (cm)	16.0	14.0															

2. Construct a similar table for a rectangle having each of the following fixed values of perimeter, beginning with a length equivalent to $\left(\frac{\text{Perimeter}}{2} - 1\right)$ cm until 1 cm.

(a) 40 cm (b) 48 cm (c) 56 cm

Open the file Rectangle fixed perimeter.ggb or the file Perimeter and area.xls from the folder downloaded from page vii for help.

3. Study the number patterns obtained in Table (a).

(a) How does the area of a rectangle change in relation to a fixed perimeter?

(b) When will the area of the rectangle be the biggest?

B **4.** Copy and complete Table (b) by stating the values for the length and width of a rectangle which has an area of 49 cm^2.

Table (b)

Length (cm)	14	13	12	11	10	9	8	7	6	5	4	3	2	1
Width (cm)	3.500	3.769												
Perimeter (cm)	35.000	33.538												
Difference between length and width (cm)	10.500	9.231												

5. Construct a similar table for a rectangle having each of the following fixed values of area, beginning with a length equivalent to an approximate integer of $2 \times \sqrt{\text{Area}}$ until 1 cm.

(a) 81 cm^2 (b) 144 cm^2 (c) 225 cm^2

Open the file Rectangle fixed area.ggb or the file Perimeter and area.xls from the folder downloaded from page vii for help.

6. Study the number patterns obtained in Table (b).
   (a) How does the perimeter of a rectangle change in relation to a fixed area?
   (b) When will the perimeter of the rectangle be the smallest?
7. Discuss with your friends your findings and state all the conclusions that can be made.

From the results of Exploration Activity 6, it is found that for rectangles with;

(a) the same perimeter,
- the area will decrease if the difference between the length and the width increase.
- the area will be the largest when the rectangle is a square.

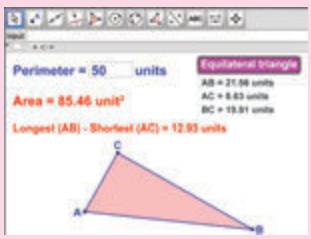
(b) the same area,
- the perimeter will increase if the difference between the length and the width decrease.
- the perimeter will be the smallest when the rectangle is a square.

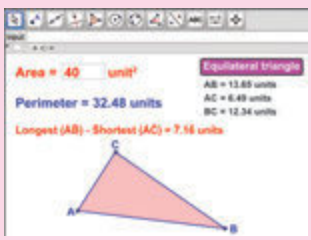
Let's Discuss

Open the file *Triangle fixed perimeter.ggb* and *Triangle fixed area.ggb* from the folder downloaded from page vii using *GeoGebra*. Explore the relationship between the perimeter and the area of a triangle like what was done in Exploration Activity 6 for rectangles. Discuss with your friends and explain your findings.

(a) How does the area of a triangle change when its perimeter is fixed?
(b) How does the perimeter of a triangle change when its area is fixed?
(c) Does a triangle show the same pattern of change as a rectangle?

Present your findings in class during the lesson.

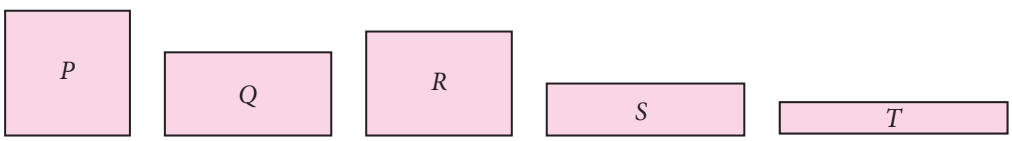




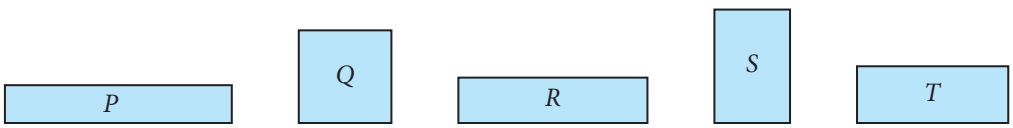
CHAPTER 10

Self Practice 10.3a

1. The rectangles P, Q, R, S and T as shown below have the same perimeter. Arrange the areas of the rectangles in ascending order. Explain your answer.



2. The rectangles P, Q, R, S and T as shown below have the same area. Arrange the perimeters of the rectangles in descending order. Explain your answer.



How do you solve problems?

LEARNING STANDARDS

Solve problems involving perimeter and area of triangles, rectangles, squares, parallelograms, kites, trapeziums and the combinations of these shapes.

Example 4

Given the area of a square garden is 500 m^2, find the perimeter of the garden.

Solution

Let the length of the side of the garden = x m

Then, $x^2 = 500$ ← Area of the square is 500 m^2.

Thus, $x = \sqrt{500}$

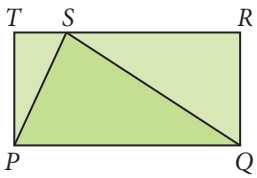
$= 22.36$ ← 2 decimal places

Hence, the perimeter of the garden $= 22.36 \times 4$

$= 89.44$ m

Example 5

The diagram shows a triangle PQS inscribed in a rectangle $PQRT$. The perimeter of the rectangle is 42 cm and the length of the rectangle is twice its width. Find the area of triangle PQS.



Solution

Let the length of the rectangle = y cm and the width of the rectangle = x cm

Perimeter = 42 cm

Thus, $2x + 2y = 42$ ..... ①

$y = 2x$ ..... ② ← The length is twice its width.

Substitute ② into ①, $2x + 2(2x) = 42$

$6x = 42$

$x = \frac{42}{6}$

$= 7$

$y = 2(7)$

$= 14$

Hence, the area of triangle $PQS = \frac{1}{2} \times 14 \times 7$

$= 49$ cm^2

Check

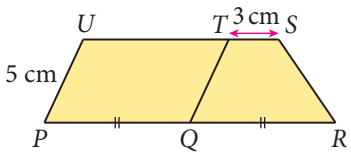
Perimeter of the rectangle $PQRT$
$= 2(7) + 2(14)$
$= 14 + 28$
$= 42$ cm

Self Practice 10.3b

1. Given the perimeter of the square floor of a hall is 82 m, find the area of the floor of the hall.

2. The length of a rectangle is 5 cm more than its width. If the perimeter of the rectangle is 40 cm, find the area of the rectangle.

3. In the diagram, $PQTU$ is a parallelogram with a perimeter of 24 cm and an area of 28 cm^2. Given that UTS and PQR are straight lines, find the area of the whole diagram.



CHAPTER 10

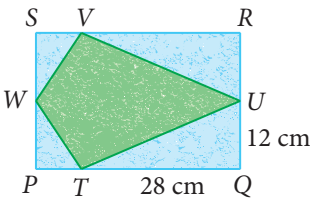
Mastery Q 10.3

Open the folder downloaded from page vii for extra questions of Mastery Q 10.3.

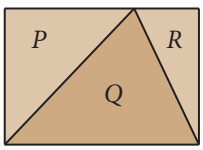
1. The width of a rectangular cardboard is 24 cm and its area is 960 cm^2. Find the perimeter of the cardboard.

2. The perimeter of a rectangle is 48 cm. Explain how you should draw the rectangle so that the area is the largest. What is the largest area?

3. The diagram shows a piece of rectangular tile *PQRS*. A kite shape *TUVW* is inscribed in the rectangle. Given that the perimeter of *PQRS* is 120 cm, find the area of *TUVW*.



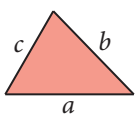
4. A plot of rectangular land is divided into 3 sections, *P*, *Q* and *R* to plant three types of vegetables. The perimeter of the land is 170 m and the length of the land is 15 m more than its width. Find the area of the largest plot for planting the vegetables.



SUMMARY

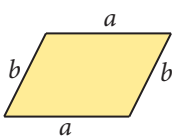
PERIMETER

Triangles



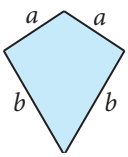
$P = a + b + c$

Parallelograms



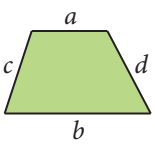
$P = 2a + 2b$

Kites



$P = 2a + 2b$

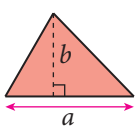
Trapeziums



$P = a + b + c + d$

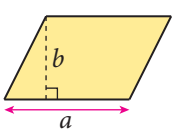
AREA

Triangles



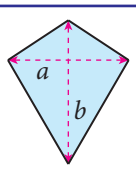
$A = \frac{1}{2}ab$

Parallelograms



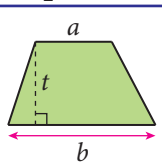
$A = ab$

Kites



$A = \frac{1}{2}ab$

Trapeziums



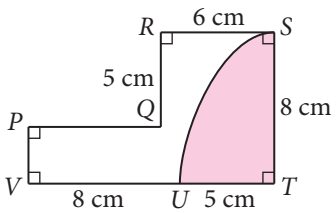
$A = \frac{1}{2}(a + b)t$

At the end of this chapter, I can...	Very good	Work harder
determine the perimeter of various shapes when the side lengths are given or need to be measured.		
estimate the perimeter of various shapes and then evaluate the accuracy of estimation.		
solve problems involving perimeter.		
estimate the area of various shapes using various methods.		
derive the formulae of the area of triangles, parallelograms, kites and trapeziums based on the area of rectangles.		
solve problems involving areas of various shapes and the combinations of these shapes.		
make and verify the conjecture about the relationship between perimeter and area.		
solve problems involving perimeter and area of various shapes and the combinations of these shapes.		

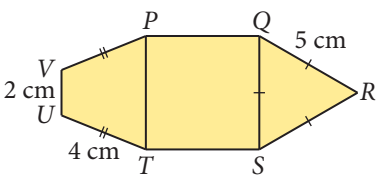
Let's PRACTISE

Test Yourself

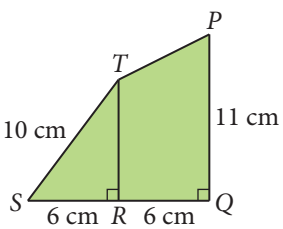
1. In the diagram, the perimeter of the shaded region is 25 cm. Find the perimeter of the region which is not shaded.



2. The diagram shows a square *PQST*, a triangle *QRS* and a trapezium *PTUV*. Find the perimeter of the whole diagram.

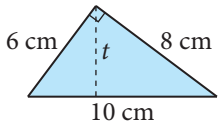


3. The diagram shows a triangle *SRT* and a trapezium *PQRT*. The perimeter of the triangle *SRT* is 24 cm. Find the area of the whole diagram.

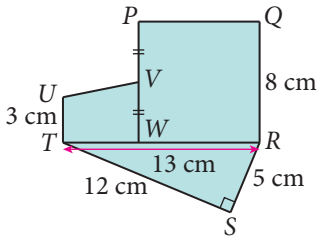


Self Mastery

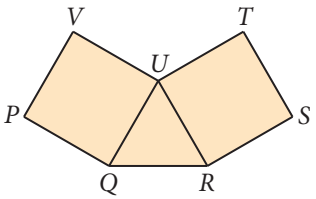
4. Based on the information given in the diagram,
(a) find the area of the triangle.
(b) find the value of t.



5. In the diagram, $PQRW$ is a square, RST is a triangle and $TWVU$ is a trapezium. Find the area of the whole diagram.

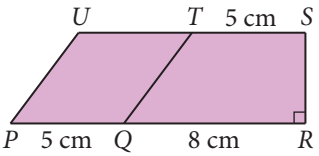


6. In the diagram, QRU is an equilateral triangle, $PQUV$ and $RSTU$ are two squares. Given the area of $PQUV$ is 36 cm^2, find the perimeter of the whole diagram.

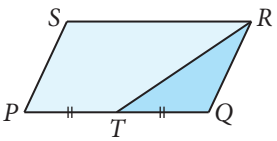


Challenge Yourself

7. The diagram shows a rhombus $PQTU$ and a trapezium $QRST$. PQR and UTS are straight lines. Given that the area of the whole diagram is 46 cm^2,
(a) find the height of the rhombus $PQTU$.
(b) hence, find the perimeter of the whole diagram.

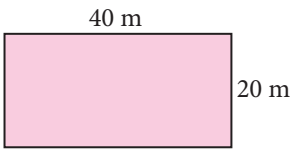


8. In the diagram, $PQRS$ is a parallelogram with an area of 100 cm^2. Find the area of triangle TQR.



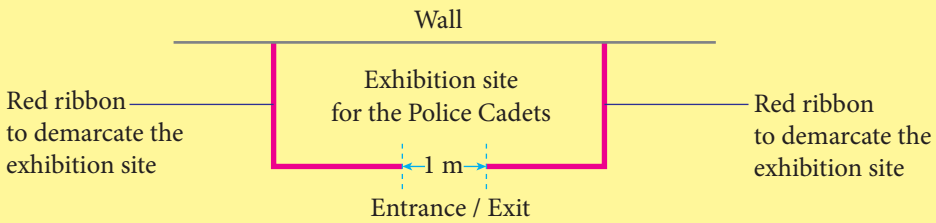
9. You are given a piece of string of length 30 cm. Explain how you would form a rectangle with the biggest area.

10. The diagram shows a rectangular plot of land with a length of 40 m and a width of 20 m. You are required to fence up a region on the land with an area of 450 m^2. Explain how you would fence the land such that you use the shortest fence. What would be the shortest length of the fence required?



ASSIGNMENT

Your school is going to have an exhibition in conjunction with *Hari Wawasan*. The Police Cadets will be taking part in the exhibition. You have been assigned to plan the exhibition site.



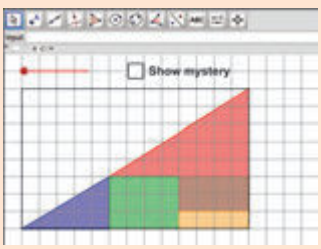
The Police Cadets are given a space beside the wall as their exhibition site. The exhibition site is demarcated with red ribbon except for a 1 m space as the entrance and exit for the exhibition site, as shown in the diagram.

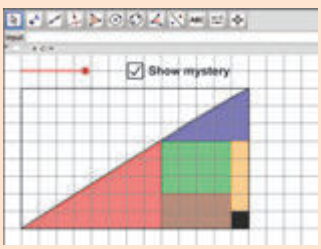
You are supplied with a piece of 20 m long red ribbon. Explain how you would demarcate the exhibition site with the red ribbon so that the exhibition site has the largest area. Write a report and present it to your teacher who is the advisor of the Police Cadets.

Open the file Assignment.ggb from the folder downloaded from page vii using GeoGebra to help you.

Exploring MATHEMATICS

1. Open the file *Mystery area.ggb* from the folder downloaded from page vii using *GeoGebra*.





2. Click and drag the slider to the end, towards the right.

3. Observe the movements of the shapes on display.

4. Explain why there is a difference of 1 unit^2 of area when the shapes are rearranged although all the shapes maintain their respective areas.

5. Click and drag the slider towards left and right a few times or click at 'Show mystery' for help.

6. Present the result of your exploration in class during the lesson.

CHAPTER 11

Introduction of Set

What will you learn?

- Set
- Venn Diagrams, Universal Sets, Complement of a Set and Subsets

Why study this chapter?

Learn the methods to represent a group of objects with specific characteristics and hence help us to understand the relationship between them. Discuss daily life examples that involve the classification of objects according to specific characteristics.

The Ministry of Urban Wellbeing, Housing and Local Government (UHLG) strives to improve the quality of the people's life by having a clean and healthy environment.

In line with this goal, UHLG always encourages us to practise the 3R programme, which means reduce, reuse and recycle. How can the classification of solid waste be carried out effectively?

Walking through Time

Georg Cantor

Georg Cantor (1845 – 1918) was a German mathematician who first introduced set theory. He published his first article about set theory in the year 1874. Set theory became popular and was recognised as a branch of mathematics in the late 19th century.

For more information:

https://goo.gl/S4GWib

Word Link

• Venn diagram	• *gambar rajah Venn*
• complement of a set	• *pelengkap bagi suatu set*
• description	• *perihalan*
• set	• *set*
• empty set	• *set kosong*
• equal sets	• *set sama*
• universal set	• *set semesta*
• subset	• *subset*
• set builder notation	• *tatatanda pembina set*
• set notation	• *tatatanda set*
• element	• *unsur*

Open the folder downloaded from page vii for the audio of Word Link.

11.1 Set

What is set?

In our daily life, we can classify the recyclable materials into several categories, such as paper, plastic, glass, aluminium, fabric and so on. How can these categories of materials be represented mathematically?

LEARNING STANDARDS

Explain the meaning of set.

Exploration Activity 1

Group

Aim: To sort and classify objects.
Instruction: Perform the activity in groups of four.

1. Observe each object given below.








2. Classify the objects according to specific groups.
3. What are the common characteristics of each group?

From the results of Exploration Activity 1, the objects which have common characteristics will be classified into the same group. Each of these groups is known as **set**.

Self Practice 11.1a

1. Classify the objects in the list below as land, water and air transport.

Car	Rocket	Sampan
Lorry	Aeroplane	Boat
Ship	Van	Ferry
Helicopter	Bus	Hot-air balloon

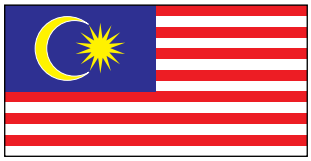
How do you describe sets?

LEARNING STANDARDS

Describe sets using:
(i) description,
(ii) listing, and
(iii) set builder notation.

Sets can be written by using description, listing and set builder notation.

For example, we can write the colours of *Jalur Gemilang* in set A as follows.



Set A		
Description	**Listing**	**Set builder notation**
A is the set of colours of *Jalur Gemilang*.	• Set A can be represented by using set notation, { }. • A = {red, white, blue, yellow} Each element in the set is separated by a comma. • Each object in the set is known as an **element**.	• Set A can also be represented by using set builder notation. • $A = \{x : x$ is a colour of *Jalur Gemilang*$\}$

Example 1

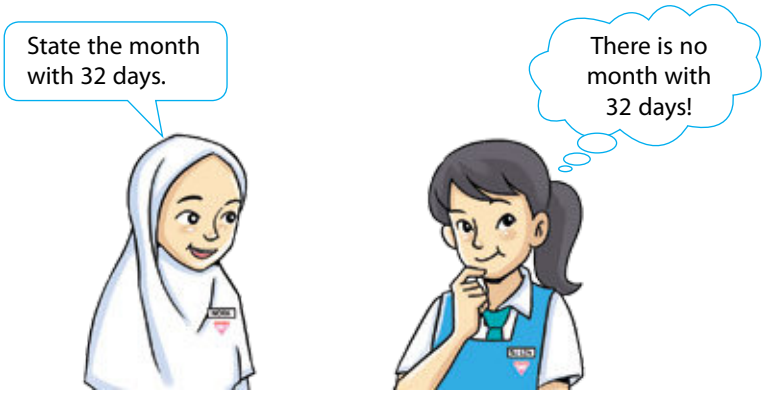
Describe each of the following sets by using description, listing and set builder notation.
(a) Letters in the word 'MALAYSIA'
(b) Odd numbers which are less than 20

Solution

(a) Let the set be represented by P.
Description: P is the set of letters in the word 'MALAYSIA'.
Listing: P = {M, A, L, Y, S, I}
The same elements need not be repeated.
Set builder notation: $P = \{x : x$ is a letter in the word 'MALAYSIA'$\}$

(b) Let the set be represented by Q.
Description: Q is the set of odd numbers which are less than 20.
Listing: Q = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}
Set builder notation: $Q = \{x : x$ is an odd number and $x < 20\}$

Empty set



Think Smart

How many months have 28 days?

If M represents the set of month with 32 days, then set M does not contain any element and is known as an **empty set**. An empty set can be represented with the symbol ϕ or $\{\ \}$.

Thus, $M = \phi$ or $M = \{\ \}$.

Did You Know

- An empty set is also called a null set.
- The symbol ϕ is read as phi.

Self Practice 11.1b

1. Describe each of the following sets by using (i) description, (ii) listing and (iii) set builder notation.

(a) Colours of the rainbow

(b) Multiples of 3 which are less than 25

2. Determine whether each of the following statements is TRUE or FALSE.

(a) If P = {triangles which have diagonals}, then $P = \phi$.

(b) If Q = {prime numbers which are also even numbers}, then $Q = \{\ \}$.

(c) If R = {odd numbers which are divisible by 2}, then $R = \phi$.

(d) If $S = \{x : x \text{ is a negative integer and } x^3 < 0\}$, then $S = \{\ \}$.

What is the element of a set?

LEARNING STANDARDS

Identify whether an object is an element of a set and represent the relation using symbol.

The elements of a set are defined according to certain characteristics. Each of the elements must satisfy the conditions of the set that is defined.

For example, set A = {local fruits in Malaysia}.

Durian is a local fruit, so durian **is an element of** set A. Symbol $\in$ is used to represent 'is an element of' the set. Therefore, durian $\in A$.

Apple is not a local fruit, so apple **is not an element of** set A. Symbol $\notin$ is used to represent 'is not an element of' the set.

Therefore, apple $\notin A$.

Did You Know

Symbol $\in$ is called epsilon.

Example 2

Given that $P = \{x : x \text{ is a prime number and } 0 \leqslant x \leqslant 20\}$, complete each of the following using the symbol $\in$ or $\notin$.

(a) 5 ☐ P (b) 8 ☐ P

(c) 19 ☐ P (d) 1 ☐ P

Solution

$P = \{2, 3, 5, 7, 11, 13, 17, 19\}$ ← List all the elements of P.

(a) 5 $\in$ P (b) 8 $\notin$ P

(c) 19 $\in$ P (d) 1 $\notin$ P

Self Practice 11.1c

1. Given that A = {states in Malaysia}, complete each of the following using the symbol $\in$ or $\notin$.

(a) Johor ☐ A (b) Langkawi ☐ A

(c) Putrajaya ☐ A (d) Selangor ☐ A

2. It is given that $P = \{x : x \text{ is an odd number and } 0 < x < 50\}$ and $Q = \{x : x \text{ is a common multiple of 4 and 6}, x < 100\}$.
Complete each of the following using the symbol $\in$ or $\notin$.

(a) 8 ☐ P (b) 17 ☐ P (c) 63 ☐ P

(d) 84 ☐ Q (e) 60 ☐ Q (f) 46 ☐ Q

How do you determine the number of elements of a set?

LEARNING STANDARDS

Determine the number of elements of a set and represent the number of elements using symbol.

Number of elements in set P can be represented by the notation $n(P)$.

For example, P = {a, b, c, d, e, f}, the number of elements in set P is 6. Thus, $n(P) = 6$.

Example 3

Given that P = {letters in the word 'WAWASAN'} and $Q = \{x : x$ is a two-digit number such that the sum of its digits is 7$\}$, find

(a) $n(P)$ (b) $n(Q)$

Solution

(a) P = {W, A, S, N} ← The number of elements is 4.
Thus, $n(P) = 4$

(b) Q = {16, 25, 34, 43, 52, 61, 70} ← The number of elements is 7.
Thus, $n(Q) = 7$

Let's Discuss

Can you determine the number of elements in set $A = \{x : x$ is a fraction and $1 < x < 2\}$? Discuss.

SMART TIPS

List all the elements in a set so that the number of elements in the set can be determined.

Self Practice 11.1d

1. Determine the number of elements in each of the following sets.
(a) A = {cone, cylinder, sphere, pyramid, cube}
(b) B = {consonants in the word 'KEMAHIRAN'}
(c) C = {colours of the traffic light}
(d) D = {multiples of 11 which are less than 100}

2. P = {integers between –5 and 2} and $Q = \{x : x < 15$ and x is an even number$\}$.
Find (a) $n(P)$ (b) $n(Q)$

What is equality of sets?

LEARNING STANDARDS

Compare and explain whether two or more sets are equal and hence, make generalisation about the equality of sets.

The sets of alphabets in a word are arranged by three students as follows.

Set A = {H, A, R, U, M}, set B = {M, U, R, A, H} and set C = {R, U, M, A, H}

Every element in sets A, B and C are the same. So, set A, set B and set C are known as equal sets and can be written as $A = B = C$.

Thus $A = B$, $A = C$ or $B = C$.

Generally, if **every element in two or more sets are the same,** then **all the sets are equal**.

SMART TIPS

The order of elements in a set is not important.

Example 4

Explain whether each of the following pairs of sets are equal sets.

(a) P = {vowels in the word 'SEKOLAH'} and
Q = {vowels in the word 'KEJOHANAN'}

(b) F = {perfect squares which are less than 50} and
G = {4, 9, 16, 25, 36, 49}

Let's Discuss

Discuss:
- If $A = B$, is $n(A) = n(B)$?
- If $n(A) = n(B)$, is $A = B$?

Solution

(a) $P = \{E, O, A\}$ and $Q = \{E, O, A\}$
Each element in set P is equal to each element in set Q.
Thus, $P = Q$

Elements 'A' need not be repeated.

Think Smart

Given $M = \phi$ and $N = \{x : x \text{ is an integer and } 1 < x < 2\}$, is $M = N$?

(b) $F = \{1, 4, 9, 16, 25, 36, 49\}$
$G = \{4, 9, 16, 25, 36, 49\}$
$1 \in F$ but $1 \notin G$.
Thus, $F \neq G$

SMART TIPS

Symbol $\neq$ means 'is not equal to'.

Self Practice 11.1e

1. Determine whether each of the following pairs of sets are equal sets.

(a) G = {letters in the word 'RAJIN'} and H = {letters in the word 'JIRAN'}

(b) $M = \{1, 2, 3, 4, 5\}$ and N = {whole numbers which are less than 6}

(c) $P = \{x : x \text{ is a multiple of 2 and } 25 \leqslant x \leqslant 45\}$ and $Q = \{x : x \text{ is a number that has the digit 2 and } 25 \leqslant x \leqslant 45\}$

(d) A = {national language of Malaysia} and B = {Malay language}

Mastery Q 11.1

Open the folder downloaded from page vii for extra questions of Mastery Q 11.1.

1. Describe each of the following sets by using description.

(a) $X = \{a, e, i, o, u\}$ (b) $Y = \{1, 4, 9, 16, 25, 36, 49\}$

2. Write each of the following sets by listing.

(a) P is the set of planets in the Solar System.

(b) Q = {prime factors of 30}

3. Write each of the following sets by using set builder notation.

(a) G = {March, May} (b) $H = \{7, 14, 21, 28, ..., 98\}$

4. Determine whether 4 is the element of each of the following sets.

(a) $P = \{1, 2, 3, 4, 5\}$ (b) $Q = \{x : x \text{ is a multiple of 8}\}$

(c) R = {prime numbers} (d) $S = \{x : x \text{ is a factor of 52}\}$

5. Sets A, B and C are defined as follows.
A = {months that begin with the letter J}
B = {states in Malaysia}
$C = \{x : x \text{ is a two-digit number such that the sum of its digits is 5}\}$
Find $n(A)$, $n(B)$ and $n(C)$.

6. Given that $P = \{7, 3, 13, x, 11, 5\}$ and Q = {prime numbers which are less than 15}, if $P = Q$, find the value of x.

11.2 Venn Diagrams, Universal Sets, Complement of a Set and Subsets

What are universal set and complement of a set?

LEARNING STANDARDS

Identify and describe universal sets and complement of a set.

A set that consists of all the elements under discussion is known as the **universal set**.

For example, the set below shows the students who participated in a mathematics quiz.

{Amir, Hazura, Laila, Sandra, Zamri, Dali, Pei San, Yana}

In this case, the 8 students are all the students under discussion. Hence, the set can be defined as universal set and is denoted by the symbol ξ.

Thus universal set, ξ = {Amir, Hazura, Laila, Sandra, Zamri, Dali, Pei San, Yana}

Among the students, Amir, Hazura, Laila, Sandra and Zamri are members of the Mathematics Society. If set A represents members of the Mathematics Society in the team, then A = {Amir, Hazura, Laila, Sandra, Zamri}

The other students in the team; Dali, Pei San and Yana are not members of the Mathematics Society. They are the other students in the universal set and are known as **complement of set** A, written as A'.

A' = {Dali, Pei San, Yana}

Example 5

Identify whether each of the following sets is the universal set of {2, 3, 5, 7}.

(a) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(b) {odd numbers which are less than 10}

Solution

(a) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} ← Contains all the elements.
Universal set

(b) {1, 3, 5, 7, 9} ← Does not contain element 2.
Not a universal set

Example 6

Given that $\xi = \{x : x \text{ is an integer and } 1 \leqslant x \leqslant 10\}$, determine the complement of each of the following sets.

(a) P = {perfect squares which are less than 10}

(b) Q = {factors of 10}

Think Smart

What is the complement of empty set?

Solution

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(a) $P = \{1, 4, 9\}$
Thus, $P' = \{2, 3, 5, 6, 7, 8, 10\}$

(b) $Q = \{1, 2, 5, 10\}$
Thus, $Q' = \{3, 4, 6, 7, 8, 9\}$

Self Practice 11.2a

1. Identify whether each of the following is the universal set of $\{1, 3, 5, 7, 9, 11, 13, 15\}$:
(a) {whole numbers}
(b) {prime numbers}
(c) $\{x : x$ is a positive integer and $x \leqslant 15\}$

2. Given that ξ = {whole numbers which are less than 10}, determine the complement of each of the following sets:
(a) P = {multiples of 3}
(b) Q = {prime numbers}

How do you represent the universal set and complement of a set by using Venn diagram?

LEARNING STANDARDS

Represent
(i) the relation of a set and universal set, and
(ii) complement of a set through Venn diagrams.

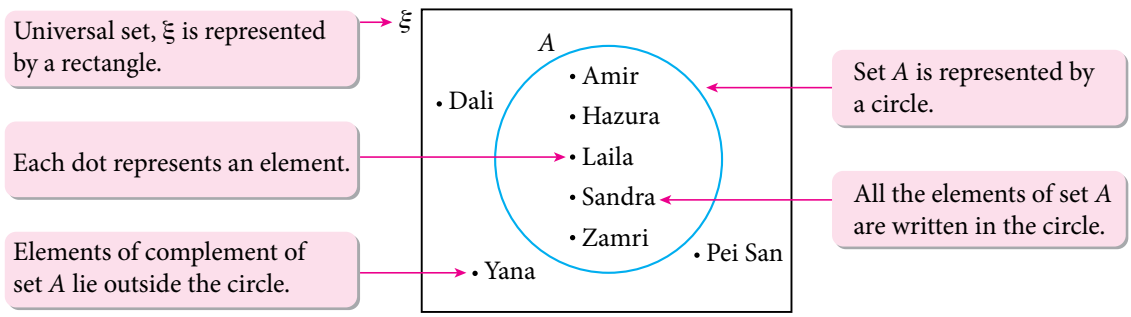
Besides description and set notation, a set can also be represented by an enclosed geometrical diagram which is known as **Venn diagram**.

For example,
ξ = {Amir, Hazura, Laila, Sandra, Zamri, Dali, Pei San, Yana}
A = {Amir, Hazura, Laila, Sandra, Zamri}
A' = {Dali, Pei San, Yana}

SMART TIPS

- A set can be represented by a circle, an oval, a rectangle or a triangle.
- The universal set is commonly represented by a rectangle.

The relationship between the sets above can be represented by the following Venn diagram.



Example 7

Given that $\xi = \{x : 10 < x < 20, x \text{ is an integer}\}$, $M = \{11, 17\}$ and $N = \{\text{odd numbers}\}$.

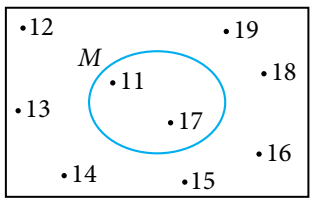
Draw a Venn diagram to represent

(a) set ξ and M, (b) N'.

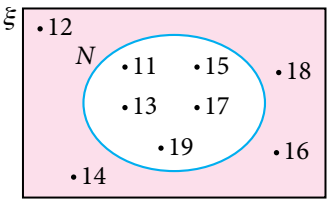
Solution

$\xi = \{11, 12, 13, 14, 15, 16, 17, 18, 19\}$
$M = \{11, 17\}$
$N = \{11, 13, 15, 17, 19\}$

(a)



(b)



The shaded region is N'.

Did You Know

John Venn (1834 – 1923), was an English mathematician who used geometrical diagrams to illustrate the relationship between sets. The Venn diagram is named after him.

Self Practice 11.2b

1. Represent the relationship between the following sets by using the Venn diagram.
(a) $\xi = \{1, 3, 5, 7, 9, 11\}$
$A = \{3, 5, 9\}$
(b) $\xi = \{x : x \text{ is an integer and } 10 < x < 20\}$
$B = \{\text{prime numbers}\}$

2. Represent the complement of each of the following sets by using Venn diagram. Shade the region that represents the complement of the set.
(a) $\xi = \{\text{multiples of 3 which are less than 30}\}$
$P = \{\text{numbers which are divisible by 6}\}$
(b) $\xi = \{\text{letters in the word 'PENGELASAN'}\}$
$Q = \{\text{consonants in the word 'PENGELASAN'}\}$

What are subsets?

Exploration Activity 2

Group

LEARNING STANDARDS

Identify and describe the possible subsets of a set.

Aim: To identify the subsets of a set.
Instruction: Perform the activity in groups of four.

1. Prepare the number cards labelled 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20.

2. Use two strings to make two circles on a table which represent the following sets respectively:
$A = \{\text{multiples of 2}\}$ $B = \{\text{multiples of 4}\}$

3. Place the number cards in the correct circle.
4. What do you notice about the positions of both the circles?
5. What is the relationship between set A and set B?

From the results of Exploration Activity 2, it is found that every element in set B is also an element of set A. Set B is known as a **subset** of set A and is written as $B \subset A$.

Let's Discuss

Referring to Exploration Activity 2, if set C represents multiples of 8, discuss the relationship between set C and set B, and the relationship between set C and set A.

Example 8

For each of the following, determine whether set A is a subset of set B.

(a) $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6\}$

(b) A = {prime numbers which are less than 20} and B = {odd numbers which are less than 20}

(c) A = {letters in the word 'SOPAN'} and B = {letters in the word 'KESOPANAN'}

Think Smart

What is the relationship between P and Q if $P \subset Q$ and $Q \subset P$?

Solution

(a) $A \subset B$ ← Every element of A is found in B.

(b) $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$
$B = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$ ← Element 2 is not in B.
$A \not\subset B$

(c) $A \subset B$ ← Every element of A is found in B.

SMART TIPS

'Is not a subset of' can be denoted using the symbol $\not\subset$. For example, 'P is not a subset of Q' can be written as $P \not\subset Q$.

On the first day of athletics championships, three track events will be held and they include 100 m, 200 m and 400 m. Each student can participate in one, two or all the three events.

If set A represents the track events held on the first day of athletics championships, then A = {100 m, 200 m, 400 m}.

The events that a student may participate in are {100 m}, {200 m}, {400 m}, {100 m, 200 m}, {100 m, 400 m}, {200 m, 400 m}, {100 m, 200 m, 400 m}. The student may also not participate in any event and it is represented by the empty set, { }.

Thus, each set of the listed events is a subset of set A.

SMART TIPS

- Empty set, ϕ is a subset of any set.
- Set itself is a subset of any set.

Example 9

List all the possible subsets of each of the following sets.

(a) $\{3, 4\}$ (b) $\{a, b, c\}$

Solution

(a) $\phi, \{3\}, \{4\}, \{3, 4\}$

(b) $\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$

SMART TIPS

If a set contains n elements, then the possible number of subsets is 2^n. For example, the number of subsets of $\{a, b, c\} = 2^3 = 8$.
(Use a scientific calculator to help you in your calculation.)
Press
2 ^ 3 =

Self Practice 11.2c

1. Complete each of the following with the symbol $\subset$ or $\not\subset$.

(a) $A = \{a, u\}$ and $B = \{a, e, i, o, u\}$

A ☐ B

(b) $E = \{\text{multiples of 4}\}$ and $F = \{\text{positive integers which are divisible by 2}\}$

E ☐ F

(c) $M = \{\text{odd numbers which are less than 50}\}$ and
$N = \{\text{multiples of 5 which are less than 50}\}$

N ☐ M

(d) $P = \{\text{quadrilaterals}\}$ and $Q = \{\text{rectangle, rhombus, pentagon}\}$

Q ☐ P

2. List all the possible subsets of each of the following sets.

(a) $\{p, q\}$

(b) $\{\text{prime numbers which are less than 10}\}$

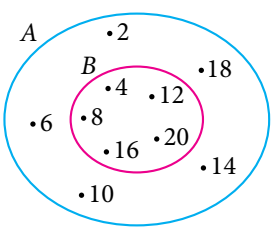
How do you represent subsets using Venn diagrams?

LEARNING STANDARDS

Represent subsets using Venn diagrams.

Given that $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$
and $B = \{4, 8, 12, 16, 20\}$.

The relationship of $B \subset A$ can be represented using the Venn diagram as shown below.



Think Smart

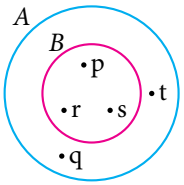
If $F \subset G$ and $H \subset G$, is $F = H$? Explain your answer with the help of a Venn diagram.

Example 10

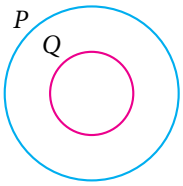
Represent the relationship of each of the following pairs of sets, using Venn diagrams.
(a) $A = \{p, q, r, s, t\}$ and $B = \{p, r, s\}$
(b) $P = \{\text{whole numbers}\}$ and $Q = \{\text{prime numbers}\}$

Solution

(a)



(b)



For an infinite set, its elements need not be written.

Self Practice 11.2d

1. Represent the relationship of each of the following pairs of sets, using Venn diagrams.
(a) $A = \{10, 20, 30, 40, 50, 60, 70\}$ and $B = \{20, 40, 60\}$
(b) $M = \{\text{letters in the word 'BIJAK'}\}$ and
$N = \{\text{vowels in the word 'BIJAK'}\}$

How do you represent the relationships between sets using Venn diagrams?

LEARNING STANDARDS

Represent the relations between sets, subsets, universal sets and complement of a set using Venn diagrams.

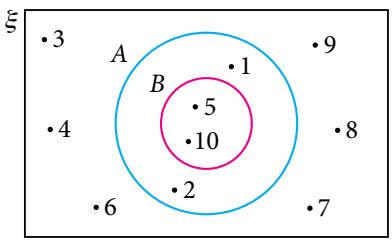
The relationships between sets, subsets, universal sets and complement of a set can be illustrated clearly by using Venn diagrams.

Example 11

Represent the relationship between the following sets, using a Venn diagram.
$\xi = \{x : x \leq 10, x \text{ is a positive integer}\}$
$A = \{\text{factors of } 10\}$
$B = \{\text{numbers which are divisible by } 5\}$

Solution

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 2, 5, 10\}$
$B = \{5, 10\}$



Self Practice 11.2e

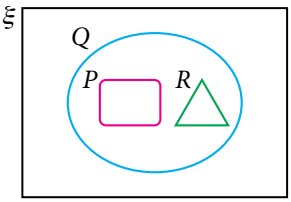
1. Represent the relationship between the following sets, using a Venn diagram.
ξ = {a, b, c, d, e, f, g, h}
P = {a, b, c, d}
Q = {b}
R = {f, g}

Mastery Q 11.2

Open the folder downloaded from page vii for extra questions of Mastery Q 11.2.

1. Given that ξ = {whole numbers which are less than 10}, A = {factors of 18} and B = {0, 4, 5, 7, 8}. With the help of a Venn diagram, determine whether $A' = B$.

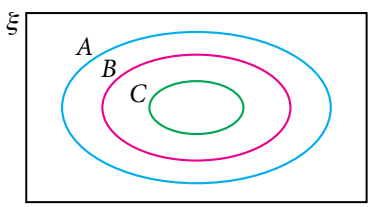
2. Based on the Venn diagram, state the relationship between
(a) P and Q,
(b) Q and R.



3. Given that K = {perfect squares which are less than 20},
(a) write all the possible subsets of K.
(b) if L = {1, 2, 3, ..., 20}, draw a Venn diagram to represent the relationship between K and L.

4. In a class, some of the girls wear spectacles while all of the boys do not wear spectacles. Sets ξ, P, Q and R are defined as follows:
ξ = {students in the class}
P = {girls}
Q = {boys}
R = {students who wear spectacles}
Represent the relationship between sets ξ, P, Q and R, using a Venn diagram.

5.



Based on the Venn diagram, write the relationship between sets ξ, A, B and C.

SUMMARY

SETS

Empty set

ø or { }

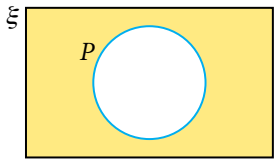
Equal sets

Example:
$A = \{M, A, S, A\}$
$B = \{S, A, M, A\}$

Set A = Set B

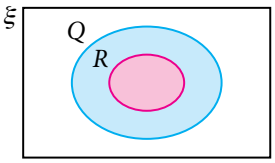
Universal set, ξ

Complement of a set



- Shaded region is P'.
- Complement of P is P'.

Subset



$R \subset Q$

At the end of this chapter, I can...	Very good	Work harder
explain the meaning of set.		
describe sets using description, listing and set builder notation.		
identify whether an object is an element of a set and represent the relation using symbol.		
determine the number of elements of a set and represent the number of elements using symbol.		
compare and explain whether two or more sets are equal, hence, make generalisation about the equality of sets.		
identify and describe universal sets and complement of a set.		
represent the relation of a set and universal set, and complement of a set using Venn diagrams.		
identify and describe the possible subsets of a set.		
represent subsets using Venn diagrams.		
represent the relations between sets, subsets, universal sets and complement of a set using Venn diagrams.		

Let's PRACTISE

Test Yourself

1. Describe set P = {square, rectangle, trapezium, parallelogram, kite, rhombus} by using description.

2. $A = \{x : x$ is neither a positive integer nor a negative integer$\}$.
What is the element of A?

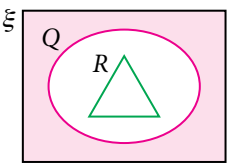
3. Explain whether each of the following pairs of sets are equal sets.
(a) P = {even numbers}; Q = {multiples of 2}
(b) $A = \{0\}$; $B = \phi$
(c) E = {factors of 15}; F = {numbers which are divisible by 15}

Self Mastery

4. If $\xi = \{x : 10 \leqslant x \leqslant 30, x$ is an even number$\}$ and P = {multiples of 4}, find $n(P')$.

5. Given that universal set ξ, $A \subset B$ and $C \subset A$, draw a Venn diagram to represent sets ξ, A, B and C.

6. Based on the Venn diagram,
(a) what is the relationship between Q and R?
(b) what is represented by the shaded region?



Challenge Yourself

7. Set P = {positive integers} and set Q = {prime numbers that are greater than 2}.
(a) Which set is the universal set? Justify your answer.
(b) If $Q \subset R \subset P$, define set R by description.

8. A training centre offers three courses such as cooking, design and computer. If set K represents the courses offered by the training centre and each possible choice a student makes is a subset of K, determine the number of possible choices a student can make.

9. The following is a conversation between Yazid and Mei Li.

Yazid : If the universal set is {students in Class 1 Bakti} and set A = {female prefects}, what is the complement of set A?

Mei Li : Complement of set A is the girls who are not prefects.

Is the statement made by Mei Li true or not true?
Justify your answer.

ASSIGNMENT

The campaign of separating household solid waste is one of the steps to reduce the amount of waste generated and delivered to landfills. By practising the classification of solid waste, we can reduce pollution and hence conserve the natural resources.

The classification of household solid waste involves the classification of solid waste according to waste composition such as paper, plastic and other recyclable materials and residues. By using the concept of set and Venn diagram, write a report regarding the classification of household solid waste.

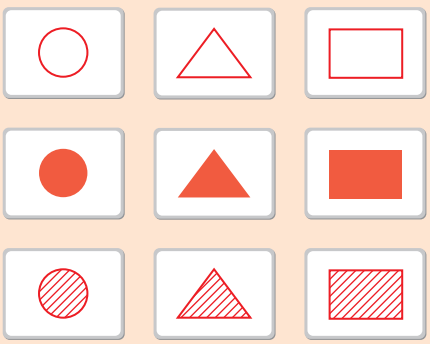
Scan the QR Code or visit **http://goo.gl/2rMjGC** to obtain the information regarding classification of household solid waste.

Exploring MATHEMATICS

Game

Method to prepare the materials

1. Prepare nine cards whereby each card is drawn with a circle, a triangle and a rectangle respectively. Each shape has three types of patterns such as empty, coloured and striped as shown in the illustration.



2. By using the same method, prepare nine similar cards for three different colours such as blue, green and yellow. Thus, the total number of cards obtained is 36.

Method to play

1. A game for four players.
2. Shuffle all the cards and distribute equally to all the players.
3. The cards for each player should be covered and cannot be seen by other players.
4. Each player picks a card at random from the player on his right.
5. The players need to collect four cards with the same shape or same pattern or same colour as one set.
6. The player who succeeds in collecting the most sets is the winner.

CHAPTER 12

Data Handling

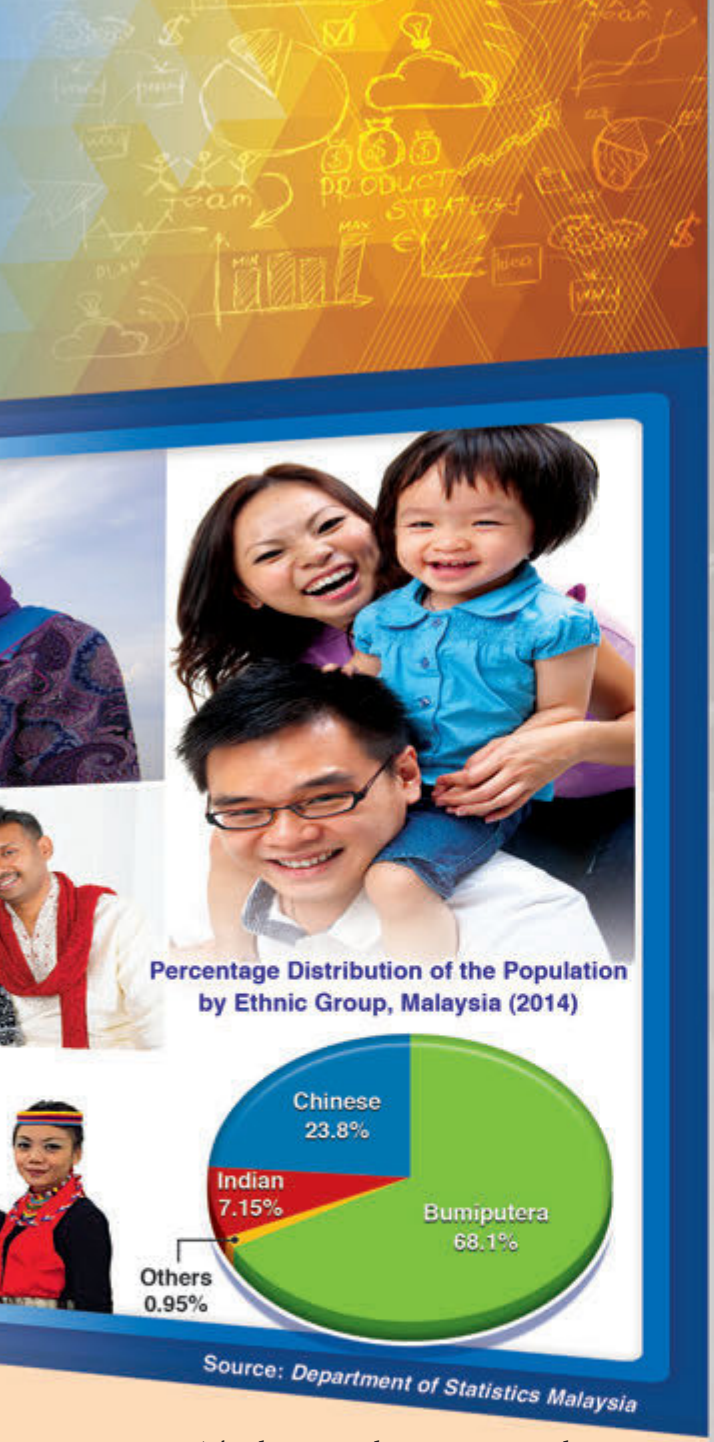
What will you learn?

- Data Collection, Organisation and Representation Process, and Interpretation of Data Representation

Why study this chapter?

As a basic knowledge in the field of statistics. Data handling involves skills like collecting, organising, representing, analysing and interpreting data, and then communicating the results of the data. Data handling is important as it enables us to understand the usage of data in newspapers, television and higher educational institutions as well as in the careers that we will pursue in the future. Discuss with your teacher other daily situations that involve data handling.

Malaysia is a multiracial country whereby all the people live together in peace and harmony. According to the Current Population Estimates 2014, the total population in Malaysia is 30.6 million; the number of males is 15.8 million and the number of females is 14.8 million.



The largest ethnic group is the *Bumiputera* who make up 68.1 percent of the total population, followed by Chinese (23.8%), Indian (7.15%) and others (0.95%).

Other than the data above, what other data can you obtain from the population distribution in Malaysia? How can this data be obtained?

Walking through Time

John Graunt

In ancient times, statistics were used by rulers to gather information about the population during their reign. However, statistics were only recorded in printed form in the 18th century by an English statistician, John Graunt. Two other English statisticians who actively contributed to the early development of statistics were Karl Pearson (1857 – 1936) and Ronald Fisher (1890 – 1962).

For more information:

http://goo.gl/Nx43ay

Word Link

• pie chart	• *carta pai*
• bar chart	• *carta palang*
• categorical data	• *data kategori*
• numerical data	• *data numerik*
• line graph	• *graf garis*
• histogram	• *histogram*
• frequency table	• *jadual kekerapan*
• displaying data	• *memaparkan data*
• analysing data	• *menganalisis data*
• classifying data	• *mengklasifikasikan data*
• organising data	• *mengorganisasikan data*
• collecting data	• *mengumpulkan data*
• interpreting data	• *mentafsir data*
• representing data	• *mewakilkan data*
• stem-and-leaf plot	• *plot batang-dan-daun*
• dot plot	• *plot titik*
• frequency polygon	• *poligon kekerapan*
• statistical question	• *soalan statistik*

Open the folder downloaded from page vii for the audio of Word Link.

12.1 Data Collection, Organisation and Representation Process, and Interpretation of Data Representation

How do you generate statistical questions and collect relevant data?

LEARNING STANDARDS

Generate statistical questions and collect relevant data.

To collect relevant data, we must generate statistical questions. What is a statistical question?

A **statistical question** is a question that can be answered by collecting data and there will be variability or diversity in the data related to the question. For example,

Statistical question	Reason
"How tall are the students in Class 1 Amanah?"	There is variability in the height of the students from that class, for example, 150 cm, 156 cm, 164 cm and so on.
"What is the favourite food of the students in Class 1 Amanah?"	There is variability in the types of food chosen by the students from that class, for example, *nasi lemak*, fried noodles, *laksa* and so on.

Information about the height of students and their favourite food are obtained through data collection.

Is each of the following questions a statistical question? Explain.

- "*How tall is Rosmee*?"
- "*Do the students in Class 1 Amanah like nasi lemak more than fried noodles*?"

Communication Corner

Generate statistical questions and collect data for each of the following in your class.

- Favourite sports of the students
- Modes of transport of students to school

After generating the statistical questions, the next step is to determine the method of data collection. Data can be collected by various methods.

DATA COLLECTION METHOD

Interview

For example, to find out how the students travel to school.

Survey

For example, to find out the favourite television programmes of the students.

Observation

For example, to count how many cars pass through the junction every hour.

Experiment

For example, to record the temperature of hot water as it cools down every five minutes.

Let's Discuss

Have a discussion with your friends about choosing the most suitable method to collect data. State the advantages and disadvantages of each method by giving examples to support your answers.

How do you classify data and construct frequency tables?

LEARNING STANDARDS

Classify data as categorical or numerical and construct frequency tables.

After collecting the data, the next step is classifying the data. Data can be classified into categorical data and numerical data.

DATA

Categorical	Numerical
• Measures characteristic • Cannot be measured numerically but can be described. For example, the gender of a person, colours of cars, flavours of sweets, blood groups	• Measures quantity • Measured numerically. For example, the number of books read in a week, the height of the badminton players, the time spent on exercise

SMART TIPS

Numerical data consists of
- **discrete data** that is measured in a whole unit. For example, the number of family members is 6 people.
- **continuous data** that is measured on a continuous scale. For example, the mass of the students are 53 kg, 56.2 kg and 66.5 kg.

Example 1

Classify the following data as categorical data or numerical data:
(a) The body temperature of each student
(b) The number of trees planted in each district
(c) The causes of road accidents

Solution

(a) Numerical data (b) Numerical data (c) Categorical data

Among the numerical data above, which is a discrete data and which is a continuous data?

After classifying the data, the next step is to organise the ungrouped data by constructing a frequency table.

SMART TIPS

Ungrouped data is an unprocessed raw data.

Example 2

The data shows the number of children in each family for 20 families. Organise the data by constructing a frequency table.

2	0	1	1	2	1	3	0	4	3
2	4	1	0	2	1	0	2	2	3

Solution

Number of children	Tally	Frequency
0	IIII	4
1	𝍸	5
2	𝍸 I	6
3	III	3
4	II	2
	Total	20

Self Practice 12.1a

1. Classify the following data as categorical data or numerical data:
(a) Number of stamps each student collects
(b) Time spent on the Internet
(c) Ability to play *sepak takraw*
(d) Colour of cars
(e) Length of earthworms
(f) Number of tourists visiting the National Museum each month
(g) Language spoken at home
(h) Annual income

2. The data shows the T-shirt size of Form 1 Cekal students. Organise the data by constructing a frequency table.

XL	L	XL	M	M	L	M	L	M
M	M	M	XL	XL	L	XL	L	M
M	L	M	L	L	S	M	M	L

How do you construct data representations?

LEARNING STANDARDS
Construct data representation for ungrouped data and justify the appropriateness of a data representation.

Data shown in the form of a table can also be presented graphically to make it easier to read and understand. The suitability of a data representation depends on the type of data collected and the purpose of acquiring the information. The data can be represented by a bar chart, a pie chart, a line graph, a dot plot and a stem-and-leaf plot.

(a) Bar chart

A bar chart is a type of data representation which represents data by using bars. It is suitable for showing comparisons between categories.

Example 3

The frequency table shows the activities during leisure time for Form 1 Bakti students. Construct a bar chart to represent the data and justify the appropriateness of the data representation.

Activity	Frequency
Reading	8
Watching television	9
Surfing the Internet	7
Exercising	6
Listening to music	4

SMART TIPS

The bars in a bar chart can be drawn horizontally or vertically.

Solution

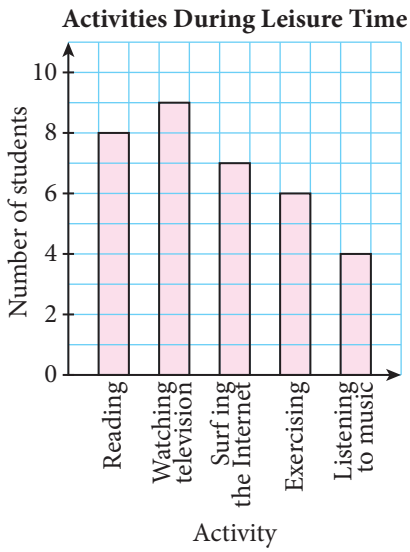
Steps to construct a bar chart:

Draw the horizontal and vertical axes on a grid.

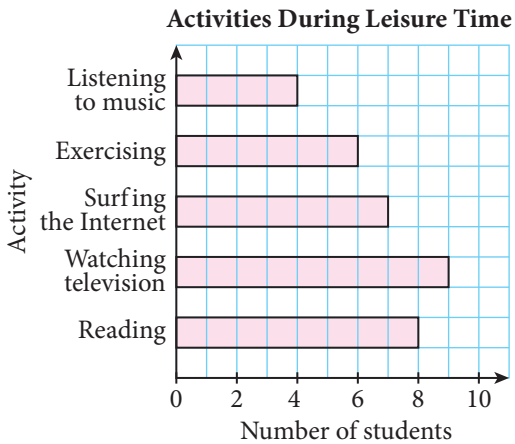
Choose one of the axes to mark a suitable scale and label the axis with the number of students. Label the other axis with types of activities.

Draw the bars such that the height of each bar corresponds to the frequency of the category it represents.

Write down the title of the bar chart.



or



These bar charts are suitable for comparing the number of students with the different leisure activities.

Scan the QR Code or visit **https://goo.gl/UxjF6c** and open the file *various bar charts_pdf* regarding data representation using various types of bar charts.

SMART TIPS

When representing data using a bar chart:

- the width of each bar must be uniform.
- the bars need to be evenly spaced.

Example 4

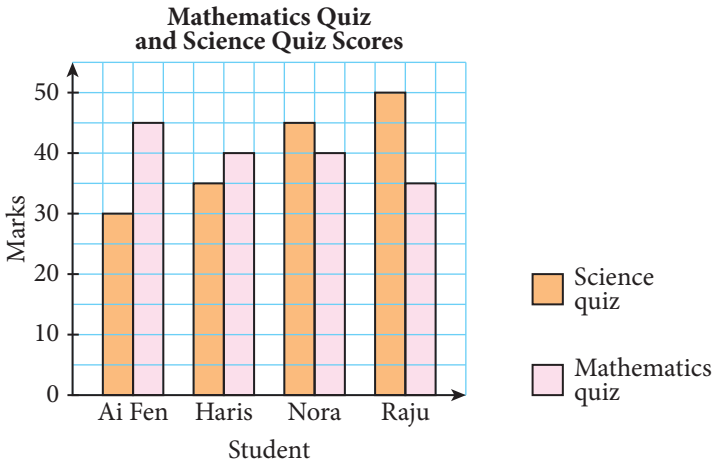
The table shows the scores obtained by a group of students in the Science quiz and the Mathematics quiz. Construct a bar chart to represent these two sets of data.

Student	Marks	
	Science quiz	Mathematics quiz
Ai Fen	30	45
Haris	35	40
Nora	45	40
Raju	50	35

Let's Discuss

Discuss whether a bar chart is suitable for representing data that has one or two categories that dominate the findings.

Solution



SMART TIPS

A dual bar chart is suitable for comparing two sets of data. For example, the performance of students in two tests, the price of hotel rooms at normal rate and during school holidays.

(b) Pie chart

A pie chart is a data representation that uses sectors of a circle to show the portion of each category of the whole data.

Example 5

The table shows the number of cars of Dynamic model sold by a car dealer. Construct a pie chart to represent the data and justify the appropriateness of the data representation.

Colour of cars	Red	Yellow	White	Blue
Number of cars	9	12	10	5

Let's Discuss

Discuss whether a pie chart is suitable for representing data that has many categories or the fraction of each category is fairly equal.

Solution

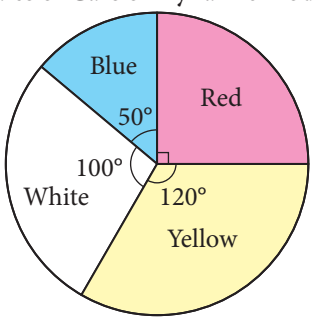
Steps to construct a pie chart:

Find the angle of sector for each category. → Draw a circle and divide it into different sectors based on the angles calculated. → Label each sector. → Write down the title of the pie chart.

Colour of cars	Number of cars	Fraction of the circle	Angle of the sector
Red	9	$\frac{9}{36}$	$\frac{9}{36} \times 360° = 90°$
Yellow	12	$\frac{12}{36}$	$\frac{12}{36} \times 360° = 120°$
White	10	$\frac{10}{36}$	$\frac{10}{36} \times 360° = 100°$
Blue	5	$\frac{5}{36}$	$\frac{5}{36} \times 360° = 50°$
Total	36	1	360°

Angle of the sector $= \frac{\text{Frequency of data}}{\text{Total frequency}} \times 360°$

Sales of Cars of Dynamic Model



This pie chart is suitable for comparing each colour of the cars with the total number of cars.

(c) Line graph

A line graph is a data representation used to display changes of data over a period of time. The data is represented by points which are connected in a straight line.

Example 6

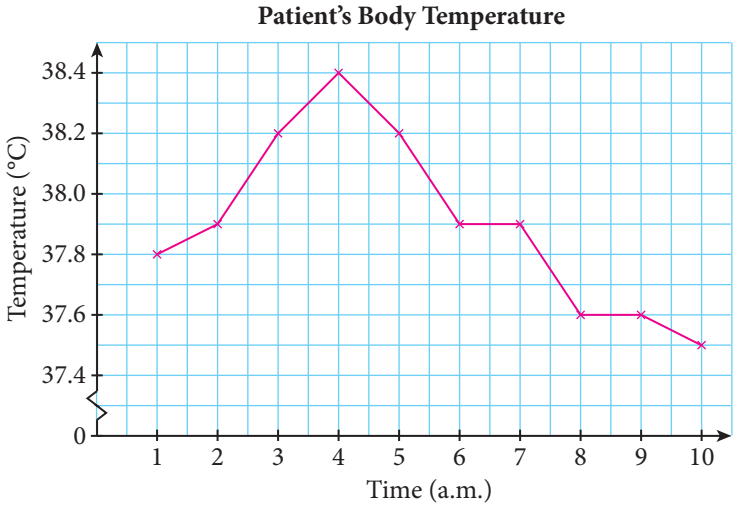
The table shows the temperature of a patient over a particular period of time. Construct a line graph to represent the data and justify the appropriateness of the data representation.

Time (a.m.)	1	2	3	4	5	6	7	8	9	10
Temperature (°C)	37.8	37.9	38.2	38.4	38.2	37.9	37.9	37.6	37.6	37.5

Solution

Steps to construct a line graph:

Draw the horizontal and vertical axes on a grid.	Choose a suitable and uniform scale for both axes. The vertical axis represents data. The horizontal axis represents time.	Plot the points and connect the points in a straight line.	Write down the title of the line graph.



This line graph is suitable for displaying changes in a patient's body temperature over a period of 10 hours.

SMART TIPS

For a line graph, the horizontal axis usually represents the time duration and the vertical axis usually represents the frequency values.

Let's Discuss

Discuss whether a line graph is suitable for
- predicting data trends.
- showing clearly the fluctuations of the data before and after a certain point of time.

(d) Dot plot

A dot plot shows the distribution of data on a number line. The data are either clustered around certain values or spread out evenly on a number line. Dot plot can help us to visualise data patterns, draw inferences and make decisions. Dot plot can also detect unusual observations, that is, the extreme values in the data. If there is extreme value in the data, we may need to investigate further to find out the cause for the unusual observations.

Career in Mathematics

A basketball coach uses a dot plot to evaluate the performance of each player under his supervision.

Example 7

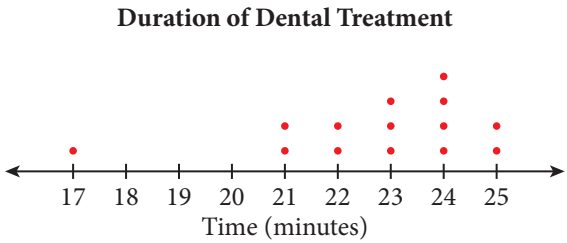
The duration of dental treatment (in minutes) provided by a dentist to 14 patients is as shown below. Represent the data in a dot plot and justify the appropriateness of the data representation.

23	24	21	24	25	24	25
24	22	17	21	23	22	23

Solution

Steps to construct a dot plot:

Draw a horizontal number line that covers the range of the given data. → Plot the individual data with a dot over their values on the number line. → Write down the title of the dot plot.



This dot plot is suitable for displaying the duration of the dental treatment which ranged from 17 minutes to 25 minutes.

Let's Discuss

Discuss whether a dot plot is suitable for
- illustrating the data frequency.
- displaying a large amount of data.
- representing categorical data or numerical data.

(e) Stem-and-leaf plot

A stem-and-leaf plot is a data representation that separates the data values into stem and leaf according to their place value. The leaf usually is the last digit of the number. The stem is the remaining digit or digits on the left of the number. The plot retains the original data values. Therefore, we are able to do arithmetic calculation on these values for the purpose of data analysis.

Walking through Time

The American mathematician John W. Tukey (1915-2000) introduced the stem-and-leaf plot in the 1960s. Since then, the stem-and-leaf plot has become a popular way of representing data for analysis.

Example 8

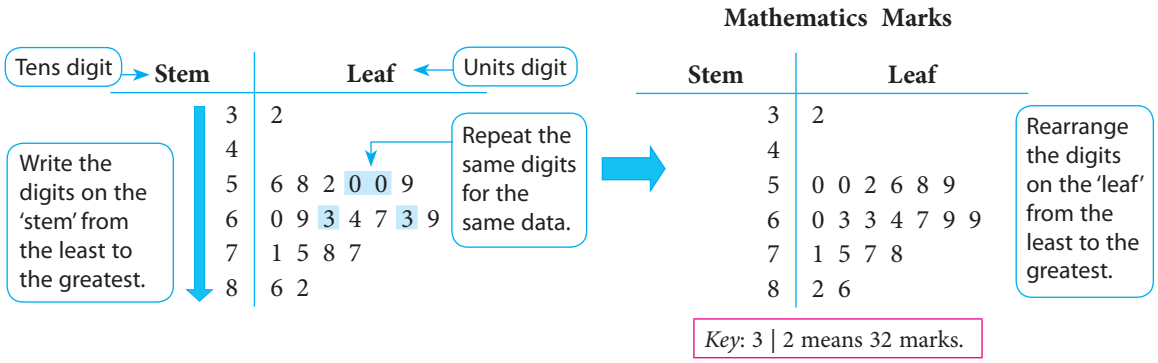
The data shows the Mathematics marks for a class of 20 students. Represent the data by using a stem-and-leaf plot and justify the appropriateness of the data representation.

60	56	69	32	63	58	71	86	52	64
50	67	82	63	75	50	69	78	77	59

Solution

Steps to construct a stem-and-leaf plot:

Write each data one by one and take the tens digit of each data value as the stem. → The last digit of the number is written on the leaf. → Rearrange the leaves in ascending order. → Write a key and the title. The key indicates the unit for the stem and leaf.



This stem-and-leaf plot is suitable for displaying the marks of each student in the class.

Smart Technology

Open the folder downloaded from page vii to open the file *Bar Chart.xls* for the data representation in Example 3. Hence, continue your exploration with other types of data representation.

Let's Discuss

Discuss whether the stem-and-leaf plot is suitable for
- displaying large data
- representing categorical data or numerical data

Self Practice 12.1b

1. A survey was carried out in a class to find out how the students travel to school, and the results are as shown in the table below. Construct a bar chart to represent the data and justify the appropriateness of the data representation.

Transportation	Car	School bus	Public bus	Bicycle	Walk
Frequency	8	10	7	2	5

2. The table shows the prices of four types of accommodation around the Historical City of Melaka at normal rate and during holiday season. Construct a bar chart to represent the two sets of data and justify the appropriateness of the data representation.

Type of accommodation	Hotel	Homestay	Budget hotel	Hostel
Normal rate (RM)	300	250	150	100
Holiday season (RM)	350	300	200	100

3. The table shows the favourite songs of a group of children.

Favourite song	Rasa Sayang	Ikan Kekek	Bangau Oh Bangau	Geylang Si Paku Geylang	Lompat Si Katak Lompat	Dayung Sampan
Number of children	30	40	20	15	10	5

Construct a pie chart to represent the data and justify the appropriateness of the data representation.

4. The table shows Kamil's height over a period of six years. Construct a line graph to represent the data and justify the appropriateness of the data representation.

Year	2011	2012	2013	2014	2015	2016
Height (cm)	145	150	153	160	164	167

5. The data shows the number of text messages sent by a group of students on a particular day. Represent the data by using a dot plot and justify the appropriateness of the data representation.

3	4	8	7	11
6	5	7	6	3
9	6	5	11	8

6. In a survey, the ages of 24 readers of a magazine are recorded as follows. Represent the data by using a stem-and-leaf plot and justify the appropriateness of the data representation.

44	53	33	65	51	30	42	34
57	36	51	32	39	44	25	31
58	47	31	22	58	38	60	47

How do you convert a data representation to another representation?

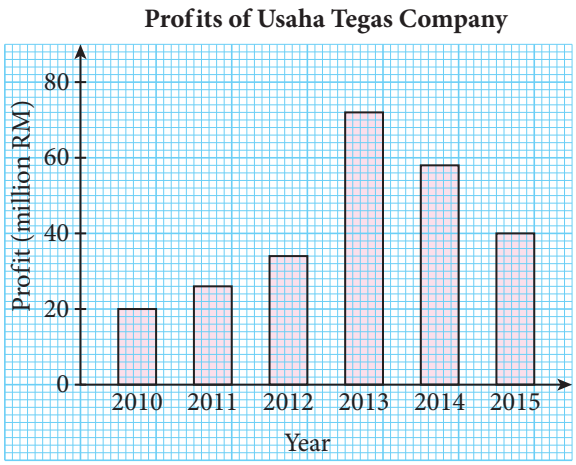
LEARNING STANDARDS

Convert a data representation to other suitable data representations with justification.

A data representation can be converted to other suitable representations for further analysis.

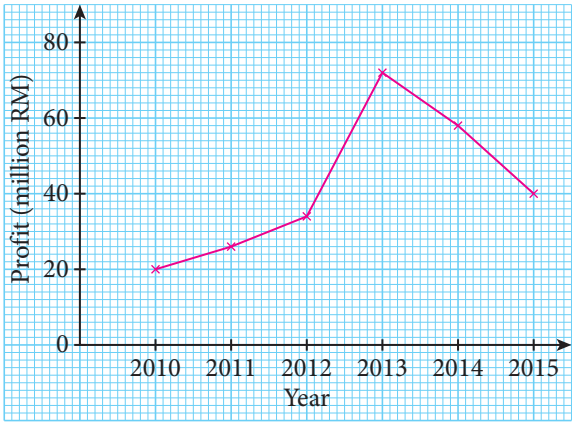
Example 9

The bar chart shows the profits obtained by Usaha Tegas Company from 2010 to 2015. Convert the bar chart representation to another suitable representation and justify the conversion for this representation.



Solution

Profits of Usaha Tegas Company



The bar chart representation is converted to a line graph representation because the line graph is suitable for displaying data collected over a certain period, that is, the profit performance of Usaha Tegas Company over a period of six years.

Example 10

The stem-and-leaf plot shows the results of a survey on the pulse rates per minute of patients treated at a community polyclinic. Convert the representation to a dot plot and justify the conversion for this representation.

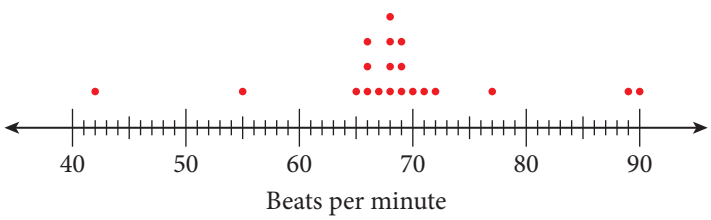
Pulse Rates Per Minute of Patients

Stem	Leaf
4	2
5	5
6	5 6 6 6 7 8 8 8 8 9 9 9
7	0 1 2 7
8	9
9	0

Key: 4 | 2 means 42 beats per minute.

Solution

Pulse Rates Per Minute of the Patients

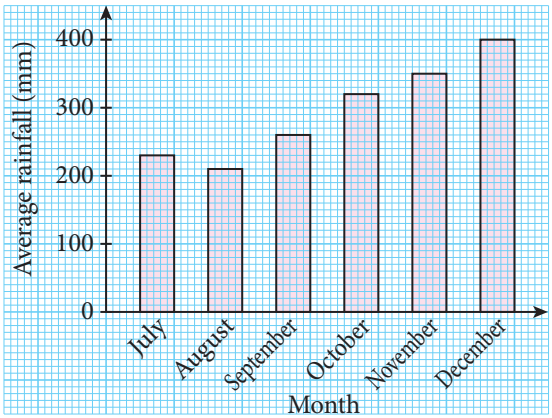


The stem-and-leaf plot is converted to a dot plot because both of these representations are suitable for displaying numerical data distribution and at the same time retain the original data values.

Self Practice 12.1c

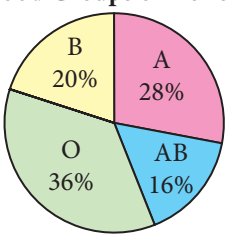
1. The bar chart shows the average monthly rainfall of Impian City from July to December 2015. Convert the representation to another suitable data representation and justify the conversion.

Average Monthly Rainfall of Impian City



2. A blood donation campaign organised by the Malaysian Red Crescent has received an overwhelming response from the public. The pie chart shows the blood groups of 25 donors in the first three hours. Convert this representation to another suitable data representation and justify the conversion.

Blood Groups of Donors



How do you interpret data representations?

LEARNING STANDARDS

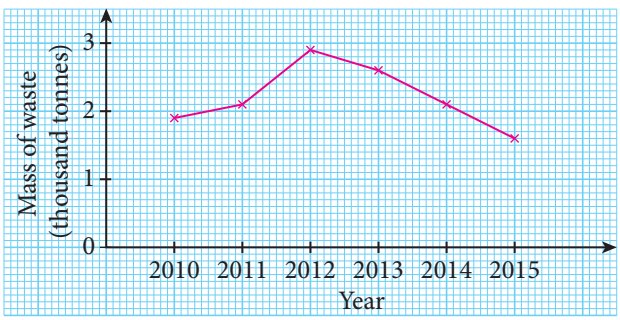
Interpret various data representations including making inferences or predictions.

By interpreting data representations, we can obtain information and hence make inferences and predictions.

Example 11

The line graph shows the mass of waste, in thousand tonnes, produced in a city from 2010 to 2015.

Mass of Waste Produced



(a) What is the mass of waste produced in 2010?

(b) What can you say about the mass of waste produced in 2011 and 2014?

(c) Find the mean mass of the waste produced over a period of six years.

(d) State one inference based on the line graph given.

(e) Based on the trend of the line graph, predict the mass of waste produced in 2016.

Solution

(a) 1 900 tonnes

(b) The mass of waste produced in 2011 and 2014 are the same.

(c) Total mass of waste produced over a period of 6 years
$= 1\,900 + 2\,100 + 2\,900 + 2\,600 + 2\,100 + 1\,600$
$= 13\,200$ tonnes

$$\text{Mean mass of waste} = \frac{13\,200}{6}$$
$$= 2\,200 \text{ tonnes}$$

(d) The mass of waste produced decreases each year after the year 2012.

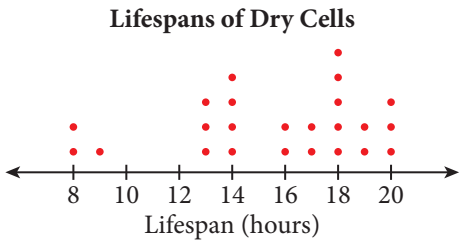
(e) 1 100 tonnes

Communication Corner

Think and state the possible reasons for the gradual decrease in waste production in a city year by year.

Example 12

In a quality control laboratory, the lifespans (to the nearest hours) of 24 dry cells are tested. The data obtained is represented by a dot plot as shown.



(a) State the maximum and minimum lifespans of the dry cells tested.

(b) State one inference based on the data from the dot plot.

(c) The quality control laboratory has decided that the dry cells with a lifespan of less than 10 hours will be considered defective and thus will be rejected. Find the percentage of the dry cells that will be rejected.

(d) It is known that 50% of the dry cells have a lifespan of at least x hours. Find the value of x.

Solution

(a) The maximum lifespan = 20 hours
The minimum lifespan = 8 hours

(b) Most of the dry cells have a lifespan from 13 hours to 20 hours.

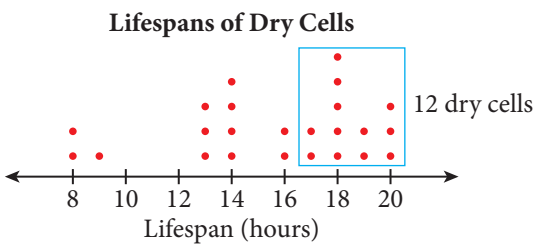
(c) Number of dry cells with a lifespan of less than 10 hours = 3

$$\text{Percentage of dry cells rejected} = \frac{3}{24} \times 100\%$$
$$= 12.5\%$$

(d) 50% of the number of dry cells

$$= \frac{50}{100} \times 24$$
$= 12$ dry cells

From the dot plot, there are 12 dry cells with a lifespan of at least 17 hours.
Thus, $x = 17$

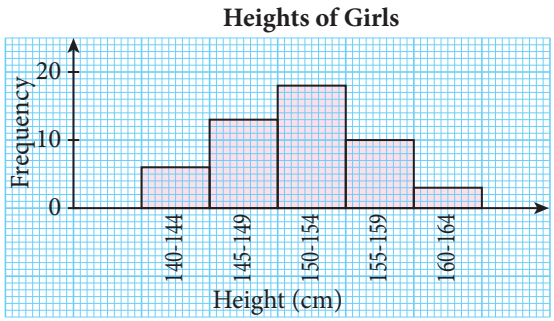


Histogram

A histogram is a data representation that displays grouped data. Grouped data is data that is collected in intervals.

Example 13

The histogram shows the heights of 50 girls.



(a) Find the number of girls with heights from 155 cm to 159 cm.

(b) A girl with a height of 160 cm and above is eligible to participate in the volleyball team. Find the number of girls who are eligible to participate in the volleyball team.

(c) By observing the shape of the histogram, make an inference based on the distribution of the heights of the girls.

Solution

(a) 10 girls

(b) 3 girls ← There are 3 girls with heights of 160 – 164 cm.

(c) Most of the girls have heights of 145 cm to 159 cm.

Did You Know

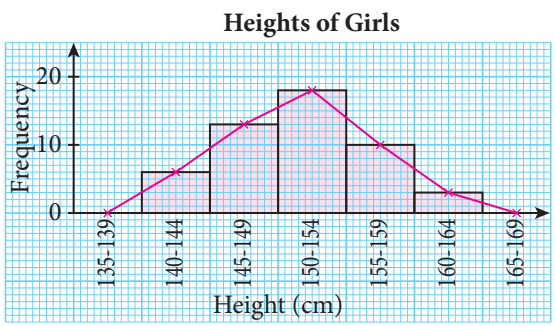
- The width of each bar in a histogram represents a specific interval. For example, the interval 140-144 covers the range of heights from 140 cm to 144 cm.
- The height of each bar represents the frequency in each interval.

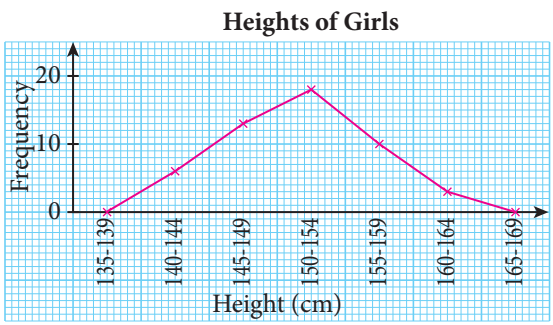
SMART TIPS

- A histogram does not display the actual values of the data but displays values in a certain interval.
- A histogram can provide a display of large data sets because the data is represented in class intervals.

Frequency polygon

A frequency polygon is a graph formed by joining the midpoint of the top of each bar in a histogram with straight lines. Based on Example 13, a frequency polygon can be drawn from the histogram and is shown below.

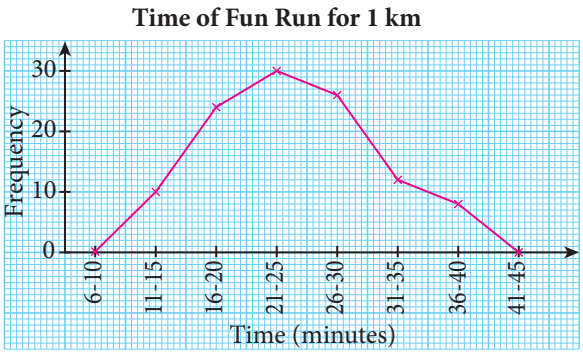




Example 14

The frequency polygon shows the time taken by a group of participants to complete the fun run in a park.

(a) Find the total number of participants in the fun run.
(b) Find the number of participants who recorded a time of 16 minutes to 20 minutes to complete the run.
(c) By observing the shape of the frequency polygon, make an inference based on the distribution of time taken by the participants.



Solution

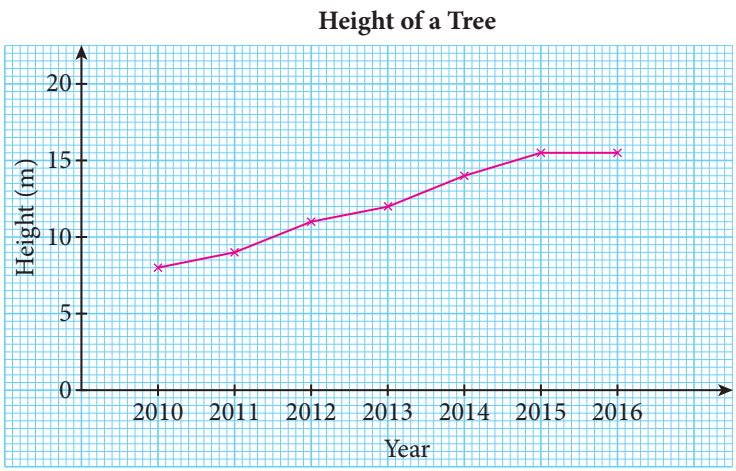
(a) Total number of participants = 10 + 24 + 30 + 26 + 12 + 8
= 110 participants

(b) 24 participants ← 24 participants recorded a time of 16 – 20 minutes

(c) Most of the participants recorded a time of 16 minutes to 30 minutes to complete the run.

Self Practice 12.1d

1. A botanist studies the height of a tree in a tropical rainforest. The line graph shows the height of the tree over a period of seven years.



(a) How tall was this tree initially?
(b) What is the increase in height of the tree over a period of seven years?
(c) In which year will the height of the tree be 12 m?
(d) State an inference based on the line graph given.
(e) Based on the trend of the line graph, predict the height of the tree in 2017.

2. The stem-and-leaf plot shows the diameters of axles manufactured by a machine.

Diameters of Axles

Stem	Leaf
24	5 6 6 8
25	0 1 1 2 3 6
26	0 0 0 1 3 3 4 5 8 9
27	3 4 4 5 7 8 8
28	2 3 6

Key: 24 | 5 means 24.5 mm.

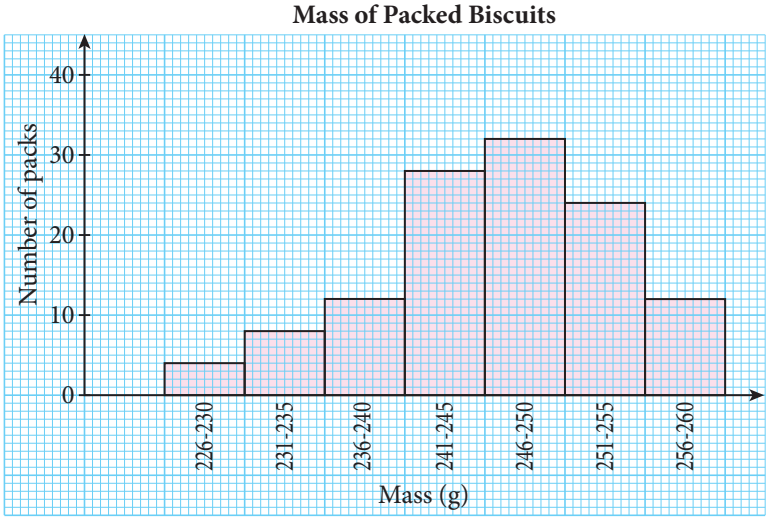
(a) Determine the total number of axles manufactured.

(b) Find the largest and the smallest diameters of the axles manufactured.

(c) A mechanic notices that he has to reduce the axles with diameters exceeding 27.5 mm so that the axles can fit into wheels. Calculate the percentage of axles that need to have their diameters reduced.

(d) State an inference based on the distribution of the diameters of axles in this stem-and-leaf plot.

3.



A quality control supervisor wants to determine whether the batch of biscuits produced by the packaging department follows the specifications of the standard mass. The histogram shows the mass of packed biscuits in a few sample tests.

(a) How many packs of biscuits are tested in the sample?

(b) How many packs of biscuits have a mass from 236 g to 240 g?

(c) Based on the specifications, if 75% of the sample have mass from 241 g to 260 g, then the batch of biscuits produced would be packaged and sent for shipment. Does this batch of biscuits produced satisfy the specifications? Show your calculations.

What is the importance of representing data ethically?

Data representations help us to analyse and interpret data much easier. We need to represent the data ethically to avoid confusion.

To represent the data ethically,
- the scale used in the representation must be consistent and start at 0.
- the data displayed must be accurate.

LEARNING STANDARDS

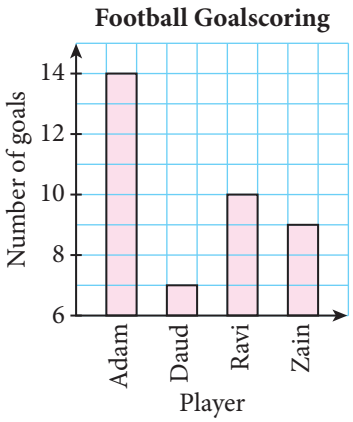
Discuss the importance of representing data ethically in order to avoid confusion.

Example 15

The bar chart shows the number of goals scored by four football players in the Permai District Football League.

(a) Is the number of goals scored by Adam twice the number of goals scored by Ravi? Explain your answer.

(b) In your opinion, does this bar chart clearly represent the number of goals scored by the players?



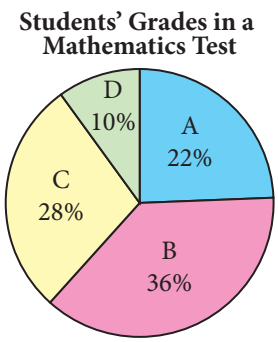
Solution

(a) Number of goals scored by Adam = 14
Number of goals scored by Ravi = 10
Thus, the number of goals scored by Adam is not twice the number of goals scored by Ravi.

(b) No, the information displayed is misleading. The scale in the vertical axis should start from 0.

Example 16

The pie chart shows the grades obtained by a group of students in a Mathematics test. Does this pie chart display data accurately? Explain your answer.



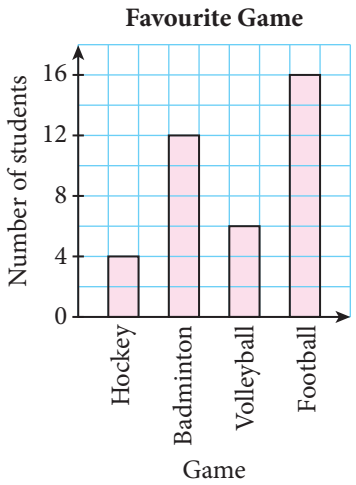
Solution

Total percentage = 22 + 36 + 28 + 10
= 96

No, because the total percentage in the pie chart is not equal to 100.

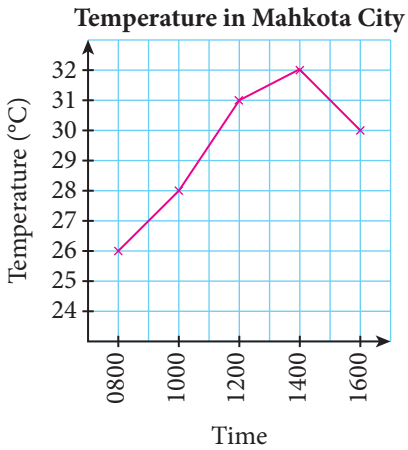
Self Practice 12.1e

1.



Zurini conducts a survey on 40 students to find out their favourite games. The results of the survey are shown in the bar chart. Does this bar chart display data accurately? Explain your answer.

2.

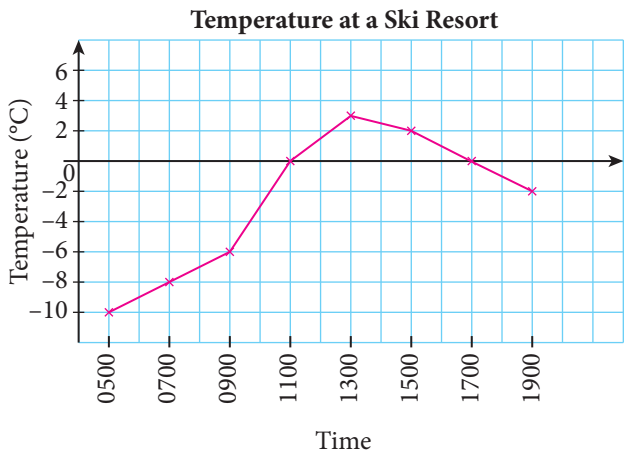


The line graph shows the temperatures in Mahkota City from 0800 to 1600. Does this line graph clearly display the data? Explain your answer.

Mastery Q 12.1

Open the folder downloaded from page vii for extra questions of Mastery Q 12.1.

1.



The line graph shows the temperatures recorded at a ski resort on a particular day during the ski season.

(a) Estimate the temperature at 1000.

(b) Predict the temperature at 2100.

(c) At what time was the temperature 0 °C?

2.

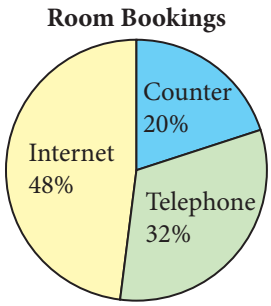
Animal	Cheetah	Zebra	Lion	Horse	Deer	Ostrich
Speed (km/h)	110	65	80	75	70	95

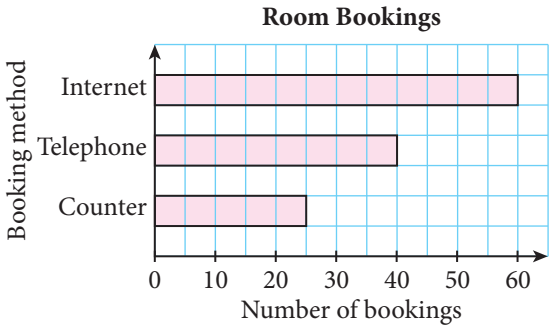
The frequency table shows the maximum speed, in km/h, of several animals.

(a) Represent the above data in a
  (i) bar chart,
  (ii) stem-and-leaf plot.

(b) Between the two types of data representations constructed in (a), which representation is more suitable? Explain your answer.

3.





A hotel has received 125 room bookings via three methods on a certain day. The data is displayed by using a pie chart and a bar chart.

(a) What is the difference between the number of room bookings via the Internet and the number of room bookings over the counter? Which representation shows this data more clearly? Explain your answer.

(b) Almost half of the room bookings are made via the Internet. Which representation shows this data more clearly? Explain your answer.

(c) Which representation clearly shows the number of room bookings made via the Internet? Justify your answer.

(d) Do you think a line graph is suitable for displaying this data? Explain your answer.

(e) What other representation is suitable for displaying this data?

4. The data shows the daily allowances (in RM) of a group of factory workers.

20	25	21	24	22	23
22	22	23	30	25	22

(a) Represent the data in a dot plot.

(b) Describe briefly
  (i) the data distribution for the daily allowances of the group of factory workers,
  (ii) the value where most of the data are clustered around,
  (iii) whether there is any extreme value in the data.

5. Diagram (a) and Diagram (b) show two histograms which represent the Science and Mathematics marks of Form 1 students.

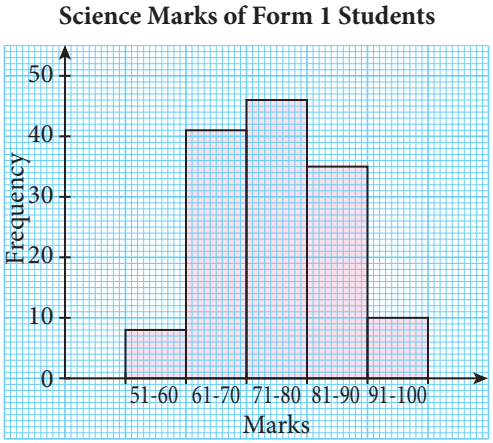


Diagram (a)

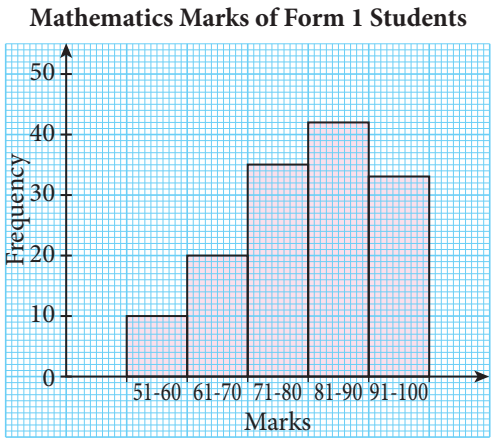


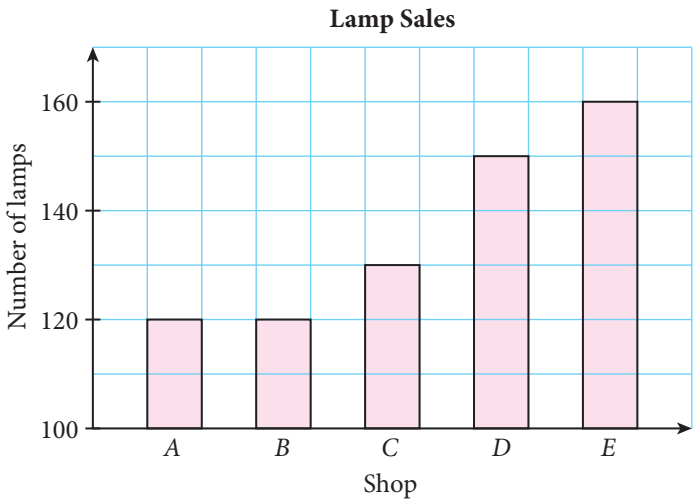
Diagram (b)

(a) Find the number of students who obtained more than 80 marks in each test.

(b) Describe briefly the distribution of marks in each test.

(c) Compare the distributions of the Science and Mathematics marks. What inference can be made?

6. The bar chart shows the number of lamps sold by five shops in a month.



(a) What can you say about the number of lamps sold by shop *A* and shop *B*?

(b) Shop *E* claims that the number of lamps it sold is twice the number of lamps sold by Shop *C*. Is this claim valid? Explain your answer.

SUMMARY

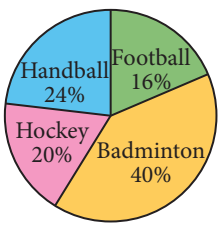
DATA HANDLING

- **Data**
  - Categorical
  - Numerical
- **Frequency**
  - Frequency table

DATA REPRESENTATION

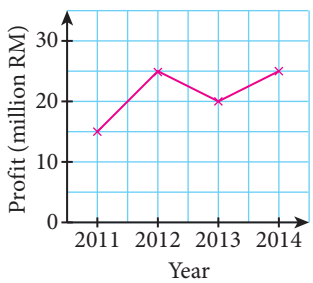
Pie chart

Favourite Game



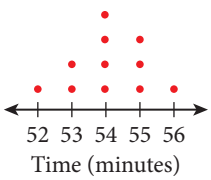
Line graph

Profits of Maju Company



Dot plot

Training Time for a Cross-country Run



Stem-and-leaf plot

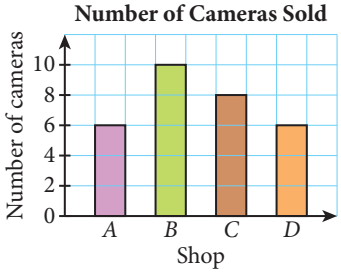
Basketball Points

Stem	Leaf
0	2
1	2 2 2 5 5 8
2	0 0 1 1 1 4 6 6 7 8
3	0 0

Key: 1 | 5 means 15 points

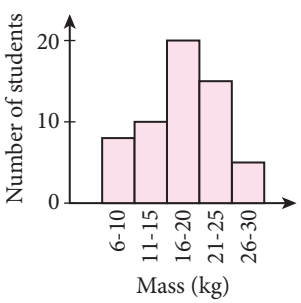
Bar chart

Number of Cameras Sold



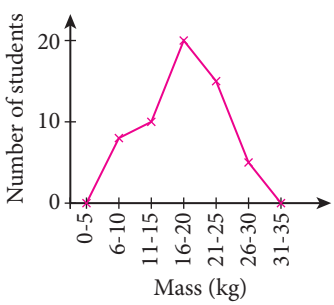
Histogram

Mass of Old Newspapers Collected



Frequency polygon

Mass of Old Newspapers Collected



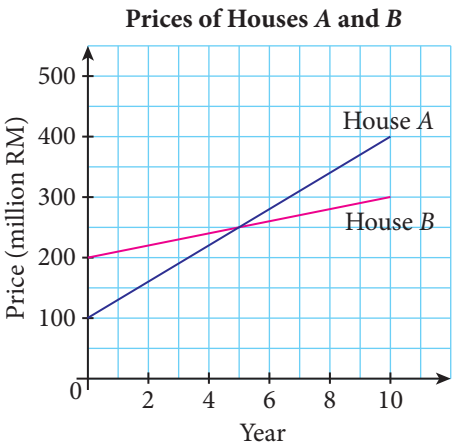
CHAPTER 12

At the end of this chapter, I can...	Very good	Work harder
generate statistical questions and collect relevant data.		
classify data as categorical or numerical and construct frequency tables.		
construct data representation for ungrouped data and justify the appropriateness of a data representation.		
convert a data representation to other suitable data representations with justification.		
interpret various data representations including making inferences or predictions.		
discuss the importance of representing data ethically in order to avoid confusion.		

Let's PRACTISE

Test Yourself

1. The line graph shows the initial selling prices of house *A* and house *B* and the rate of increase in prices over a 10-year period.



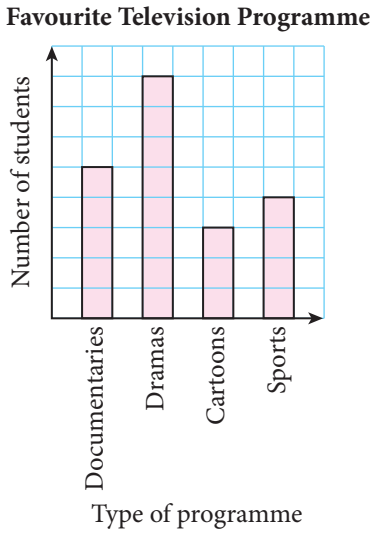
(a) Which house has a higher rate of increase in price?

(b) In which year will the prices of house *A* and house *B* be equal?

2. The bar chart shows four types of favourite television programmes of a group of students.

(a) State the most favourite television programme of the group of students.

(b) Represent all the information on the bar chart by using a pie chart.

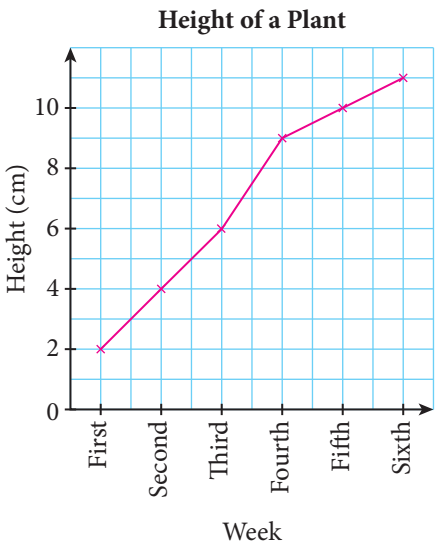


3. In an experiment, Johari measures the height of a plant every week over a six-week period. He then presents the data collected in a line graph.

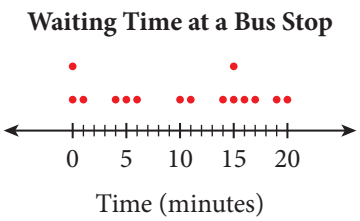
(a) State which weeks the plant will grow at the same rate.

(b) Between which two weeks will the plant attain the largest increase in height?

(c) What is the advantage of this representation?



4. A survey was conducted to study the waiting time (to the nearest minutes) of a group of passengers at a bus stop. The data obtained from the survey is represented in a dot plot. Based on the dot plot, determine whether the following statements are TRUE or FALSE:



(a) 20 passengers were involved in this survey.

(b) The shortest waiting time was 0 minute.

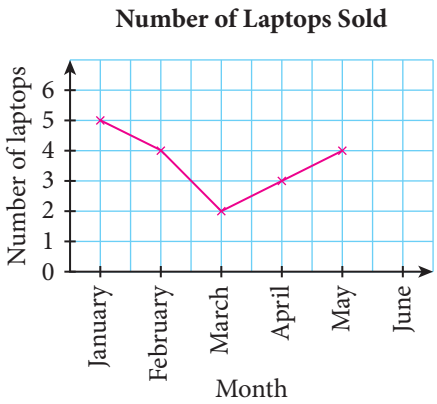
(c) 40% of the passengers faced a waiting time of at least 15 minutes.

Self Mastery

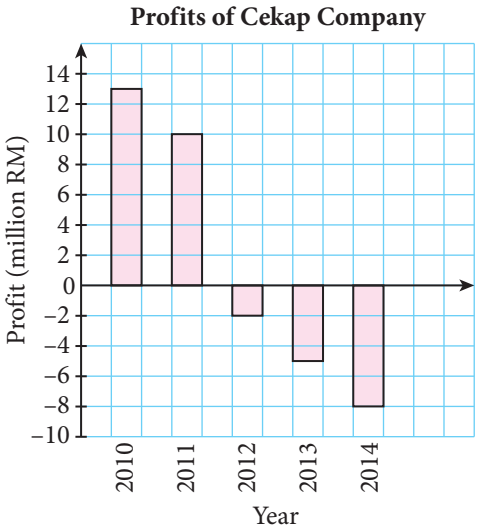
5. The incomplete line graph shows the sales of laptops in a computer shop from January to May. The number of laptops sold in June is not shown.

(a) The total number of laptops sold from January to June is 24 units. Copy and complete the line graph for June.

(b) Convert the line graph to another suitable representation and justify the conversion of this representation.



6. The bar chart shows the profits of Cekap Company from 2010 to 2014.



(a) In which year did Cekap Company obtain maximum profit? What was the maximum profit?

(b) In which year did Cekap Company start incurring losses? What was the loss in that year?

(c) What was the profit or loss in 2014?

(d) (i) In 2015, given that Cekap Company recorded an increase of RM11 million in profit compared to 2014. Based on the information, complete the bar chart for 2015 on the same diagram.

(ii) Hence, calculate the company's total profits or losses over a six-year period.

Challenge Yourself

7. The time taken (in minutes) by a plumber to fix 30 leaking pipes are shown in the stem-and-leaf plot below.

Time Taken to Fix Leaking Pipes

Stem	Leaf
1	2 5 5 6 7 8
2	1 4 5 7 8
3	0 0 1 3 4 5 6 7 7 8 9
4	1 2 3 5 7 8
5	2 6

Key: 1 | 2 means 12 minutes.

(a) List all the data displayed in the stem-and-leaf plot above.

(b) State the shortest time taken by the plumber to fix a leaking pipe.

(c) What inference can you make regarding the times taken to fix the leaking pipes?

8.

Ticket Price of Desa Mutiara Theme Park

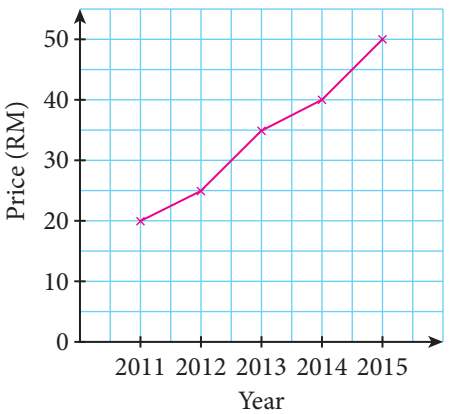


Diagram (a)

Ticket Price of Desa Mutiara Theme Park

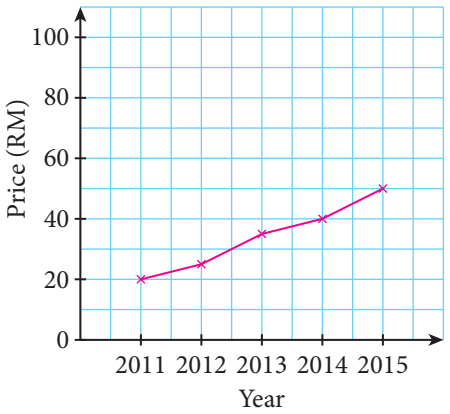


Diagram (b)

The line graphs in Diagram (a) and Diagram (b) display the same data for the ticket prices of Desa Mutiara theme park from 2011 to 2015.

(a) Which line graph shows a higher increase in price over a five-year period?

(b) Which line graph might the theme park manager use to show that the price increase is not significant? Is this an ethical representation? Explain your answers.

ASSIGNMENT

In this assignment, you are to collect, display, analyse and interpret data regarding the media channels used to promote a new product among teenagers. The media channels include newspapers, television, radio, Internet, social media, magazines, catalogues, pamphlets and others.

Write a report to suggest the media channels that can extensively promote the new product among teenagers, and predict the media channels that would become increasingly popular in the future. To support your suggestion, your report must include questionnaires, frequency tables, suitable data representations using software, data interpretations and conclusions.

Exploring MATHEMATICS

Read the following article and discuss the questions posed.

PUTRAJAYA: The National Consumer Complaints Centre (NCCC) revealed that in the year 2012, they had received the highest number of complaints from the telecommunications category with 5 985 complaints. The second highest source of complaints was from the general consumer products category with 5 764 complaints followed by the utility-based category with 5 568 complaints.

No	Category of Complaints	Number of complaints
1	Telecommunications	5 985
2	General Consumer Products	5 764
3	Utility-based	5 568
4	Banking	5 555
5	Automobile	2 986
6	Travel and Leisure	2 034
7	Health and Fitness	1 839
8	Housing and Property	1 734
9	Vehicles Inspection – Workshops	1 695
10	Consumer Services – Retail	1 617

Source: Generasi Pengguna, August 2013, jointly published by the Ministry of Domestic Trade, Co-operatives and Consumerism, Malaysia and FOMCA

1. State the three categories that received the most number of complaints.
2. A statistician commented that "there is a possibility that the general consumer products category received more complaints than the telecommunications category". Discuss why this comment may be true.
   Note: You can think of how the data (number of complaints) have been arranged.
3. What can you tell about this data organisation?

CHAPTER 13 The Pythagoras' Theorem

What will you learn?

- The Pythagoras' Theorem
- The Converse of Pythagoras' Theorem

Why study this chapter?

As the fundamental knowledge for solving problems involving right-angled triangles. Discuss the fields that involve solving problems related to right-angled triangles.

Right angles exist in many objects around us. In the construction of buildings, how does a civil engineer ensure that the corners of the walls of the building are built at right angles?

Walking through Time

Pythagoras

Pythagoras (569 B.C. – 475 B.C.) was a mathematician and also a philosopher who had contributed substantially to the development of mathematics today. He was the first person who proved the Pythagoras' theorem.

For more information:

http://goo.gl/r4JZ

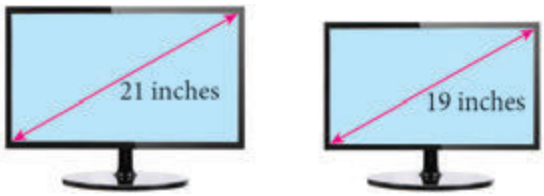
Word Link

• converse of Pythagoras' theorem	• *akas teorem Pythagoras*
• hypotenuse	• *hipotenus*
• Pythagoras' theorem	• *teorem Pythagoras*

Open the folder downloaded from page vii for the audio of Word Link.

13.1 The Pythagoras' Theorem

What is a hypotenuse?



LEARNING STANDARDS

Identify and define the hypotenuse of a right-angled triangle.

We are often told about the size of the monitor screen of a computer as 19 inches, 21 inches or 24 inches and so forth. The size is measured according to the length of the monitor diagonally. What is the relationship between the size of the monitor screen and the length and width of the monitor screen?

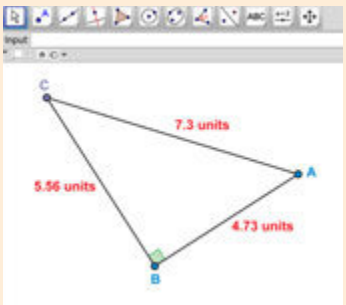
Exploration Activity 1

Group

Aim: To identify the hypotenuse of a right-angled triangle.

Instruction:
- Perform the activity in groups of four.
- Open the folder downloaded from page vii.

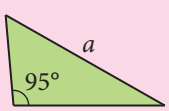
1. Open the file *Hypotenuse.ggb* using *GeoGebra*. The screen displayed shows a right-angled triangle with the length of each side.
2. Identify and record the longest side.
3. Click and drag points *A*, *B* or *C* to change the shape of the triangle and repeat the exploration in Step 2.
4. Discuss your findings with your friends.

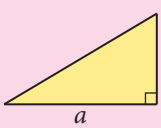


From the results of Exploration Activity 1, it is found that the longest side of a right-angled triangle is always the side opposite to the right angle.

The longest side opposite to the right angle is known as the **hypotenuse** of the right-angled triangle.

Let's Discuss

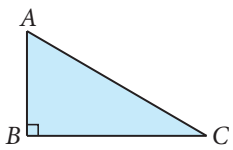


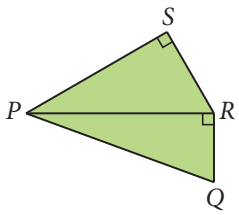


Discuss with your friends and explain why the side labelled as *a* is not the hypotenuse.

Example 1

For each of the following, identify the hypotenuse.

(a) 

(b) 

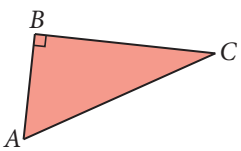
Solution

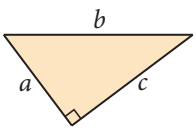
(a) AC is the hypotenuse. ← The side opposite to the right angle.

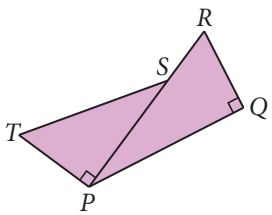
(b) PR is the hypotenuse of triangle PSR.
PQ is the hypotenuse of triangle PRQ.

Self Practice 13.1a

1. For each of the following, identify the hypotenuse.

(a) 

(b) 

(c) 

What is the relationship between the sides of a right-angled triangle?

LEARNING STANDARDS

Determine the relationship between the sides of a right-angled triangle. Hence, explain the Pythagoras' theorem by referring to the relationship.

Exploration Activity 2

Flip Classroom

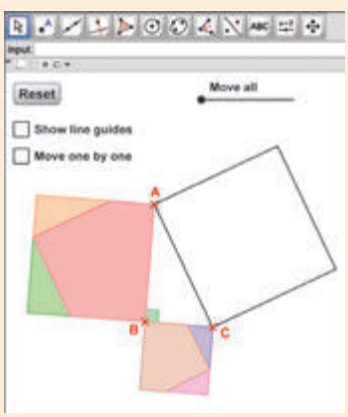
Aim: To explore and explain the Pythagoras' theorem.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Pythagoras.ggb* using *GeoGebra*. The screen displayed shows a right-angled triangle ABC with a square drawn on each side of the triangle.

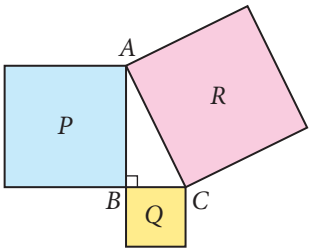
2. Click and drag the coloured shapes in the square on sides AB and BC, and move them into the square on side AC. Do all the coloured shapes fill up the square on side AC perfectly?

3. Click and drag the slider 'Move all' or click at the checkboxes 'Show line guides' and 'Move one by one' for help.



4. Click and drag points A, B and C to change the shape of the right-angled triangle and repeat your exploration.
5. Discuss your findings with your friends.
6. Consider the area of a square, state the relationship between sides AB, BC and AC.

From the results of Exploration Activity 2, it is found that the area of the square on the hypotenuse is equal to the total area of the squares on the other two sides.



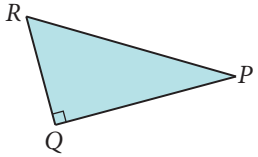
Area of R = Area of P + Area of Q

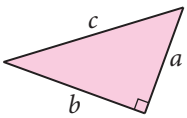
$AC^2 = AB^2 + BC^2$

This relationship is known as the **Pythagoras' theorem**.

Example 2

For each of the following, state the relationship between the lengths of sides of the given right-angled triangle.

(a) 

(b) 

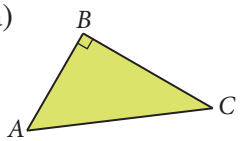
Solution

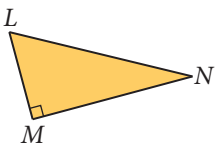
(a) $PR^2 = QR^2 + PQ^2$

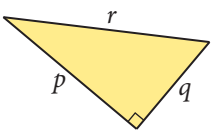
(b) $c^2 = a^2 + b^2$

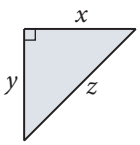
Self Practice 13.1b

1. For each of the following, state the relationship between the lengths of sides of the given right-angled triangle.

(a) 

(b) 

(c) 

(d) 

How do you determine the length of the unknown side of a right-angled triangle?

Pythagoras' theorem can be used to determine the length of an unknown side in a right-angled triangle if the lengths of two other sides are given.

LEARNING STANDARDS

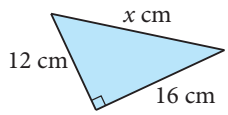
Determine the lengths of the unknown side of
(i) a right-angled triangle.
(ii) combined geometric shapes.

CHAPTER 13

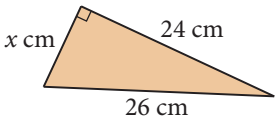
Example 3

For each of the following, calculate the value of x.

(a)



(b)



Solution

(a) $x^2 = 12^2 + 16^2$
$= 144 + 256$
$= 400$
$x = \sqrt{400}$
$= 20$

(b) $26^2 = x^2 + 24^2$
$x^2 = 26^2 - 24^2$
$= 676 - 576$
$= 100$
$x = \sqrt{100}$
$= 10$

Smart

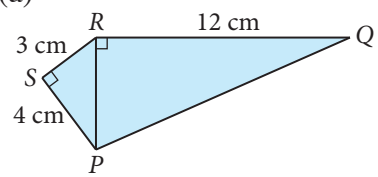
The length of the hypotenuse can be calculated using the Pol function. For instance, Example 3(a),

press [Pol(] [1] [2] [,] [1] [6] [)] [=]

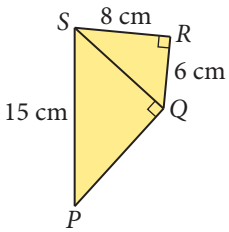
Example 4

Calculate the length of PQ in each of the following diagrams.

(a)



(b)



Did You Know

Three numbers a, b and c that satisfy $c^2 = a^2 + b^2$ are known as a Pythagorean triples (or triplets). For example, (3, 4, 5), (5, 12, 13), (7, 24, 25) and so forth.

Solution

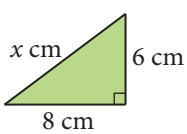
(a) $PR^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$
$PQ^2 = PR^2 + RQ^2$
$= 25 + 12^2$
$= 169$
$PQ = \sqrt{169}$
$= 13$ cm

(b) $QS^2 = 6^2 + 8^2$
$= 36 + 64$
$= 100$
$PS^2 = QS^2 + PQ^2$
$PQ^2 = PS^2 - QS^2$
$= 15^2 - 100$
$= 125$
$PQ = \sqrt{125}$
$= 11.18$ cm (2 decimal places)

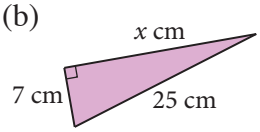
Self Practice 13.1c

1. For each of the following, calculate the value of x. Give your answer correct to two decimal places if necessary.

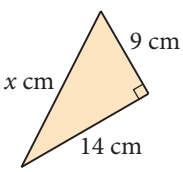
(a)



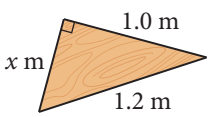
(b)



(c)

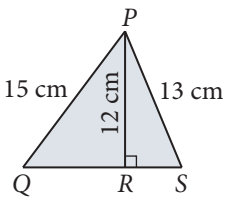


(d)

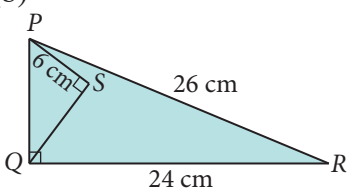


2. Calculate the length of QS in each of the following diagrams. Give your answer correct to two decimal places if necessary.

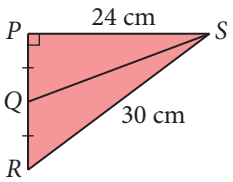
(a)



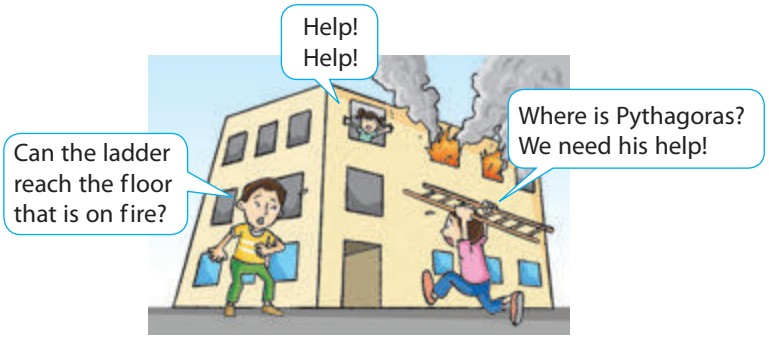
(b)



(c)



How do you solve problems?



LEARNING STANDARDS

Solve problems involving the Pythagoras' theorem.

MATHEMATICS APPLICATION ZONE

A fireman climbs up a ladder to save a child who is trapped on the third floor as shown in the diagram. The third floor is 6 m high from the horizontal ground. The base of the ladder is 4.5 m away from the wall of the building. How long is the ladder?

Solution

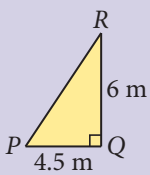
Understanding the problem

- Distance of the third floor from the horizontal ground = 6 m
- Distance of the base of the ladder from the building = 4.5 m
- Find the length of the ladder.

Devising a plan

- Draw a right-angled triangle PQR to represent the given information.
- Use Pythagoras' theorem.

Implementing the strategy



$$PR^2 = PQ^2 + QR^2$$
$$= 4.5^2 + 6^2$$
$$= 20.25 + 36$$
$$= 56.25$$
$$PR = 7.5 \text{ m}$$

Thus, the length of the ladder is 7.5 m.

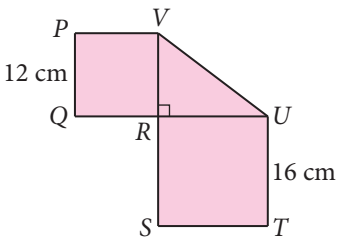
Doing reflection

$7.5^2 = 56.25$

$4.5^2 + 6^2 = 56.25$

Example 5

In the diagram, $PVRQ$ and $RUTS$ are squares. Calculate the perimeter of the whole diagram.



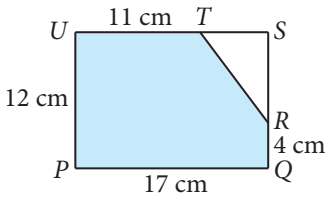
Solution

$VU^2 = VR^2 + RU^2$
$= 12^2 + 16^2$
$= 400$
$VU = \sqrt{400}$
$= 20$ cm

Perimeter of the whole diagram
$= 20 + 16 + 16 + 16 + 12 + 12 + 12$
$= 104$ cm

Self Practice 13.1d

1. In the diagram, $PQSU$ is a rectangle. Calculate the perimeter of the shaded region.

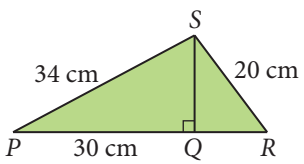


2. Ship A is at 34 km north of ship B. Ship C is at 10 km west of ship A. Calculate the distance between ship B and ship C, correct to two decimal places.

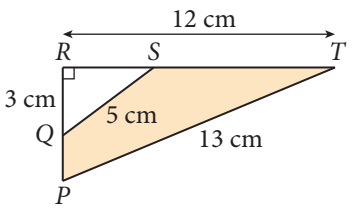
Mastery Q 13.1

Open the folder downloaded from page vii for extra questions of Mastery Q 13.1.

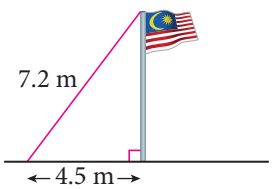
1. In the diagram, PQR is a straight line. Calculate the length of QR.



2. In the diagram, PQR and RST are straight lines. Calculate the perimeter of the shaded region.



3. One end of a rope of length 7.2 m is tied to the tip of a flag pole. The other end of the rope is tied to a spot on the horizontal ground 4.5 m away from the base of the flag pole. Calculate the height of the flag pole and give your answer correct to two decimal places.



4. A ship departs from point O and sails towards southwest for a distance of 300 km and then towards northwest for a distance of 450 km. Calculate the final distance of the ship from point O and give your answer correct to two decimal places.

13.2 The Converse of Pythagoras' Theorem

LEARNING STANDARDS

Determine whether a triangle is a right-angled triangle and give justification based on the converse of the Pythagoras' theorem.

How do you determine whether a triangle is a right-angled triangle?

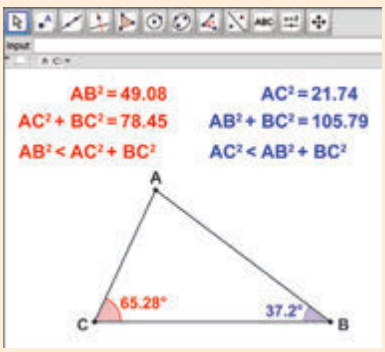
Exploration Activity 3

Flip Classroom

Aim: To explore the converse of Pythagoras' theorem.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Converse of Pythagoras.ggb* using *GeoGebra*. The screen displayed shows a triangle ABC with angles at vertex B and vertex C.
2. Click and drag point A towards left or right and observe the change in the information displayed in red colour. Copy and record your observations in the following table for a few sets of values. Click and drag point B or C to change the shape of the triangle if necessary.

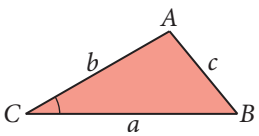


Comparative value (red)	Size of angle (red)
$AB^2 > AC^2 + BC^2$	
$AB^2 < AC^2 + BC^2$	
$AB^2 = AC^2 + BC^2$	

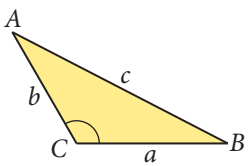
3. Repeat Step 2 for the information displayed in blue colour.
4. Discuss your findings with your friends.
5. What are the conclusions that can be made?

Flashback
- An acute angle is an angle less than 90°.
- An obtuse angle is an angle more than 90° but less than 180°.

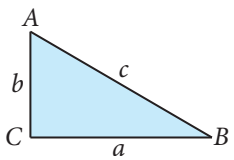
From the results of Exploration Activity 3, it is found that



If $c^2 < a^2 + b^2$, then the angle opposite to side c is an **acute angle**.



If $c^2 > a^2 + b^2$, then the angle opposite to side c is an **obtuse angle**.

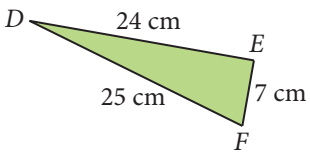


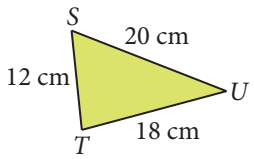
If $c^2 = a^2 + b^2$, then the angle opposite to side c is a **right angle**.

The **converse of Pythagoras' theorem** states that:
If $\boldsymbol{c^2 = a^2 + b^2}$, then the angle opposite to side c is a **right angle**.

Example 6

Determine whether each of the following triangles is a right-angled triangle.

(a) 

(b) 

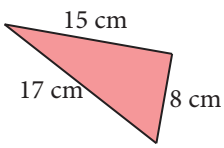
Solution

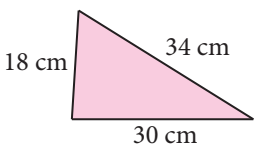
(a) The longest side = 25 cm
Thus, $25^2 = 625$
$24^2 + 7^2 = 576 + 49$
$= 625$
Hence, *DEF* is a right-angled triangle.

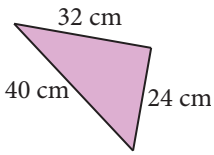
(b) The longest side = 20 cm
Thus, $20^2 = 400$
$18^2 + 12^2 = 324 + 144$
$= 468$
Hence, *STU* is not a right-angled triangle.

Self Practice 13.2a

1. Determine whether each of the following triangles is a right-angled triangle.

(a) 

(b) 

(c) 

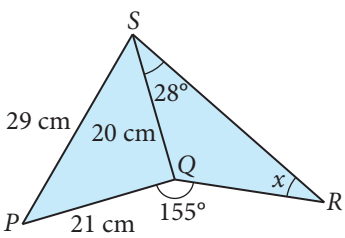
How do you solve problems?

LEARNING STANDARDS

Solve problems involving the converse of the Pythagoras' theorem.

Example 7

In the diagram, calculate the value of x.



Solution

20 cm, 21 cm and 29 cm satisfy $c^2 = a^2 + b^2$.
Thus, $\angle PQS = 90°$
$\angle SQR = 360° - 90° - 155°$
$= 115°$
$x = 180° - 115° - 28°$
$= 37°$

Career in Mathematics

A housing contractor and a civil engineer will use the Pythagoras' theorem to solve problems involving right angles in the construction of buildings.

Example 8

Sheila is given three straws to form a frame in the shape of a right-angled triangle. The straws are 15 cm, 20 cm and 25 cm long respectively. Will she be able to form the frame in the shape of a right-angled triangle?

Solution

$15^2 + 20^2 = 225 + 400$

$= 625$

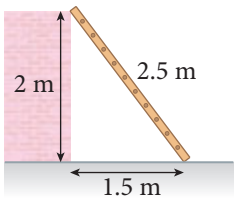
$25^2 = 625$

$15^2 + 20^2 = 25^2$

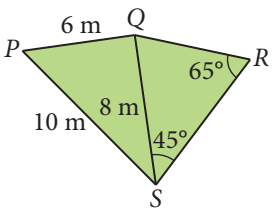
Therefore, Sheila will be able to form the frame in the shape of a right-angled triangle.

Self Practice 13.2b

1. A 2.5 m long ladder leans against the wall of a building. The base of the ladder is 1.5 m away from the wall. Explain how you would determine whether the wall is vertical.



2. In the diagram, find $\angle PQR$.



Mastery Q 13.2

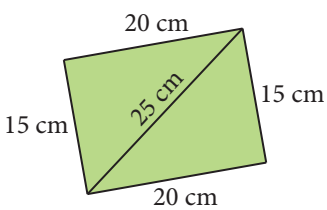
Open the folder downloaded from page vii for extra questions of Mastery Q 13.2.

1. Explain whether the lengths of sides in each of the following can form a right-angled triangle:

(a) 9 cm, 40 cm, 41 cm
(b) 27 m, 45 m, 35 m
(c) 2.5 cm, 6 cm, 6.5 cm
(d) 13 m, 84 m, 85 m

2. A carpenter wishes to fix a triangular piece of wood measuring 12 cm, 16 cm and 20 cm onto a L-shaped wooden structure, as shown in the diagram. Explain whether the triangular piece of wood can be fixed perfectly onto the L-shaped structure.

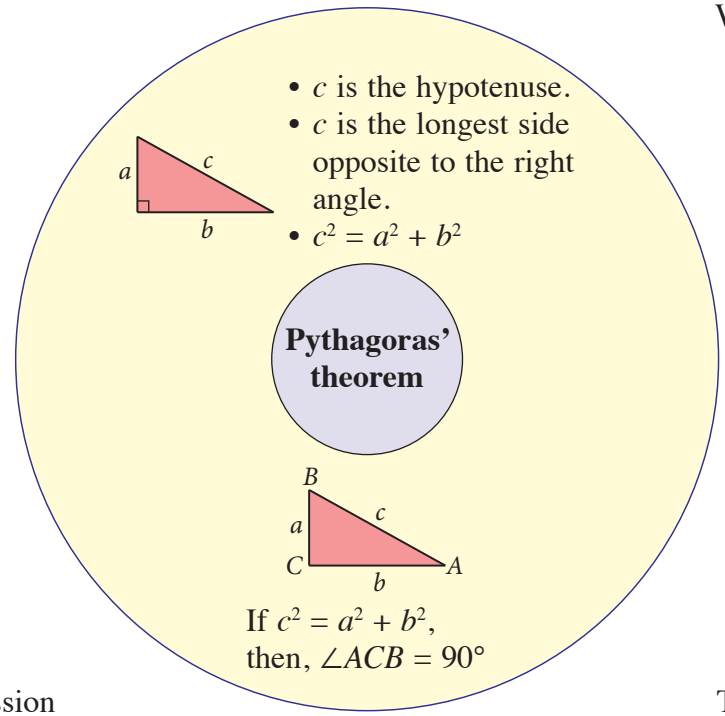
3. Kanang draws a quadrilateral with measurements as shown in the diagram. What is the name of the quadrilateral he has drawn? Explain.



SUMMARY

Book

Website



Discussion

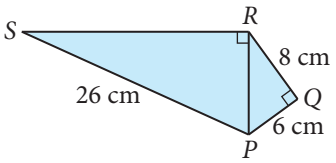
Teacher

At the end of this chapter, I can...	Very good	Work harder
identify and define the hypotenuse of a right-angled triangle.		
determine the relationship between the sides of a right-angled triangle. Hence, explain the Pythagoras' theorem by referring to the relationship.		
determine the lengths of the unknown side of (i) a right-angled triangle. (ii) combined geometric shapes.		
solve problems involving the Pythagoras' theorem.		
determine whether a triangle is a right-angled triangle and give justification based on the converse of the Pythagoras' theorem.		
solve problems involving the converse of the Pythagoras' theorem.		

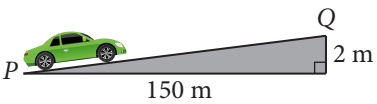
Let's PRACTISE

Test Yourself

1. In the diagram, calculate the length of
(a) PR (b) SR

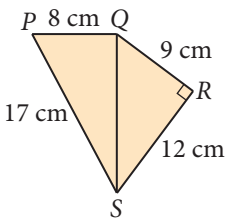


2. A car moves up a slope from P to Q. When the car reaches Q, the horizontal distance and the vertical distance it has covered are 150 m and 2 m respectively. Explain how you would calculate the actual distance the car has moved, correct to two decimal places.

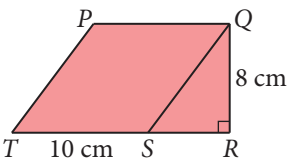


Self Mastery

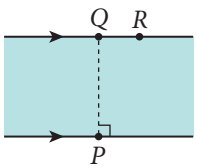
3. Based on the diagram,
(a) calculate the length of QS.
(b) explain whether PQS is a right-angled triangle.



4. In the diagram, $PQST$ is a rhombus and TSR is a straight line. Calculate the area of the whole diagram.

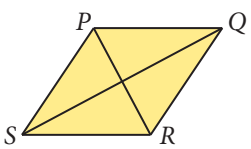


5. The width of a river is 18 m. Imran swims across the river from point P to point Q, as shown in the diagram. Due to strong water currents, Imran eventually lands at point R which is 6 m away from Q. Explain how you would calculate the actual distance Imran has swum, correct to two decimal places.

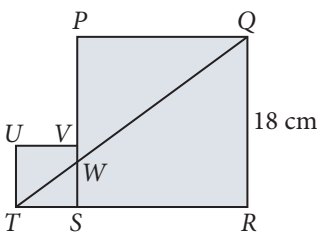


Challenge Yourself

6. In the diagram, $PQRS$ is a rhombus. PR and SQ are 16 cm and 30 cm long respectively. Explain how you would calculate the length of side SR.

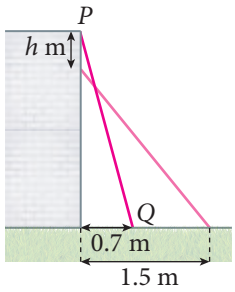


7. In the diagram, $PQRS$ and $UVST$ are squares. Given TQ = 30 cm, find the area of $UVST$.

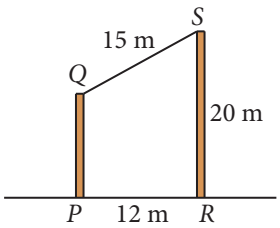


CHAPTER 13

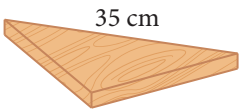
8. The diagram shows a ladder PQ leaning against the wall. The ladder is 2.5 m long and the base of the ladder is 0.7 m away from the wall. When the top of the ladder slides down a distance of h m, the base of the ladder becomes 1.5 m away from the wall. Find the value of h.



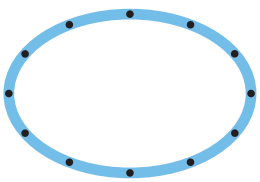
9. The distance between two poles, PQ and RS, on a horizontal ground is 12 m. A piece of 15 m long string is tied to both tips of the poles. If the height of pole RS is 20 m, explain how you would find the height of pole PQ.



10. Ali wishes to saw a piece of wood into the shape of a right-angled triangle. The hypotenuse of the wood must be 35 cm long and the lengths of the two other sides of the wood must be in the ratio 3 : 4. Explain how you would find the lengths of the other two sides and hence help Ali saw the wood.



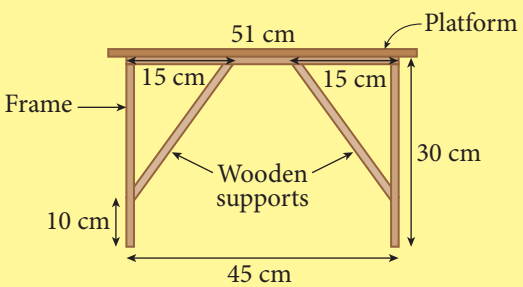
11. A loop of thread is marked with 12 points so that the adjacent points are of the same distance from one another. Explain how you would form a right-angled triangle using the loop of thread.



ASSIGNMENT

Making a wooden stool

A carpenter wishes to make a wooden stool according to the design shown in the diagram.



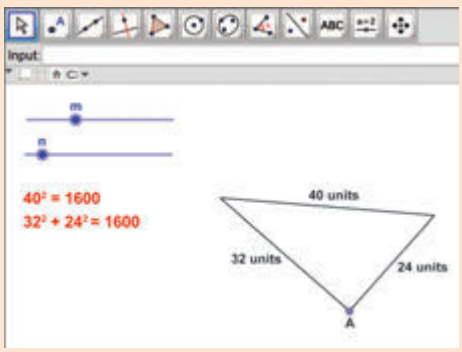
The wooden stool he wants to make consists of three parts. Part 1 is the frame; part 2 is the platform top for sitting on while part 3 are the supports for the corners of the frame. Each end of the supports is fixed at 15 cm from the corners of the frame, and the other end is fixed at 10 cm from the legs of the wooden stool. The platform is 51 cm long. Explain how you would use the Pythagoras' theorem to help the carpenter make the wooden stool.

Exploring MATHEMATICS

A The lengths of the three sides of a right-angled triangle such as (3, 4, 5) and (8, 15, 17) are known as Pythagorean triples.
You can explore the Pythagorean triples through the following activity.

Open the folder downloaded from page vii for this activity.

1. Open the file *Triples.ggb* using *GeoGebra*.
2. Is the triangle shown a right-angled triangle?
3. Click and drag the slider *m* and the slider *n* and observe the changes on the screen displayed.
4. Do the lengths of sides shown constitute a set of triples?
5. Click and drag the slider *m* and the slider *n* for other combinations.
6. Explain what you have observed.
7. Present your findings in class.



B Constructing a Pythagorean Tree

A Pythagorean tree is a pattern constructed based on right-angled triangles and squares related to Pythagoras' theorem. The Pythagorean tree was invented by a Dutch mathematics teacher in 1942.

Construct your own Pythagorean tree beginning from a right-angled triangle with squares drawn on each side of the triangle. Then similar triangles are drawn on the sides of the squares. Then squares are drawn on the sides of the new triangles and so on.

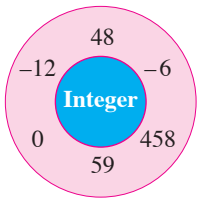
ANSWERS

Chapter 1 Rational Numbers

Self Practice 1.1a

1. (a) +1 000 m, –250 m
   (b) +RM2 000, –RM500

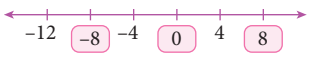
Self Practice 1.1b

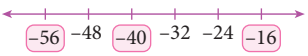
1. ✗ ✓ ✓ ✗ ✓ ✓ ✗ ✓ ✗ ✓
2. 

Self Practice 1.1c

1. (a)


(b)


2. (a)


(b)


Self Practice 1.1d

1. –6, –4, –2, 0, 1, 3, 5
2. 4, 3, 2, –1, –3, –4, –5

Mastery Q 1.1

1. (a) 20 m below sea level
   (b) a movement of 90 m to the south
   (c) –800
   (d) –1 000
2. (a) –80
   (b) +76
3. (a) –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4
   (b) –12, –11, –10, –9, –8, –7, –6, –5, –4, –3, –2
4. Integer: –14, 12, –26, 85, 0, –2
   Not integers: 3.9
5. –4 °C, –3 °C, 1 °C, 2 °C, 4 °C

Self Practice 1.2a

1. (a) 8 (b) –7
   (c) 1 (d) 2
   (e) –10 (f) 3
   (g) 13 (h) –2

Self Practice 1.2b

1. (a) 18 (b) –14
   (c) –32 (d) –48
   (e) –4 (f) 3
   (g) –3 (h) –5

Self Practice 1.2c

1. (a) –18 (b) –24
   (c) 1 (d) –3
   (e) 39 (f) $3\frac{1}{11}$

Self Practice 1.2d

1. (a) 426 (b) 56 700
   (c) 452 120 (d) 1 380
   (e) 6 (f) 4 495

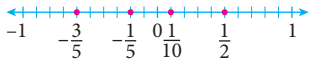
Self Practice 1.2e

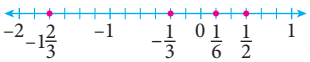
1. A loss of RM9 200
2. (a) 26°C (b) 32°C

Mastery Q 1.2

1. –12 + (–2);
   6 × (–2) – 2;
   5 – 11 – 8;
   –2 × (–3) – 15 – 5;
   8 × (–2) + 2;
   or other valid answers
2. (a) × , –
   (b) ÷ , –
3. (a) –5, –1, 3
   (b) –8, 64, –128
4. (a) –23°C
   (b) –11°C
5. Below sea level = –50 m
   After 2 minutes
   $= -50 + \frac{120}{5} \times 2$
   = –2 m
   The diver had not reached the sea surface after 2 minutes.
6. (a) Cheque valued at RM1 730
   (b) RM1 382

Self Practice 1.3a

1. (a)


(b)


Self Practice 1.3b

1. (a) $-\frac{5}{6}, -\frac{5}{12}, -\frac{1}{4}, \frac{7}{24}, \frac{3}{8}$
   (b) $-\frac{5}{6}, -\frac{13}{18}, -\frac{2}{3}, -\frac{15}{24}, \frac{1}{3}, \frac{5}{8}$
2. (a) $\frac{5}{6}, \frac{3}{5}, -\frac{1}{8}, -\frac{7}{20}, -\frac{5}{12}$
   (b) $\frac{11}{18}, \frac{2}{9}, -\frac{7}{18}, -\frac{1}{2}, -\frac{5}{9}, -\frac{7}{12}$

Self Practice 1.3c

1. (a) $1\frac{13}{120}$ (b) $-4\frac{13}{18}$
   (c) $\frac{7}{12}$ (d) $7\frac{2}{7}$
   (e) $1\frac{7}{8}$ (f) $-2\frac{71}{240}$

Self Practice 1.3d

1. $22\frac{1}{2}$ cups
2. RM1 080

Mastery Q 1.3

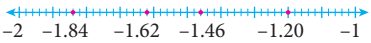
1. $\frac{1}{5} - \frac{4}{5} + \frac{1}{10}$;
   $\frac{3}{10} \div \frac{2}{5} \times \left(-\frac{2}{3}\right)$;
   $\frac{3}{2} \div \frac{6}{5} - 1\frac{3}{4}$;
   or other valid answers
2. (a) $\frac{2}{3}$ (b) $\frac{5}{216}$
3. (a) $-2\frac{1}{3}$ (b) –8
4. $3\frac{7}{30}$ m
5. 128 m*l*

Self Practice 1.4a

1. (a)

(b)

2. (a)



(b)

Self Practice 1.4b

1. (a) −1.48, −1.23, −0.034, 0.34, 1.034

(b) −1.654, −1.546, −1.456, 1.456, 1.564

2. (a) 2.522, 2.452, −2.005, −2.052, −2.505

(b) 0.621, 0.065, −0.068, −0.639, −0.647

Self Practice 1.4c

1. (a) 2.36 (b) −43.75
(c) 1.68 (d) −27.72
(e) 1.77 (f) 2.23

Self Practice 1.4d

1. RM19.85

2. 30.2°C

Mastery Q 1.4

1. 1.2 + 1.5 − 5.2;
0.4 − 2.1 + (−0.8);
−0.2 × 4.5 ÷ 0.36;
or other valid answers

2. (a) −0.6, 0 (b) 4.2, −33.6

3. (a) −1.84 (b) −6.2

4. (a) RM0.70 (b) RM4.10

5. 15.64 m

Self Practice 1.5a

1. $\frac{-2}{4}, \frac{8}{7}, \frac{-12}{15}, \frac{153}{20}, \frac{12}{5}, \frac{-21}{5}$

Thus, $\frac{-2}{4}, \frac{8}{7}, \frac{-1.2}{1.5}$, 7.65, $2\frac{2}{5}$, −4.2 are rational numbers.

Self Practice 1.5b

1. (a) $-1\frac{4}{5}$ (b) $\frac{1}{16}$

(c) $3\frac{409}{648}$ (d) $-3\frac{5}{24}$

Self Practice 1.5c

1. RM10.05 million

2. 0.925 m

Mastery Q 1.5

1. (a) $-20\frac{5}{6}$

(b) $17\frac{11}{12}$

2. (a) −5.7, −6.8

(b) $-\frac{1}{8}$

3. (a) 1 238

(b) $\frac{77}{100}$

4. Jia Kang is at a level 2.2 m lower than Ishak.
Suresh is at a level $1\frac{17}{60}$ m lower than Ishak.

Let's Practise

1. B

2. (a) $\frac{1}{100}$

(b) −4.3

(c) 2.5

3. −2

4. 348 years

5. Mass of alms for each victim
$= 2 + \frac{1}{2} + 0.4$
$= \frac{29}{10}$ kg

Mass of alms in three vans
$= 80 \times \frac{29}{10}$
= 232 kg

Mass of alms in a van
= 232 ÷ 3
= 77.33 kg

6. (a) −, +
(b) +, −

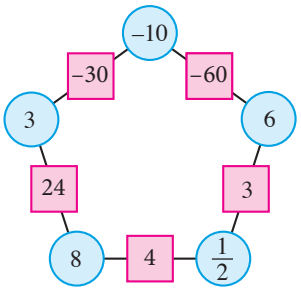
7. −7°C

8. 0.75 m towards right of *O*

9. Moves 5.6 m towards west

10. 14.7 m below level H

11.



or other valid answers

Chapter 2 Factors and Multiples

Self Practice 2.1a

1. (a) No (b) Yes
(c) Yes (d) Yes
(e) Yes (f) Yes
(g) Yes (h) No

2. (a) 1, 3, 5, 15
(b) 1, 2, 4, 8, 16, 32
(c) 1, 2, 4, 5, 8, 10, 20, 40
(d) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
(e) 1, 3, 17, 51
(f) 1, 3, 29, 87
(g) 1, 2, 7, 14, 49, 98
(h) 1, 2, 4, 31, 62, 124

Self Practice 2.1b

1. (a) 3 and 5 are prime factors of 30.
7 is not a prime factor of 30.
(b) 3 is a prime factor of 54.
5 and 9 are not the prime factors of 54.

2. (a) 2, 3 (b) 2, 3
(c) 2, 29 (d) 3, 11

3. (a) 42 = 2 × 3 × 7
(b) 96 = 2 × 2 × 2 × 2 × 2 × 3
(c) 120 = 2 × 2 × 2 × 3 × 5
(d) 135 = 3 × 3 × 3 × 5

Self Practice 2.1c

1. (a) Yes (b) Yes
(c) No (d) Yes
(e) Yes (f) No

2. (a) 1, 2, 3, 6
(b) 1, 2, 3, 4, 6, 8, 12, 24
(c) 1, 5, 7, 35
(d) 1, 2
(e) 1, 5
(f) 1, 2, 3, 4, 6, 12
(g) 1, 2
(h) 1, 2, 3, 4, 6, 12
(i) 1, 2, 5, 10

Self Practice 2.1d

1. (a) 24 (b) 18
(c) 12 (d) 6
(e) 18 (f) 6
(g) 4 (h) 3

Self Practice 2.1e

1. 6 boxes
2. 15 plates

Mastery Q 2.1

1. 15
2. $1\,968 = 2 \times 2 \times 2 \times 2 \times 3 \times 41$
$4\,968 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 23$
HCF = 24
3. 4, 8, 16, 20, 28, 32
4. 2 segments
5. 20 cm

Self Practice 2.2a

1. (a) Yes (b) No
(c) Yes (d) Yes
(e) Yes (f) Yes
2. (a) 10, 20, 30, 40, 50
(b) 15, 30, 45, 60, 75
(c) 198, 396, 594, 792, 990
(d) 120, 240, 360, 480, 600
(e) 24, 48, 72, 96, 120
(f) 120, 240, 360, 480, 600
(g) 120, 240, 360, 480, 600
(h) 180, 360, 540, 720, 900

Self Practice 2.2b

1. (a) 144 (b) 30
(c) 24 (d) 180
(e) 90 (f) 224
(g) 252 (h) 60

Self Practice 2.2c

1. 36 seconds
2. 30 pieces

Mastery Q 2.2

1. 40
2. 72
3. 24 days
4. 150 cm

Let's Practise

1. 60
2. 2
3. 60
4. 48
5. 27
6. These two numbers do not have common factors except 1. For instance, 2 and 3.
7. 6 and 60
8. 5:15 p.m.
9. 30 cm × 30 cm
10. The second Saturday
11. (a) 6 pages
(b) 4 photographs and 7 newspaper cuttings.

Chapter 3 Squares, Square Roots, Cubes and Cube Roots

Self Practice 3.1a

1. (a) No (b) Yes
(c) No (d) Yes

Self Practice 3.1b

1. (a) $\sqrt{5 \times 5} = 5$
(b) $\sqrt{8 \times 8} = 8$
(c) $\sqrt{24^2} = 24$

Self Practice 3.1c

1. (a) 64 (b) $\frac{25}{36}$
(c) 1.96
2. (a) 841 (b) $\frac{81}{121}$
(c) 234.09

Self Practice 3.1d

1. (a) 9 (b) 7
(c) 11 (d) 30
(e) $\frac{7}{9}$ (f) $2\frac{2}{3}$
(g) $\frac{5}{8}$ (h) 1.5

Self Practice 3.1e

1. (a) 6.56 (b) 6.15
(c) 0.68 (d) 3.58

Self Practice 3.1f

1. (a) 3 600 (b) 400
(c) 81 (d) 0.04
(e) 6 (f) 4
(g) 11 (h) 0.9

Self Practice 3.1g

1. (a) 92 cm to 110 cm
(b) Enough. It is because the maximum length of the white lace needed is 4.4 m

Mastery Q 3.1

1. (a) No
(b) No
(c) Yes
1024: $(2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2)$

2. $100 = (2 \times 5) \times (2 \times 5)$
$\sqrt{100} = 10$

3. 6, 10, 14, 19, 22

4. (a) 36 (b) $\frac{4}{49}$
(c) $\frac{169}{9}$ (d) 65.61
(e) 19 (f) $\frac{3}{7}$
(g) $\frac{8}{5}$ (h) 1.1

5. (a) 16 129 (b) 1 197.16
(c) 0.009409 (d) $\frac{441}{64}$
(e) 8.72 (f) 10.41
(g) 0.63 (h) 1.61

6. Length of side of the base of the pyramid
$= \sqrt{52\,900}$
$= 230$ m

7. (a) 90 000 (b) 2 500
(c) 0.0016 (d) 64
(e) 4 (f) 15
(g) 3 (h) 0.7

8. (a) 10 (b) 6, 8

Self Practice 3.2a

1. (a) $27 = 3 \times 3 \times 3$
27 is a perfect cube.
(b) $45 = 3 \times 3 \times 5$
45 is not a perfect cube.
(c) $215 = 5 \times 43$
215 is not a perfect cube.
(d) $343 = 7 \times 7 \times 7$
343 is a perfect cube.

Self Practice 3.2b

1. (a) $\sqrt[3]{8 \times 8 \times 8}$
$= 8$

(b) $\sqrt[3]{0.3 \times 0.3 \times 0.3}$
$= 0.3$

(c) $\sqrt[3]{\left(-\frac{1}{2}\right)^3}$
$= -\frac{1}{2}$

Self Practice 3.2c

1. (a) 216 (b) –343
(c) $-\frac{8}{729}$ (d) –0.027
(e) $\frac{2\,197}{125}$

2. (a) 17 576 (b) –132.651
(c) 0.027 (d) $-\frac{5\,832}{1\,331}$
(e) $\frac{13\,824}{125}$

Self Practice 3.2d

1. 21

2. 14

3. (a) 3 (b) –5
(c) 7 (d) –10

4. (a) $\frac{2}{5}$ (b) $-\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{5}{4}$
(e) 0.1 (f) –0.4
(g) –0.6 (h) 0.07

Self Practice 3.2e

1. (a) 2.47 (b) –4.20
(c) 5.48 (d) 0.92
(e) –1.12

Self Practice 3.2f

1. (a) 8
(b) –1 000
(c) 8 000
(d) –64 000

2. (a) 2 (b) 4
(c) –5 (d) –3

Self Practice 3.2g

1. Not enough.
The length of the wire needed is 156 cm.

Self Practice 3.2h

1. (a) 16 (b) 0.75
(c) $-\frac{4}{125}$ (d) 0.4
(e) –1 (f) 0.018
(g) $-\frac{9}{4}$ (h) $\frac{56}{3}$
(i) –3

Mastery Q 3.2

1. (a) No
(b) Yes
$343 = 7 \times 7 \times 7$
(c) Yes
$1\,000 = 2\times5 \times 2\times5 \times 2\times5$

2. $3\,375 = 3\times5 \times 3\times5 \times 3\times5$
$\sqrt[3]{3\,375} = 15$

3. (a) –125 (b) $\frac{64}{125}$
(c) $-\frac{343}{216}$ (d) –32.768
(e) 5 (f) –8
(g) 9 (h) –30
(i) $\frac{2}{5}$ (j) $-\frac{4}{7}$
(k) –0.8 (l) 1.1

4. (a) 8 242 408
(b) –5 451.776
(c) 0.000068921
(d) $-\frac{4\,913}{343}$
(e) 3.26 (f) 6.00
(g) –0.98 (h) –0.86

5. (a) 27 (b) –1 000
(c) 3 375 (d) –1
(e) 4 (f) 7
(g) –9 (h) 2

6. (a) 50 mm (b) 50^3

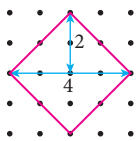
7. (a) 2.09 (b) –80
(c) $\frac{24}{25}$ (d) 12
(e) –225 (f) $-\frac{19}{21}$

Let's Practise

1. 4, 81, 49

2. $\sqrt{\frac{36}{25}} - (-0.1)^3$
$= \frac{6}{5} - (-0.001)$
$= \frac{1\,201}{1\,000}$

3.



Area of the square
$= 2 \times \frac{1}{2} \times 4 \times 2$
$= 8$ unit^2
Length of side of the square
$= \sqrt{8}$ unit

4. $400 = 2 \times 2 \times 5 \times 2 \times 2 \times 5$
Length of side $= 2 \times 2 \times 5$
$= 20$ m

5. (a) $512 = 2\times2\times2 \times 2\times2\times2 \times 2\times2\times2$
Prime factors can be grouped into three identical groups.

(b) $\sqrt{512}$
$= 2\times2\times2\times2\times2\times2\times2\times2\times2$
Prime factors cannot be grouped into two identical groups.

6. 6 m

7. (a) 10 cm
(b) 100 cm^2

8. (a) (i) RM3.20
(ii) RM20.00

(b) Number of 20 sen coins with an amount of RM60
$= \frac{60}{0.2}$
$= 300$
300 is between 289 and 324, that is, $\sqrt{300}$ is between $\sqrt{289}$ and $\sqrt{324}$.
$\sqrt{300}$ is between 17 and 18.
Therefore, the measurement of the largest square that can be arranged is 17 × 17.

9. (a) 112
(b) No.
112 is not a perfect square.

10. 10, 13, 17

Chapter 4 Ratios, Rates and Proportions

Self Practice 4.1a

1. (a) 14 : 16 : 7
(b) 2 : 1 : 5
(c) 2 : 1 : 48
(d) 2 : 3 : 4

2. 56 : 12 : 3

Self Practice 4.1b

1. 36 : 48 : 90,
0.6 : 0.8 : 1.5,
$\frac{2}{5} : \frac{8}{15} : 1$

2. (a) 1 : 3 and 2 : 6
(b) 3 : 8, 6 : 16 and 9 : 24

3. 2 : 4 : 6, 1 : 2 : 3
or other valid answers.

Self Practice 4.1c

1. (a) 3 : 20 (b) 5 : 7 : 8
(c) 9 : 20 (d) 3 : 4 : 8

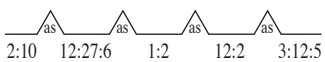
Mastery Q 4.1

1. (a) (i) 2 : 3 : 4
(ii) 8 : 12 : 16
(iii) 4 : 9 : 16
(b) 2 : 3 : 4 is equivalent to 8 : 12 : 16.
(c) The numbers in the ratio of areas are the squares of the numbers in the ratio of lengths.

2. 25 : 1

3. (a) 1 Amanah and 1 Cekap
(b) 16 : 20 : 12
(c) 12 : 7

4.



or other valid answers.

5. (a) 5 : 9 is equivalent to 15 : 27
(b) 5 : 12 is equivalent to 10 : 24 and 15 : 36

6. 6 : 4 : 2, 18 : 12 : 6
or other valid answers.

Self Practice 4.2a

1. (a) Rate = $\frac{\text{RM154}}{\text{2 passengers}}$;
amount of money (RM) and number of passengers (person).

(b) Rate = $\frac{\text{20 litres}}{\text{1 time}}$;
volume (litre) and number of times the water is pumped.

(c) Rate = $\frac{\text{RM240}}{\text{4 subjects}}$;
amount of money (RM) and number of subjects.

(d) Rate = $\frac{\text{RM500}}{\text{10 hectares}}$;
amount of money (RM) and area (hectare).

(e) Rate = $\frac{\text{600 revolutions}}{\text{3 seconds}}$;
number of revolutions and time (second).

2. Object *A*

3. 30 g per 100 cm^2

4. 35 g per m^2

Mastery Q 4.2

1. 2.7 g per cm^3

2. (a) (i) 1 : 21; 1 : 11
(ii) $\frac{\text{6 kg}}{\text{60 days}}$; $\frac{\text{7 kg}}{\text{60 days}}$
(b) mass and time

3. (a) $\frac{\text{RM2.25}}{\text{250 m}l}$, $\frac{\text{RM4.00}}{\text{500 m}l}$, $\frac{\text{RM7.50}}{\text{1000 m}l}$
(b) RM8.00
(c) 1 *l* carton of milk. It has the lowest price compared to 250 m*l* carton of milk and 500 m*l* carton of milk priced at RM9 per litre and RM8 per litre respectively.

Self Practice 4.3a

1. (a) $\frac{\text{RM5}}{3} = \frac{\text{RM20}}{12}$

(b) $\frac{\text{24 cm}}{4} = \frac{\text{78 cm}}{13}$

(c) $\frac{\text{13 boys}}{\text{15 girls}} = \frac{\text{65 boys}}{\text{75 girls}}$

Self Practice 4.3b

1. 50 players

2. 100 times

3. 510 chilli plants

Mastery Q 4.3

1. (a) $\frac{\text{175 m}l\text{ olive oil}}{\text{50 m}l\text{ vinegar}} = \frac{\text{300 m}l\text{ olive oil}}{x\text{ m}l\text{ vinegar}}$

(b) $\frac{\text{7.8 mm}}{\text{3 hours}} = \frac{\text{11.7 mm}}{t\text{ hours}}$

2. 12 times

3. RM60

Self Practice 4.4a

1. 2 : 9 : 7

2. 8 : 12 : 3

Self Practice 4.4b

1. RM48

2. 63 kg

3. RM200

4. 35 questions

5. 3 : 1 : 2

Self Practice 4.4c

1. 58 calories

2. RM49.00

3. 2 cups

4. USD50

Self Practice 4.4d

1. 300 fish

2. (a) 12 times
(b) 4 : 3
(c) 15 times

Mastery Q 4.4

1. 8 teachers

2. 3 : 4 : 7

3. Hardtail scad and one finlet scad or hardtail scad and Indian mackerel
Pomfret: RM15/kg
Hardtail scad: RM6.40/kg
One finlet scad: RM9.00/kg
Indian mackerel: RM13.50/kg

4. (a) Bacteria *A*, 40 million per minute
(b) 20 million
(c) 3.3 minutes

5. (a) 3 : 4 : 2
(b) Nitrogen: 4.5 kg
Phosphorus: 6 kg
Potassium: 3 kg

Self Practice 4.5a

1. 75%
2. 7 : 13

Self Practice 4.5b

1. (a) 25%
   (b) 40%
   (c) 7 500 000%
2. RM11.25

Self Practice 4.5c

1. (a) 20%
   (b) 18 : 50 or 9 : 25

Mastery 4.5

1. (a) $\frac{3}{8}$; 0.375
   (b) 37.5%
2. (a) 3 : 2
   (b) Same
   (c) The length of each side of $P'Q'R'$ is 1.5 times the length of the corresponding side of PQR.
3. Buying online is a better choice. The online price is 50 sen cheaper than the price offered by the bookshop.

Let's Practise

1. 6 : 9 → 2 : 3; 6 : 90 → 1 : 15; 9 : 60 → 3 : 20
2. (a) 4 : 5
   (b) For aquarium A, add 4 fish. For aquarium B, add 5 fish.
3. (a) 36
   (b) 9 : 8
   (c) Nurin, 84
4. 1.8 l bottle
5. 0900
6. 10 m
7. (a) 0.804 litre
   (b) Current house:
   Total cost of monthly rental and petrol
   = RM338.40
   New house:
   Total cost of monthly rental and petrol
   = RM368.80
   Lai Huat should not consider moving to the new house because he can save RM30.40 while staying in his current house.
8. (a) RM3.60
   (b) Mr Ong should go back to his office to get his parking ticket. The parking charge is RM10.60 compared to the lost ticket penalty of RM20.

Chapter 5 Algebraic Expressions

Self Practice 5.1a

1. (a) k represents the mass of each student in the class. k has a varied value because the mass of each student is different.
   (b) x represents Zaini's marks in a Mathematics test. x has a fixed value because it is Zaini's score for the Mathematics test.
   (c) h represents the distance between Arman's house and his school. h has a fixed value because the distance between Arman's house and his school is the same for every trip.
   (d) c represents the temperature at the peak of Gunung Kinabalu in a day. c has a varied value because the temperature at the peak of Gunung Kinabalu is always changing within a day.

Self Practice 5.1b

1. (a) $x - 7$ (b) $\frac{y+z}{9}$
   (c) $4x$ (d) $mp + nq$
   (e) $h - 2k$

Self Practice 5.1c

1. (a) 12 (b) 30
   (c) 45 (d) 9
2. (a) $8x + 4y$
   (b) 80 kg
3. (a) $p(m - n)$ or $p(n - m)$
   (b) RM3

Self Practice 5.1d

1. (a) $6k$ and $2k$
   (b) x^2 and $9xy$
   (c) $\frac{ab}{3}$, $2a$ and $5b$
   (d) $4pq$, $\frac{7x}{2}$, $8p^2q$ and 1
2. (a) -8 (b) $-y^2$
   (c) $-8x$ (d) $8y^2$

Self Practice 5.1e

1. (a) Like terms
   (b) Unlike terms
   (c) Like terms
   (d) Unlike terms

Mastery 5.1

1. k is the amount of money invested and d is the dividend given.
   k has a fixed value because it is the initial amount invested and d has a varied value because the dividend rate will vary based on economic conditions every year.
2. $\left(\frac{m-5}{2}\right)$ kg
3. $(xy - 225)$ m^2
4. (a) $4\frac{1}{2}$ (b) 142
5. (a) $n - 4x$
   (b) RM22.20
6. (a) $p - 3x$
   (b) 35
7.

Algebraic term	Coefficient	Variable
$-10abc$	$-10ab$	c
	$-10ac$	b
	$-10b$	ac
	-10	abc

   or other possible answers.

Self Practice 5.2a

1. (a) $8x + 7y$
(b) $5ab - bc + 12$
(c) $17xy + 7k - 7$
(d) $6p - 3q - 3pq$
(e) $fg - 6mn$

Self Practice 5.2b

1. (a) $(pq)^3$
(b) $(6a - 1)^2$
(c) $(8x + 3y)^3$

2. (a) $(2 + 7x)(2 + 7x)$
(b) $(h - 4k)(h - 4k)(h - 4k)$
(c) $(5p+q)(5p+q)(5p+q)(5p+q)$

Self Practice 5.2c

1. (a) $15x^4$ (b) $-28m^3n$
(c) $4p^5qr$

2. (a) $4x^5y$ (b) $\frac{2b^2}{3a}$
(c) $-\frac{6p^4r}{5q}$

3. (a) $\frac{10m^3}{3n^2}$ (b) $-\frac{3p^5y^2}{2x}$

Mastery Q 5.2

1. (a) $\frac{4}{3}x + \frac{3}{2}pq - 8y + 11$
(b) $8ab - 11mn$

2. $(2d + 7y)$ cm

3. $4n + 4$

4. $n = 3$, $a = 9$, $b = -2$

5. $6(2 + 3p)^2$ cm^2

6. (a) $\frac{12y^4}{z}$ (b) $2pq^3$

7. (a) $5pr^2$ (b) $14x^3y^3z^2$

8. $\frac{12a^3b^2}{3ab} = 4a^2b$ cm

Let's Practise

1. $a = 8$, $b = 4$, $c = 4$

2. RM$(120 - 12x - 7y)$

3. -11

4. RM4.80

5. $2k + 10$

6. (a) $\frac{mx}{5} + 2my$
(b) RM33.40

7. Let n represent a number.

$$n + 7 = x$$
$$\frac{n}{2} = y$$
$$x + y = (n + 7) + \left(\frac{n}{2}\right)$$
$$= n + 7 + \frac{n}{2}$$
$$x + y = \frac{3}{2}n + 7$$

8. RM60$(3h + 4k - 2p - 3q)$

9. $(6xy + 8x + 18)$ m

10. 27°F

Chapter 6 Linear Equations

Self Practice 6.1a

1. (a) Yes, because the equation has one variable m and the power of m is 1.
(b) Yes, because the equation has one variable p and the power of p is 1.
(c) No, because the equation has two variables, x and y.
(d) No, because the highest power of the variable k is 2.

Self Practice 6.1b

1. (a) $\frac{x}{6} = 12$
(b) $5y = 40$
(c) $4x + 1\,000 = 1\,400$

2. (a) Subtract 1 from a number, the result is 6.
(b) When Edri's score is added to 10, it will be 78.
(c) The total mass of four packs of rice is 50 kg.

Self Practice 6.1c

1. (a) 13 (b) 2
(c) 12 (d) 10
(e) 3 (f) –15

Self Practice 6.1d

1. 53 marks

2. 266 cm^2

Mastery Q 6.1

1. (a) $x - 8 = 15$
(b) $14y = 42$
(c) $p + 34 = 3p$

2. (a) 16 (b) 4 (c) 2

3. (a) $-\frac{1}{2}$ (b) -4 (c) 24

4. (a) $\frac{10}{3}$ (b) 7 (c) 6

5. 20 boys

6. 1 hour and 30 minutes

7. Small table = 2.16 m^2
Large table = 4.32 m^2

Self Practice 6.2a

1. (a) Yes, because the equation has two variables, h and k, and the power of each variable is 1.
(b) No, because the equation has only one variable.
(c) Yes, because the equation has two variables, x and y, and the power of each variable is 1.
(d) No, because the highest power of variable p is 2.

Self Practice 6.2b

1. (a) $x + y = 258$
(b) $p - q = 15$
(c) $8x + 5y = 265$
(d) $x + 2y = 40$

2. (a) The total number of aluminium cans and glass bottles collected during the recycling campaign was 465.
(b) The difference between the length and width of a rectangle is 3 cm.

Self Practice 6.2c

1. (a) (7, 0), (8, 1), (9, 2)
(b) (0, 1), (2, –3), (4, –7)
(c) (2, 0), (6, 24), (1, – 6)
or other valid answers

2. 1 gold medal and 3 bronze medals;
2 gold medals and 2 bronze medals;
3 gold medals and 1 bronze medal.

Self Practice 6.2d

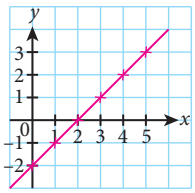
1. Diagram (a) and Diagram (c).

Mastery Q 6.2

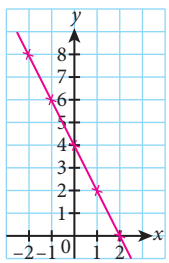
1. (a) $30x - 20y = 8$
(b) $x + 2y = 130$
(c) $x = 2y$

2. (a) (4, 3), (2, 5)
(b) (1, 7), (2, 12)
or other valid answers

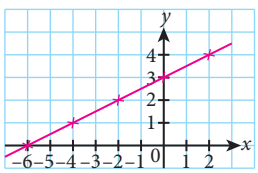
3. (a)



(b)

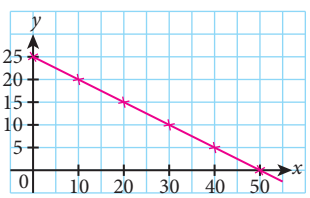


(c)



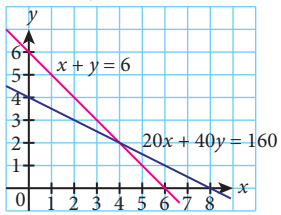
4. 1 shirt and 6 pairs of pants;
2 shirts and 4 pairs of pants;
3 shirts and 2 pairs of pants.
The maximum number: 3 shirts

5. $10x + 20y = 500$

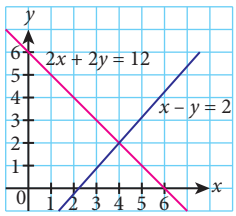


Self Practice 6.3a

1. (a) $x + y = 6$
$20x + 40y = 160$



(b) $2x + 2y = 12$
$x - y = 2$



Self Practice 6.3b

1. (a) $x = 2, y = 7$
(b) $x = 9, y = -4$
(c) $x = 2, y = 0$
(d) $x = -26, y = 68$

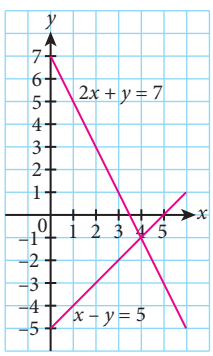
Self Practice 6.3c

1. 500 booklets of RM30 coupon and 300 booklets of RM50 coupon

2. $p = 50$ m, $q = 25$ m

Mastery Q 6.3

1. $x - y = 5$
$2x + y = 7$



Unique solution.

2. (a) $x = 2, y = 3$
(b) $m = 5, n = -2$
(c) $p = 9, q = 1$
(d) $f = \frac{10}{3}, g = \frac{20}{3}$

3. First part of the wire = 24 cm
Second part of the wire = 24 cm
Third part of the wire = 52 cm

4. Lai Yee: 45 stamps
Khadijah: 15 stamps

5. Devaki's father: 30 years old
Devaki: 6 years old

6. Sarah: RM35
Hui Chin: RM15

Let's Practise

1. 16 sweets

2. 220 points

3. Ella = RM600
Zahida = RM960

4. RM310

5. 187 cm^2

6. A cup of coffee: RM1.80
A piece of curry puff: RM0.80

7. 3 children

8. Asnita: 25 years old
Reslynna: 19 years old

9. Year 2013
Population of trees = 25

10. 12.7s

11. Pineapple: RM2
Watermelon: RM1.50

12. P : RM287 500
Q : RM108 000

Chapter 7 Linear Inequalities

Self Practice 7.1a

1. (a) $<$
-6 is less than 0.
(b) $<$
$\frac{1}{7}$ is less than $\frac{1}{4}$.
(c) $>$
0.42 is greater than 0.072.
(d) $>$
4.5 is greater than $\sqrt{4.5}$.
(e) $>$
10 cm is greater than 50 mm.
(f) $<$
1 200 g is less than 1.6 kg.

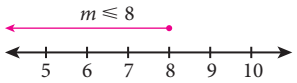
Self Practice 7.1b

1. 12 is greater than y.
$12 > y$

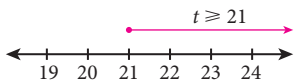
2. 3 is less than b.
$3 < b$

Self Practice 7.1c

1. (a) m is less than or equal to 8.
$m \leqslant 8$



(b) t is greater than or equal to 21.
$t \geqslant 21$



Self Practice 7.1d

1. (a) $14 > 5$
(b) $-8 < 8$
(c) $-32 < -23$
(d) $1.5 < 6.7$
(e) $\frac{1}{14} < \frac{1}{13}$
(f) $-11.8 < -2$

2. (a) $-2 < 10$
(b) $-15 < 0$
(c) $-4.56 < 2.01$
(d) $-17\frac{2}{9} < 20\frac{1}{3}$
(e) $\frac{1}{20} < \frac{1}{7}$
(f) $-8 < \frac{1}{9}$

Self Practice 7.1e

1. (a) (i) $<$ (ii) $<$ (iii) $<$
(b) (i) $>$ (ii) $>$ (iii) $>$
(c) (i) $>$ (ii) $>$ (iii) $>$

2. (a) (i) $<$ (ii) $<$ (iii) $<$
(b) $8c < 16c$
$\frac{8}{c} < \frac{16}{c}$
(c) $16c > 8c$
$\frac{16}{c} > \frac{8}{c}$

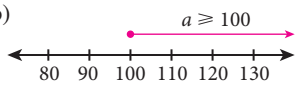
3. (a) (i) $<$ (ii) $>$ (iii) $>$
(iv) $>$ (v) $>$
(b) $6d > 12d$
$\frac{6}{d} > \frac{12}{d}$
(c) $12d < 6d$
$\frac{12}{d} < \frac{6}{d}$

Mastery Q 7.1

1. (a) $<$ (b) $<$ (c) $<$
(d) $>$ (e) $>$ (f) $>$

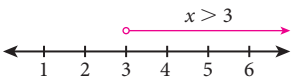
2. (a) y is greater than x.
(b) $y > x$

3. (a) t = deposit
t is greater than or equal to 100.
(b)

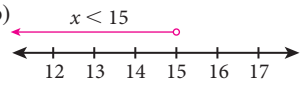


$a \geqslant 100$

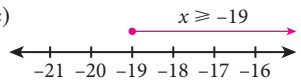
4. (a)



(b)

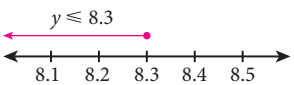


(c)

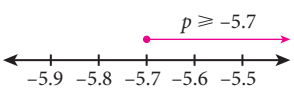


(d)

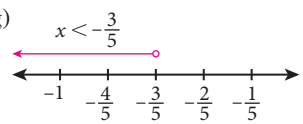
(e)



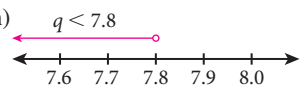
(f)



(g)



(h)



5. (a) $>$ (b) $<$ (c) $>$
(d) $>$ (e) $<$ (f) $<$

Self Practice 7.2a

1. (a) $x \geqslant 450\,000$
(b) $y < 50$
(c) $k \leqslant 30$
(d) $q \geqslant 600$

2. (a) The number of passengers in a taxi should not exceed 4.
(b) The area of an apartment is more than 1 000 m^2.
(c) The total expenditure of four customers who patronize the restaurant is at least RM60.

Self Practice 7.2b

1. (a) $x \geqslant 7$
(b) $x > -9$
(c) $x < -21$
(d) $x \geqslant 4$

2. At least 200 canned drinks in an hour.

3. 10 months

4. 5 hours

Self Practice 7.2c

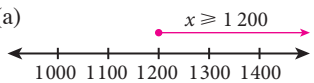
1. (a) $x > 5$
(b) $x \leqslant -3$
(c) $-\frac{3}{2} < x \leqslant 2$
(d) $-1 \leqslant x < 1$
(e) $x < 1\frac{5}{7}$
(f) $x > 3$

Mastery Q 7.2

1. (a) $x \geqslant 18\,000$
(b) $t \leqslant 8$
(c) $h > 700$

2. (a) The speed of the vehicle should not exceed 30 km/h when approaching the school area.
(b) The mass of a car is greater than 1 100 kg.
(c) A student whose parents' salary is less than RM900 qualifies to apply for a scholarship.

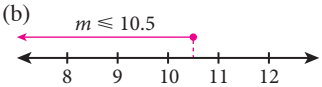
3. (a)



(b) $x \geqslant 1\,200$

4. 2 packets

5. (a) $m \leqslant 10.50$
(b)



6. 21 boys

7. 7 months

8. (a) $x < -\frac{5}{6}$

(b) $-5 \leqslant x < 8$

(c) $x \leqslant -1$

Let's Practise

1. (a) $<$ (b) $<$ (c) $>$

2. (a) $x > 18$

(b) $x + 3 > 21$

(c) $x - 5 > 13$

3. $20 - 1.20x > 5$

4. (a) $170 + n$

(b) 1, 2, 3, 4, 5, 6, 7

5. (a) $x < 4$ (b) $x < 2$

6. 14 g

7. 17 hours

8. (a) $15n + 200 > 290$

(b) 7

9. RM751

10. (a) $-4 \leqslant x \leqslant 3$

(b) $x > \frac{8}{3}$

(c) $3 \leqslant x \leqslant 7$

(d) $\frac{5}{2} \leqslant x < 3$

Chapter 8 Lines and Angles

Self Practice 8.1a

1. (a) Congruent; $PQ = RS$

(b) Not congruent; $PQ \neq RS$

(c) Congruent; $PQ = RS$

2. (a) Congruent; $\angle PQR = \angle ABC$

(b) Not congruent; $\angle PQR \neq \angle ABC$

(c) Congruent; $\angle PQR = \angle ABC$

Self Practice 8.1b

1. (a) 4 cm. Use a 4 cm long eraser.

(b) 6 cm. Use a 3 cm long paper clip.

(or other objects of known lengths)

2. (a) 4.3 cm

(b) 5.8 cm

3. (a) Appears slightly more than a right angle.
Thus, estimated size is about 100°.

(b) Appears slightly less than a right angle.
Thus, estimated size is about 80°.

(c) Appears much more than a right angle.
Thus, estimated size is about 130°.

4. (a) 52°

(b) 115°

(c) 146°

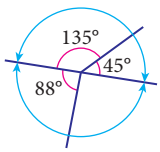
Self Practice 8.1c

1. (a) Reflex angle

(b) The angle on a straight line

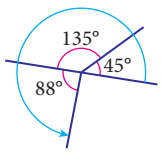
(c) The angle of one whole turn

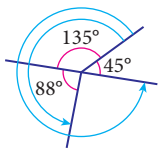
2. (a) (i)

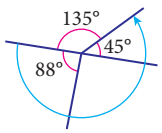


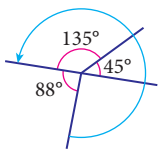
$135° + 45° = 180°$

(ii)



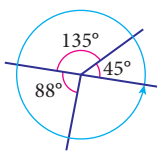






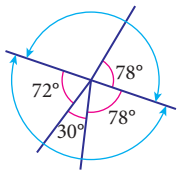
A reflex angle is more than 180° and less than 360°.

(iii)



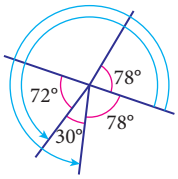
The angle of one whole turn is 360°.

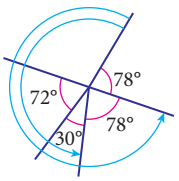
(b) (i)

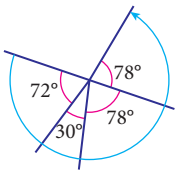


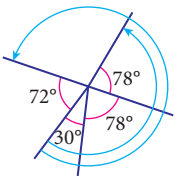
$72 + 30° + 78° = 180°$

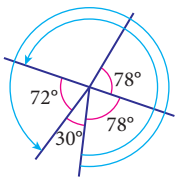
(ii)





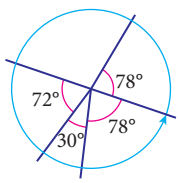






A reflex angle is more than 180° and less than 360°.

(iii)



The angle of one whole turn is 360°.

Self Practice 8.1d

1. (a) True (b) False
(c) True (d) True
(e) True

Self Practice 8.1e

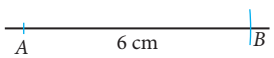
1. $a = 44°$, $b = 136°$, $c = 77°$, $d = 57°$

2. $p = 116°$, $q = 64°$

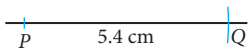
Self Practice 8.1f

The following diagrams are not drawn to actual scale.

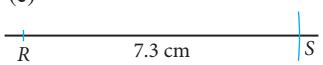
1. (a)



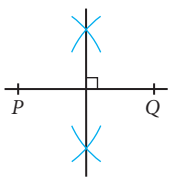
(b)



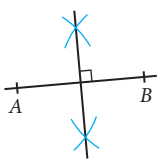
(c)



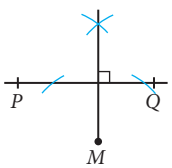
2. (a)



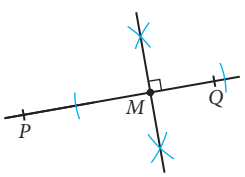
(b)



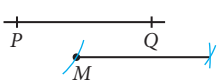
3. (a)



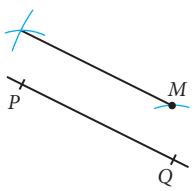
(b)



4. (a)

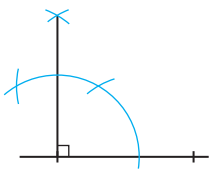


(b)

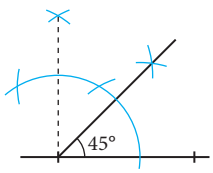


Self Practice 8.1g

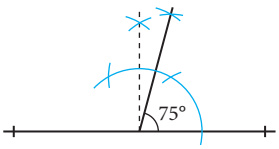
1. (a)



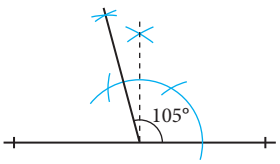
(b)



(c)



(d)



Mastery Q 8.1

1. (a) True (b) False
(c) True (d) True
(e) True

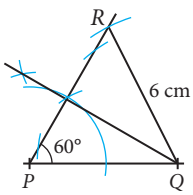
2. $p = 45°$, $q = 45°$

3. 7 cm

4. $p = 105°$, $q = 75°$

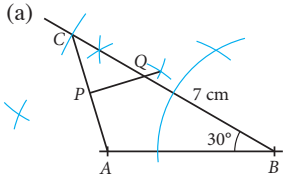
5. $p = 72°$, $q = 288°$

6. (a)



(b) 5 cm

7. (a)

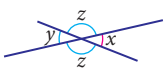


(b) 47°

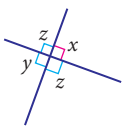
Self Practice 8.2a

1. (a) (i) Vertically opposite angles: $\angle p$, $\angle q$
(ii) Adjacent angles at intersecting lines: $\angle q$ and $\angle r$; $\angle p$ and $\angle r$
(b) (i) Vertically opposite angles: $\angle a$, $\angle b$
(ii) Adjacent angles at intersecting lines: $\angle a$ and $\angle c$; $\angle b$ and $\angle c$
(c) (i) Vertically opposite angles: $\angle t$, $\angle r$
(ii) Adjacent angles at intersecting lines: $\angle t$ and $\angle s$; $\angle r$ and $\angle s$

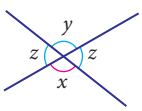
2. (a)



(b)



(c)



Self Practice 8.2b

1. (a) $x = 140°$, $y = 100°$
(b) $x = 24°$, $y = 148°$

Self Practice 8.2c

1. $x = 66°$, $y = 58°$

2. $x = 90°$, $y = 25°$

Mastery Q 8.2

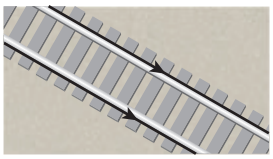
1. $x = 34°$, $y = 62°$

2. 15°

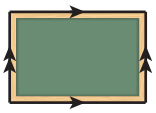
3. 28°

Self Practice 8.3a

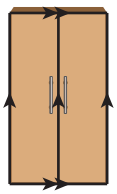
1. (a)



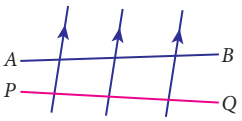
(b)



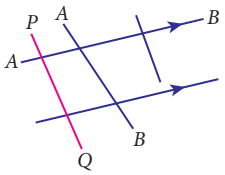
(c)



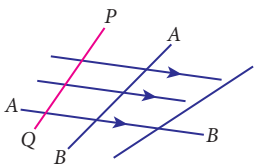
2. (a)



(b)



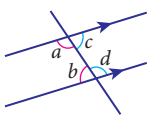
(c)



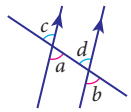
3. (a) Yes
(b) Yes
(c) No

Self Practice 8.3b

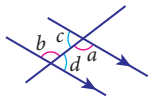
1. (a) Interior angles;
$\angle a + \angle b = 180°$



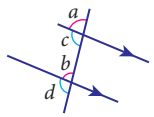
(b) Corresponding angles;
$\angle a = \angle b$



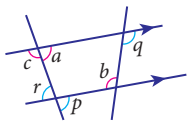
(c) Alternate angles; $\angle a = \angle b$



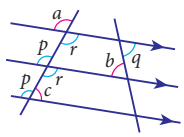
(d) Corresponding angles;
$\angle a = \angle b$



2. (a)



(b)



Self Practice 8.3c

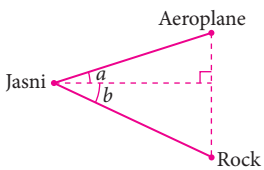
1. (a) Parallel
(b) Parallel
(c) Not parallel

Self Practice 8.3d

1. $a = 76°, b = 70°, c = 70°, d = 70°$
2. $a = 67°, b = 42°$

Self Practice 8.3e

1.



Self Practice 8.3f

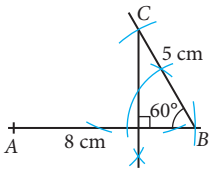
1. (a) $x = 117°, y = 88°$
(b) 88°

Mastery Q 8.3

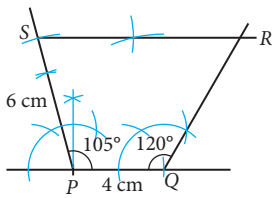
1. $x = 45°, y = 93°$
2. $x = 106°, y = 58°$
3. $x = 125°, y = 100°$
4. (a) $\angle QOR$
(b) $\angle TOU$
5. (a) $x = 70°, y = 48°$
(b) 250°
(c) 70°
6. $x = 33°, y = 58°$

Let's Practise

1. $x = 65°, y = 118°$
2. $x = 60°, y = 84°$
3.



4. $x = 139°, y = 94°$
5. $x = 72°, y = 108°$
6. (a) $x = 75°, y = 45°$
(b)



7. $x = 64°, y = 32°$
8. $x = 72°, y = 120°, z = 60°$
9. $x = 90°, y = 52°$
10. (a) 46° (b) 42°

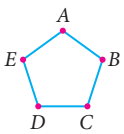
Chapter 9 Basic Polygons

Self Practice 9.1a

1. (a) Number of vertices: 6
Number of diagonals: 9
(b) Number of vertices: 9
Number of diagonals: 27
(c) Number of vertices: 12
Number of diagonals: 54
(d) Number of vertices: 20
Number of diagonals: 170

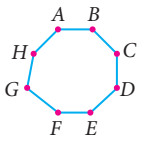
Self Practice 9.1b

1. (a)



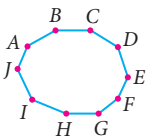
Pentagon *ABCDE*

(b)



Octagon *ABCDEFGH*

(c)

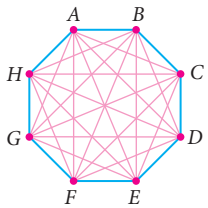


Decagon *ABCDEFGHIJ*

Mastery Q 9.1

1. (a) True (b) True

2.



Name of polygon: octagon
Number of diagonals: 20

Self Practice 9.2a

1. (a) Obtuse-angled triangle; or scalene triangle
(b) Equilateral triangle
(c) Right-angled triangle
(d) Acute-angled triangle; or scalene triangle

Self Practice 9.2b

1. (a) 35° (b) 47°
(c) 27° (d) 42°

2. (a) 123° (b) 54°
(c) 82° (d) 76°

Self Practice 9.2c

1. $x = 56°$, $y = 32°$

2. $x = 60°$, $y = 89°$

Mastery Q 9.2

1. (a) 1 (b) 3
(c) 1 (d) 1

2. (a) Equilateral triangle
(b) Isosceles triangle
(c) Obtuse-angled triangle; or scalene triangle
(d) Right-angled triangle

3. $x = 56°$, $y = 118°$

4. $x = 108°$, $y = 144°$

5. $x = 160°$, $y = 62°$

Self Practice 9.3a

1. All sides have the same length; opposite sides are parallel; diagonals are perpendicular bisectors of each other.

2.

Rectangle	Parallelogram
Opposite sides are of the same length and parallel.	Opposite sides are of the same length and parallel.
All of its interior angles are 90°.	Opposite angles are equal.
Diagonals are of equal length and are bisectors of each other.	Diagonals are bisectors of each other.
It has two axes of symmetry.	It does not have any axes of symmetry.

Self Practice 9.3b

1. 121°

2. $x = 84°$, $y = 90°$

Self Practice 9.3c

1. $x = 152°$, $y = 121°$

2. $x = 132°$, $y = 138°$

Self Practice 9.3d

1. $x = 72°$, $y = 108°$

2. $x = 30°$, $y = 72°$

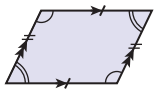
Mastery Q 9.3

1. **Similarities:**
- Opposite sides are of equal length and parallel.
- Opposite angles are equal.

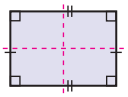
Differences:
Parallelogram
- Opposite sides are of equal length.
- Diagonals are bisectors of each other.
- It does not have any axes of symmetry.

Rhombus
- All sides have the same length.
- Diagonals are perpendicular bisectors of each other.
- It has 2 axes of symmetry.

2. (a)



(b)



3. $x = 164°$, $y = 74°$

4. $x = 34°$, $y = 66°$

Let's Practise

1. (a) ✓
(b) ✓
(c) ✗
(d) ✓

2. (a) Rectangle, rhombus
(b) Scalene triangle
(c) Square, rhombus
(d) Rectangle, square

3. (a) 85°
(b) 24°
(c) 50.5°
(d) 15°

4. $x = 53°$, $y = 53°$

5. $x = 15°$, $y = 60°$

6. The largest number in the ratio is 6; 120°

7. 75°

8. $x = 112°$, $y = 44°$

9. $x = 30°$, $y = 74°$

10. $x = 56°$, $y = 62°$

11. $x = 53°$, $y = 127°$

Chapter 10 Perimeter and Area

Self Practice 10.1a

1. (a) 74 cm
(b) 71 cm
(c) 13 cm

Self Practice 10.1b

The estimated answers depend on the students.

1. (a) Estimation: 10 cm
Perimeter: 9.9 cm
The difference between the estimated value and the actual value is small, therefore the estimated value is accurate.

(b) Estimation: 11.5 cm
Perimeter: 12.3 cm
The difference between the estimated value and the actual value is small, therefore the estimated value is accurate.

Self Practice 10.1c

1. 76 m
2. RM1 720
3. 22 cm

Mastery Q 10.1

1. 9 m
2. 42 cm
3. Perimeter of shape A
$= x + y + 6 + (y - 5) + (x - 6) + 5$
$= (2x + 2y)$ cm
Perimeter of shape B
$= x + y + 3 + (y - 2) + (x - 3) + 2$
$= (2x + 2y)$ cm
Thus, perimeters of both the shapes A and B are equal.
4. RM24 000
5. 6 cm

Self Practice 10.2b

1. (a) $\frac{1}{2}mn$ (b) tk
(c) $\frac{1}{2}(p + q)r$ (d) $\frac{1}{2}st$

Self Practice 10.2c

1. (a) 15 cm^2 (b) 24.5 cm^2
(c) 16.8 cm^2 (d) 24 cm^2
2. 146 m^2

Mastery Q 10.2

2. 1 550 m^2
3. 122.5 cm^2
4. 75.25 cm^2

Self Practice 10.3a

1. T, S, Q, R, P
The bigger the difference between the length and the width of the rectangle, the smaller the area.
2. P, R, T, S, Q
The bigger the difference between the length and the width of the rectangle, the bigger the perimeter.

Self Practice 10.3b

1. 420.25 m^2
2. 93.75 cm^2
3. 48 cm^2

Mastery Q 10.3

1. 128 cm
2. Draw a square with length of side of 12 cm. The area is 144 cm^2.
3. 432 cm^2
4. 875 m^2

Let's Practise

1. 41 cm
2. 30 cm
3. 81 cm^2
4. (a) 24 cm^2
(b) 4.8 cm
5. 111.5 cm^2
6. 42 cm
7. (a) 4 cm
(b) 32 cm
8. 25 cm^2
9. Form a square with length of side of 7.5 cm.
10. Fence up a region with a length of 22.5 m and a width of 20 m; 85 m

Chapter 11 Introduction of Set

Self Practice 11.1a

1. Land transport: car, lorry, van, bus
Water transport: sampan, boat, ship, ferry
Air transport: rocket, aeroplane, helicopter, hot-air balloon

Self Practice 11.1b

1. (a) (i) P is the set of colours of the rainbow
(ii) P = {red, orange, yellow, green, blue, indigo, violet}
(iii) $P = \{x : x \text{ is a colour of the rainbow}\}$
(b) (i) Q is the set of multiples of 3 which are less than 25.
(ii) $Q = \{3, 6, 9, 12, 15, 18, 21, 24\}$
(iii) $Q = \{x : x \text{ is a multiple of 3 and } x < 25\}$
2. (a) True
(b) False
(c) True
(d) False

Self Practice 11.1c

1. (a) $\in$ (b) $\notin$
(c) $\notin$ (d) $\in$
2. (a) $\notin$ (b) $\in$
(c) $\notin$ (d) $\in$
(e) $\in$ (f) $\notin$

Self Practice 11.1d

1. (a) $n(A) = 5$ (b) $n(B) = 5$
(c) $n(C) = 3$ (d) $n(D) = 9$
2. (a) 6 (b) 7

Self Practice 11.1e

1. (a) Yes (b) No
(c) No (d) Yes

Mastery Q 11.1

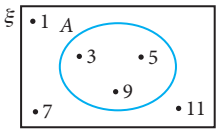
1. (a) X is the set of vowels.
(b) Y is the set of perfect squares which are less than 50.
2. (a) P = {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune}
(b) $Q = \{2, 3, 5\}$
3. (a) $G = \{x : x \text{ is a month which begins with the letter 'M'}\}$
(b) $H = \{x : x \text{ is a multiple of 7 and } 1 \leqslant x \leqslant 100\}$
4. (a) Yes
(b) No
(c) No
(d) Yes
5. $n(A) = 3, n(B) = 13, n(C) = 5$
6. 2

Self Practice 11.2a

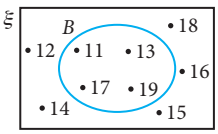
1. (a) Universal set
(b) Not a universal set
(c) Universal set
2. (a) $P' = \{0, 1, 2, 4, 5, 7, 8\}$
(b) $Q' = \{0, 1, 4, 6, 8, 9\}$

Self Practice 11.2b

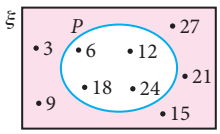
1. (a)



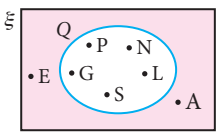
(b)



2. (a)



(b)



Self Practice 11.2c

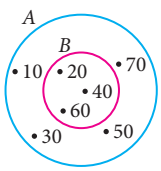
1. (a) $\subset$ (b) $\subset$
(c) $\not\subset$ (d) $\not\subset$

2. (a) ø, {p}, {q}, {p, q}

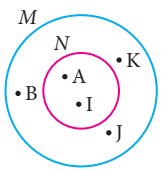
(b) ø, {2}, {3}, {5}, {7}, {2, 3}, {2, 5}, {2, 7}, {3, 5}, {3, 7}, {5, 7}, {2, 3, 5}, {2, 3, 7}, {2,5,7},{3,5,7},{2,3,5,7}

Self Practice 11.2d

1. (a)

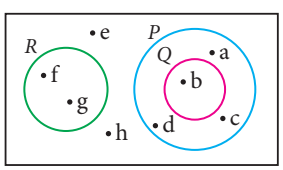


(b)



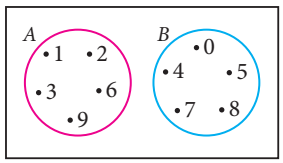
Self Practice 11.2e

1.



Mastery Q 11.2

1.

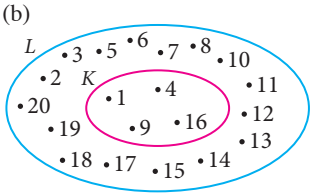


$\therefore A' = B$

2. (a) $P \subset Q$
(b) $R \subset Q$

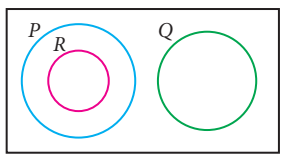
3. (a) { }, {1}, {4}, {9}, {16}, {1, 4}, {1, 9}, {1, 16}, {4, 9}, {4, 16}, {9, 16}, {1,4,9},{1,4,16},{1,9,16}, {4, 9, 16}, {1, 4, 9, 16}

(b)



$K \subset L$

4.



5. $C \subset B \subset A \subset \xi$

Let's Practise

1. P is the set of quadrilaterals.

2. {0}

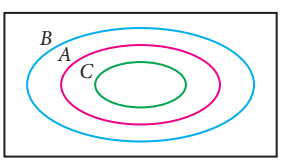
3. (a) $P = \{2, 4, 6, 8, \ldots\}$
$Q = \{2, 4, 6, 8, \ldots\}$
$P = Q$

(b) Set A has an element '0', therefore set A is not an empty set.
$A \neq B$

(c) $E = \{1, 3, 5, 15\}$
$F = \{15, 30, 45, 60, \ldots\}$
$E \neq F$

4. 6

5.



6. (a) $R \subset Q$ (b) Q'

7. (a) Set P is a universal set. It is because set P consists of all the elements under discussion.

(b) R is the set of odd numbers.

8. 8

9. Not true. The complement of set A is the girls who are not prefects and the boys in Class 1 Bakti.

Chapter 12 Data Handling

Self Practice 12.1a

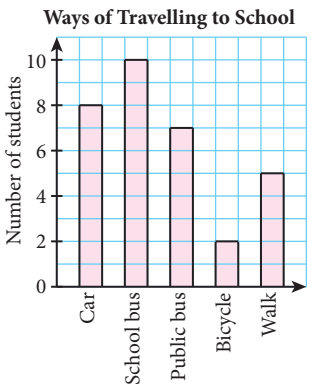
1. (a) Numerical data
(b) Numerical data
(c) Categorical data
(d) Categorical data
(e) Numerical data
(f) Numerical data
(g) Categorical data
(h) Numerical data

2.

T-shirt size	Tally	Frequency
S	|	1
M	𝍸 𝍸 ||	12
L	𝍸 ||||	9
XL	𝍸	5
	Total	27

Self Practice 12.1b

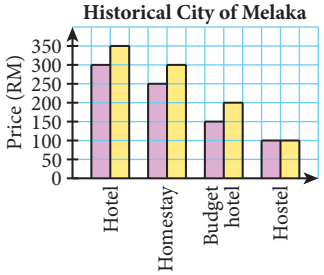
1.



Way of travelling

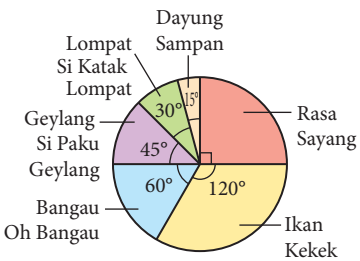
The bar chart is suitable for comparing the number of students with how they travel to school.

2.



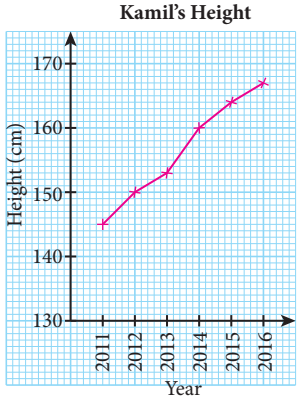
The dual bar chart is suitable for comparing different types of accommodation with the normal and holiday season rates.

3.



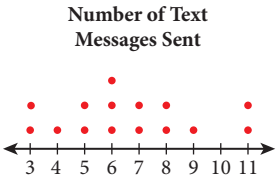
The pie chart is suitable for comparing the favourite songs of a group of children.

4.



The line graph is suitable for displaying the changes in Kamil's height over a period of 6 years.

5.



The dot plot is suitable for displaying the number of text messages sent by a group of students.

6. **Ages of Magazine Readers**

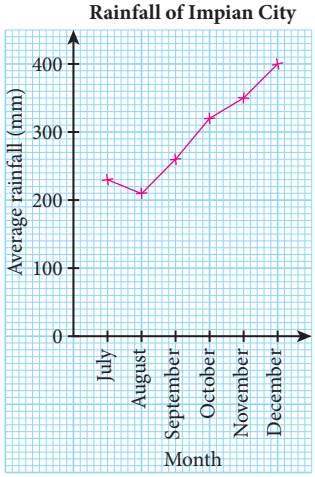
Stem	Leaf
2	2 5
3	0 1 1 2 3 4 6 8 9
4	2 4 4 7 7
5	1 1 3 7 8 8
6	0 5

Key: 2|2 means 22 years old

The stem-and-leaf plot is suitable for displaying the age of each reader of the magazine in the survey.

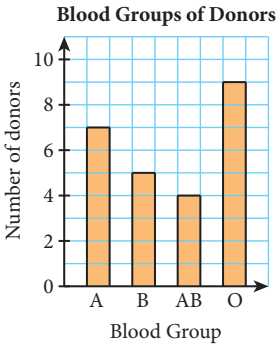
Self Practice 12.1c

1.



The line graph is suitable for displaying the changes in the average monthly rainfall of Impian City over a six-month period.

2.



The bar chart is suitable as the data of the blood groups of donors is a categorical data.

Self Practice 12.1d

1. (a) 8 m
(b) 7.5 m
(c) Year 2013
(d) The tree has a nearly uniform growth rate from 2010 to 2015 or there is no increase in height from 2015 to 2016.
(e) 16 m

2. (a) 30
(b) Largest diameter = 28.6 mm
Smallest diameter = 24.5 mm
(c) 20%
(d) The distribution of the diameters of the axles is quite symmetric since most of the diameters range from 26.0 mm to 26.9 mm.

3. (a) 120
(b) 12
(c) Number of tested samples with the mass of 241 g to 260 g
$= 28 + 32 + 24 + 12$
$= 96$
Percentage of tested samples with the mass of 241 g to 260 g
$= \frac{96}{120} \times 100\%$
$= 80\%$
Thus, this batch of biscuits produced satisfies the required specifications.

Self Practice 12.1e

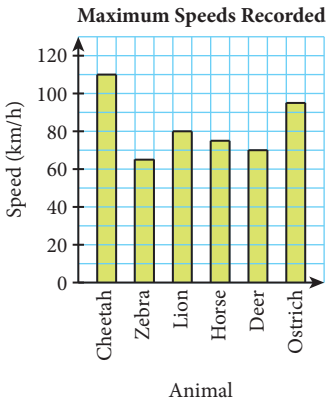
1. No, because not all the results of the survey are shown in the bar chart, that is:
$16 + 6 + 12 + 4 < 40$

2. No, because the line graph seems to show a rapid change in the temperature every 2 hours in the day. The scale on the vertical axis should start at zero so that the information displayed will not be confusing.

Mastery Q 12.1

1. (a) −3°C
(b) −4°C
(c) 1100 and 1700

2. (a) (i)



(ii) **Maximum Speed Recorded**

Stem	Leaf
6	5
7	0 5
8	0
9	5
10	
11	0

Key: 615 means 65 km/h

(b) A bar chart is more suitable as the data displayed is a categorical data. The stem-and-leaf plot displays only the maximum speeds but does not display the types of animals.

3. (a) 35 bookings. Bar chart. The difference in room bookings can be obtained by calculating the difference in length of the bars for the Internet and counter.

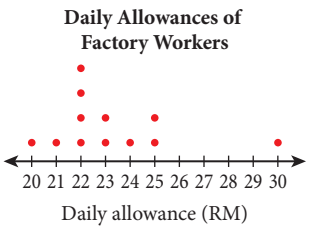
(b) A pie chart because the pie chart shows 48% of the rooms are booked via the Internet and that is almost half of the total room bookings.

(c) A bar chart because the number of rooms booked via the Internet can be read directly from the length of the bar for the Internet. For a pie chart, the number of room bookings can only be known by calculation.

(d) No, it is not suitable as the data does not show the changes in the number of room bookings over a certain period.

(e) Pictogram

4. (a)



(b) (i) The distribution of the data ranges from 20 to 30.

(ii) The data clusters around 22.

(iii) 30 deviates greatly from other values on the distribution of data, thus this data has one extreme value, that is, 30.

5. (a) Science test
= 45 students
Mathematics test
= 75 students

(b) In Diagram (a), most of the taller bars lie in the middle of the histogram. It shows that most students obtained average marks in the Science test.

In Diagram (b), most of the taller bars lie on the right-hand side of the histogram. It shows that most students obtained higher marks in the Mathematics test.

(c) More students obtained higher marks in the Mathematics test than in the Science test.

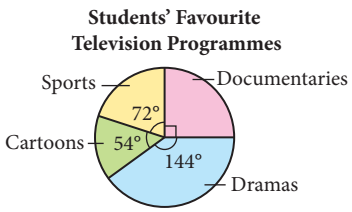
6. (a) Shop *A* and shop *B* sold the same number of lamps, which was 120.

(b) Not valid because shop *E* sold 30 more lamps than shop *C*.

Let's Practise

1. (a) House *A*
(b) 5th year

2. (a) Dramas
(b)



3. (a) Between the first week and the second week, and between the second week and the third week.

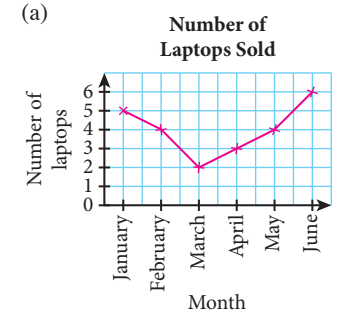
Between the fourth week and the fifth week, and between the fifth week and the sixth week.

(b) Between the third week and the fourth week.

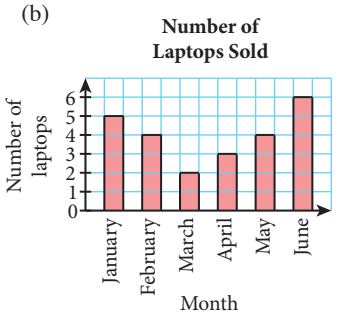
(c) The line graph can display the changes in height of the plant over the six-week period.

4. (a) False
(b) True
(c) True

5. (a)

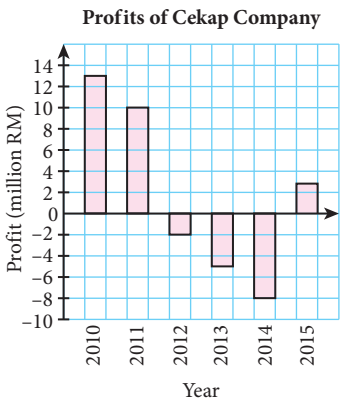


(b)



The bar chart is more suitable in representing this data because the number of laptops sold can be read from the height of the bars.

6. (a) Year 2010, RM13 million
(b) Year 2012, RM2 million
(c) Loss of RM8 million
(d) (i)



(ii) Profit of RM11 million

7. (a) 12, 15, 15, 16, 17, 18, 21, 24, 25, 27, 28, 30, 30, 31, 33, 34, 35, 36, 37, 37, 38, 39, 41, 42, 43, 45, 47, 48, 52, 56
(b) 12 minutes
(c) Much of the time taken to fix the leaking pipes ranges from 30 minutes to 39 minutes.

8. (a) Diagram (a)
(b) Diagram (b). Not ethical. The scale used on the vertical axis of this line graph is larger.

Chapter 13 The Pythagoras' Theorem

Self Practice 13.1a

1. (a) AC
(b) b
(c) PR is the hypotenuse of ΔPQR.
ST is the hypotenuse of ΔSPT.

Self Practice 13.1b

1. (a) $AC^2 = AB^2 + BC^2$
(b) $LN^2 = LM^2 + MN^2$
(c) $r^2 = p^2 + q^2$
(d) $z^2 = x^2 + y^2$

Self Practice 13.1c

1. (a) 10
(b) 24
(c) 16.64
(d) 0.66

2. (a) 14 cm
(b) 8 cm
(c) 25.63 cm

Self Practice 13.1d

1. 54 cm

2. 35.44 km

Mastery Q 13.1

1. 12 cm

2. 28 cm

3. 5.62 m

4. 540.83 km

Self Practice 13.2a

1. (a) Yes
(b) No
(c) Yes

Self Practice 13.2b

1. $1.5^2 + 2.0^2 = 2.5^2$
Thus, the wall is vertical.

2. 160°

Mastery Q 13.2

1. (a) Yes; $9^2 + 40^2 = 41^2$
(b) No; $27^2 + 35^2 \neq 45^2$
(c) Yes; $2.5^2 + 6^2 = 6.5^2$
(d) Yes; $13^2 + 84^2 = 85^2$

2. $12^2 + 16^2 = 20^2$
$\therefore$ The triangle is a right-angled triangle. Therefore, the triangular piece of wood can be fixed perfectly onto the L-shaped structure.

3. $15^2 + 20^2 = 25^2$
All angles at the vertices of the quadrilateral are right angles, thus the quadrilateral is a rectangle.

Let's Practise

1. (a) 10 cm
(b) 24 cm

2. $\sqrt{2^2 + 150^2} = 150.01$ m

3. (a) 15 cm
(b) Right-angled triangle;
$8^2 + 15^2 = 17^2$

4. 104 cm^2

5. $\sqrt{18^2 + 6^2} = 18.97$ m

6. $8^2 + 15^2 = 17^2$
$SR = 17$ cm

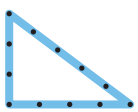
7. 36 cm^2

8. 0.4

9. $20 - \sqrt{15^2 - 12^2} = 11$ m

10. Using the concept of Pythagorean triples, (3, 4, 5)
$7 \times 5 = 35$ cm
$7 \times 3 = 21$ cm
$7 \times 4 = 28$ cm

11. Using the concept of Pythagorean triples,
$3 + 4 + 5 = 12$



GLOSSARY

adjacent angles (*sudut bersebelahan*) Two angles that are next to each other and whose sum is 180°.

bar chart (*carta palang*) A type of data representation that uses bars to represent data.

common factor (*faktor sepunya*) A number that is a factor of a few other numbers.

complement of a set (*pelengkap bagi suatu set*) The elements in the universal set that are not the elements of the set.

complementary angles (*sudut pelengkap*) Two angles with a sum of 90°.

congruent angles (*sudut kongruen*) Angles having the same size.

congruent line segments (*tembereng garis kongruen*) Line segments having the same length.

conjugate angles (*sudut konjugat*) Two angles with a sum of 360°.

element (*unsur*) Each object in a set.

empty set (*set kosong*) A set that contains no elements.

equal sets (*set sama*) Sets in which every element of the sets are the same.

factor (*faktor*) A number that divides another number completely.

geometrical construction (*pembinaan geometri*) Method of using geometric tools or geometry software to make drawings of accurate measurements.

hypotenuse (*hipotenus*) The longest side of a right-angled triangle which is opposite to the right angle.

inequality (*ketaksamaan*) A relationship between two quantities of different values.

integer (*integer*) Positive and negative whole numbers including zero.

like terms (*sebutan serupa*) Terms that have the same variables raised to the same power.

line graph (*graf garis*) A type of data representation that displays changes of data over a period of time.

linear inequality in one variable (*ketaksamaan linear dalam satu pemboleh ubah*) An unequal relationship between a number and a variable to the power of one.

negative number (*nombor negatif*) A number having a value less than zero which is written with a negative sign '–'. For example, –8, –45, –200.

pie chart (*carta pai*) A type of data representation that uses sectors of a circle to represent data.

positive number (*nombor positif*) A number having a value more than zero which is written with a positive sign '+' or without a sign. For example, +3, +56, +100 or 3, 56, 100.

prime factorisation (*pemfaktoran perdana*) Process of expressing a number as the product of its prime factors.

proportion (*kadaran*) A relationship that states that two ratios or two rates are equal.

rate (*kadar*) A special case of ratio that involves two quantities of different units.

rational number (*nombor nisbah*) A number that can be written in fractional form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

reflex angle (*sudut refleks*) An angle whose size is more than 180° but less than 360°.

set (*set*) A group of objects which have the common characteristics and classified in the same group.

simultaneous linear equations (*persamaan linear serentak*) Two linear equations which have the same two variables.

statistics (*statistik*) The branch of mathematics that involves collecting, organising, recording, representing, interpreting and analysing data.

subset of a set (*subset bagi suatu set*) A set whereby all of its elements are the elements of another set.

supplementary angles (*sudut penggenap*) Two angles with a sum of 180°.

transversal (*garis rentas lintang*) A line that intersects two or more straight lines.

universal set (*set semesta*) A set that consists of all the elements under discussion.

unlike terms (*sebutan tidak serupa*) Terms that do not have the same variables raised to the same power.

Venn diagram (*gambar rajah Venn*) A geometrical diagram used to represent sets.

vertically opposite angles (*sudut bertentang bucu*) Two angles opposite to each other at an intersection of two lines and are equal in size.

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Dengan ini **SAYA BERJANJI** akan menjaga buku ini dengan baik dan bertanggungjawab atas kehilangannya serta mengembalikannya kepada pihak sekolah pada tarikh yang ditetapkan

Skim Pinjaman Buku Teks

Sekolah _____

Tahun	Tingkatan	Nama Penerima	Tarikh Terima

Nombor Perolehan: _____

Tarikh Penerimaan: _____

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