KURIKULUM STANDARD SEKOLAH MENENGAH

# MATHEMATICS
# FORM 2


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# CONTENTS

# INTRODUCTION

The Form 2 Mathematics textbook consists of 13 chapters that are formulated and designed based on The Framework of Secondary School Standard-based Curriculum for Mathematics Form 2.

Each chapter contains creative activities which aim to stimulate students' thinking. Learning objectives and word links are also provided to highlight the content of the chapter.

**The special features of this textbook are:**

| | |
|---|---|
| WHAT WILL YOU LEARN? | Contains learning standards of the chapter. |
| WORD LINK | Words used in the chapter. |
| WALKING THROUGH TIME | Looks at contribution of past mathematicians and the origin of words used in the chapter. |
| WHY STUDY THIS CHAPTER? | Knowledge obtained in this chapter and the vocation or jobs related to the chapter. |
| *CREATIVE* ACTIVITY | Induction activities that stimulate discussion and understanding among students. |
| COGNITIVE STIMULATION (Individual, Group) | Helps students to understand basic mathematical concepts through individual and group activities. |
| FLASHBACK | Reflects on previous knowledge and skills. |
| REMEMBER | Exposes students to extra knowledge and facts that are important in the chapter. |
| ATTENTION | Draws students attention to extra facts, common errors that students should avoid. |

THINK SMART → Questions that encourage mental stimulation, creativity and critical thinking.

SELF PRACTICE → Questions at the end of the chapter that test the students' understanding about the chapter.

QR CODE → Quick Response Code that uses URL in application.

→ Two dimensional bar code that is readable by mobile devices to watch videos, perform activities and obtain extra information.

TIPS → Exposes students to extra knowledge that is important.

DO YOU KNOW ? → Relevant general knowledge related to the chapter.

GENERATING EXCELLENCE → Summative exercises that enhance and enrich students at the end of the chapter.

→ Higher Order Thinking Skills (HOTS) that test the students' ability.

CHAPTER SUMMARY → Summary of the chapter.

SELF REFLECTION → Determines whether learning standards have been achieved.

MINI PROJECT → Beyond classroom activities that enhance understanding and creativity of the students at the end of the chapter.



# SYMBOLS AND FORMULAE

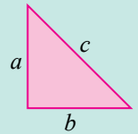
## SYMBOLS

| | | | |
|---|---|---|---|
| $\sqrt{\ }$ | square root | $\angle$ | angle |
| $\sqrt[3]{\ }$ | cube root | $T_n$ | $n^{th}$ term |
| $=$ | is equal to | $\sum$ | sum of total |
| $\neq$ | is not equal to | $\geqslant$ | is greater than or equal to |
| $\Delta$ | triangle | $\leqslant$ | is less than or equal to |
| $n$ | number of term | $n(A)$ | number of elements of set $A$ |
| $\pi$ | pi | | |

## FORMULAE

Sum of interior angles of a polygon $= (n - 2) \times 180°$

Pythagoras theorem:




$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$a^2 = c^2 - b^2$

Circumference $= 2\pi r$

Area of circle $= \pi r^2$

$\frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{360°}$

$\frac{\text{Arc length}}{2\pi r} = \frac{\theta}{360°}$

Surface area of cylinder $= 2\pi r^2 + 2\pi rh$

Surface area of cone $= \pi r^2 + \pi rs$

Surface area of sphere $= 4\pi r^2$

Volume of prism = area of cross section × height

Volume of cylinder $= \pi r^2 h$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of sphere $= \frac{4}{3}\pi r^3$

Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$

$\text{Gradient, } m = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = -\frac{y\text{-intercept}}{x\text{-intercept}}$

$\text{Mean} = \frac{\text{Total value of data}}{\text{Number of data}}$

$\text{Probability of an event} = \frac{\text{The number of ways event can occur}}{\text{The total number of possible outcome}}$

$P(A) = \frac{n(A)}{n(S)}$

Complement of an event, $P(A') = 1 - P(A)$

Download the free QR Code scanner from Google Play, App Store or other platforms to your mobile devices. Scan QR Code or visit the website http://rimbunanilmu.my/mat_t2e/msvii to download files, videos, GeoGebra, electronic spreadsheets and additional training questions. Then, save the downloaded file for offline use.

Note: *Students can download the free GeoGebra software programme to open the related files.*

http://www.geogebra.org/

# CHAPTER 1

# Patterns and Sequences

## WHAT WILL YOU LEARN?



## WORD LINK

| | |
|---|---|
| • Number pattern | • *Pola nombor* |
| • Odd number | • *Nombor ganjil* |
| • Even number | • *Nombor genap* |
| • Fibonacci Number | • *Nombor Fibonacci* |
| • Pascal's Triangle | • *Segi Tiga Pascal* |
| • Sequence | • *Jujukan* |
| • Algebraic expression | • *Ungkapan algebra* |
| • Term | • *Sebutan* |

The sunflower is a unique flower in terms of the arrangements of its seeds. The seeds are arranged in a spiral pattern and follow a particular direction. The number of seeds in the spirals can be arranged in a number pattern known as Fibonacci Numbers. The seeds are usually arranged into 2 types of spiral patterns. For example, 21 spirals follow the clockwise pattern and 34 spirals follow the anti-clockwise pattern. The numbers 21 and 34 are found in the Fibonacci sequence.

## WALKING THROUGH TIME

The Fibonacci Numbers began with a question posed by the Italian mathematician, Leonardo of Pisa or Fibonacci in his book, 'Liber Abaci' about the population of rabbits.
The question posed was that if a pair of female and male rabbits were placed in an enclosed space, how many pairs of rabbits will be reproduced in a year? If every pair of rabbits reproduce a new pair every month, then the increase in the population of the rabbits will produce a number sequence as follows 0, 1, 1, 2, 3, 5, 8, ... . These numbers are known as Fibonacci Numbers. The Fibonacci Numbers are arranged by adding the number before it. For example, the pairs of rabbits are 1 + 1, then the population of the rabbits becomes 2. Consequently, by adding the preceding number 1 and 2, the population of the rabbits becomes 3 and so forth.

For more information

http://rimbunanilmu.my/mat_t2e/ms001

## WHY STUDY THIS CHAPTER?

- The concept of number pattern and sequence can be applied in architecture, fashion design, science, astronomy, chemistry, physics and technology.

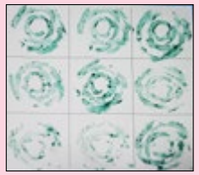
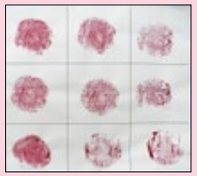
**CREATIVE ACTIVITY**

**Aim:** Recognising patterns
**Materials:** Potato, onion, mustard stem, drawing paper and water colour
**Steps:**

1. Take a piece of drawing paper.
2. Cut the potato, onion and mustard stem as shown in the pictures below.



3. Use the materials and do stamping on the drawing paper.
4. Dry the printout.

5. State the pattern produced.

From the activity above, students will be able to recognise the different type of patterns in our natural surroundings. These patterns become attractive formations.

## 1.1 Patterns

### 1.1.1 Recognising number patterns

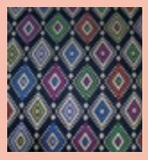
**LEARNING STANDARD**

Recognise and describe patterns of various number sets and objects based on real life situations, and hence make generalisation on patterns.

**COGNITIVE STIMULATION** Group

**Aim:** Recognising patterns
**Materials:** Batik cloth
**Steps:**

1. Look at the picture, that shows the patterns on some traditional Malaysian fabric.



**Discussion:**
(i) What patterns do you see?
(ii) What are the arrangements of the patterns like?

From the activity above, the patterns seen are repetitions of a polygon.



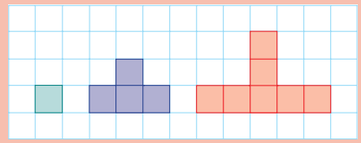




**COGNITIVE STIMULATION** Group

**Aim:** Recognising patterns
**Materials:** Colour pencil, ruler, pencil and grid paper
**Steps:**

1. Work in groups.
2. Open the file MS003 file for the grid paper.
3. Draw and colour the patterns as shown below.
4. Then continue to draw and colour the 4th, 5th and 6th patterns.
5. Fill up the table below.



**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms003 to get grid paper.

| Pattern Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Number of squares | 1 | 4 | 7 | | | | | |

6. Present your answers.

**Discussion:**
(i) State the pattern that you can observe.
(ii) Calculate the number of squares for pattern number 7 and 8.

From the activity above, the number of squares in the pattern 1, 4, 7, ... is determined by adding 3 to the number before it. Addition of 3 is the **pattern** for this sequence.

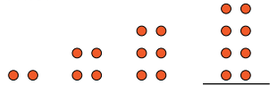
> Patterns are list of numbers or objects arranged based on a rule or design.

**EXAMPLE 1**

Draw the next object. State its pattern.

(a) 

*Solution:*

(a) 

Pattern: Add two dots to the previous object.

(b) 

(b) 

Pattern: Add a triangle to the previous object.

**EXAMPLE 2**

Determine the patterns for the following.

(a) $-10, -4, 2, 8, \ldots$

(b) $17, 7, -3, -13, \ldots$

(c) $2, 6, 18, 54, \ldots$

(d) $81, 27, 9, 3, \ldots$

(e) $1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$

(f) $-2.3, -2.6, -2.9, -3.2, \ldots$

*Solution:*

(a) $-10, \underset{+6}{\overset{}{-4}}, \underset{+6}{\overset{}{2}}, \underset{+6}{\overset{}{8}}, \ldots$

Pattern: Add 6 to the previous number.

(b) $17, \underset{-10}{\overset{}{7}}, \underset{-10}{\overset{}{-3}}, \underset{-10}{\overset{}{-13}}, \ldots$

Pattern: Subtract 10 from the previous number.

(c) $2, \underset{\times 3}{\overset{}{6}}, \underset{\times 3}{\overset{}{18}}, \underset{\times 3}{\overset{}{54}}, \ldots$

Pattern: Multiply the previous number by 3.

(d) $81, \underset{\div 3}{\overset{}{27}}, \underset{\div 3}{\overset{}{9}}, \underset{\div 3}{\overset{}{3}}, \ldots$

Pattern: Divide the previous number by 3.

(e) $1, \underset{+\frac{1}{2}}{\overset{}{\frac{3}{2}}}, 2, \underset{+\frac{1}{2}}{\overset{}{\frac{5}{2}}}, \ldots$

Pattern: Add $\frac{1}{2}$ to the previous number.

(f) $-2.3, -2.6, -2.9, -3.2, \ldots$

$\underset{-0.3}{\quad}\ \underset{-0.3}{\quad}\ \underset{-0.3}{\quad}$

Pattern: Subtract 0.3 from the previous number.

**Even and Odd numbers**

**EXAMPLE 3**

Given a series of numbers 7, 12, 17, 22, 27, …, 67. Identify and state the pattern for the sequence of

(i) odd numbers

(ii) even numbers

**FLASHBACK**

**Even numbers:** numbers that are divisible by 2, e.g. 2, 4, 6, 8, …

**Odd numbers:** numbers that are not divisible by 2. e.g. 1, 3, 5, 7, 9, ...

*Solution:*

7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67

(i) **Odd numbers:** $7, \underset{+10}{\overset{}{17}}, \underset{+10}{\overset{}{27}}$, 37, 47, 57 and 67

These odd numbers were obtained by adding 10 to the previous number.

(ii) **Even numbers:** $12, \underset{+10}{\overset{}{22}}, \underset{+10}{\overset{}{32}}$, 42, 52 and 62

These even numbers were obtained by adding10 to the previous number.

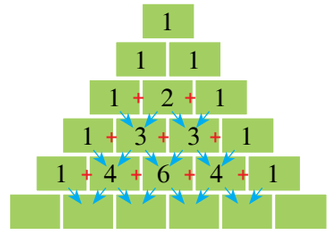






**Pascal's Triangle**

The diagram below shows a Pascal's Triangle. Based on the triangle, the numbers in the row can be determined by adding the numbers in the previous row.



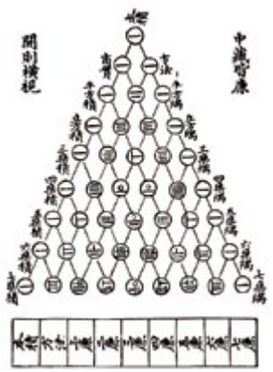

The Pascal's Triangle above starts with the number 1. The next row is 1, 1. All the rows start and end with 1. The other numbers can be obtained by adding the two numbers above.

The number 2 (row 3) is found by adding the two numbers 1, 1 (row 2). Likewise the number 3 on the fourth row is found by adding the number 1 and 2 from the previous row. Number 6 is found by adding the two numbers 3 and 3 from the previous row.

Fill in the last row.

From the triangle above, various number series and certain patterns can be observed:

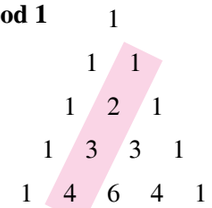
**DO YOU KNOW ?**



Yang Hui Triangle

Pascal's Triangle was known as Yang Hui's Triangle by the Chinese and is illustrated using magic squares and magic circles.

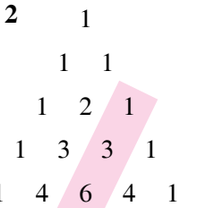
**Method 1**




Sequence: 1, 2, 3, 4, ...

Pattern: Add 1

**Method 2**




Sequence: 1, 3, 6, ...

Pattern: Add 2, 3, 4, ...

Pattern of a set of numbers is a sequence of numbers that are arranged according to a rule.

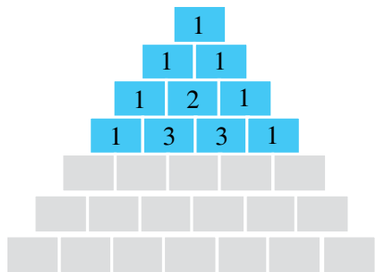
**THINK SMART**

| | |
|---|---|
| 1 × 1 | 1 |
| 11 × 11 | 121 |
| 111 × 111 | 12321 |
| 1111 × 1111 | 1234321 |
| 11111 × 11111 | 123454321 |

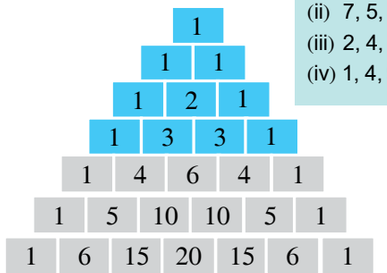
Determine the value of the next two terms.

**EXAMPLE 4**

Complete the Pascal's Triangle below.




*Solution:*




**THINK SMART**

State the next two terms.

(i) 3, 8, 15, 24, 35, ...

(ii) 7, 5, 8, 4, 9, 3, ...

(iii) 2, 4, 5, 10, 12, 24, 27, ...

(iv) 1, 4, 9, 18, 35, ...

**Fibonacci Numbers**

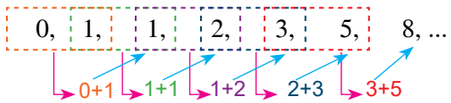
Fibonacci Numbers are a pattern of numbers in a sequence.

0, 1, 1, 2, 3, 5, 8, ...

0+1 1+1 1+2 2+3 3+5

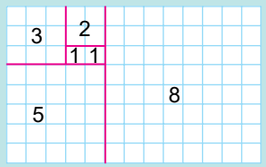
This sequence starts with 0, 1, 1 and the next term is obtained by adding the previous two terms.

Example:




**THINK SMART**

How will you form more Fibonacci squares?




**EXAMPLE 5**

Complete the number sequence below .

(a) 0, 1, 1, ☐, ☐, ☐, 8, 13, ☐, ...

(b) 1, 3, ☐, ☐, 11, ...

*Solution:*

(a) 0, 1, 1, 2, 3, 5, 8, 13, 21, ...

(b) 1, 3, 4, 7, 11, ...

**QR CODE**

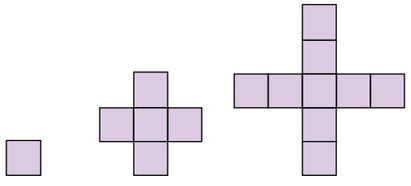
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms006 on one of the Fibonacci series.

Patterns are list of numbers or objects arranged following a rule or design. A pattern in a list of numbers is obtained from addition, subtraction, multiplication or division of the previous numbers. Whereas pattern in objects is obtained by observing the arrangements of the previous objects.

**SELF PRACTICE 1.1**

1. Draw the following patterns for the diagrams below.

(a)



_____ _____ _____ _____

(b)

_____ _____ _____ _____







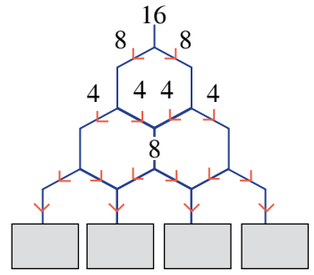
2. State the pattern for the following sequence.
   (a) 5, 12, 19, 26, ...
   (b) −1, −4, −7, −10, ...
   (c) −4, 0, 4, 8, ...
   (d) 144, 72, 36, 18, ...
   (e) $\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \ldots$
   (f) 11.2, −33.6, 100.8, −302.4, ...

3. For the number sequence 28, 37, 46, 55, … , 145, state the number pattern for
   (a) odd numbers
   (b) even numbers

4. Complete the following Fibonacci Numbers sequence.

   1, ☐, 2, ☐, ☐, ☐, ☐, ...

5. Fill in the missing number in the boxes below.




## 1.2 Sequences

### 1.2.1 Sequences

**LEARNING STANDARD**

Explain the meaning of sequence.

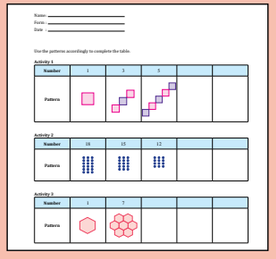
**COGNITIVE STIMULATION** Individual

**Aim:** Recognising the pattern in a number sequence

**Material:** Worksheet

**Steps:**

1. Open the file MS007 for the grid paper.
2. Complete the table by drawing the next patterns.



**Discussion:**

(i) State the pattern that you found from activity 1, 2 and 3.

(ii) List down the number sequence from activity 1, 2, and 3.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms007 to get the worksheet.

Based on the activities, the pattern can be determined by following the previous arrangement. An arrangement of numbers or objects following this pattern is known as **sequence**.

Sequence is a set of numbers or objects arranged according to a certain pattern.

## 1.2.2 Patterns of a sequence

**LEARNING STANDARD**

Identify and describe the pattern of a sequence, and hence complete and extend the sequence.

**EXAMPLE 6**

Determine whether each set of numbers is a sequence

(a) $-10, -6, -2, 2, 6, \ldots$ (b) $4, 5, -7, 10, -14, \ldots$

*Solution:*

(a) $-10, -6, -2, 2, 6, \ldots$

+4 +4 +4 +4

Pattern: Add 4
Therefore, the set of numbers is a sequence.

(b) $4, 5, -7, 10, -14, \ldots$

+1 −12 +17 −24

Pattern: None
Therefore, the set of numbers is not a sequence.

**DO YOU KNOW ?**

Astronomers use patterns to predict the path of a comet.

### Number sequence

**EXAMPLE 7**

Complete the number sequences below.

(a) $7, 13, \square, 25, \square, \square, \ldots$ (b) $88, \square, 64, 52, \square, \square, \ldots$

(c) $\square, 0.3, \square, 0.027, 0.0081, \square, \ldots$ (d) $\square, \square, \frac{1}{3}, \frac{4}{6}, \square, \ldots$

*Solution:*

(a) $7, 13, \boxed{19}, 25, \boxed{31}, \ldots$

+6 +6 +6 +6

(b) $88, \boxed{76}, 64, 52, \boxed{40}, \boxed{28}, \ldots$

−12 −12 −12 −12 −12

(c) $\boxed{1}, 0.3, \boxed{0.09}, 0.027, 0.0081, \boxed{0.00243}, \ldots$

×0.3 ×0.3 ×0.3 ×0.3 ×0.3

(d) $-\frac{1}{3}, \boxed{0}, \frac{1}{3}, \frac{4}{6}, \boxed{1}, \ldots$

$+\frac{1}{3}$ $+\frac{1}{3}$ $+\frac{1}{3}$ $+\frac{1}{3}$







**EXAMPLE 8**

Complete the number sequences below based on the given pattern.

(a) Subtract 4 from the previous number.

$96, \square, \square, \square, \square, \square, \ldots$

(b) Multiply the previous number by 3.

$7, \square, \square, \square, \square, \square, \ldots$

(c) Subtract 8 from the previous number.

$21.3, \square, \square, \square, \square, \square, \ldots$

(d) Divide the previous number by 5.

$400, \square, \square, \square, \square, \square, \ldots$

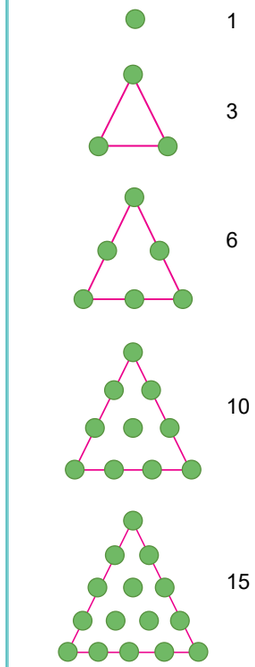
*Solution:*

(a) 92, 88, 84, 80, 76, ...
(b) 21, 63, 189, 567, 1 701, ...
(c) 13.3, 5.3, −2.7, −10.7, −18.7, ...
(d) 80, 16, 3.2, 0.64, 0.128, ...

**DO YOU KNOW ?**

Triangular numbers are numbers represented by dots to make an equilateral triangle.

1, 3, 6, 10, 15, 21, 28, 36, ...




**SELF PRACTICE 1.2**

1. Determine whether each set of the numbers is a sequence.
   (a) 3, 18, 33, 48, ... (b) 100, 116, 132, 148, ...
   (c) 1.0, −1.7, −2.4, 3.1, ... (d) −15, 30, 60, −120, ...
   (e) $\frac{1}{4}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{3}, \ldots$ (f) −0.32, −0.16, −0.8, −0.4, ...

2. Complete the number sequences below.
   (a) $34, 28, \square, 16, \square, \square, \ldots$ (b) $\square, \square, 32, 16, \square, 4, \ldots$
   (c) $0.07, \square, 1.12, \square, 17.92, \ldots$ (d) $1\frac{1}{10}, 1, \square, \square, \square, \ldots$
   (e) $0.2, 2.4, 28.8, \square, \square, \ldots$ (f) $\square, -80, -16, \square, \square, \ldots$
   (g) $\square, \frac{2}{3}, \frac{7}{12}, \square, \square, \ldots$ (h) $-8.1, \square, -4.1, -2.1, \square, \ldots$

3. Complete the number sequences below based on the given pattern.

   (a) Add 7 to the previous number.

   42, ☐, ☐, ☐, ☐, ☐, ☐, ...

   (b) Divide the previous number by 2.

   96, ☐, ☐, ☐, ☐, ☐, ☐, ...

## 1.3 Patterns and Sequences

### 1.3.1 Pattern of a sequence using numbers, words and algebraic expressions

**LEARNING STANDARD**

Make generalisation about the pattern of a sequence using numbers, words and algebraic expressions.

**EXAMPLE 9**

Describe the pattern for the number sequence 1, 9, 17, 25, 33, … by using numbers, words and algebraic expressions.

*Solution:*

(i) **Numbers**

1, 9, 17, 25, 33, ...

+8 +8 +8 +8

Therefore, the pattern is + 8.

(ii) **Words**

1, 9, 17, 25, 33, ...

+8 +8 +8 +8

Therefore, the pattern for the above sequence is add 8 to the previous number.

(iii) **Algebraic expressions**

1, 9, 17, 25, 33, ...

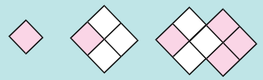
+8 +8 +8 +8

1 = 1 + 8 (0)
9 = 1 + 8 (1)
17 = 1 + 8 (2)
25 = 1 + 8 (3)
33 = 1 + 8 (4)

Therefore, the pattern for the number sequence can be expressed as $1 + 8n$ where, $n = 0, 1, 2, 3, 4, \ldots$ .

**THINK SMART**

An interior decorator wishes to arrange the tiles on a wall using the patterns below.



What is the pattern?

**FLASHBACK**

Algebraic expressions is an expression which has a combination of basic mathematical operations on numbers, variables or other mathematical entities.

Example:
$2ab + 3c$, $5a + 2b - 3c$







### 1.3.2 Terms of a sequence

**LEARNING STANDARD**

Determine specific terms of a sequence.

The $n^{th}$ term in a number sequence and is written as $T_n$ whereby $T$ is the term and $n$ is the position of the term.

$T_n = n^{th}$ term

For example,

4, 8, 12, 16, ...

From the sequence above...

$T_1 = 4$,
$T_2 = 8$,
$T_3 = 12$,
$T_4 = 16$, ...

**DO YOU KNOW ?**

The queen bee lays eggs in its nest which has a hexagonal pattern.

**EXAMPLE 10**

State the 5th term for the following number sequence.

2, 10, 18, ...

*Solution:*

Step 1: Determine the pattern for the number sequence.

2, 10, 18, ...

+8 +8

Number pattern: Add 8 to the previous number.

Step 2: List down the first 5 terms as shown below.

$T_1 = 2$ $T_4 = 26$
$T_2 = 10$ $T_5 = 34$
$T_3 = 18$

Hence, the 5th term is 34.

**THINK SMART**

$2^2 + (2 + 2 + 1) = 3^2$
$3^2 + (3 + 3 + 1) = 4^2$
$4^2 + (4 + 4 + 1) = 5^2$
$5^2 + (5 + 5 + 1) = 6^2$

(i) State the next two terms of the sequence.
(ii) State the $n^{th}$ term.

**THINK SMART**

Identify the patterns for the sequences below.
(i) 1, 4, 9, 18, 35
(ii) 23, 45, 89, 177
(iii) 5, 7, 12, 19, 31
(iv) 0, 4, 2, 6, 4, 8
(v) 4, 7, 15, 29, 59, 117

**EXAMPLE 11**

Given the number sequence 65, 60, 55, 50, ... . Determine which term in the number sequence is 40.

*Solution:*

Step 1:

65, 60, 55, 50, ...

–5 –5 –5

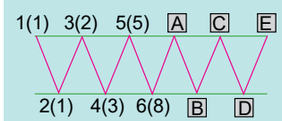
Pattern: Subtract 5 from the previous number.

Step 2:

$T_1 = 65$ $T_4 = 50$
$T_2 = 60$ $T_5 = 45$
$T_3 = 55$ $T_6 = 40$

Hence, 40 is the 6th term.

**THINK SMART**




State the suitable pairs of numbers for A, B, C, D, E.

## 1.3.3 Solving problems

**LEARNING STANDARD**

Solve problems involving sequences.

**EXAMPLE 12**

Automatic fish feeder

**Specifications**

- Container size: Moderate
- Dried Food and pellet maybe used
- A timer is used to arrange feeding time
- Use the latest technology to prevent food from getting moist or stuck in the container
- Can be operated manually or automatically
- Digital screen display

The picture shows an automatic fish feeder and its specifications. If Eng Wei decides to feed the fishes 4 times a day with the first feeding time at 7:35 a.m., at what time should he feed the fish for the third feeding?

| Understanding the problem | Planning the strategy | Implementing the strategy | Conclusion |
|---|---|---|---|
| Time for third feeding for the fishes. | 1 day = 24 hours<br>each feed $= \frac{24}{4}$<br>$= 6$ hours | Pattern: 6 hours<br>$T_1$ = 7:35 a.m.<br>$T_2$ = 7:35 a.m. + 6 hours<br>= 1:35 p.m.<br>$T_3$ = 1:35 p.m. + 6 hours<br>= 7:35 p.m. | Hence, fishes are fed for the third time at 7:35 p.m. |

**SELF PRACTICE 1.3**

1. State the pattern for the number sequences below in words.
   (a) 4, 12, 36, 108, 324, ...
   (b) 256, 128, 64, 32, 16, ...

2. Determine the pattern for the number sequences below using algebraic expressions.
   (a) 2, 4, 8, 16, ...
   (b) 5, 8, 11, 14, ...
   (c) 3, 6, 9, 12, ...
   (d) 3, 1, –1, –3, ...

3. Determine the seventh and the eleventh terms for the number sequences below.
   (a) –3, 5, 13, ...
   (b) $4, 5\frac{1}{2}, 7, \ldots$
   (c) –3.7, –4.3, –4.9, ...







4. The table below shows the timetable for buses travelling from Kuala Lumpur to Pulau Pinang.

| Bus | Departure time |
|---|---|
| *A* | 8:00 a.m. |
| *B* | 8:30 a.m. |
| *C* | 9:00 a.m. |
| *D* | |
| *E* | |

Based on the table above, answer the following questions.
(a) Calculate the interval between departure time of one bus and the next bus.
(b) What time does Bus *E* leave?
(c) What time will Bus *E* reach Pulau Pinang if the journey takes 5 hours?

**GENERATING EXCELLENCE**

1. Match the term with the suitable statement.

| | |
|---|---|
| Pascal's Triangle | Numbers that cannot be divided by 2 exactly. |
| Odd numbers | This sequence starts with 0, 1, 1 and the following terms can be determined by adding the previous two terms. |
| Fibonacci Numbers | Numbers that can be divided by 2 exactly. |
| Even numbers | Geometrical arrangements on the binomial coefficients of a triangle. |

2. Determine the pattern for the given number sequences.
   (a) 7, 13, 19, 25, ...
   (b) 54, 50, 46, 42, ...
   (c) –13, –39, –117, –351, ...
   (d) 1 296, 216, 36, 6, ...

3. Complete the table below.

| Sequence | Number | Words | Algebraic expressions |
|---|---|---|---|
| (a) 2, 4, 6, 8, ... | | | |
| (b) 100, 50, 25, 12.5, ... | | | |

4. Complete the following number sequence.

   (a) 1, 3, 5, ☐, 9, ☐, ...

   (b) ☐, ☐, −20, −10, −5, ...

   (c) 268, ☐, ☐, 169, 136, ☐, ...

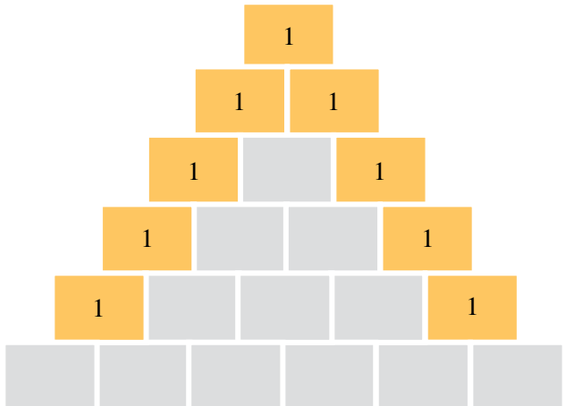
   (d) $\frac{1}{2}$, ☐, $\frac{1}{3}$, ☐, $\frac{1}{6}$, ...

5. The first four terms of a sequence are 9, $x$, −5, −12, ...
   (a) Calculate the value of $x$.
   (b) State the pattern of the sequence using
      (i) Numbers
      (ii) Words
      (iii) Algebraic expressions

6. Complete the Fibonacci Numbers shown below.

   0, 1, 1, ☐, ☐, ☐, ...

7. The diagram below shows the first five rows of the Pascal's Triangle. Complete the Pascal's Triangle. Explain how the Pascal's Triangle is formed.



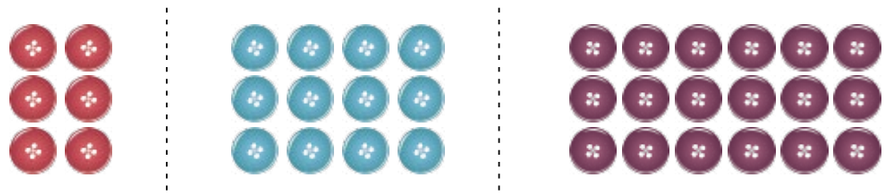

8. The first four terms for a sequence are 11, $x$, −5, −13, ...
   (a) Calculate the value of $x$.
   (b) State the tenth term, $T_{10}$





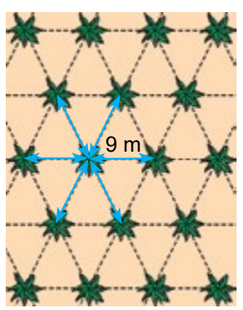


9. Nina arranged some buttons as shown below.



   (a) State the pattern for the number of buttons.
   (b) Determine the sequence for the buttons.
   (c) Draw the fourth term of the arrangement of buttons.
   (d) Calculate the value of $T_6$.

10. Encik Hamid wishes to replant the oil palm plants. The distance between each plant is 9 m and the distances are triangular shaped. Encik Hamid sketched a map of the plants as shown below.




    If Encik Hamid planted 18 oil palm plants, what is the area of his land?

11. Raiyan went to see a doctor because he had been unwell for more than three days. The doctor prescribed three types of medicines which are fever medication, antibiotics and flu medication. Help Raiyan to plot a time table for taking his medication if he starts at 8:30 a.m.

| Medicine | 1 | 2 | 3 |
|---|---|---|---|
| Fever | | | |
| Antibiotics | | | |
| Flu | | | |

Fever = 2 tablets 3 times a day
Antibiotics = 1 tablet 2 times a day
Flu = 1 tablet 1 times a day

## CHAPTER SUMMARY

**Patterns**

Patterns are list of numbers or objects arranged based on a rule or design.

**Patterns of various set of numbers**

(i) Even numbers and odd numbers

4, 9, 14, 19, ...
+5 +5 +5

**even numbers:** 4, 14, 24, ...
+10 +10

**odd numbers:** 9, 19, 29, ...
+10 +10

(ii) Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

(iii) Fibonacci Numbers

0, 1, 1, 2, 3, 5, 8, ...

**Sequences**

Sequence is a set of numbers or objects which follows a certain pattern.

**Patterns and Sequences**

The pattern of a sequence is the rule or design of the sequence.

**Patterns of a sequences**

**Numbers**

3, 6, 9, 12, 15, ...
+3 +3 +3 +3

Pattern: + 3

**Algebraic expressions**

3, 6, 9, 12, 15, ...

Is written as $3n$ where $n = 1, 2, 3, ...$

**Words**

4, 7, 10, 13, 16, ...

The sequence begins with the number 4 and the pattern is add 3 to the number before it.

**Terms of a sequence**

$-9, -11, -13, -15, -17, ...$
$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5$

First term, $T_1 = -9$
Second term, $T_2 = -11$
Third term, $T_3 = -13$
Fourth term, $T_4 = -15$
Fifth term, $T_5 = -17$







## SELF REFLECTION

| At the end of this chapter, I will be able to: | | | |
|---|---|---|---|
| 1. Recognise and describe the patterns of various set of numbers and objects in everday life. | ○ | ○ | ○ |
| 2. Explain the meaning of sequence. | ○ | ○ | ○ |
| 3. Recognise and describe the pattern of a sequence. | ○ | ○ | ○ |
| 4. Complete and extend a sequence using numbers. | ○ | ○ | ○ |
| 5. Express pattern of a sequence using numbers, words and algebraic expressions. | ○ | ○ | ○ |
| 6. Determine specific terms of a sequence. | ○ | ○ | ○ |
| 7. Solve problems involving sequences. | ○ | ○ | ○ |

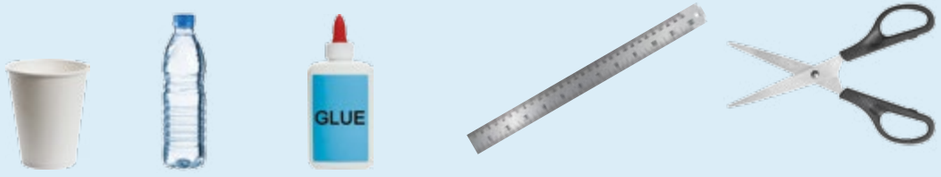
## MINI PROJECT

**Title:** Futuristic building
**Materials:** Paper cups, mineral water bottle, glue, ruler and scissors

Each group is to make building which has futuristic features using the materials.

Colour the building and name it.

In group present the product.



CHAPTER 2

# Factorisation and Algebraic Fractions

Algebra is a branch of mathematics used to explain the relationship between various units of quantity, for example distance with speed, weight and height etc. Students will be able to learn problem solving skills under different situations through these types of relationships.

## WHAT WILL YOU LEARN?

- **2.1** Expansion
- **2.2** Factorisation
- **2.3** Algebraic Expressions and Laws of Basic Arithmetic Operations

## WORD LINK

- Expansion
- Algebraic expression
- Factor
- Highest Common Factor (HCF)
- Algebraic fraction
- Perfect squares
- Cross multiplication
- Numerator
- Denominator
- Lowest term
- Lowest Common Multiple (LCM)

- *Kembangan*
- *Ungkapan algebra*
- *Faktor*
- *Faktor Sepunya Terbesar (FSTB)*
- *Pecahan algebra*
- *Kuasa dua sempurna*
- *Pendaraban silang*
- *Pengangka*
- *Penyebut*
- *Sebutan terendah*
- *Gandaan Sepunya Terkecil (GSTK)*



## WALKING THROUGH TIME

According to the book 'al-Jabr w'al-Muqabalah' written by the Persian Arabian mathematician, Muhammad Ibn Musa al-Khwarizmi, the word algebra originated from 'al-Jabr'. He was also known as the 'Father of Algebra' for his contribution in the field of Algebra.

For more information:

http://rimbunanilmu.my/mat_t2e/ms019

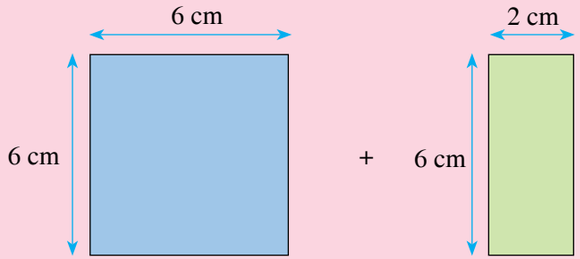
## WHY STUDY THIS CHAPTER?

- Algebra is mostly used in price comparison, buying and selling process, measurement, etc.
- Algebra is also used in certain fields of study like Chemistry, Physics and Forensics.

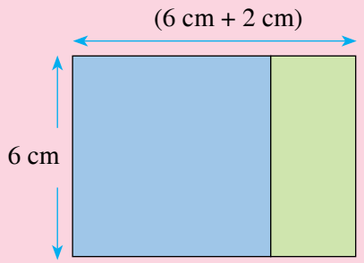
**CREATIVE ACTIVITY**

**Aim:** Determining area using algebraic tiles
**Materials:** Green and blue papers
**Steps:**
1. Cut the blue paper into a square measuring 6 cm by 6 cm.
2. Cut the green paper into a rectangle measuring 6 cm by 2 cm.
3. Calculate the area of the blue square and green rectangle using method 1 and 2.

**Method 1**: Area of blue square **+** area of green rectangle.



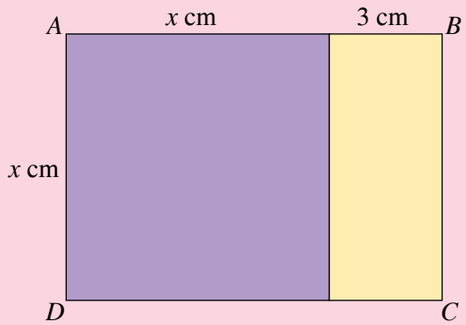

**Method 2**: Length × (width of the blue square + width of the green rectangle)




**DO YOU KNOW ?**

Algebra tiles are rectangular and square tiles used to represent algebraic principles.

4. Do the two methods give similar answers? Discuss.
5. Based on the diagram below, calculate the area of the rectangle, *ABCD*.




**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms020 to learn more on algebra tiles.







# 2.1 Expansion

## 2.1.1 Expansion of algebraic expressions

Expansion of algebraic expression is the product of multiplication of one or two expressions in brackets.

**LEARNING STANDARD**

Explain the meaning of the expansion of two algebraic expressions.

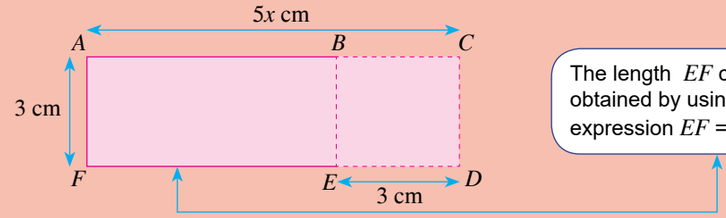
**FLASHBACK**

Algebraic expressions are expressions that combine numbers, variables or mathematical entities using mathematical operations. Example, $2a + 5$.

## 2.1.2 Expansion on two algebraic expressions

**COGNITIVE STIMULATION** Individual

**Aim:** Determining the area of rectangle *ABEF*
**Material:** Worksheet
**Steps:**
1. Calculate area *ABEF* using the two methods shown below.




The length *EF* can be obtained by using the expression $EF = (5x - 3)$ cm

**Method 1 :**
Area *ABEF*
= Area *ACDF* – Area *BCDE*
= ☐ – ☐
= ☐ cm²

**Method 2:**
Area *ABEF*
= length × width
= *EF* × *AF*
= ☐ × ☐
= ☐ cm²

**Discussion:**
Is the answer for method 2 the same as the answer for method 1? Explain.

**LEARNING STANDARD**

Expand two algebraic expressions.

When doing expansion of algebraic expressions, every term within the bracket needs to be multiplied with the term outside the bracket.

**FLASHBACK**

| | |
|---|---|
| (+) × (+) | + |
| (+) × (–) | – |
| (–) × (+) | – |
| (–) × (–) | + |

**EXAMPLE 1**

Expand the following expressions.

(a) $6(3 + 4w)$

(b) $3r(r - 2s)$

(c) $-5b(a + 3)$

(d) $-\frac{2y}{3}(9y - 3z + 6x)$

***Solution:***

(a) $6(3 + 4w)$
$= (6 \times 3) + (6 \times 4w)$
$= 18 + 24w$

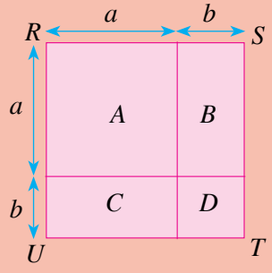
(b) $3r(r - 2s)$
$= (3r \times r) + [3r \times (-2s)]$
$= 3r^2 - 6rs$

(c) $-5b(a + 3)$
$= (-5b \times a) + (-5b \times 3)$
$= -5ab - 15b$

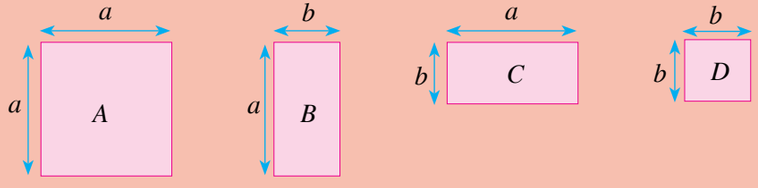
(d) $-\frac{2y}{3}(9y - 3z + 6x)$
$= \left(-\frac{2y}{1\cancel{3}} \times \cancel{9}^{3}y\right) + \left[-\frac{2y}{1\cancel{3}} \times (-\cancel{3}^{1}z)\right] + \left(-\frac{2y}{1\cancel{3}} \times \cancel{6}^{2}x\right)$
$= -6y^2 + 2yz - 4xy$

**COGNITIVE STIMULATION** Group

**Aim:** Expanding two algebraic expressions
**Material:** Worksheet
**Steps:**
1. Work in pairs.
2. Student *A* calculates the area of square *RSTU* using method 1.
3. Student *B* calculates the area of square *RSTU* using method 2.




**Method 1**




Area for the square *RSTU* = Area *A* + Area *B* + Area *C* + Area *D*

$= \boxed{(\_ \times \_)} + \boxed{(\_ \times \_)} + \boxed{(\_ \times \_)} + \boxed{(\_ \times \_)}$
$= \boxed{\phantom{00}} + \boxed{\phantom{00}} + \boxed{\phantom{00}} + \boxed{\phantom{00}}$
$= \boxed{\phantom{00}} + \boxed{\phantom{00}} + \boxed{\phantom{00}}$



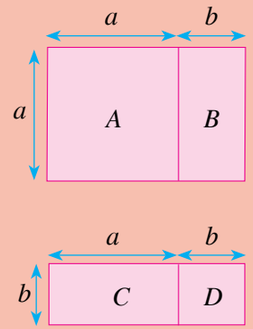




**Method 2**
Separate the square into two parts as follow.

Area of square *RSTU* = Area *A* and *B* + Area *C* and *D*

$= \boxed{(\_\_\_)(a + b)} + \boxed{(\_\_\_)(a + b)}$
$= \boxed{\phantom{00}} + \boxed{\phantom{00}} + \boxed{\phantom{00}} + \boxed{\phantom{00}}$
$= \boxed{\phantom{00}} + \boxed{\phantom{00}} + \boxed{\phantom{00}}$




**Discussion:**
Are your answers for both methods similar? Explain.

When doing expansion on two algebraic expressions in two brackets, every term in the first bracket must be multiplied with every term in the second bracket. For example,

$(a + 2)(a + 1)$
$= a(a + 1) + 2(a + 1)$
$= a^2 + a + 2a + 2$
$= a^2 + 3a + 2$

Like terms can be solved

**TIPS**

$(a + b)(a + b) = (a + b)^2$
$(a - b)(a - b) = (a - b)^2$
$(a + b)(a - b) = (a \times a) + [a \times (-b)] + (b \times a) + [b \times (-b)]$
$= a^2 - ab + ba - b^2$
$= a^2 - b^2$

**ATTENTION**

$(a + b)(a - b) = a^2 - b^2$
$(a + b)(a + b) \neq a^2 + b^2$
$(a - b)(a - b) \neq a^2 - b^2$

**EXAMPLE 2**

Expand each of the following expressions.

(a) $(y + 1)(y - 3)$ (b) $(4 + 3r)(2 + r)$
(c) $(3r + 4s)(r - 2s)$ (d) $(3p + 2)^2$

***Solution:***

(a) $(y + 1)(y - 3)$
$= y(y - 3) + 1(y - 3)$
$= y^2 - 3y + y - 3$
$= y^2 - 2y - 3$

(b) $(4 + 3r)(2 + r)$
$= 8 + 4r + 6r + 3r^2$
$= 8 + 10r + 3r^2$
$= 3r^2 + 10r + 8$

**DO YOU KNOW?**

**Alternative methods**
(i) Cross multiplication

| | | | | |
|---|---|---|---|---|
| (×) | $a$ | $+2$ | $2a$ | (+) |
| | $a$ | $+1$ | $a$ | |
| | $a^2$ | $+2$ | $3a$ | |

Hence, $a^2 + 3a + 2$

(ii) Standard form

$$\begin{array}{r} a + 2 \\ \times \quad a + 1 \\ \hline a + 2 \\ (+)\ a^2 + 2a \\ \hline a^2 + 3a + 2 \end{array}$$

(c) $(3r + 4s)(r - 2s)$
$= 3r(r - 2s) + 4s(r - 2s)$
$= (3r \times r) + [3r \times (-2s)] + (4s \times r) + [4s \times (-2s)]$
$= 3r^2 - 6rs + 4sr - 8s^2$
$= 3r^2 - 2rs - 8s^2$

Like terms can be solved

Like terms
$sr = rs$

(d) $(3p + 2)^2$
$= (3p + 2)(3p + 2)$
$= 9p^2 + 6p + 6p + 4$
$= 9p^2 + 12p + 4$

Like terms can be solved

**THINK SMART**

Relationship between repeated multiplication of Binomial expression with Pascal's Triangle.

$1 \quad (a+b)^0$
$1a + 1b \quad (a+b)^1$
$1a^2 + 2ab + 1b^2 \quad (a+b)^2$
$1a^3 + 3a^2b + 3ab^2 + 1b^3 \quad (a+b)^3$
$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4 \quad (a+b)^4$

State the next two terms.

**TIPS**

Algebraic terms are arranged from the highest power to the lowest power.

**QR CODE**

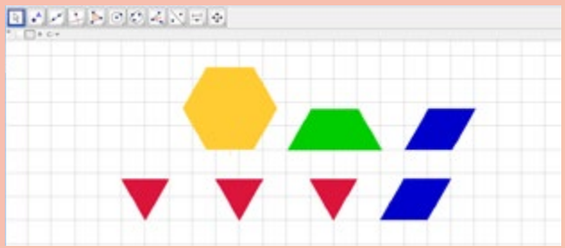
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms024a to view a video on cross multiplication method.

## 2.1.3 Combined operations including expansion

Combined operations for algebraic terms must be solved by following the 'BODMAS' rule.

**COGNITIVE STIMULATION** Group

**Aim:** Writing algebraic expression using algebra tiles
**Material:** Dynamic geometry software
**Steps:**



1. Open the file MS024B to display a yellow hexagon and other coloured shapes of red, blue and green.
2. Choose a combination of coloured shapes of red, blue or green to fit into the yellow hexagon.
3. Write out the algebraic relationship.
4. Choose other combinations of shapes to fit into the green trapezium.

**Discussion:**
Compare your findings with other groups.

**LEARNING STANDARD**

Simplify algebraic expressions involving combined operations, including expansion.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms024b to construct polygon.







**EXAMPLE 3**

Simplify.
(a) $(3w - 2)(4w - 1) - 10w$ (b) $(r - 3t)^2 + 4rt$
(c) $(x + y)(x - y) + x(x - 2y)$

***Solution:***

(a) $(3w - 2)(4w - 1) - 10w = 3w\,(4w - 1) - 2\,(4w - 1) - 10w$
$= 12w^2 - 3w - 8w + 2 - 10w$
$= 12w^2 - 3w - 8w - 10w + 2$
$= 12w^2 - 21w + 2$

(b) $(r - 3t)^2 + 4rt = (r - 3t)(r - 3t) + 4rt$
$= r^2 - 3rt - 3rt + 9t^2 + 4rt$
$= r^2 + 9t^2 - 3rt - 3rt + 4rt$
$= r^2 + 9t^2 - 2rt$

(c) $(x + y)(x - y) + x(x - 2y) = x^2 - xy + xy - y^2 + x^2 - 2xy$
$= x^2 + x^2 - y^2 - xy + xy - 2xy$
$= 2x^2 - y^2 - 2xy$

**FLASHBACK**

**B** = Brackets
**O** = Order
**D** = Division
**M** = Multiplication
**A** = Addition
**S** = Subtraction

For further information:

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms025

**FLASHBACK**

Distributive law is used when expanding bracket.

$a \times (b + c) = a \times b + a \times c$
$a \times (b - c) = a \times b - a \times c$

## 2.1.4 Solving problems

**LEARNING STANDARD**

Solve problems involving expansion of two algebraic expressions.

**EXAMPLE 4**

Puan Maria has a piece of carpet, $(3r - 2)$ metre in length and $(r + 1)$ metre in width. Calculate the area of her carpet.

***Solution:***


Area = length × width
$= (3r - 2)(r + 1)$
$= 3r^2 + 3r - 2r - 2$
$= 3r^2 + r - 2$

Hence, the size of the carpet is $(3r^2 + r - 2)$ square meters.

**EXAMPLE 5**

Ramesh received RM50 pocket money for $(y - 8)$ days. Everyday he spends RM$(x - 3)$ for a cup of coffee and RM$(x + 4)$ for a bowl of mee rebus. How much pocket money is he left with?

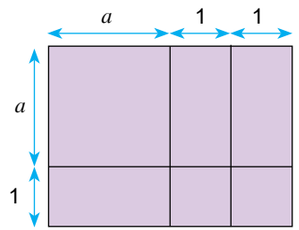
*Solution:*

| Understanding the problem | Planning the strategy | Implementing the strategy | Conclusion |
|---|---|---|---|
| Determine the total price of the coffee and mee rebus.<br><br>$(x-3)+(x+4)$<br>$=2x+1$ | Determine the total spending for $(y-8)$ days using the expansion method.<br><br>$(y-8)(2x+1)$<br>$=2xy+y-16x-8$ | Calculate the remainder money using the expansion process.<br><br>Pocket money – spending<br>$=50-(2xy+y-16x-8)$<br>$=50-2xy-y+16x+8$<br>$=58-2xy-y+16x$ | Remainder of pocket money<br><br>RM$(58-2xy-y+16x)$ |

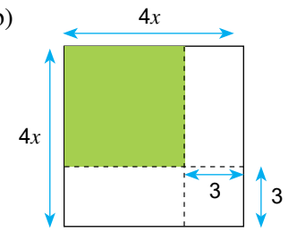
## SELF PRACTICE 2.1

1. Based on the following algebra tiles, write out the area of the shaded region in the form of multiplication of two algebraic expressions.

(a)




(b)




2. Expand the following algebraic expressions.

(a) $3(x+2)$ (b) $4(8x-3)$ (c) $2(a+5)$

(d) $p(6p-8)$ (e) $-\frac{r}{8}(2s-8)$ (f) $-2(pr-2pq)$

(g) $3(5bc-6)$ (h) $7(2ef+3e)$ (i) $8g(2+gh)$

3. Expand the following algebraic expressions.

(a) $(a+1)(a+2)$ (b) $(x-5)(x+4)$ (c) $(2+m)(5-m)$

(d) $(3p-2)(4p-1)$ (e) $(3r-2)(4r-1)$ (f) $(2r+s)(4r-3s)$

(g) $(2d-\frac{1}{2}b)(3d-\frac{1}{2}b)$ (h) $(r-3s)^2$ (i) $(4e-3)^2$

4. Simplify each of the following expressions.

(a) $(5b+3)+4(3b-a)$ (b) $3(4m-5mn)-2(8m+mn)$

(c) $(h-j)^2-2h(3h-3j)$ (d) $(x+y)(x-y)+2x(x+2y)$

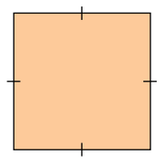




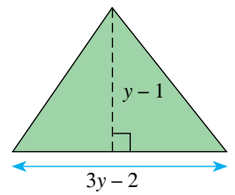


5. Determine the area for the diagrams below using algebraic expressions.

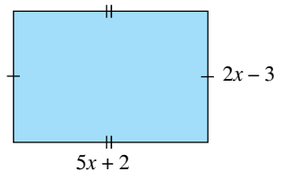
(a)



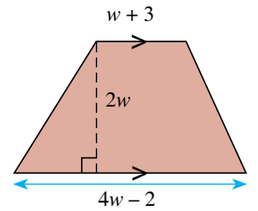

(b)




(c)



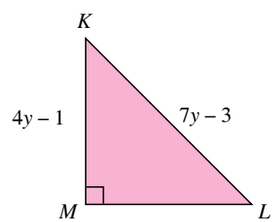

(d)




6. Hadila is 2 years younger than Kai Yee. Kai Yee's father's age is the square of Hadila's age. If Kai Yee is $p$ years old, calculate the total age of the three of them. Express your answer in the form of algebraic expression.

7. The table top is a rectangular shape with length $(5x-2)$ metre and width $(x+2)$ metre. Mr. Phillip wants to put a piece of glass over the table top. The section of the table top not covered with the glass has a width of $(x-3)$ metre. Determine the area of the table top that is not covered in the form of algebraic expressions.

8. Determine the length of $LM$ in terms of $y$.




## 2.2 Factorisation

### 2.2.1 Factors and factorisation concept

**LEARNING STANDARD**

Relate the multiplication of algebraic expression to the concept of factors and factorisation, and hence list out the factors of the product of the algebraic expressions.

**Factorisation** is the process of determining the factors of an algebraic expression or algebraic terms and when multiplied together will form the original expression. Factorisation is the reverse process of an expansion.

Example, the factors of $3p$

$1 \times 3p$ $3 \times p$

Therefore, factors of $3p$ are 1, 3, $p$ and $3p$.

## Factors, Common Factors and Highest Common Factor (HCF) for the product of algebraic expressions

Common factor is the factor of an algebra term that divides two or more other terms exactly. Highest Common Factor (HCF) is the largest of those common factors.

Examine the expression,

$$4x + 2 = 2(2x + 1)$$

2 is the common factor for the expression $4x$ and 2.

**TIPS**

Factorisation is the opposite of expansion.

**Expansion**

$a(a + b) = a^2 + ab$

**Factorisation**

**EXAMPLE 6**

List out all the common factors for each of the following terms.

(a) $6h, 4gh$ (b) $9c^2d, 3d^2e, 6def$

***Solution:***

(a) $6h = 1 \times 6h$
$2 \times 3h$
$3 \times 2h$
$h \times 6$
$4gh = 1 \times 4gh$
$4 \times gh$
$2 \times 2gh$
$2g \times 2h$
$g \times 4h$
$h \times 4g$

Therefore, common factors for $6h$ and $4gh$ are 1, 2, $h$ and $2h$.

(b) $9c^2d, 3d^2e$ and $6def$
$9c^2d = 1 \times 3 \times 3 \times c \times c \times d$
$3d^2e = 1 \times 3 \times d \times d \times e$
$6def = 1 \times 2 \times 3 \times d \times e \times f$

Common factors for $9c^2d$, $3d^2e$ and $6def$ are 1, 3, $d$ and $3d$. $3d$ is a common factor as it can divide all the above terms exactly.

**ATTENTION**

'1' is a factor for all algebraic terms.

## 2.2.2 Factorisation of algebraic expressions

**LEARNING STANDARD**

Factorise algebraic expressions using various methods.

### Using HCF

Algebraic expressions can be factorised using Highest Common Factor (HCF).

Example,

$8x$, $12x^2$ → $4x$ is HCF

Therefore, the algebraic expressions, $8x + 12x^2$ can be written as a product of two factors as in,

$$4x(2 + 3x)$$

This is known as **factorisation**.

**FLASHBACK**

Factors of 16

$16 \div 1 = 16$ $16 \div 8 = 2$
$16 \div 2 = 8$ $16 \div 16 = 1$
$16 \div 4 = 4$

Hence, factors of 16 are 1, 2, 4, 8 and 16.







**EXAMPLE 7**

1. Determine the Highest Common Factor (HCF) for each of the following the terms.
   (a) $6h, 4gh$ (b) $9c^2d, 3d^2e, 6def$
2. Factorise the expressions below.
   (a) $3x + 15$ (b) $7m + 21m^2$

***Solution:***

1. (a)
$2 \mid 6h, 4gh$
$h \mid 3h, 2gh$
$3, 2g$
HCF = $2h$

(b)
$3 \mid 9c^2d, 3d^2e, 6def$
$d \mid 3c^2d, d^2e, 2def$
$3c^2, de, 2ef$
HCF = $3d$

***Solution:***

2. (a)
$3 \mid 3x + 15$
$x + 5$
HCF = 3
Hence, $3(x + 5)$

(b)
$7 \mid 7m + 21m^2$
$m \mid m + 3m^2$
$1 + 3m$
HCF = $7m$
Hence, $7m(1 + 3m)$

**TIPS**

$4 \mid 8x, 12x^2$
$x \mid 2x, 3x^2$
$2, 3x$

HCF = $4x$

HCF can be determined by using long division.

Check your answers using the expansion method.

$= 4x(2 + 3x)$
$= 8x + 12x^2$

**FLASHBACK**

Perfect squares:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ...

### Using difference of squares of two terms

$x^2 - y^2$ is the difference of squares. $x^2 - y^2$ can be factorised by using difference of perfect squares. This method can only be used if the two algebraic terms are perfect squares.

Examine this expressions,

$$x^2 - 4 = x^2 - 2^2$$
$$= (x + 2)(x - 2)$$

**TIPS**

Check your answers using the expansion method.
$(x + 2)(x - 2)$
$= x(x - 2) + 2(x - 2)$
$= x^2 - 2x + 2x - 4$
$= x^2 - 4$

**EXAMPLE 8**

Factorise each of the following expressions.

(a) $b^2 - 1$ (b) $9m^2 - 100$
(c) $3y^2 - 147$ (d) $5k^2 - 80$

***Solution:***

(a) $b^2 - 1$
$= b^2 - 1^2$
$= (b + 1)(b - 1)$

(b) $9m^2 - 100$
$= (3m)^2 - 10^2$
$= (3m + 10)(3m - 10)$

**DO YOU KNOW ?**

| Odd numbers | Differences of squares |
|---|---|
| 1 | $1^2 - 0^2$ |
| 3 | $2^2 - 1^2$ |
| 5 | $3^2 - 2^2$ |
| 7 | $4^2 - 3^2$ |
| 9 | $5^2 - 4^2$ |
| 11 | $6^2 - 5^2$ |
| 13 | $7^2 - 6^2$ |

(c) $3y^2 - 147$
$= 3(y^2 - 49)$ ← HCF of 3 and 147 is 3
$= 3(y^2 - 7^2)$
$= 3(y + 7)(y - 7)$

(d) $5k^2 - 80$ ← HCF of 5 and 80 is 5
$= 5(k^2 - 16)$
$= 5(k^2 - 4^2)$
$= 5(k + 4)(k - 4)$

An algebraic expression as in $x^2 + 2xy + y^2$ can be factorised as $(x + y)(x + y)$.

**TIPS**

**Factoring Identities**

(a) $(x + y)^2$
$= (x + y)(x + y)$
$= x^2 + 2xy + y^2$

(b) $(x - y)^2$
$= (x - y)(x - y)$
$= x^2 - 2xy + y^2$

(c) $x^2 - y^2$
$= (x + y)(x - y)$

### Using cross multiplication

Algebraic expressions of $ax^2 + bx + c$, where by $a \neq 0$ and $a$, $b$, $c$ are integers that can be factorised.

Examine the example below with its explanation for the factorisation of the algebraic expression $x^2 + 6x + 8$.

Step 1: Compare the coefficients

$1x^2 + 6x + 8$

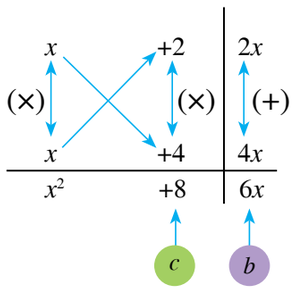
$a\,x^2 + b\,x + c$

Hence, $a = 1$, $b = 6$ and $c = 8$

Step 2: Factors of 8 are 1, 2, 4 and 8. 2 and 4 are selected because they conform to $c$, i.e. $2 \times 4 = 8$.

Step 3: 2 and 4 are chosen because they conform to $b$, i.e. $2 + 4 = 6$.

Step 4: Do cross multiplication as shown below.




Step 5: Factors for $x^2 + 6x + 8$ are $(x + 2)(x + 4)$.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms030 to learn factorisation methods using algebra tiles.

**TIPS**

| Sum of $b$ | Product of $c$ |
|---|---|
| $1 + 8 = 9$<br>$-1 + (-8) = -9$ | $1 \times 8 = 8$<br>$-1 \times (-8) = 8$ |
| $2 + 4 = 6$<br>$-2 + (-4) = -6$ | $2 \times 4 = 8$<br>$-2 \times (-4) = 8$ |

**DO YOU KNOW?**

Factorisation and division

$$x + 2 \overline{)\, x^2 + 6x + 8}$$
$$x + 4$$
$$(-)\ x^2 + 2x$$
$$4x + 8$$
$$(-)\ 4x + 8$$
$$0$$







**EXAMPLE 9**

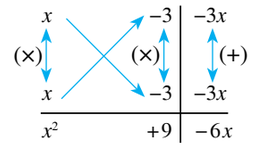
Factorise each of the following expressions.
(a) $x^2 - 6x + 9$
(b) $m^2 - 2m - 8$

***Solution:***

(a) $x^2 - 6x + 9$

Multiplication of factors of 9:
$(-1) \times (-9)$
$(-3) \times (-3)$

$-3 + (-3) = -6$

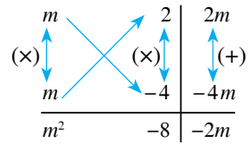



Hence, $x^2 - 6x + 9 = (x - 3)(x - 3)$.

(b) $m^2 - 2m - 8$

Multiplication of factors of 8:
$1 \times (-8)$
$-2 \times 4$
$2 \times (-4)$

$2 + (-4) = -2$



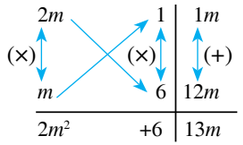

Hence, $m^2 - 2m - 8 = (m + 2)(m - 4)$.

**EXAMPLE 10**

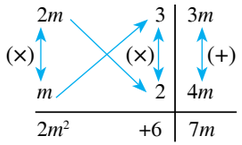
Factorise the following expressions.
$2m^2 + 7m + 6$ ← Multiplication of factors of 6: $1 \times 6$, $2 \times 3$

***Solution:***

First trial:




Second trial:




Hence, $2m^2 + 7m + 6 = (2m + 3)(m + 2)$. ← Check your answer with the expansion method

**QR CODE**

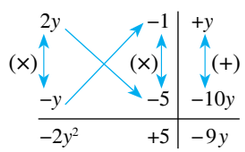
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms031 to explore factorisation using cross multiplication method.

**EXAMPLE 11**

Factorise the following expressions.
(a) $-2y^2 - 9y + 5$
(b) $-3x^2 - 8x - 5$

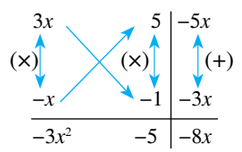
***Solution:***

(a)




Hence, $-2y^2 - 9y + 5 = (2y - 1)(-y - 5)$.

(b)




Hence, $-3x^2 - 8x - 5 = (3x + 5)(-x - 1)$.

**THINK SMART**

Solution to $-2y^2 - 9y + 5$ can be written as $(-2y + 1)(y + 5)$. Discuss.

**Using common factors involving 4 algebraic terms**

$$\begin{aligned} ab + ac + bd + cd &= (ab + ac) + (bd + cd) \\ &= a(b + c) + d(b + c) \\ &= (b + c)(a + d) \end{aligned}$$

← Distributive law

**DO YOU KNOW?**

Factorisation can be solved as follows.
$2x^2 + 7x + 3$
$= 2x^2 + 6x + x + 3$
$= 2x(x + 3) + (x + 3)$
$= 2x(x + 3) + 1(x + 3)$
$= (2x + 1)(x + 3)$

**EXAMPLE 12**

Factorise each of the following terms.

(a) $pq + qr + ps + rs$

(b) $2px + 6qy - 4py - 3qx$

*Solution:*

(a) $pq + qr + ps + rs$
$= (pq + qr) + (ps + rs)$ ← Group terms with common factors in a bracket
$= q(p + r) + s(p + r)$ ← Common factors
$= (q + s)(p + r)$

(b) $2px - 4py - 3qx + 6qy$
$= (2px - 4py) - (3qx - 6qy)$
$= 2p(x - 2y) - 3q(x - 2y)$
$= (x - 2y)(2p - 3q)$

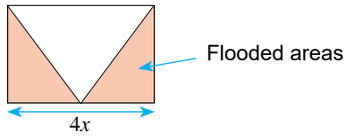
## 2.2.3 Solving problems

**LEARNING STANDARD**

Solve problems involving factorisation.

**EXAMPLE 13**

The area of a rectangular shaped football field is $(4x^2 + 16x)$ square metres. The field was flooded as shown in the diagram below. If the width of the field is $4x$ metre and the two flooded regions are right-angled triangles which are congruent, what is the area of the region that is not flooded?




*Solution:*

**Understanding the problem**

Determine the length of the field

$$\text{Length} = \frac{\text{area}}{\text{width}}$$
$$= \frac{4x^2 + 16x}{4x}$$
$$= \frac{\cancel{4x}^{1}(x + 4)}{\cancel{4x}_{1}}$$
$$= (x + 4)$$

Determine the base of the right-angled triangle

Base of the right-angled triangle
$= 4x \div 2$
$= 2x$

**Planning the strategy**

Area of two right-angled triangles

$$\text{Area} = 2 \times \left(\frac{1}{2} \times \text{base} \times \text{height}\right)$$
$$= 2 \times \left(\frac{1}{2} \times 2x \times (x + 4)\right)$$
$$= 2x^2 + 8x$$

**Implementing the strategy**

Area of the region that is not flooded
= Area of the field – area of two right-angled triangles.
$= 4x^2 + 16x - (2x^2 + 8x)$
$= 4x^2 - 2x^2 + 16x - 8x$
$= 2x^2 + 8x$

**Conclusion**

Area of the region that is not flooded
$= (2x^2 + 8x)$ m$^2$







## SELF PRACTICE 2.2

1. Determine the common factors and HCF for each of the following terms.
   (a) $8y, 12y$ (b) $2b, 3b$ (c) $3w, 5w^2$
   (d) $10m^2, 15mk$ (e) $5bc, 2c^2, 3cd$ (f) $4a^2b, 8b^2c, 6bcd$

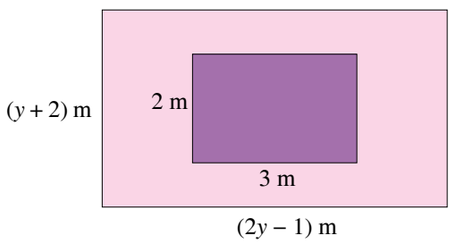
2. Factorise the following algebraic expressions.
   (a) $5e + 10$ (b) $2ab - 8a^2$ (c) $3abc + 6a^2b$
   (d) $4x - 12x^2$ (e) $ef + f^2 + fg$ (f) $2x^2 - 4xy + 6wx$

3. Factorise the following algebraic expressions.
   (a) $b^2 - 81$ (b) $a^2 - b^2$ (c) $x^2 - 1$
   (d) $16y^2 - 49$ (e) $(m + 3)^2 - 16$ (f) $4(x - 1)^2 - 9$

4. Factorise the following algebraic expressions.
   (a) $x^2 + 9x + 14$ (b) $x^2 + 7x - 18$ (c) $x^2 - 5x - 24$
   (d) $m^2 + 11m - 26$ (e) $y^2 - 2y - 15$ (f) $k^2 - 8k + 16$
   (g) $2m^2 - 11m - 6$ (h) $9f^2 - 12f + 4$ (i) $2m^2 + 4m - 16$
   (j) $2x^2 - 5x - 7$ (k) $12y^2 + 8y - 15$ (l) $5p^2 + 6p - 8$
   (m) $-5m^2 - 6m + 8$ (n) $-3p^2 + 8p - 4$ (o) $-6x^2 - x + 15$

5. Factorise the following algebraic expressions.
   (a) $pq - qr - pw + rw$ (b) $x^2 + xy + 6x + 6y$
   (c) $3ab - 9ad + bc - 3cd$ (d) $ah + aj - bh - bj$
   (e) $jm - jn + ym - yn$ (f) $9xy - 3xz + 12py - 4pz$

6. 




A rectangular carpet measuring 3 metre by 2 metre is laid on the rectangular floor of a room.
(a) Calculate the floor area that is not covered by the carpet.
(b) Felisa wants to cover the whole floor area with the carpet of that size. Determine how many pieces of carpets would she need if the value of $y = 2$.

## 2.3 Algebraic Expressions and Basic Arithmetic Operations

You have learned expansion, factorisation and problem solving. Now try solving the following combined operations with expansion and factorisation.

### 2.3.1 Addition and subtraction of algebraic expressions

**LEARNING STANDARD**

Perform addition and subtraction of algebraic expressions involving expansion and factorisation.

**EXAMPLE 14**

Simplify.

(a) $2x^2 - 2(4x + 5)$ (b) $4w\,(w - 2) - 5$

*Solution:*

(a) $2x^2 - 2(4x + 5) = 2x^2 - 8x - 10$
$= 2(x^2 - 4x - 5)$
$= 2(x - 5)(x + 1)$

(b) $4w\,(w - 2) - 5 = 4w^2 - 8w - 5$
$= (2w - 5)(2w + 1)$

#### Addition and subtraction of algebraic fractions with the same denominators

**EXAMPLE 15**

Simplify each of the following.

(a) $\frac{4a}{5} + \frac{3a}{5}$ (b) $\frac{y}{2x} - \frac{3y}{2x}$ (c) $\frac{x+2}{5w} - \frac{x-5}{5w}$

*Solution:*

(a) $\frac{4a}{5} + \frac{3a}{5}$
$= \frac{7a}{5}$

(b) $\frac{y}{2x} - \frac{3y}{2x}$
$= \frac{y - 3y}{2x}$
$= -\frac{2y}{2x}$
$= -\frac{y}{x}$

$-\frac{y}{x} = \frac{-y}{x}$
Negative sign is usually not attached to the denominator

(c) $\frac{x+2}{5w} - \frac{x-5}{5w}$
$= \frac{x + 2 - (x - 5)}{5w}$
$= \frac{x + 2 - x + 5}{5w}$
$= \frac{7}{5w}$

$(-) \times (-) = +$

**FLASHBACK**

Before solving fractions, the first step is to make sure the denominators are of the same value.

(a) $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$

(b) $\frac{3y}{5} + \frac{8y}{5} = \frac{11y}{5}$

(c) $\frac{7x}{5} - \frac{x}{10}$
$= \frac{7x \times 2}{5 \times 2} - \frac{x}{10}$
$= \frac{14x}{10} - \frac{x}{10}$
$= \frac{13x}{10}$

(d) $\frac{4}{xy^2} - \frac{x}{y}$
$= \frac{4}{xy^2} - \frac{x \times xy}{y \times xy}$
$= \frac{4}{xy^2} - \frac{x^2y}{xy^2}$
$= \frac{4 - x^2y}{xy^2}$







#### Addition or subtraction of algebraic fractions with different denominators

**One of the denominators is a multiple of the other denominators**

**EXAMPLE 16**

Simplify the following expressions.

(a) $\frac{3}{4y} - \frac{1}{2y}$ (b) $\frac{4}{rs} - \frac{2r}{s}$

*Solution:*

(a) $\frac{3}{4y} - \frac{1 \times 2}{2y \times 2} = \frac{3 - 2}{4y}$
$= \frac{1}{4y}$

Equating the denominators

(b) $\frac{4}{rs} - \frac{2r \times r}{s \times r}$
$= \frac{4 - 2r^2}{rs}$

**FLASHBACK**

$\frac{1}{2} \frac{\times 2}{\times 2} - \frac{1}{4} = \frac{2 - 1}{4}$
$= \frac{1}{4}$

**Denominators of fractions with denominators that are not common factors**

**EXAMPLE 17**

Simplify each of the following expressions.

(a) $\frac{5x}{3} - \frac{3x}{2}$ (b) $\frac{2a}{3} + \frac{b}{2c}$

*Solution:*

(a) $\frac{5x \times 2}{3 \times 2} - \frac{3x \times 3}{2 \times 3}$
$= \frac{10x - 9x}{6}$
$= \frac{x}{6}$

(b) $\frac{2a}{3} + \frac{b}{2c}$
$= \frac{2a \times 2c}{3 \times 2c} + \frac{b \times 3}{2c \times 3}$
$= \frac{4ac + 3b}{6c}$

**FLASHBACK**

$\frac{3}{4} - \frac{1}{3} = \frac{3 \times 3}{4 \times 3} - \frac{1 \times 4}{3 \times 4}$
$= \frac{9}{12} - \frac{4}{12}$
$= \frac{5}{12}$

**Denominators of fractions that are of common factors.**

**EXAMPLE 18**

Simplify each of the following expressions.

(a) $\frac{1}{4p} + \frac{4}{6p}$ (b) $\frac{m}{4r} - \frac{5m}{14rs}$

*Solution:*

(a) $\frac{1}{4p} + \frac{4}{6p} = \frac{1 \times 3}{4p \times 3} + \frac{4 \times 2}{6p \times 2}$
$= \frac{3}{12p} + \frac{8}{12p}$
$= \frac{11}{12p}$

$2p \mid 4p, 6p$
$2, 3$
LCM $= 2p \times 2 \times 3 = 12p$

(b) $\frac{m}{4r} - \frac{5m}{14rs} = \frac{m \times 7s}{4r \times 7s} - \frac{5m \times 2}{14rs \times 2}$
$= \frac{7ms - 10m}{28rs}$

$2r \mid 4r, 14rs$
$2, 7s$
LCM $= 2r \times 2 \times 7s = 28rs$

**FLASHBACK**

LCM or Lowest Common Multiple

## 2.3.2 Multiplication and division of algebraic expressions

To multiply and divide algebraic expressions, you need to factorise the expression and simplify similar expressions of the numerators and denominators.
Example,

$(2p+4) \div (p^2-4)$ can be written as $\frac{2p+4}{p^2-4}$.

$$\frac{2p+4}{p^2-4} = \frac{2(p+2)}{p^2-2^2}$$ ← Factorise the numerator

$$= \frac{2(p+2)}{(p+2)(p-2)}$$ ← Simplify similar expressions or terms

$$= \frac{2}{p-2}$$

This process of simplification requires factorisation skills that you have learned.

**EXAMPLE 19**

Simplify.

(a) $\frac{a^2-1}{2ab} \times \frac{b^2}{1+a}$

(b) $\frac{(h+k)^2}{2k-h} \times \frac{6k-3h}{h^2-k^2}$

(c) $\frac{5a}{a+2b} \div \frac{2ab}{3a+6b}$

(d) $\frac{a^2-b}{10a-5b} \div \frac{(a-b)^2}{8a-4b}$

***Solution:***

(a) $\frac{a^2-1}{2ab} \times \frac{b^2}{(1+a)}$

$= \frac{(a+1)(a-1)}{2ab} \times \frac{b(b)}{(1+a)}$

$= \frac{b(a-1)}{2a}$ Simplify similar expressions

(b) $\frac{(h+k)^2}{2k-h} \times \frac{6k-3h}{h^2-k^2}$ ← Factorise

$= \frac{(h+k)(h+k)}{2k-h} \times \frac{3(2k-h)}{(h+k)(h-k)}$

$= \frac{3(h+k)}{h-k}$ Simplify similar expressions

(c) $\frac{5a}{a+2b} \div \frac{2ab}{3a+6b}$

$= \frac{5a}{(a+2b)} \times \frac{3(a+2b)}{2ab}$

$= \frac{15}{2b}$ Simplify similar expressions

(d) $\frac{a^2-b^2}{10a-5b} \div \frac{(a-b)^2}{8a-4b}$

$= \frac{(a+b)(a-b)}{5(2a-b)} \times \frac{4(2a-b)}{(a-b)(a-b)}$

$= \frac{4(a+b)}{5(a-b)}$ Simplify similar expressions

**LEARNING STANDARD**

Perform multiplication and division of algebraic expressions involving expansion and factorisation.

**FLASHBACK**

$\frac{m}{mn} = \frac{1}{n}$

$\frac{2s^2}{8sp} = \frac{2(s)(s)}{8(s)(p)}$

$= \frac{s}{4p}$

**FLASHBACK**

$a^2 + 2ab + b^2 = (a+b)^2$
$a^2 - 2ab + b^2 = (a-b)^2$
$a^2 - b^2 = (a+b)(a-b)$

**TIPS**

$a + 1 = 1 + a$
$a - b = -(b - a)$
$(p-q)^2 = (q-p)^2$

**FLASHBACK**

$\frac{1}{x} \div \frac{1}{x}$ Mutual $\frac{1}{x}$ is $x \div 1$ and change the operation ÷ to ×

$= \frac{1}{x} \times \frac{x}{1}$

$= 1$

**TIPS**

$\frac{3}{4} \div \frac{5}{4}$

$= \frac{3}{4} \times \frac{4}{5}$

$= \frac{3}{5}$







## 2.3.3 Combined operations of algebraic expressions

**EXAMPLE 20**

Solve the following using combined operations.

(a) $\frac{2}{5b}(15a+25b) + \frac{a}{b}$

(b) $\frac{9k^2-12k+4}{(3k+2)(3k-2)}$

(c) $\frac{12m-18m^2}{4n^2-16n} \times \frac{n}{m}$

(d) $\frac{a-b}{3a+b} \div \frac{(a-b)^2}{6a+2b}$

***Solution:***

(a) $\frac{2}{5b}(15a+25b) + \frac{a}{b}$

$= \frac{2}{5b} \times 5(3a+5) + \frac{a}{b}$

$= \frac{2(3a+5b)}{b} + \frac{a}{b}$

$= \frac{6a+10b}{b} + \frac{a}{b}$

$= \frac{7a+10b}{b}$

(b) $\frac{9k^2-12k+4}{(3k+2)(3k-2)}$

$= \frac{(3k-2)(3k-2)}{(3k+2)(3k-2)}$

$= \frac{3k-2}{3k+2}$

(c) $\frac{12m-18m^2}{4n^2-16n} \times \frac{n}{m}$

$= \frac{6m(2-3m)}{4n(n-4)} \times \frac{n}{m}$

$= \frac{3(2-3m)}{2(n-4)}$

(d) $\frac{a-b}{3a+b} \div \frac{(a-b)^2}{6a+2b}$

$= \frac{a-b}{3a+b} \times \frac{6a+2b}{(a-b)^2}$

$= \frac{(a-b)}{(3a+b)} \times \frac{2(3a+b)}{(a-b)(a-b)}$

$= \frac{2}{a-b}$

**LEARNING STANDARD**

Perform combined operations of algebraic expressions involving expansion and factorisation.

**ATTENTION**

Factoring two, three or four terms:

**Two terms**

$a^2 - b^2 = (a+b)(a-b)$
Example:
$x^2 - 16$
$= (x+4)(x-4)$

**Three terms**

Factor in two brackets
( )( )
Example:
$x^2 - 4x - 21$
$= (x-7)(x+3)$

**Four terms**

$6xy + 2y + 9x + 3$
Example:
$(6xy + 2y) + (9x + 3)$
$= 2y(3x+1) + 3(3x+1)$
$= (2y+3)(3x+1)$

## SELF PRACTICE 2.3

1. Simplify each of the following.
   (a) $4(b-1)^2 - 9$
   (b) $(m+3)^2 - 16$
   (c) $(p-5)^2 - 49$
   (d) $7x(x-1) - 3$
   (e) $(2c-1)^2 + 2(4+c)$

2. Simplify each of the following.
   (a) $\frac{3y}{5} + \frac{3y}{5}$
   (b) $\frac{3m+2n}{m-2n} - \frac{m-5n}{m-2n}$
   (c) $\frac{4r-3s}{2r+3s} - \frac{3r-4s}{2r+3s}$

3. Simplify each of the following.
   (a) $\frac{5}{p} - \frac{2}{p^2}$
   (b) $\frac{2s}{3} - \frac{4s}{9}$
   (c) $\frac{3}{x+y} - \frac{3z}{4(x+y)}$

4. Simplify each of the following.
   (a) $\frac{3u}{4} + \frac{5v}{3}$
   (b) $\frac{1}{6s} - \frac{2}{5t}$
   (c) $\frac{2}{r-2} + \frac{4}{3s}$

5. Simplify each of the following.
   (a) $\frac{m}{9}+\frac{n}{12}$ (b) $\frac{3}{3mn}+\frac{n}{6m^2}$ (c) $\frac{4}{d^2g}+\frac{3}{5dg}$

6. Simplify.
   (a) $\frac{x^2-x}{xy}$ (b) $\frac{6a+15}{12}$ (c) $\frac{m+n}{m^2-n^2}$ (d) $\frac{2k-1}{4k^2-1}$ (e) $\frac{c^2-9}{2c+6}$

7. Simplify.
   (a) $\frac{2}{a-3}\times\frac{3}{3+a}$ (b) $\frac{h}{k-2}\times\frac{y}{h+3}$
   (c) $\frac{3m}{(m-n)}\times\frac{2mn}{(n-2m)}$ (d) $\frac{2r}{s-2}\times\frac{s-4}{r+5}$

8. Simplify.
   (a) $\frac{m}{x+2}\times\frac{2(x+2)}{m^2(x-a)}$ (b) $\frac{2r^2}{rs-s^2}\times\frac{5r-5s}{2r-4r^2}$
   (c) $\frac{x}{x+2}\times\frac{x^2+5x+6}{5x^2}$ (d) $\frac{e+2f}{5e-2f}\times\frac{4f^2-10ef}{3e^2-9ef}$

9. Simplify.
   (a) $\frac{5a}{2a+3}\div\frac{3b}{a+b}$ (b) $\frac{4}{n-3}\div\frac{8a}{3n-9}$
   (c) $\frac{6y^2}{x^2+xy}\div\frac{18xy}{x+y}$ (d) $\frac{f-1}{eg+2e}\div\frac{fg-g}{g+2}$

10. Solve the following combined operations.
    (a) $\frac{x^2+x}{x^2-y^2}\times\frac{xy-y^2}{x+y}$ (b) $\frac{4p^2-1}{p^2-1}\times\frac{pq+q}{4p-2}$
    (c) $\frac{pq-pr}{r^2-1}\div\frac{q^2-r^2}{r^2+r}$ (d) $\frac{st+tu}{4t^2-1}\div\frac{s^2-u^2}{4t^2+4t+1}$

## GENERATING EXCELLENCE

1. Expand each of the following expressions.
   (a) $\frac{1}{2}(6a+12b)$ (b) $(n+2)(n-5)$ (c) $(a+2b)^2$
   (d) $(4x-y)^2$ (e) $\left(2v-\frac{1}{3w}\right)\left(3v+\frac{2}{3w}\right)$ (f) $(h-k)^2-4h(2k-3h)$

2. Factorise the following expressions.
   (a) $12m-18m^2$ (b) $y^2-81$ (c) $4ab-8a^2b$
   (d) $x^2-16y^2$ (e) $(s-3)^2-1$ (f) $x^2+4x+3$
   (g) $x^2+2x-15$ (h) $x^2+6x+8$ (i) $6cd-2ce-3bd+be$

3. Simplify each of the following expressions.
   (a) $\frac{a+2}{4v}+\frac{a-b}{2v}$ (b) $\frac{3e}{5ab}-\frac{5d}{4c}$ (c) $\frac{4}{f^2g}-\frac{3}{5fg}$







(d) $\frac{n+2}{m^2}+\frac{n}{mp}$ (e) $\frac{5x}{8yz}+\frac{y-1}{12xz}$ (f) $\frac{rs}{4y}+\frac{2-r}{18yz}$

4. Grandma has a piece of chocolate with a length of $(k^2-16)$ cm and she wants to divide it equally among her $(k-4)$ grandchildren. What length of chocolate will each of her grandchildren receive?

5. Gurdip and Jumrang are part-time workers in a grocery shop. Gurdip is paid RM3 per hour less than twice Jumrang's pay. If Jumrang is paid RM$x$ per hour, how much is Gurdip's pay if he works $(x+2)$ hours and Jumrang's pay if he works $(2x+3)$ hours. State your answers in algebraic form.

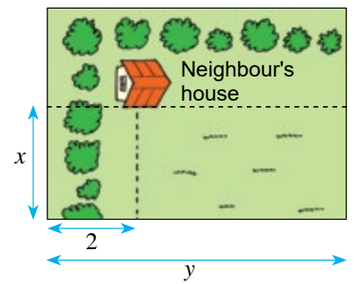
6. The ground area of a piece of land of a supermarket used for parking cars is $25(x^2-8x+16)$ square metres.
   (i) If the area of a parking lot for a vehicle is $(x-4)^2$ square metres, how many cars can be parked there?
   (ii) If 4 units of the parking lot have been booked by the supermarket, how many parking lots are left?

7. Khairul wants to cover a wall measuring $(x+5)$ metres long and $(3x-2)$ metres wide with decorative papers.
   (i) What is the area of the wall that will be covered with decorative paper if there is a door measuring $(x-1)$ metres long by $x$ metres wide.
   (ii) If the cost of the decorative papers is RM $8x$ per square metres, how much will Khairul have to spend?

8. Swee Lee should have finished $(28+16x)$ Mathematics questions in 4 hours.
   (i) How many questions would have been done in 30 minutes?
   (ii) If Swee Lee could only finish $(14+8x)$ questions, how much time did she spend?

9. Azimah bakes a square layered cake measuring $(3x+2)$ cm long and $(x+2)$ cm wide. She cuts the cake into 6 equal parts along the length and 3 equal parts along the width. Determine the area of each piece of cake in the form of algebraic expressions.

10. Encik Hanapi intends to build a single storey bungalow on a piece of land measuring $x$ metres wide and $y$ metres long. He needs to reserve part of the land which is 2 metres wide for a road for his neighbour.
    (i) What is the total area of his piece of land?
    (ii) What is the remaining size of the land after reserving some parts for the road?
    (iii) If the cost of land is RM18 per square metres, what is the total cost of the remaining land?



## CHAPTER SUMMARY

### Factorisation and Algebraic Factors

#### Expansion

Multiplication of an algebraic expression with a term or an algebraic expression

- $a(x + y) = ax + ay$
- $(a + b)(x + y) = ax + ay + bx + by$
- $b(c + d) = bc + bd$
- $(b + c)(d + e) = bd + be + cd + ce$
- $(b + c)^2 = b^2 + 2bc + c^2$
- $(b - c)^2 = b^2 - 2bc + c^2$
- $(b + c)(b - c) = b^2 - c^2$

#### Factorisation

Factorisation is the method of writing an algebraic expression as a product of two or more algebraic terms or algebraic expressions.
Factorisation is the reverse process of an expansion.

- $2a - a^2 = a(2 - a)$
- $a^2 + 4a + 3 = (a + 1)(a + 3)$
- $a^2 - 7a + 10 = (a - 5)(a - 2)$
- $a^2 - 36 = (a^2 - 6^2) = (a - 6)(a + 6)$
- $ab + ac + bd + cd = (b + c)(a + d)$
- $a^2 - 2ab + b^2 = (a - b)^2$

#### Addition and Subtraction

Before adding or subtracting two algebraic fractions, check the denominators first. If they are not the same, you need to express all fractions in terms of common denominators.

- $\frac{a}{4} + \frac{b}{4} = \frac{a+b}{4}$
- $\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$
- $\frac{1}{2a} - \frac{1}{ab} = \frac{1 \times b}{2a \times b} - \frac{1 \times 2}{ab \times 2}$
  $= \frac{b-2}{2ab}$

#### Multiplication and Distribution

Factorise expressions before division or multiplication when it is necessary.

$$\frac{m+n}{x-y} \div \frac{(m+n)^2}{x^2-y^2} = \frac{\cancel{m+n}^{1}}{\cancel{x-y}_{1}} \times \frac{(x+y)(\cancel{x-y}^{1})}{(\cancel{m+n}_{1})(m+n)}$$
$$= \frac{x+y}{m+n}$$







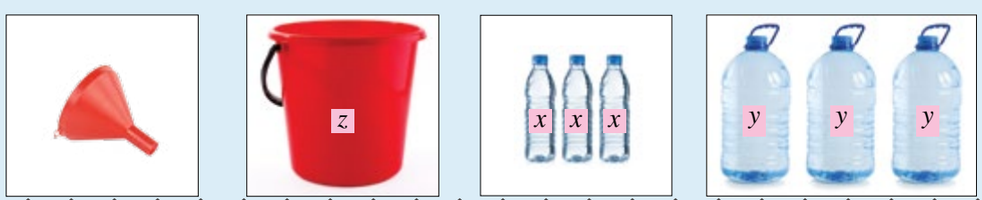
## SELF REFLECTION

| At the end of this chapter, I will be able to: | |
|---|---|
| 1. Explain the meaning of expansion of two algebraic expressions. | |
| 2. Work out expansion of two algebraic expressions. | |
| 3. Simplify algebraic expressions involving combined operations, including expansion. | |
| 4. Solve problems involving expansion of two algebraic expressions. | |
| 5. Relate the multiplications of algebraic expressions to the concept of factors and factorisation, and hence list out the factors of the product of the algebraic expressions. | |
| 6. Factorise algebraic expressions using various methods. | |
| 7. Solve problems involving factorisation. | |
| 8. Perform addition and subtraction of algebraic expressions involving expansion and factorisation. | |
| 9. Perform multiplication and division of algebraic expression involving expansion and factorisation. | |
| 10. Perform combined operations of algebraic expression involving expansion and factorisation. | |

## MINI PROJECT

**Title:** What is the volume of this pail of water?
**Materials:** A pail of water (labelled $z$), a few small mineral water bottles (labelled $x$), a few big mineral water bottles (labelled $y$) and a funnel

Each group is given a few empty mineral water bottles of different size and a funnel. Students fill up the empty bottles with the water. Then they write out the algebraic expression to express the volume of water. Every group presents their answers. Are they the same? Can you determine the volume of water?



# CHAPTER 3

# Algebraic Formulae

## WHAT WILL YOU LEARN?

**3.1** Algebraic Formulae

## WORD LINK

- Algebraic formula — *Rumus algebra*
- Variable — *Pemboleh ubah*
- Coefficient — *Pekali*
- Subject of formula — *Perkara rumus*

A wholesale store sells clothes for RM$y$. During the festive season, the store discounted the price of the clothes as shown below.


As a computer programmer, you are asked to develop a programme that contains the formula for calculating the selling price of the clothes.


## WALKING THROUGH TIME

Al-Khwarizmi introduced negative and decimal numbers. He also founded a mathematical programme using a set of instructions to complete a complex calculation.

For more information:

http://rimbunanilmu.my/mat_t2e/ms043

## WHY STUDY THIS CHAPTER?

- The algebraic formulae is applied by engineers, statisticians, mathematicians and astronomers in their respective jobs.

**CREATIVE ACTIVITY**

**Aim:** Identifying formula
**Material:** School Calendar
**Steps:**

1. Students carry out the activity in pairs.
2. Calculate the amount of money that can be saved from the following situations (assume that the calculation starts from first to the last day of each month).

   **Situation 1**
   Badrul is a form 2 student who likes to save. On each school day, he receives RM5 as his pocket money and spends RM4.50. What is the amount of Badrul's savings in January?

   **Situation 2**
   Sedthu saves RM15 per month. If he receives RM10 as pocket money, calculate Sedthu's expenditure for one day in April.

3. State the method of calculating the savings.

From the situations above, write an equation for the total savings in relation to pocket money, money spent and to the number of days using basic mathematical operations to get the total amount of savings. Pocket money, money spent and number of days are variables. You can determine the amount saved by changing the value of the variables.

## 3.1 Algebraic Formulae

Algebraic expression is a combination of two or more algebraic terms. The algebraic formulae combines an algebraic expression using addition, subtraction, multiplication or division and is written in the form of an equation.

### 3.1.1 Forming formula

**LEARNING STANDARD**

Write a formula based on a situation.

**COGNITIVE STIMULATION** Group

**Aim:** Forming algebraic formulae
**Material:** Worksheets
**Steps:**

1. Students carry out this activity in groups.

   A cultural club will perform at a school-level cultural night. The table shows the number of dancers according to the types of dance and race represented by an alphabet.







| Types of dance | Race | | |
|---|---|---|---|
| | Malay | Chinese | Indian |
| Sumazau | $a$ | $2c$ | $2a$ |
| Kuda Kepang | $2b$ | $b$ | $5b$ |
| Lion | $2c$ | $3a$ | $c$ |

The alphabet $a$, $b$ and $c$ are known as variables.

2. Write a formula for each of the following subject.
   (a) $s$, number of Chinese dancers.
   (b) $d$, number of Kuda Kepang dancers.
   (c) $w$, number of Indian and Malay dancers.

**Discussion:**
(i) Difference in formula between the groups in your class.
(ii) Conclusion from the activity above.

The formula is expressed as $s = 2c + b + 3a$, $d = 8b$, $w = 3a + 7b + 3c$. From the activity above, the formula is formed by the relationship among a few variables.

**EXAMPLE 1**

Suzi sold two types of cakes of different prices. The chocolate cake sold at RM3 a slice. The cheese cake was sold at twice the price of the chocolate cake. In conjunction with the opening of a new branch, she gave 10% discount for all cakes. Determine a formula to calculate the selling price of the cake, if $m$ slices of chocolate cake and $n$ slices of cheese cake were sold.

***Solution:***

Price of cheese cake = twice the price of the chocolate cake
$= 2 \times \text{RM3}$
$= \text{RM6}$

Selling price, $z$ = [(number of chocolate cake × price) + (number of cheese cake × price)] × discount
$= [(m \times \text{RM3}) + (n \times \text{RM6})] \times (100\% - 10\%)$
$= (\text{RM3}m + \text{RM6}n) \times 90\%$
$= (3m + 6n) \times 0.9$

Let, $z$ = Selling price
$m$ = number of chocolate cake
$n$ = number of cheese cake

The algebraic formula is $z = 0.9\,(3m + 6n)$
$= 2.7m + 5.4n$

**DO YOU KNOW?**

The Sumazau dance is known as the traditional dance of the Kadazan Dusun tribe in Sabah. The Sumazau dance is performed during the Tadau Kaamatan festival and is celebrated every year in May.

http://www.jkkn.gov.my/pemetaanbudaya/

**TIPS**

In the activity on the left, $s$, $d$ and $w$ are subjects of formula. They can be written on the left or right side.

**TIPS**

A variable in a formula can be represented by letters $a$ to $z$ (in example 1, $m$ and $n$ represents variables). $z$, written on the left is known as subject of formula.

**THINK SMART!**

Is this equation called a formula?

(i) $a \times (b + c) = (a \times b) + (a \times c)$

(ii) $\frac{p}{a} + \frac{q}{a} = \frac{b}{a}$

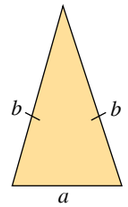
Discuss.

## 3.1.2 Changing the subject of formula

**LEARNING STANDARD**

Change the subject of formula of an algebraic equation.

The subject of a formula can be a variable for the algebraic formula and the variable can be the subject of an algebraic formula.




Perimeter, $P$ for an equilateral triangle can be expressed in $a$ and $b$.
Hence, $P = a + 2b$

The subject of formula of the equation above can be changed as shown.

(i) $a = P - 2b$ (ii) $b = \dfrac{P-a}{2}$

**TIPS**

Coefficient for the subject of formula must be 1.

**FLASHBACK**

$1 \times p = p$

$-1 \times p = -p$

$0 \times p = 0$

$\dfrac{1}{3} \times p = \dfrac{p}{3}$

$-\dfrac{1}{3} \times p = -\dfrac{p}{3}$

**EXAMPLE 2**

State $m$, as the subject of formula.

(a) $q = m + p$ (b) $b = 2s - m$

(c) $a = \dfrac{5}{2m}$ (d) $t = \dfrac{m-n}{-3}$

*Solution:*

(a) $m + p = q$

$m + p - p = q - p$

Then, $m = q - p$

$m$ in terms of $p$ and $q$

(b) $2s - m = b$

$2s - 2s - m = b - 2s$

$-m = b - 2s$

$\dfrac{1}{-1} \times (-m) = \dfrac{1}{-1}(b - 2s)$

$m = -b + 2s$

$m$ in terms of $b$ and $s$ → Then, $m = 2s - b$

**TIPS**

Subject of formula should be on the left side of the equation.

(c) $a = \dfrac{5}{2m}$

$a \times 2m = \dfrac{5}{2\cancel{m}_{1}} \times \cancel{2m}^{1}$

$2am = 5$

$\dfrac{\cancel{2a}^{1}m}{\cancel{2a}_{1}} = \dfrac{5}{2a}$

Then, $m = \dfrac{5}{2a}$ ← $m$ in terms of $a$

(d) $\dfrac{m-n}{-3} = t$

$\dfrac{m-n}{\cancel{-3}_{1}} \times \cancel{-3}^{1} = t \times (-3)$

$m - n = -3t$

$m - n + n = -3t + n$

$m = -3t + n$

Then, $m = n - 3t$

$m$ in terms of $n$ and $t$

**FLASHBACK**

You have learned to solve linear equation using the following three methods:
(a) Trial and improvement
(b) Application of equality concept
(c) Back track







**EXAMPLE 3**

State $p$, as the subject of formula.

(a) $q = \sqrt{p}$ (b) $s = p^2$

(c) $w = \sqrt{\dfrac{p}{3}}$ (d) $t = \dfrac{1}{p^2}$

*Solution:*

(a) $\sqrt{p} = q$

$(\sqrt{p})^2 = (q)^2$ ← Both sides of the equation are squared

$p = q^2$

(b) $p^2 = s$

$\sqrt{p^2} = \sqrt{s}$

$p = \sqrt{s}$

(c) $\sqrt{\dfrac{p}{3}} = w$

$\left(\sqrt{\dfrac{p}{3}}\right)^2 = w^2$

$\dfrac{p}{3} = w^2$

$\dfrac{p}{\cancel{3}_{1}} \times \cancel{3}^{1} = w^2 \times 3$

$p = 3w^2$

(d) $t = \dfrac{1}{p^2}$

$t \times p^2 = \dfrac{1}{\cancel{p^2}_{1}} \times \cancel{p^2}^{1}$

$tp^2 = 1$

$p^2 = \dfrac{1}{t}$

$p = \sqrt{\dfrac{1}{t}}$

**TIPS**

$(\sqrt{a^2})^2 = a^2$

$\sqrt{a^2} = a$

**ATTENTION**

$\sqrt{x} = x^{\frac{1}{2}}$

$(\sqrt{x})^2 = (x^{\frac{1}{2}})^2$

$= x^{\frac{1}{2} \times 2}$

$= x$

**ATTENTION**

**Reciprocals**

$\dfrac{1}{x} = a,\ x = \dfrac{1}{a}$

**Squares**

$(\sqrt{x})^2 = a^2,\ x = a^2$

**Square root**

$\sqrt{x^2} = \sqrt{a},\ x = \pm\sqrt{a}$

## 3.1.3 Determining the value of variable

**LEARNING STANDARD**

Determine the value of a variable when the value of another variable is given.

The value of a subject of a formula can be obtained when all variable values are given. On the other hand, the value of a variable can be obtained when the value of subject of the formula and variable is given.

**EXAMPLE 4**

Given $w = 7t - 5u$, calculate the following.

(a) value $w$ when $t = 3$ and $u = -2$

(b) value $t$ when $w = 15$ and $u = 4$

*Solution:*

(a) Substitute $t = 3$ and $u = -2$ into the formula.

$w = 7(3) - 5(-2)$

$= 21 + 10$

$= 31$

**FLASHBACK**

$-a + a = 0$

$-a - a = -2a$

$-a \times a = -a^2$

$(-a) \times (-a) = a^2$

$-a \div a = -1$

$(-a) \div (-a) = 1$

(b) Substitute $w = 15$ and $u = 4$ into the formula.

$$7t - 5u = w$$
$$7t - 5(4) = 15$$
$$7t = 15 + 20$$
$$t = \frac{35}{7}$$
$$t = 5$$

**EXAMPLE 5**

Given $m = \frac{1}{4}(p-q)^2$, calculate the value $q$ if $m = 16$ and $p = 3$.

***Solution:***

$$m \times 4 = \frac{1}{4}(p-q)^2 \times 4$$
$$4m = (p-q)^2$$
$$\sqrt{4m} = \sqrt{(p-q)^2}$$ ← Square root both sides of the equation
$$p - q = \sqrt{4m}$$
$$-q = \sqrt{4m} - p$$
$$(-q) \times \frac{1}{-1} = (\sqrt{4m} - p) \times \frac{1}{-1}$$ ← Multiply both sides of the equation by $\frac{1}{-1}$
$$q = -\sqrt{4m} + p$$
$$q = p - \sqrt{4m}$$
$$q = 3 - \sqrt{4(16)}$$ ← Replace $m = 16$ and $p = 3$
$$q = 3 - 8$$
$$q = -5$$

**FLASHBACK**

**Algebraic formulae**

**Variables**
Variables is a quantity where the value is not known or can be changed.

**Constant**
Constant is a quantity where the value is fixed.

**Algebraic formulae**
Algebraic formulae involve equations that connect a few variables.

**Subject of formula**
Subject of formula is a dependant variable expressed in terms of an independent variable of a formula. The subject of formula always has coefficient 1. The algebraic formulae involves
(a) One of the basic mathematical operations.
(b) Squares and square root
(c) A combination of basic and square operations or square root.

**TIPS**

**Alternative method**

Substitute $m = 16$ and $p = 3$

$$16 = \frac{1}{4}(3-q)^2$$
$$64 = (3-q)^2$$
$$\sqrt{64} = (3-q)$$
$$8 = 3 - q$$
$$q = 3 - 8$$
$$q = -5$$

## 3.1.4 Solving problems

**LEARNING STANDARD**

Solving problems involving formulae.

**EXAMPLE 6**

The price of a fried chicken at a school canteen is twice the price of a bun. With RM5, Azman bought two buns and a piece of chicken. The balance of RM1 is saved. If Azman has RM12 and decides to buy the same number of buns, how many pieces of fried chickens will he be able to buy?







***Solution:***

**Understanding the problem**

Number of fried chicken that can be bought by Azman for RM12.

**Planning the strategy**

Determine the price of a bun.

(a) Represent the price of bun and chicken with $x$.
Price of bun = RM$x$
Price of chicken = RM2$x$

(b) The total price of bun + The total price of chicken + RM1 = Total expenditure

$$2(\text{RM}x) + \text{RM}2x + \text{RM}1 = \text{RM}5$$
$$2x + 2x + 1 = \text{RM}5$$
$$4x + 1 = 5$$
$$x = \frac{5-1}{4}$$
$$= 1$$

Thus, the price of a bun is RM1 and the price of a piece of chicken is RM2.

**Conclusion**

Azman gets to buy 5 pieces of fried chicken.

**Implementing the strategy**

(a) Represent the number of fried chicken with $y$.

(b) Total price of bun + Total price of chicken = RM12
$$(\text{RM}1 \times 2) + (\text{RM}2 \times y) = \text{RM}12$$
$$2 + 2y = 12$$
$$y = \frac{12-2}{2}$$
$$= 5$$

**SELF PRACTICE 3.1**

1. Express the letters in the brackets as subject of formula.

(a) $z = m - qp$ [ $m$ ]
(b) $v = u + 2$ [ $u$ ]
(c) $3y = \frac{7w}{x}$ [ $x$ ]
(d) $3a = \frac{4}{5+b}$ [ $b$ ]
(e) $5q = \frac{3}{u} - 5$ [ $u$ ]
(f) $2w = -4 + \frac{5}{v}$ [ $v$ ]
(g) $2a = \sqrt{3b} + 5$ [ $b$ ]
(h) $(-5t)^2 = \frac{25w^2}{36}$ [ $w$ ]
(i) $(-3m)^2 = 4p - 8$ [$m$]
(j) $\sqrt{(9r^2)} = 4s - 7$ [ $r$ ]

2. The price of a shirt is RM35, while the cost of a pair of trousers is RM45. A discount of 15% is given on the price of a shirt, while a discount of 10% is given on the price of a pair of trousers. Write the formula for the total expenditure, $z$, if Syamsul wants to buy $x$ shirts and $y$ trousers.

3. Solve the following.

(a) Given $c = 4d + 8$, calculate
   (i) value $c$ when $d = 2$
   (ii) value $d$ when $c = 10$

(b) Given $4p = 18 - 5q$, calculate
   (i) value $p$ when $q = 2$
   (ii) value $q$ when $p = 2$

(c) Given $\frac{1}{3}m = \frac{2}{3}n + 8$, calculate
   (i) value $m$ when $n = -15$
   (ii) value $n$ when $m = 30$

(d) Given $\sqrt{4m} = \frac{n^2 - 5}{2}$, calculate
   (i) value $n$ when $m = 4$
   (ii) value $m$ when $n = 2$

(e) Given $3u = 4r + s$, calculate
   (i) value $u$ when $r = 5$ and $s = -2$
   (ii) value $r$ when $u = 3$ and $s = 3$
   (iii) value $s$ when $u = 2$ and $r = \frac{1}{2}$

(f) Given $\frac{3}{5}p = \frac{2}{3}q - \frac{1}{4}r$, calculate
   (i) value $p$ when $q = 3$ and $r = 8$
   (ii) value $q$ when $r = -12$ and $p = 10$
   (iii) value $r$ when $p = -15$ and $q = -15$

(g) Given $\sqrt{3a} = 9b - 4c$, calculate
   (i) value $a$ when $b = \frac{1}{3}$ and $c = \frac{1}{2}$
   (ii) value $b$ when $c = 3$ and $a = 12$
   (iii) value $c$ when $a = 3$ and $b = 3$

(h) Given $1\frac{1}{2}s = \frac{3}{5}t^2 + \frac{1}{3}u^2$, calculate
   (i) value $s$ when $t = -5$ and $u = 3$
   (ii) value $t$ when $u = -6$ and $s = 28$
   (iii) value $u$ when $s = \frac{4}{6}$ and $t = \frac{5}{6}$

4. Write the algebraic formula based on the following situations.

(a) The total price RM$z$ that needs to be paid by a buyer who bought $x$ workbook and $y$ geometry set. The workbook and the geometry set each costs RM5.90 and RM3.60 respectively.

(b) In a class party, a teacher buys $p$ carton of canned drinks to be distributed to the $q$ students. From the total number of canned drinks, seven cans were distributed to the subject teachers. If a carton contains 24 cans of drinks, calculate the number of cans received by each student, $b$ in terms of $p$ and $q$.

(c) Shoe $A$ is sold at RM35 a pair, while shoe $B$ costs RM76 a pair. Beautiful Shoe Shop offers a 15% discount on purchases of two pairs of shoes. Mei Ling buys $m$ pairs of shoe $A$ and $n$ pairs of shoe $B$. Calculate the price payable, $P$ in terms of $m$ and $n$.

(d) A car is able to travel as far as 10 km with a litre of petrol. Express the cost, RM$x$ of the petrol that needs to be filled for $s$ km if a litre of petrol costs RM$t$.

## GENERATING EXCELLENCE

1. Write the algebraic formula from the following situation.
   (a) $A$ represents area, $x$ represents the length of a square. Write the formula that relates $A$ to $x$.







(b) The rental fee of a sepak takraw court is RM5 for the first hour. Payment for the next hour is RM3. Write the formula that relates the amount of payment $p$, and the hours used, $h$.

(c) Acceleration, $a$ is defined as the difference between the final velocity, $v_2$ and initial velocity $v_1$ divided by time, $t$. Write the relationship between $a$, $v_2$, $v_1$ and $t$.

2. Express the letters in the brackets as a subject of formula.

(a) $m = -3q + p$ [$q$]
(b) $x = -p - w$ [$w$]
(c) $2e = 4g + 3h$ [$g$]
(d) $\frac{3}{4}m - 6p = \frac{3}{4}q$ [$q$]
(e) $w = 3v^2$ [$v$]
(f) $2m = \frac{3}{4}n^2$ [$n$]
(g) $3w = \frac{(v+1)^{\frac{1}{2}}}{2}$ [$v$]
(h) $\frac{5}{4}f = \frac{5}{\sqrt{k-7}}$ [$k$]

3. Calculate the following value.

(a) Given $w = \frac{x+y}{1+x}$, calculate the value
   (i) $w$, when $x = 2$ and $y = -8$
   (ii) $x$, when $w = 20$ and $y = 5$
   (iii) $y$, when $w = 5$ and $x = 6$

(b) Given $6b = \sqrt{\frac{c-d^2}{9}}$, calculate the value
   (i) $b$, when $c = 20$ and $d = 2$
   (ii) $c$, when $b = \frac{1}{9}$ and $d = 2$
   (iii) $d$, when $b = \frac{1}{2}$ and $c = 90$

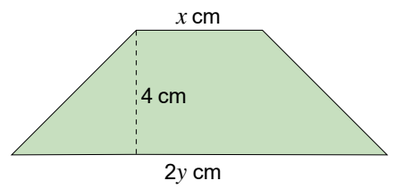
(c) Given $-2p = \frac{(q+1)}{(r+q)}$, calculate the value
   (i) $p$, when $q = 3$ and $r = 3q$
   (ii) $q$, when $p = 3$ and $r = 2q$
   (iii) $r$, when $p = -\frac{1}{3}$ and $q = 2p$

(d) Given $4s^2 = \left(\frac{3t - 4u}{5}\right)^2$, calculate the value
   (i) $s$, when $t = s - 1$ and $u = 2s$
   (ii) $t$, when $s = -5u$ and $u = 3$
   (iii) $u$, when $s = \frac{1}{3}t$ and $t = 2 - u$

4. The salary of fast food store branch manager is 3 times more than a part time employee salary, RM$x$ per day. Working hours for part time employees are half of the manager's working time, $y$ within a month. If they work 26 days in a month, write the formula for the difference in salary, RM$z$ between the two workers in terms of $x$ and $y$.

5. Julia takes 40 seconds to walk as far as 50 metres. Write a formula to help Julia calculate the duration of the trip, $t$ in minutes from her home to the school that is $s$ kilometre away.

6. The area of the trapezium below is 36 cm$^2$. If $x + y = 11$ cm, calculate the value of $x$ and $y$.



## CHAPTER SUMMARY

### Algebraic Formulae

Algebraic formulae connect the algebraic expressions through addition, subtraction, multiplication and division in an equation form.

1. $y = 3x - 5$
2. $w = \frac{6 - 7v}{v}$
3. $L = \frac{1}{2}th$
4. $A = \pi r^2$

Subject of formula is represented by a letter.

Subject of formula can be changed according to the value of the variable.

$$w = -6 - 8t$$

$$t = \frac{-6 - w}{8}$$

A variable in the subject of formula can be obtained when the value of the other variables is given.

Example:

Given $Q = \frac{2v}{-v + u}$, calculate value $u$,

if $v = 2,\ Q = 4$

Thus, $u = 3$

Solving problem involves changing the subject of a formula, combination of basic mathematical operations, square and square root.

## SELF REFLECTION

**At the end of the chapter, I am able to:**

| | | | |
|---|---|---|---|
| 1. Write a formula based on a situation. | ○ | ○ | ○ |
| 2. Change the subject of formula for an algebraic equation. | ○ | ○ | ○ |
| 3. Determine the value of a variable when the value of another variable is given. | ○ | ○ | ○ |
| 4. Solve problems involving formulae. | ○ | ○ | ○ |







## MINI PROJECT

**Title:** Counting board

**Materials:** Manila card, used card board, coloured paper, glue and scissors

**Steps:**

1. Create a counting board to calculate the price that needs to be paid by the student to purchase three items.
2. Example of the things that needs to be purchased are pen, mineral water, and note book.
3. Price of the pen, mineral water and note book is determined by the students according to the current price.

| Items |  |  |  |
|---|---|---|---|
| Number | ☐ *a* | ☐ *b* | ☐ *c* |
| Price | *a* × RM ☐ | *b* × RM ☐ | *c* × RM ☐ |
| Total | ☐ (i) | ☐ (ii) | ☐ (iii) |
| Overall total | ☐ (i) + ☐ (ii) + ☐ (iii) | | |

Example of a counting board

# CHAPTER 4

# Polygon

In our daily life, there is a combination of polygons around us especially in the designs of buildings. The combinations of polygons produce interesting and diverse forms of art. This geometric pattern can be seen at Tanjung Bungah Floating Mosque, Penang whereby it has a unique combination of local and Western Asian architecture.

## WHAT WILL YOU LEARN?

4.1 Regular Polygons

4.2 Interior Angles and Exterior Angles of Polygons

## WORD LINK

| | |
|---|---|
| • Polygon | • *Poligon* |
| • Regular polygon | • *Poligon sekata* |
| • Irregular polygon | • *Poligon tak sekata* |
| • Axis of symmetry | • *Paksi simetri* |
| • Side | • *Sisi* |
| • Interior angle | • *Sudut pedalaman* |
| • Exterior angle | • *Sudut peluaran* |
| • Supplementary angle | • *Sudut penggenap* |
| • Origami | • *Origami* |

## WALKING THROUGH TIME

Polygon refers to the words 'poly' which means many and 'gon' which means angle. Polygon is named by the number of sides. For larger polygons, mathematicians name the polygon according to the number of sides for example 17-gon.

For more information:

http://rimbunanilmu.my/mat_t2e/ms055

## WHY STUDY THIS CHAPTER?

- Creating logos, murals on school walls and creating symmetry on drawing.
- In the field of technology, knowledge of polygon is used in building architecture, roofing, interior designing, fabric design and more.
- Careers involved in this field are surveyors, technicians, engineers, architects, graphic designers and many others.

## CREATIVE ACTIVITY

**Aim:** Producing a pentagon using paper folding technique (origami)

**Materials:** Square paper and scissors

**Steps:**

1. Fold the square paper into two sections.
2. Label each vertex with *PQRS*.
3. Fold *P* towards line *QR*. Press the fold down. Open the fold.
4. Fold *Q* towards line *PS*. Press the fold down. Open the fold.
5. There should be fold marks shaped *X* in the middle. Label it as *X*.
6. Bring *S* to the centre marked *X*. Press the fold down.
7. Bring the vertex that touches *X* and fold it back so that this side rests on the furthest side.
8. Take *P* and fold it to line *TU*. Fold this shape back to get Diagram *D*.
9. Now, cut off the top as shown in Diagram *D*.
10. Open the folds. State the shape of the origami.

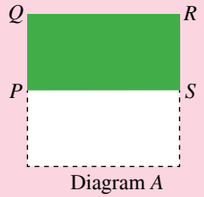


Diagram *A*

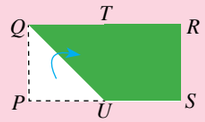


Diagram *B*

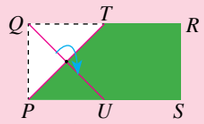


Diagram *C*

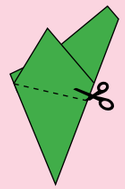


Diagram *D*

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms056 to view the tutorial video of the pentagon shape origami.

## 4.1 Regular Polygon

### 4.1.1 Geometric properties of regular polygon

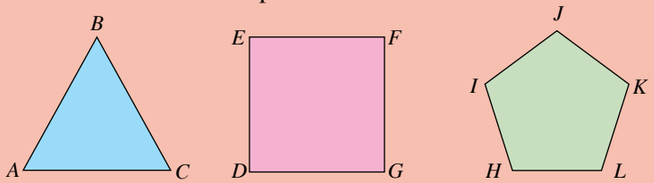
A regular polygon is a polygon that has sides with equal length and interior angles of the same size.

#### Identifying regular polygon

**COGNITIVE STIMULATION** Individual

**Aim:** Exploring geometrical characteristics of regular polygon

**Materials:** Ruler and compasses




**LEARNING STANDARD**

Describe the geometric properties of a regular polygons using various representations.

**DO YOU KNOW?**

Origami originated from Japan that means
'ori' = art, 'gami' = paper

**FLASHBACK**

A polygon is a closed form on a plane that is bounded by three or more straight lines as the sides.







**Steps:**

1. Measure the length of the side and interior angle for all the polygons.
2. Complete the table below.

| Triangle *ABC* | | | |
|---|---|---|---|
| Length of side | | Measurement of angle | |
| *AB* | | ∠*CAB* | |
| *BC* | | ∠*ABC* | |
| *CA* | | ∠*BCA* | |

| Square *DEFG* | | | |
|---|---|---|---|
| Length of side | | Measurement of angle | |
| *DE* | | ∠*GDE* | |
| *EF* | | ∠*DEF* | |
| *FG* | | ∠*EFG* | |
| *GD* | | ∠*FGD* | |

| Pentagon *HIJKL* | | | |
|---|---|---|---|
| Length of side | | Measurement of angle | |
| *HI* | | ∠*HIJ* | |
| *IJ* | | ∠*IJK* | |
| *JK* | | ∠*JKL* | |
| *KL* | | ∠*KLH* | |
| *LH* | | ∠*LHI* | |

Conclusion: _____

Conclusion: _____

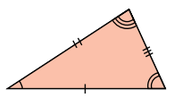
Conclusion: _____

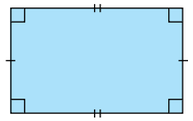
**Discussion:**
Your findings from the activity above.

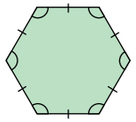
**Regular polygons** are polygons for which all sides are equal and all interior angles are of the same size. Regular polygons have congruent interior angles. **Irregular polygons** are polygons with irregular sides.

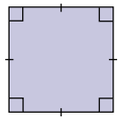
**EXAMPLE 1**

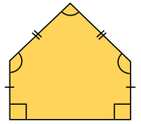
Based on the diagram, which one is a regular polygon and which is an irregular polygon?

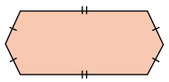
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

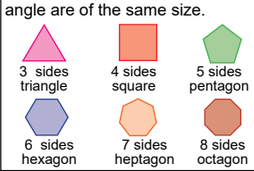
*Solution:*

(a) Irregular polygon
(b) Irregular polygon
(c) Regular polygon
(d) Regular polygon
(e) Irregular polygon
(f) Irregular polygon

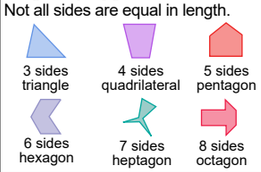
**DO YOU KNOW?**

**Determining types of polygon.**
A polygon can have 3 or more sides.

**Regular polygon**
All sides are equal. All interior angle are of the same size.




**Irregular polygon**
Not all sides are equal in length.

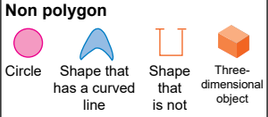



**Concave polygon**
Has at least one angle that is more than 180°.

**Convex polygon**
No interior angle more than 180°.

**Complex polygon**
Has lines that intersects in the polygon.

**Non polygon**



## Determining axis of symmetry

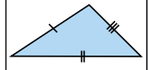
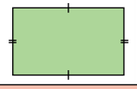
**COGNITIVE STIMULATION** Group

**Aim:** Describing the axis of symmetry of a regular polygon
**Materials:** Dynamic geometry software and scissors
**Steps:**

1. Open the file MS058A and print the worksheet.
2. Divide the class into two groups.
3. The first group cuts the regular polygons and second group cuts the irregular polygons.
4. By folding the polygon, determine the axis of symmetry for all the regular polygons and irregular polygons.
5. Complete the table below.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms058a to print the worksheet.

| | | Number of sides | Number of axis of symmetry |
|---|---|---|---|
| Regular polygons |  | | |
| |  | | |
| |  | | |
| |  | | |
| Irregular polygons |  | | |
| |  | | |

**Discussion:**

(i) What is the relationship between the number of sides with the number of axis of symmetry?
(ii) Conclusion from the findings of the first group and second group.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms058b to find out names of the multi-sided polygon.

The number of axis of symmetry for a regular polygon is equals to the number of sides of the polygon.

For irregular polygons the number of axis of symmetry should be explored using the folding method.





## 4.1.2 Constructing a regular polygon

LEARNING STANDARD

Construct regular polygons using various methods and explain the rationales for the steps of construction.

Regular polygon can be constructed by using various methods. Explore the activity below.

**COGNITIVE STIMULATION** Individual

**Aim:** Creating regular polygon
**Materials:** Dynamic geometry software, paper and scissors
**Steps:**

1. Open the file MS059A.
2. Click on the *polygon* instructions and choose *regular polygon*.
3. Click any points on the Cartesian plane.
4. Click any second point.
5. On the window of the *regular polygon*, at the *vertices* enter the number of edges that has to be built. For example, pentagon has five vertices.
6. Repeat the same steps for regular hexagon, regular heptagon, regular octagon and regular nonagon.
7. Print out the shapes.
8. Paste your work in your book.

**Discussion:**

Your findings from the activity above.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms059a for cognitive stimulation.

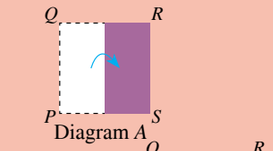
**COGNITIVE STIMULATION** Individual

**Aim:** Producing a regular octagon using origami
**Materials:** Dynamic geometry software, printer, square shaped coloured paper and scissors
**Steps:**

1. Open the file MS059B to watch the tutorial on making an octagon shaped origami.
2. Fold the paper into two parts. Open the fold.
3. Fold the diagonal part of the paper into two parts.
4. Take the centre point of the folded line and bring it close to the diagonal line through the centre point.
5. Cut away the extra paper.
6. Open the fold, then an octagon is produced.

**Discussion:**

Your findings from the activity above.



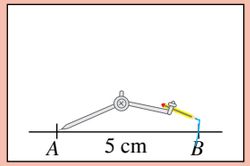

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms059b to watch the tutorial on creating an octagon shaped origami.

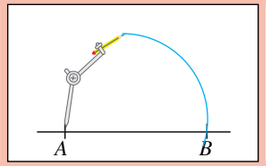
**COGNITIVE STIMULATION** Individual

**Aim:** Building a regular polygon using geometry tools
**Materials:** Pencil, ruler, A4 paper and compasses

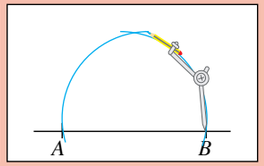
**Activity 1:**
Construct an **equilateral triangle** with the sides 5 cm



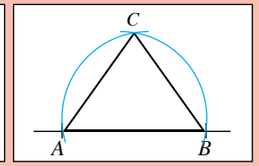

(a) Construct a line segment $AB$ with length 5 cm.




(b) Construct a curve with the radius 5 cm from point $A$.

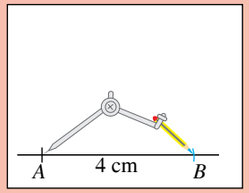



(c) Construct a curve with radius 5 cm from point $B$ so that it intersects with the first curve. The intersection point is labelled $C$.

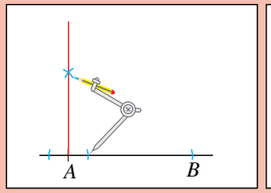



(d) Draw a line from $A$ to $C$ and $B$ to $C$. An equilateral triangle is created.

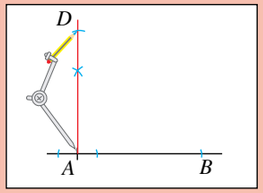
**Activity 2:**
Construct a **square** with sides 4 cm



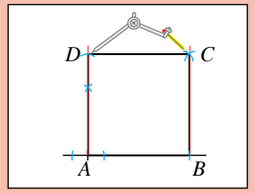

(a) Construct a line segment $AB$ with the length 4 cm.




(b) Construct a perpendicular line with $AB$ through point $A$.

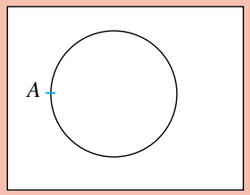



(c) Construct a curve with a distance 4 cm from point $A$ so that it intersects with the perpendicular line. The intersection point is labelled $D$.

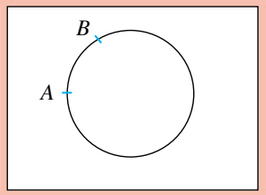



(d) Construct two curves with a distance of 4 cm from $B$ and $D$ so that both the curves intersect. The intersection point is labelled $C$.

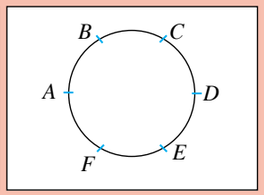
**Activity 3:**
Construct a regular **hexagon** with sides 3.5 cm



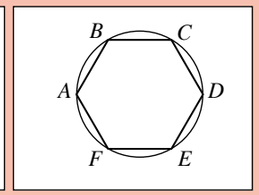

(a) Construct a circle with the radius of 3.5 cm. Mark one point at the circumference and label it $A$.




(b) Construct a curve with radius 3.5 cm from $A$ and label it $B$.




(c) Construct a curve with the distance 3.5 cm from $B$ and label it $C$. Repeat the steps until $F$.




(d) Draw line $AB$, $BC$, $CD$, $DE$, $EF$ and $FA$ to form a regular hexagon.

**Discussion:**
Your findings from the activity above.

From all the activities that have been done, the most accurate method in constructing regular polygons is by using dynamic geometry software.

**TIPS**

Regular polygons can also be constructed by dividing the angles at the centre of a circle according to the number of sides.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms060 to construct regular polygon using geometry tools.



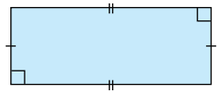




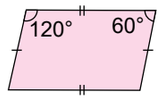
## SELF PRACTICE 4.1

1. Determine whether each polygon is a regular polygon or irregular polygon.

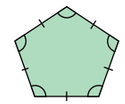
(a)



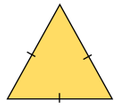
(b)



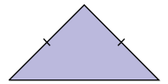

(c)



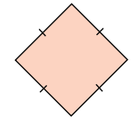
(d)



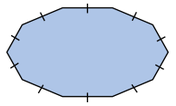
(e)



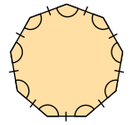
(f)



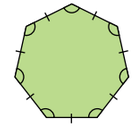
(g)



(h)

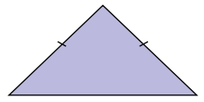


(i)

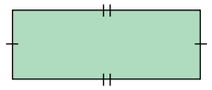


2. Trace the following diagrams. Determine the number of axis of symmetry for each diagram.

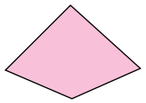
(a)



(b)

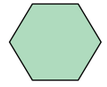
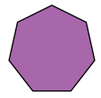
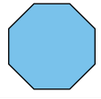
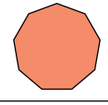


(c)



(d)

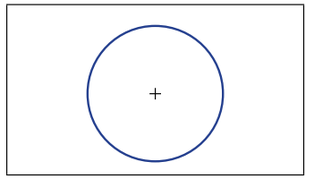
3. Complete the table below with the characteristics of the polygons.

| Regular polygon | Polygon name | Number of sides | Number of vertices | Number of axis of symmetry |
|---|---|---|---|---|
|  | | | | |
|  | | | | |
|  | | | | |
|  | | | | |

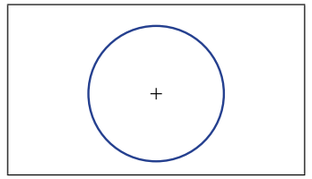
4. Construct the following regular polygons by using a ruler and compasses.
   (a) Equilateral triangle with side length 3.4 cm.
   (b) Square with sides 3.6 cm.
   (c) Regular hexagon with sides 4 cm.
   (d) Regular heptagon with sides 4.2 cm.
   (e) Regular octagon with sides 4.5 cm

5. Draw the following regular polygon by dividing the vertices at the centre equally.

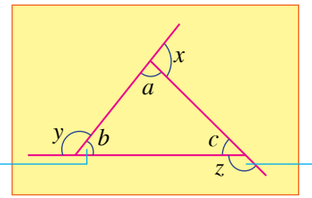
(a) Regular pentagon



(b) Regular hexagon



## 4.2 Interior Angles and Exterior Angles of Polygons




**Interior angle** is an angle that is shaped by two adjacent sides of a polygon.

Angle $a$, $b$, and $c$ are interior angles.

**Exterior angle** is an angle that is formed when one side of the polygon is extended. It is the supplementary angle to the interior angle of the adjacent side.

Angle $x$, $y$, and $z$ are exterior angles.

### 4.2.1 Total sum of an interior angle

There is a relationship between the number of vertices of a polygon with the sum of the interior angle. Explore the activity below:

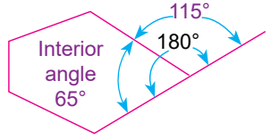
**COGNITIVE STIMULATION** Individual

**Aim:** Exploring the number of triangle in a polygon
**Materials:** Paper and protractor
**Steps:**

1. Open the file MS062 to obtain information about polygon shapes.
2. Print the riangle, square, pentagon, hexagon, heptagon, octagon and nonagon.

**DO YOU KNOW ?**

Exterior angle + Interior angle = 180°.




**FLASHBACK**

Total sum of interior angles of a triangle is 180°.


$a + b + c = 180°$

**LEARNING STANDARD**

Derive the formula for the sum of interior angles of a polygon.

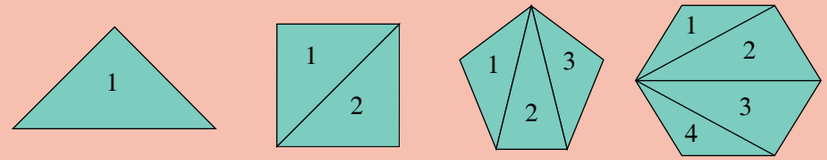
**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms062 to obtain the worksheets on polygon shapes.





3. Connect the edges of each polygon to form a triangle in the polygon as shown below.




4. Complete the table below.

| Polygon | Number of sides ($n$) | Number of triangles | Total sum of interior angles |
|---|---|---|---|
| Triangle | 3 | 1 | $1 \times 180° = 180°$ |
| Square | 4 | 2 | $2 \times 180° = 360°$ |
| Pentagon | | | |
| Hexagon | | | |
| Heptagon | | | |
| Octagon | | | |
| Nonagon | | | |
| Decagon | | | |

**Discussion:**

(i) What is the relationship between the number of sides, $n$ with the number of triangles?
(ii) What is the relationship between the number of sides in a triangle with the total sum of interior angles?

5. Total sum of interior angles of a polygon
= number of triangles × 180°
= ☐ × 180°

the $n^{th}$ term

Total sum of interior angles of a polygon = $(n - 2) \times 180°$.

**EXAMPLE 2**

State the number of triangles formed for each of the following polygon.

(a) 13 sided polygon (b) 18 sided polygon

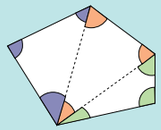
*Solution:*

(a) Number of triangles = 13 – 2
= 11

(b) Number of triangles = 18 – 2
= 16

**THINK SMART**

Pentagon can be divided into 3 triangles. State the number of interior angles of a pentagon.



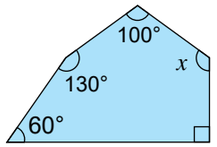
**DO YOU KNOW ?**

| Number of sides | Polygon Name |
|---|---|
| 12 | dodecagon |
| 13 | tridecagon |
| 14 | tetradecagon |
| 15 | pentadecagon |
| 16 | hexadecagon |
| 17 | heptadecagon |
| 18 | octadecagon |
| 19 | enneadecagon |
| 20 | icosagon |

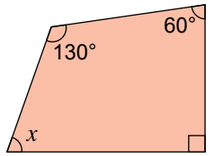
**EXAMPLE 3**

Calculate the value $x$ for the following.

(a)




(b)




*Solution:*

(a) Total sum of interior angles,

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$x + 100° + 130° + 60° + 90° = 540°$

$x + 380° = 540°$

$x = 540° - 380°$

$x = 160°$

(b) Total sum of interior angles,

$= (n - 2) \times 180°$

$= (4 - 2) \times 180°$

$= 360°$

$x + 130° + 60° + 90° = 360°$

$x + 280° = 360°$

$x = 360° - 280°$

$x = 80°$

## 4.2.2 Total sum of exterior angles of polygons

**LEARNING STANDARD**

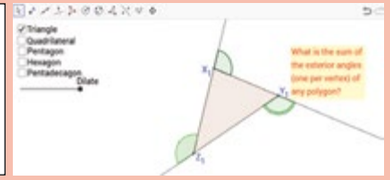
Make and verify conjectures about the sum of exterior angles of a polygon.

**COGNITIVE STIMULATION** Group

**Aim:** Exploring the total sum of exterior angle

**Material:** Dynamic geometry software

| Polygon | $n$ | The output angle enhancement | |
|---|---|---|---|
| | | Conjecture | Verify (Yes / No) |
| | | | |




**Steps:**

1. Open the file MS064 and print the file.
2. Do a conjecture for each of the polygon in the space provided in worksheet.
3. Open the file MS064 to view total sum of exterior angle.
4. Explore each polygon that is available.
5. Drag the slider *dilate* to change the size of the polygon sides that is being displayed.
6. State the total sum of the exterior angles of a polygon.

**Discussion:**

The sum of exterior angles of polygon.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms064 to obtain the worksheet.

**TIPS**

A conjecture is a proposition or a theorem that looks right. Conjecture decisions are not formally proven. The conjecture allows students to speculate based on a mathematical situation. For example, if we add 2 positive numbers then the result is always greater than the number.

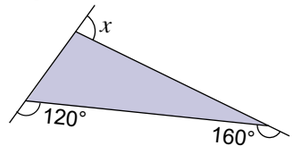
The total sum of exterior angles of a polygon is 360°.



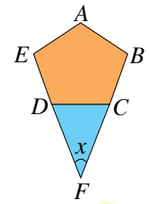


**EXAMPLE 4**

(a) Calculate the value $x$ for the diagram below.




(b) In the diagram below, $ABCDE$ is a regular pentagon. $BCF$ and $EDF$ are straight lines. Calculate the value $x$.




*Solution:*

(a) Total sum of exterior angles = 360°

$x + 160° + 120° = 360°$

$x + 280° = 360°$

$x = 360° - 280°$

$x = 80°$

(b) $\angle FCD = \frac{360°}{5}$

$= 72°$

$x = 180° - 72° - 72°$

$= 36°$

**TIPS**

Exterior angle of a regular polygon $= \frac{360°}{n}$

Interior angle = 180° − exterior angle

## 4.2.3 Total sum of exterior angles of polygons

**LEARNING STANDARD**

Determine the values of interior angles, exterior angles and the number of sides of a polygon.

**EXAMPLE 5**

Calculate the value of the interior angle for a regular hexagon.

*Solution:*

Number of sides of regular hexagon, $n = 6$

Total sum of interior angles $= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 4 \times 180°$

$= 720°$

$$\text{Interior angle} = \frac{\text{Total sum of interior angles}}{\text{Number of sides}}$$

$$= \frac{720°}{6}$$

$= 120°$

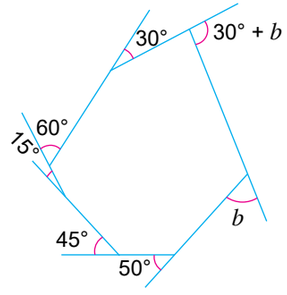
**TIPS**

Interior angle of regular polygon

$= \frac{(n - 2) \times 180°}{n}$

**EXAMPLE 6**

Calculate the value of $b$ for the diagram on the right.




*Solution:*

$360° = (30° + b + b + 50° + 45° + 15° + 60° + 30°)$

$360° = 230° + 2b$

$2b = 360° - 230°$

$2b = 130°$

$b = 65°$

**EXAMPLE 7**

Calculate the value of the exterior angle of a regular octagon.

***Solution:***

Number of sides of a regular octagon, $n = 8$

Total sum of exterior angles $= 360°$

Exterior angle $= \dfrac{360°}{8}$

$= 45°$

**EXAMPLE 8**

Calculate the number of sides of the following regular polygon when given the value of interior angle.

(a) $108°$ (b) $144°$

***Solution:***

(a) Exterior angle $= 180° - 108°$

$= 72°$

Number of sides, $n = \dfrac{360°}{\text{exterior angle}}$

$n = \dfrac{360°}{72°}$

$n = 5$

(b) Exterior angle $= 180° - 144°$

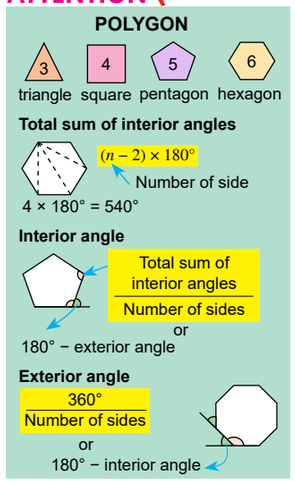
$= 36°$

Number of sides, $n = \dfrac{360°}{\text{exterior angle}}$

$n = \dfrac{360°}{36°}$

$n = 10$

**ATTENTION**




## 4.2.4 Solving problems

**LEARNING STANDARD**

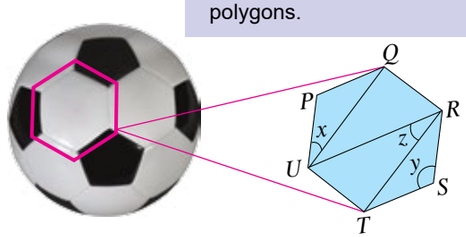
Solve problems involving polygons.

**EXAMPLE 9**

The diagram on the right is a regular hexagon that is enlarged from the design of a football.

(a) Calculate angle $y$.

(b) Calculate the difference between $y$ and $(x + z)$.




***Solution:***

**Understanding the problem**

Calculating angle $y$ using formula $\dfrac{(n-2) \times 180°}{n}$

Angle $x$ is in the equilateral triangle

$\angle UPQ = \angle TSR = y$

$\dfrac{180° - \angle UPQ}{2}$

**Planning the strategy**

(a) $y = \dfrac{(6-2) \times 180°}{6}$

$y = 120°$

(b) $x = \dfrac{180° - 120°}{2}$

$x = 30°$

$z = 30°$ (alternate angle)

**Implementing the strategy**

(b) Difference between $y$ and $(x + z)$

$= 120° - (30° + 30°)$

$= 60°$

**Conclusion**

(a) $y = 120°$

(b) $y - (x + z) = 60°$



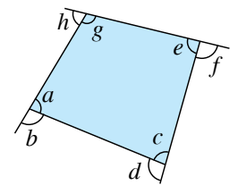
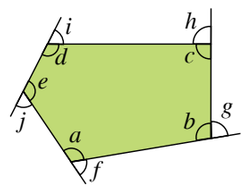


**SELF PRACTICE 4.2**

1. State the number of triangles that can be found in the polygon below and calculate the total sum of the exterior angles.

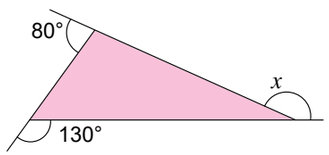
| Polygon | Number of triangles in the polygon | Total sum of exterior angles |
|---|---|---|
| Pentagon | | |
| Hexagon | | |
| Heptagon | | |
| Octagon | | |
| Nonagon | | |

2. Name all the interior angles and exterior angles for each of the following polygons.

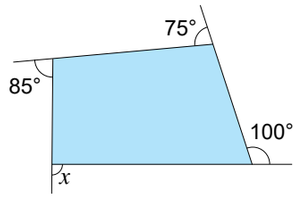
| (a) | (b) |
|---|---|
|   |   |
| Interior angles: | Interior angles: |
| Exterior angles: | Exterior angles: |

3. Calculate the value $x$ for each of the following diagrams.

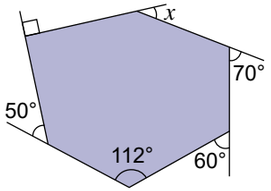
(a)



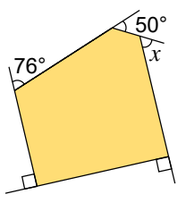

(b)




(c)

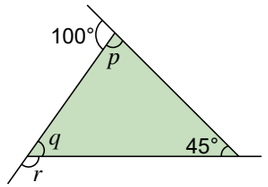



(d)

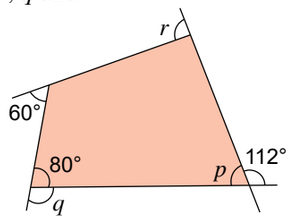


4. For each of the diagram below, calculate the value $p$, $q$ and $r$.

(a)

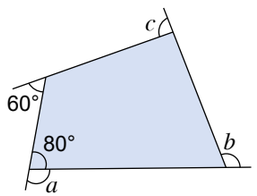



(b)

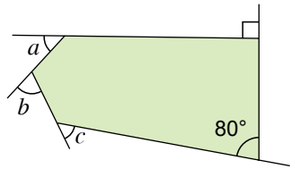



5. Calculate the value $a + b + c$.

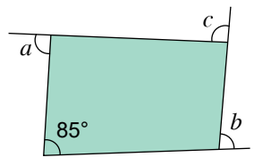
(a)



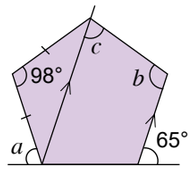

(b)




(c)



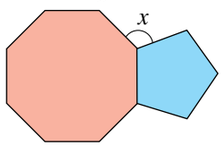

(d)




6. Determine the number of sides for a polygon if the total sum of exterior angles is
   (a) 900°  (b) 1 080°  (c) 1 260°

7. Zaidi has a vegetable garden that is shaped like a regular polygon. The dotted lines is the axis of symmetry of his garden.
   (a) What is the actual shape of Zaidi's garden?
   (b) Calculate the value $y$.


8. The diagram shows two swimming pools at a sports centre in the shape of a regular octagon and pentagon. What is the value of angle $x$?




## GENERATING EXCELLENCE

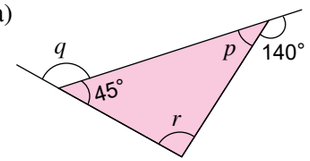
1. Construct the following polygons using compasses and a ruler.
   (a) Equilateral triangle $ABC$ with sides 4 cm.
   (b) Square $PQRS$ with sides 3 cm.



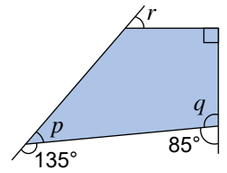


2. Calculate the value $p$,$q$ and $r$ in the following.

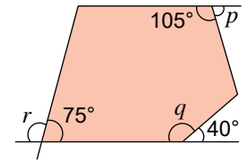
(a)




(b)

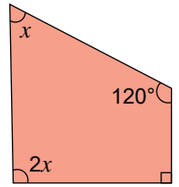



(c)

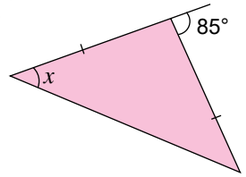



3. Calculate the value $x$ for the following.

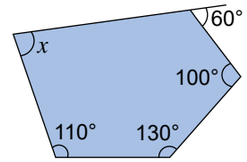
(a)




(b)

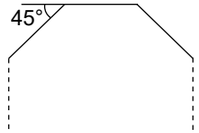



(c)

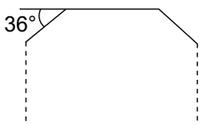



4. Calculate the number of sides for the following.

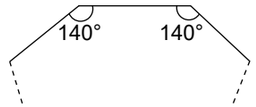
(a)



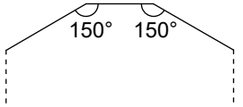

(b)



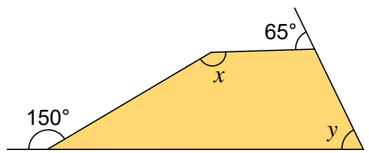

(c)



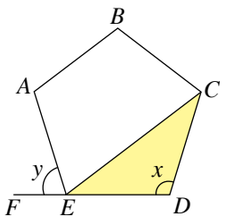

(d)



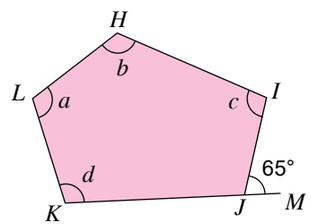

5. (a) Calculate the value $x + y$ in the diagram below.




(b) The diagram shows a logo in the shape of a regular pentagon. $FED$ is a straight line. Calculate the value $x + y$.



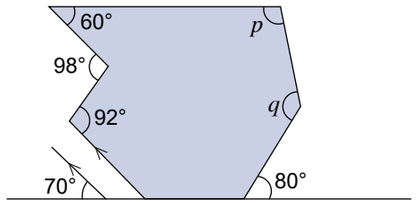

(c) In the diagram below, $HIJKL$ is a pentagon. $KJM$ is a straight line. Calculate the value $a + b + c + d$.



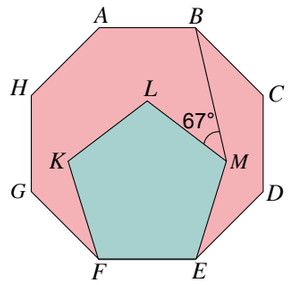
6. Azreen wants to draw a logo for the Peers Counselling Club at her school. She chooses to draw a regular hexagon with the radius 4 cm. Help Azreen draw her logo using a ruler, protractor and compasses.

7. Total sum of all interior angles of a regular polygon is 2 700°. State the number of sides of this polygon.

8. In the diagram below, calculate the value $p + q$.

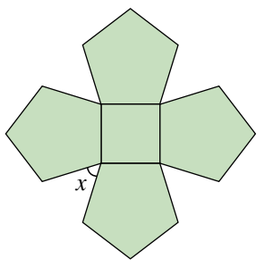



9. In the diagram below, *ABCDEFGH* is a regular octagon and *EFKLM* is a regular pentagon. Calculate $\angle CBM$.




10. The exterior angle of a regular polygon is $2h$ and the interior angle of the same polygon is $7h$.
    (a) Calculate the value of $h$.
    (b) Calculate the value of the interior angle and exterior angle.
    (c) Calculate the number of sides of the polygon and name the polygon.

11. The diagram below shows 4 regular pentagons and a square. Calculate the value $x$.



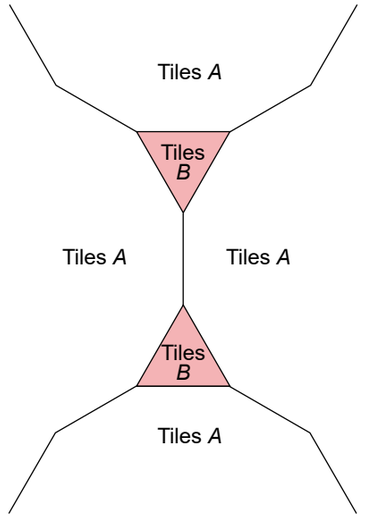




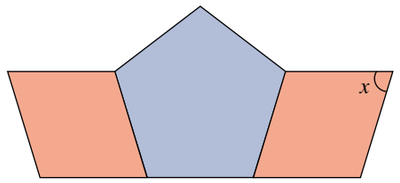



12. Bahar wants to construct a polygon that has an interior angle of 300°. Can Bahar construct the polygon? Justify your answer.

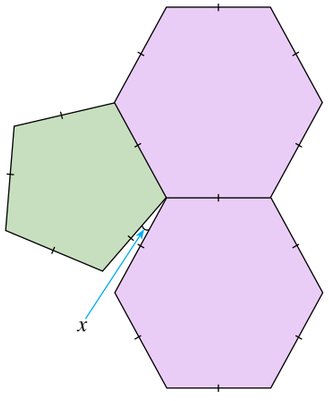
13. The diagram below shows a partial design that has been formed from combining two tiles. There are two types of tiles. They are tile *A* and tile *B* that are regular polygons. Calculate the number of sides of tile *A*.




14. Devaa is a graphic design student at a local university. Help Devaa calculate the value of $x$ to construct a photo frame that has the characteristics of combined polygons.




15. Calculate the value of $x$.



## CHAPTER SUMMARY

The number of axis of symmetry in a regular polygon with $n$ side and $n$ axis of symmetry.

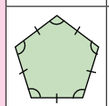
### Regular polygon

- Interior angle $= \frac{(n-2) \times 180°}{n}$
- Exterior angle $= \frac{360°}{n}$

Total sum of interior angles $= (n-2) \times 180°$

Total sum of exterior angles $= 360°$

Regular polygon is a polygon where all its sides have the same length and all interior angles are equal. Examples

| | Exterior angle | Interior angle |
|---|---|---|
|  | $= \frac{360°}{3}$ | $= \frac{(3-2) \times 180°}{3}$ |
|  | $= \frac{360°}{4}$ | $= \frac{(4-2) \times 180°}{4}$ |
|  | $= \frac{360°}{5}$ | $= \frac{(5-2) \times 180°}{5}$ |

### Irregular polygon

Exterior angle of a polygon is a complement to an interior angle of the polygon.

Exterior angle + Interior angle = 180°

Total sum of exterior angle = 360°

Irregular polygon is a polygon where not all sides are of the same length.





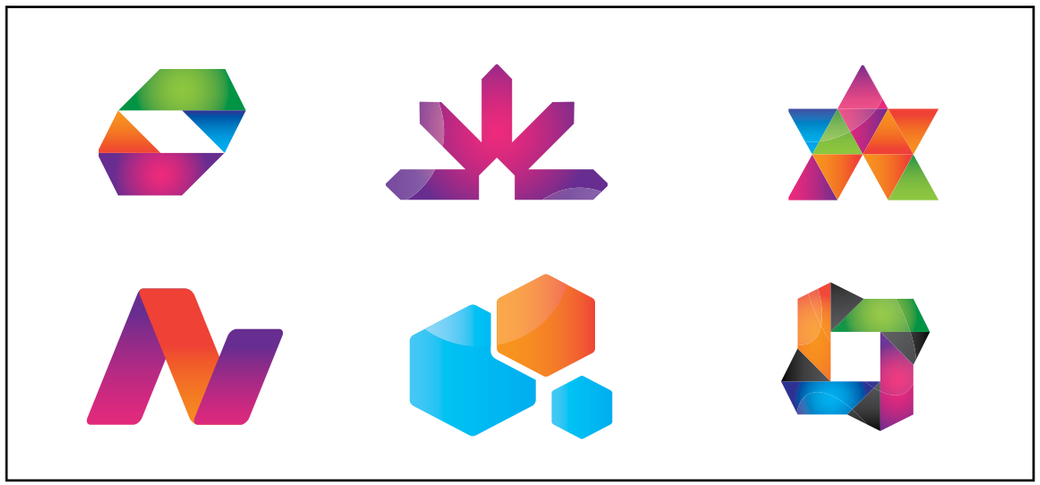


## SELF REFLECTION

| At the end of this chapter, I will be able to: |  |
|---|---|
| 1. Describe the geometric properties of regular polygons using various representations. | |
| 2. Construct regular polygons using various methods and explain the rationales for the steps of construction. | |
| 3. Derive the formula for the sum of interior angles of a polygon. | |
| 4. Make and verify a conjecture about the sum of exterior angles of a polygon. | |
| 5. Determine the values of interior angles, exterior angles and the number of sides of a polygon. | |
| 6. Solve problems involving polygons. | |

### MINI PROJECT

You are a food stall owner. Create your business logo using the combination of two or three polygons. You can use the dynamic geometry software, geometry tools or origami to design your logo. Present the rationale for the choice of your logo in the class.



Examples of logo

CHAPTER 5

# Circles

The clockwise movement of the hand of a clock forms a circle as it completes a 360° rotation. This is called 'kirkos' in Greek, which means to rotate and to become a curve.


## WHAT WILL YOU LEARN?

**5.1** Properties of Circles

**5.2** Symmetrical Properties of Chords

**5.3** Circumference and Area of a Circle

## WORD LINK

| | |
|---|---|
| • Circle | • *Bulatan* |
| • Circumference | • *Lilitan* |
| • Radius | • *Jejari* |
| • Centre | • *Pusat* |
| • Diameter | • *Diameter* |
| • Chord | • *Perentas* |
| • Segment | • *Tembereng* |
| • Sector | • *Sektor* |
| • Minor sector | • *Sektor minor* |
| • Major sector | • *Sektor major* |
| • Minor segment | • *Tembereng minor* |
| • Major segment | • *Tembereng major* |
| • Symmetry | • *Simetri* |



## WALKING THROUGH TIME

A circle is a curved pathway locus of a point that is equidistant from a fixed point. This fixed point is known as the centre and the distance from this fixed point to the pathway is called the radius. A circle is also a curve that is joined which is known as circumference. A mathematician named Euclid was the first person to study circles. He is also known as the 'Father of Geometry' due to his research.

For more information:

http://rimbunanilmu.my/mat_t2e/ms075

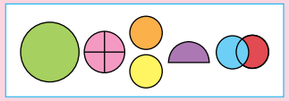
## WHY STUDY THIS CHAPTER?

- The application of this chapter is in the field of architecture, astronomy, design and astrology.

**CREATIVE ACTIVITY**

**Aim:** Getting to know circles
**Materials:** Coloured paper, glue, scissors, string and punch
**Steps:**

1. Students form groups.
2. Each group is required to draw circles of various sizes.
   Examples are like the ones in the diagram on the right.



3. The circles will be used to decorate the class.
4. Write down the mathematical formulae of area of a rectangle, area of triangle, volume of cube, volume of cuboid, Pythagoras theorem and so on in the circles.



## 5.1 Properties of Circles

### 5.1.1 Getting to know the parts of the circle

**LEARNING STANDARD**

Recognise parts of a circle and explain the properties of a circle.

**COGNITIVE STIMULATION** Group

**Aim:** Knowing parts of a circle
**Material:** Dynamic geometry software
**Steps:**

1. Open the file MS076.
2. The perimeter of a circle is called ____ .
3. Drag point *A* in the all directions.
   (i) Point *A* is called the ____ of the circle.
4. Drag point *B* around the circle.
   (i) The line from the centre of the circle to any point on the circumference of the circle is called ____ .
5. Drag point *C* around the circle.
   (i) The *CC*' line that goes through the centre and touches the circumference is called ____ .
6. Drag point *D* and the point *E* around the circle.
   (i) The line that connects two points on the circumference is called ____ .
   (ii) The region is called ____ .
7. Drag points *C* and *D*.
   (i) Name two lines generated. Lines *AC* and ____ .
   (ii) The region bounded by these two radius is called ____ .

**QR CODE**

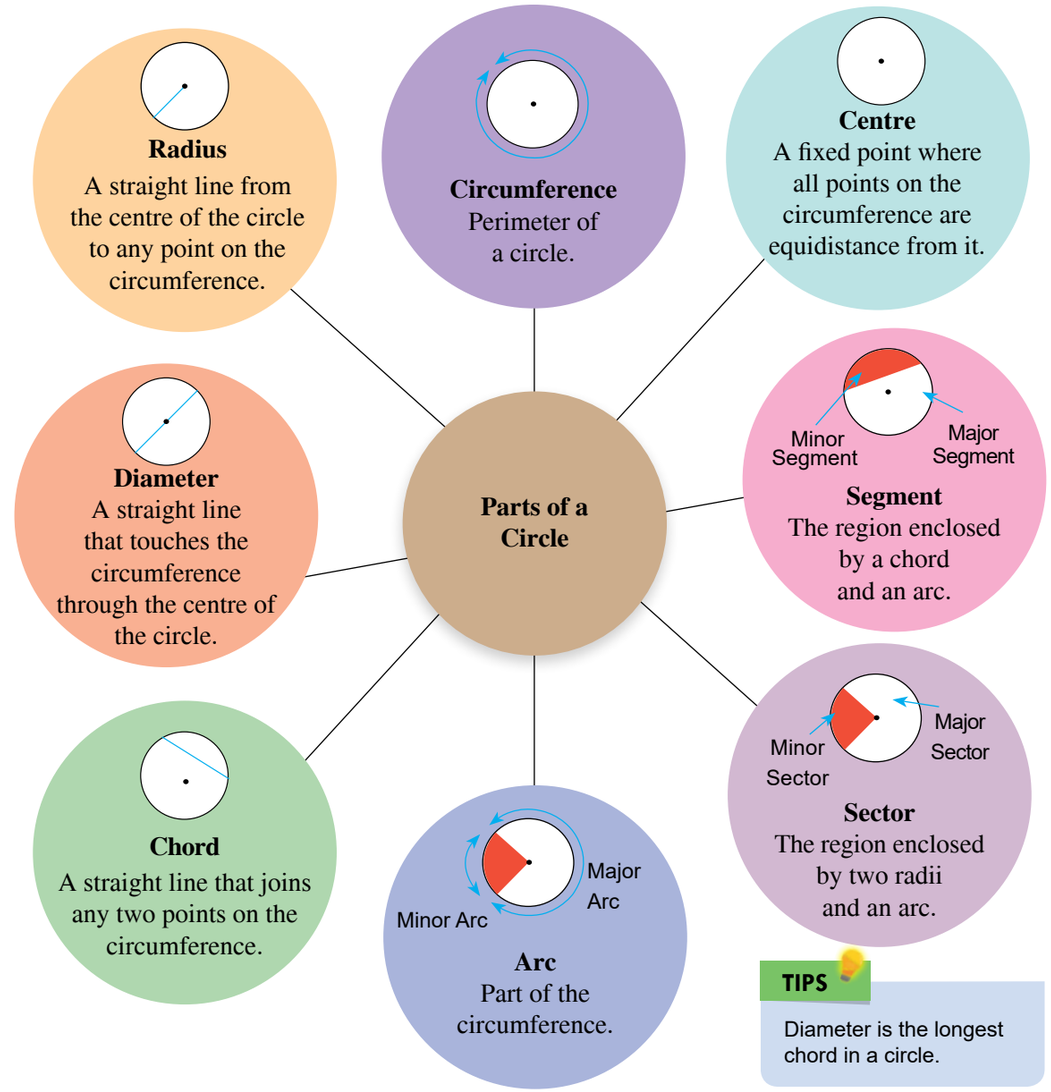
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms076 to explore parts of the circle.

**Discussion:**
Make a conclusion on your explorations.

From the activity above, several parts of the circle have been identified as in the diagrams on the next page.








**TIPS**

Diameter is the longest chord in a circle.

**TIPS**

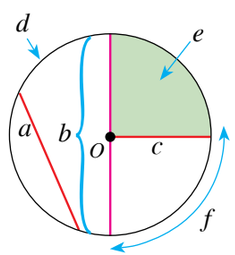
A circle is a curved pathway of a point that is equidistant from a fixed point.

**THINK SMART**

Why are balls, globes and marbles not considered as circles?

**EXAMPLE 1**

Identify the following parts of a circle.




*Solution:*

*a*, Chord
*b*, Diameter
*c*, Radius
*d*, Circumference
*e*, Sector
*f*, Arc

## 5.1.2 Constructing a circle

LEARNING STANDARD

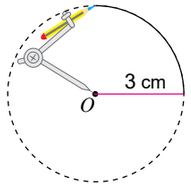
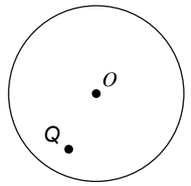
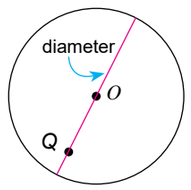
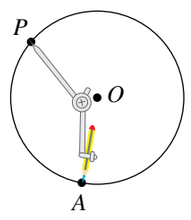
Construct a circle and parts of the circle based on the conditions given.

COGNITIVE STIMULATION Group

**Aim:** Constructing a circle and parts of the circle based on the conditions given

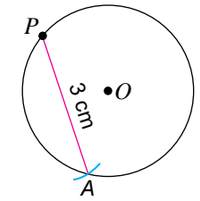
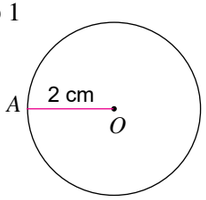
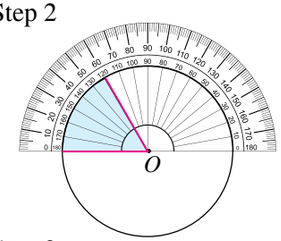
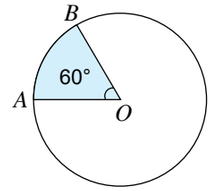
**Materials:** Compasses, protractor, ruler and pencil

**Steps:**

| Conditions | Steps | Solution |
|---|---|---|
| (a) Construct a circle with a radius of 3 cm from the centre $O$. | 1. Mark point $O$.<br>2. Using compasses measure 3 cm on a ruler.<br>3. Place the sharp point of the compasses on point $O$ and draw a circle with the radius of 3 cm. |  |
| (b) Construct a diameter that passes through point $Q$ in a circle with the centre $O$. | 1. Join points $O$ and $Q$ with a straight line using a ruler.<br>2. Extend the line until it touches the circumference. The extended straight line that passes through $Q$ and centre $O$ is the **diameter**. | Step 1<br>  <br>Step 2<br>  |
| (c) Construct two chords of 3 cm in length from point $P$ on a circle. | 1. Using compasses measure 3 cm on a ruler.<br>2. Place the sharp point of the compasses on point $P$.<br>3. Draw the arc that cuts on the circumference and label it as point $A$. | Step 1<br>  |





| | | |
|---|---|---|
| | 4. Join point $P$ to point $A$ that has been marked on the circumference.<br>5. Thus, the line $PA$ is a **chord**. | Step 2<br>  |
| (d) Construct a sector with an angle of 60° at the centre of a circle with a radius of 2 cm. | 1. Draw a circle with the radius $OA$ 2 cm.<br>2. Measure 60° with a protractor as shown.<br>3. Draw the radius $OB$. $AOB$ is the **sector of the circle**. | Step 1<br>  <br>Step 2<br>  <br>Step 3<br>  |

**Discussion:**

From the activity above, what parts of the circle has been constructed?

From the activities above, students are able to

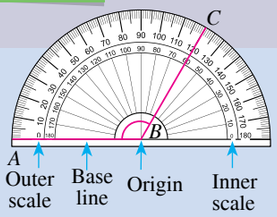
(a) construct a circle when the radius or diameter is given.

(b) construct a diameter through a given point in a circle.

(c) construct a chord through a given point when the length of the chord is given.

(d) construct a sector when angle of the sector and the radius is given.

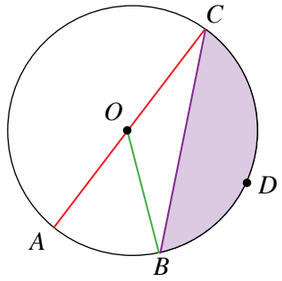
TIPS




To measure the $ABC$ angle, place the protractor's centre point on the vertex of that angle. Make sure the line with the value 0 is located on the $AB$ line. Read the angle using the external scale. Thus, the angle for $ABC$ is 120°.

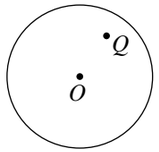
## SELF PRACTICE 5.1

1. Name the following parts
   (i) point $O$
   (ii) line $AOC$
   (iii) dector $AOB$
   (iv) line $OA$
   (v) arc $AB$
   (vi) line $BC$
   (vii) the shaded area $BCD$.

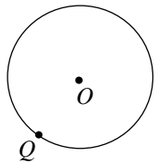



2. Construct a circle with radius
   (a) 3 cm
   (b) 4.5 cm
   (c) 2.5 cm
   (d) 6 cm

3. Construct a diameter that passes through point $Q$ for each of the circles with the centre $O$.
   (a)




   (b)




4. Construct the chord of a circle with radius and length given below.

| | Radius | Length of Chord |
|---|---|---|
| (a) | 3 cm | 4 cm |
| (b) | 4.5 cm | 6.7 cm |

5. Using a protractor, construct the sector $AOB$ with $O$ as the centre of the circle. The radius and $\angle AOB$ as given below.

| | Radius | $\angle AOB$ |
|---|---|---|
| (a) | 3 cm | 70° |
| (b) | 3.6 cm | 120° |





# 5.2 Symmetry and Chords

## 5.2.1 Features in a circle

**COGNITIVE STIMULATION** Group

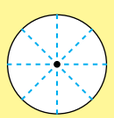
**Aim:** Verifying
(i) properties of the diameter of a circle.
(ii) the relationship of a radius with chords.

**Material:** Dynamic geometry software

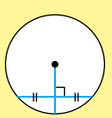
**Steps:**
1. Open the file MS081.
2. Click on the *Activity* box.
3. Drag point $Q$ to points $P$, $T$, $U$, $B_1$, $V$ and $Z$.
   (i) Name the diameter of the circle. Lines [ ] .
   (ii) Observe the value of angle at the centre when the diameter $QQ'$ is moved. Does it produce the same value? Are the resulting shapes similar?
   (iii) If you fold the circle on the line $QQ'$, do the shapes overlap each other perfectly?
   (iv) The diameter of a circle is known as [ ] .
4. Click the *Activity* box for the next activity.
5. Drag the slider *Drag Me* until the end.
   (i) Radius that bisects a chord is [ ] to the chord.
   (ii) Radius that is perpendicular to the chord [ ] the chord.
   (iii) Equal chords produce [ ] arc.

**Discussion:**
State the conclusions for all the activities above.

The diameter of a circle is the axis of symmetry of the circle.



A radius which is perpendicular to the chord bisects the chord.



**LEARNING STANDARD**

Verify and explain that
(i) diameter of a circle is an axis of symmetry of the circle;
(ii) a radius that is perpendicular to a chord bisects the chord and vice versa;
(iii) perpendicular bisectors of two chords intersect at the centre;
(iv) chords that are equal in length produce arcs of the same length and vice versa;
(v) chords that are equal in length are equidistant from the centre of the circle and vice versa.

**DO YOU KNOW?**

The circle has an infinite number of axes of symmetry because any straight line that passes through the centre is the axes of symmetry of the circle.

**QR CODE**

Scan the QR Code below or visit http://rimbunanilmu.my/mat_t2e/ms081 for the properties of symmetric chord 1.

**TIPS**

The diameter is the chord that passes through the centre of the circle.

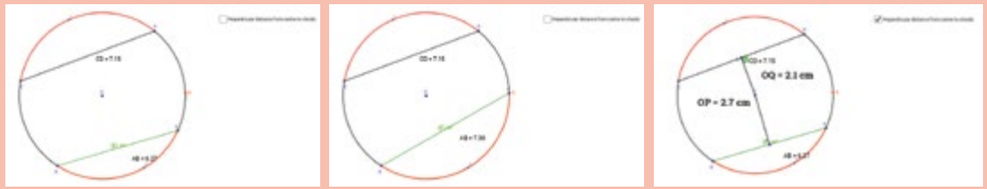
## COGNITIVE STIMULATION

Individual

**Aim:** Verifying
(i) properties of the bisector of the two chords.
(ii) properties of equal chords in a circle.

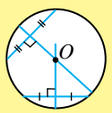
**Material:** Dynamic geometry software

**Steps:**
1. Open the file MS082.
2. Drag point *A* where, *AB* = *CD*.
3. Click on the box *length of the perpendicular line from the centre of the circle*.
4. Repeat steps 1 and 2 to get the others values.

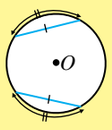



**Discussion:**
(i) Where do lines *OP* and *OQ* meet?
(ii) Is the length of arc *AGB* and *CID* the same?
(iii) If the length of *AB* = *CD*, the distance of *OP* = the distance of ☐.
(iv) Is the distance of *OP* and *OQ* the same?

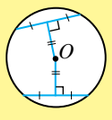
Perpendicular bisectors of two chords meet at the centre of the circle.




Equal chords or chords of the same length produce arc of the same length.




Equal chords are equidistant from the centre of the circle.




**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms082 for the properties of symmetric chord 2.

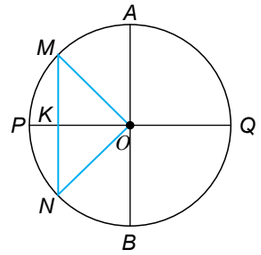
**THINK SMART**

How many axes of symmetry are there in half a circle?





## EXAMPLE 2

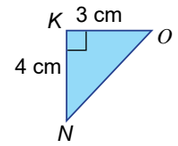



The diagram above shows a circle with centre *O* and the line *MN* is the chord.
(a) Name the axes of symmetry of this circle.
(b) Given *OK* = 3 cm and *NK* = 4 cm, calculate length of *ON*.
(c) Name the angle that is equal to ∠*ONK*.

***Solution:***

(a) *AOB* and *POQ*

(b)




$ON^2 = 4^2 + 3^2$

$ON = \sqrt{(16 + 9)}$

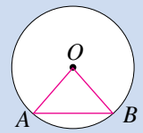
$ON = \sqrt{25}$ ← *ON* = *OM*

$ON = 5$

Therefore, length of *ON* is 5 cm.

(c) ∠*OMK*

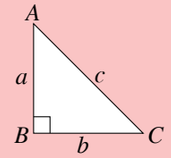
**TIPS**




Two radii and a chord forms an isosceles triangle.

**FLASHBACK**

**Pythagoras theorem**

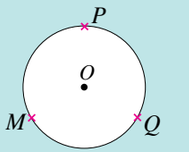



$AB^2 + BC^2 = AC^2$

or

$a^2 + b^2 = c^2$

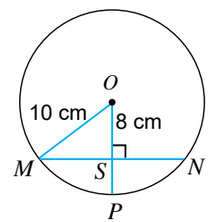
**THINK SMART**




*O* is the centre of the circle. What is the relationship between *OP*, *OQ* and *OM*?

## EXAMPLE 3

The diagram on the right shows a circle with the radius *OP* that is perpendicular to the chord *MN*.
(a) Is the length *MS* equal to length of *SN*? Explain.
(b) If the radius of the circle is 10 cm and *OS* = 8 cm, calculate the length of the chord *MN*.




***Solution:***

(a) Yes, *MS* = *SN*
The radius *OP* which is perpendicular bisects *MN*.

(b) $MS = \sqrt{10^2 - 8^2}$

$MS = \sqrt{100 - 64}$

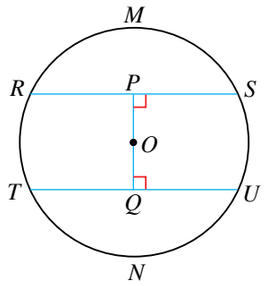
$MS = \sqrt{36}$

$MS = SN = 6$

Therefore, *MN* = 12 cm.

**EXAMPLE 4**

The diagram on the right shows two equal chords, $RS$ and $TU$. $POQ$ is a straight line passing through the centre of the circle $O$. Given $OP = 5$ cm and $RS = 24$ cm.
(a) Calculate the length of $PR$.
(b) Are minor arcs $RMS$ and $TNU$ equal in length? Explain.
(c) Calculate the radius of the circle.



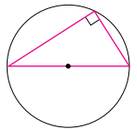

***Solution:***

(a) A radius that is perpendicular to the chord bisects the chord into two equal lengths. Length of $PR = 24 \div 2$ cm $= 12$ cm.

(b) Yes, chords that are equal in length produce arc of the same length.

(c) $OR = \sqrt{PR^2 + OP^2}$ ← Chords $RS$ and $TU$ are equal in length
$= \sqrt{12^2 + 5^2}$
$= \sqrt{144 + 25}$ ← $OR$, $OS$, $OT$ and $OU$ are radii of a circle
$= \sqrt{169}$
$= 13$ cm

**DO YOU KNOW ?**



Angle at the circumference of a semicircle is 90°.

## 5.2.2 Centre and radius of a circle

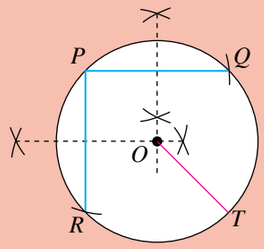
**LEARNING STANDARD**

Determine the centre and radius of a circle by geometrical construction.

**COGNITIVE STIMULATION** Group

**Aim:** Determining the centre and radius of the circle
**Materials:** Compasses, rulers, pencils and rounded material
**Steps:**
1. Trace a circle on a piece of paper.
2. Construct two chords, $PQ$ and $PR$ from point $P$.
3. Construct a perpendicular line for the chords $PQ$ and $PR$.
4. The intersection point of two perpendicular lines is indicated by $O$.
5. Draw a line from $O$ to the circumference and label it as $OT$.

**Discussion:**
(i) Properties of point $O$.
(ii) Properties of line $OT$.




A perpendicular bisector for any chord will always intersect at the centre of the circle.





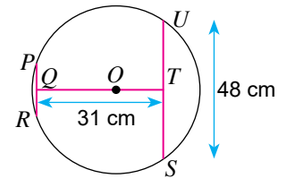
## 5.2.3 Solving problems

**LEARNING STANDARD**

Solve problems involving symmetrical properties of chords.

**EXAMPLE 5**

A blacksmith was asked to build a round-shaped window frame as shown below. The window is 50 cm in diameter. Three iron rods, $PR$, $US$ and $QT$ that are not equal in length are used to support the window. Calculate the length of $PR$.




***Solution:***

**Understanding the problem**
Diameter of window = 50 cm
$QT = 31$ cm
$US = 48$ cm
Calculate length $PR$.

**Planning the strategy**
Radius $= \frac{\text{diameter}}{2}$
$= \frac{50}{2}$
$= 25$ cm

$OT = \sqrt{OU^2 - UT^2}$
$OQ = QT - OT$
$PQ = \sqrt{OP^2 - OQ^2}$
$PR = PQ \times 2$

**Conclusion**
Therefore, $PR$ is 14 cm.

**Implementing the strategy**
$OT = \sqrt{25^2 - 24^2}$
$= \sqrt{625 - 576}$
$= \sqrt{49}$
$= 7$ cm

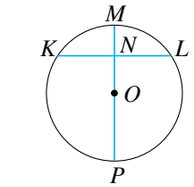
$OQ = 31 - 7$
$= 24$ cm

$PQ = \sqrt{25^2 - 24^2}$
$= \sqrt{625 - 576}$
$= \sqrt{49}$
$= 7$ cm

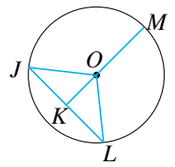
$PR = 7 + 7$
$= 14$ cm

**SELF PRACTICE 5.2**

1. In the diagram on the right, $O$ is the centre of the circle. $MNOP$ and $KNL$ are straight lines. Given that $MN = 8$ cm and $NP = 18$ cm. Calculate the length of $KL$.




2. The diagram on the right shows a circle with the centre $O$. $JKL$ and $KOM$ are straight lines. Given that $JK = KL = 15$ cm and radius of the circle is 25 cm. Calculate the length of $KOM$.



## 5.3 Circumference and Area of a Circle

### 5.3.1 Relationship between circumference and diameter

**LEARNING STANDARD**

Determine the relationship between circumference and diameter of a circle, and hence define $\pi$ and derive the circumference formula.

Circumference is the measurement around a circle. The diagram shows a round table that needs to be lined with skirting for a wedding. What is the length of the skirting needed?

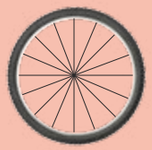
The length for the skirting can be calculated using the formula that involves $\pi$(pi).

**COGNITIVE STIMULATION** Group

**Aim:** Determining the relationship between circumference and diameter
**Materials:** Stopwatch, pail, bicycle tyre, measuring tape, pencil or any circular material.
**Steps:**
1. Measure the circumference of the stopwatch, pail and bicycle tyre with the measuring tape. Record the results in the table below.
2. Measure the diameter of the three items and record them in the table.
3. Complete the table below.

| Material | Circumference (cm) | Diameter (cm) | $\frac{\text{Circumference}}{\text{Diameter}}$ |
|---|---|---|---|
| 1. Stopwatch | | | |
| 2. Pail | | | |
| 3. Bicycle tyre | | | |





**Discussion:**
(i) The relationship between diameter and circumference.
(ii) What is the ratio of the circumference to the diameter?

From the above activities, the ratio of circumference to diameter, is $\pi$ of a circle which is 3.142 or $\frac{22}{7}$.

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

**REMEMBER**

Diameter = 2 × radius





The circumference of a circle is $\pi$ multiplied by the diameter.

$$\text{Circumference} = \pi \times \text{diameter}$$
$$= \pi d$$

The circumference of a circle can also be expressed as follows.

$$\text{Circumference} = \pi \times 2 \times \text{radius}$$
$$= 2\pi r$$

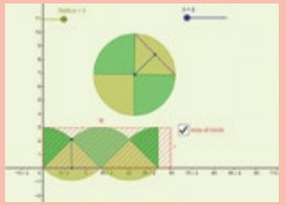
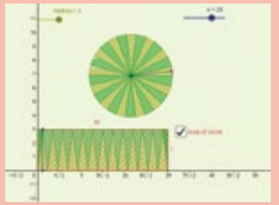
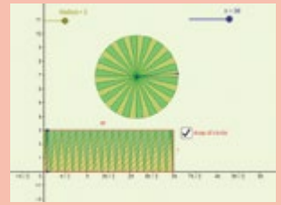
### 5.3.2 Formula for area of a circle

**LEARNING STANDARD**

Derive the formula for the area of a circle.

**COGNITIVE STIMULATION** Group

**Aim:** Expressing formula of a circle
**Material:** Dynamic geometry software
**Steps:**
1. Open the file MS087.
2. Drag the radius up to value 3, and drag the $n$ until it reaches the value of 6. Take note of the changes.
3. Repeat step 2 by changing the value of radius and $n$. Take note of the changes.

**Discussion:**
(i) The [ ] the sector of the circle is divided into the clearer is the rectangular shape produced.
(ii) Height of rectangles = [ ] of the circle.
(iii) Rectangular base = [ ] of the circle.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms087 to explore area of a circle.

From the activity above,
Area of circle = area of rectangle

$$= \text{base} \times \text{height}$$
$$= \frac{1}{2} \times \text{circumference} \times \text{height}$$
$$= \frac{1}{2} \times 2\pi r \times r$$
$$= \pi r^2$$

Therefore, area of circle = $\pi r^2$




CHAPTER 5

## 5.3.3 Circumference, area of a circle, length of arc and area of sector

**LEARNING STANDARD**

Determine the circumference, area of a circle, length of arc, area of a sector and other related measurements.

### To determine the circumference of a circle

**EXAMPLE 6**

Calculate the circumference of a circle, if

(a) diameter, $d = 14$ cm (Use $\pi = \frac{22}{7}$)

(b) Radius, $r = 21.3$ cm (Use $\pi = 3.142$)

*Solution:*

(a) Radius $= \pi d$

$= \frac{22}{7} \times 14$

$= 44$ cm

(b) Circumference $= 2\pi r$

$= 2 \times 3.142 \times 21.3$

$= 133.85$ cm

**EXAMPLE 7**

(a) Given the circumference of a circle is 88 cm, calculate the diameter of the circle in cm. (Use $\pi = \frac{22}{7}$)

(b) Given the circumference of a circle is 36.8 cm, calculate the radius of the circle in cm and round off the answer to two decimal places. (Use $\pi = 3.142$)

*Solution:*

(a) Circumference $= \pi d$

$88 = \frac{22}{7} \times d$

$d = 88 \times \frac{7}{22}$

$d = 28$ cm

(b) Circumference $= 2\pi r$

$2\pi r = 36.8$

$2 \times 3.142 \times r = 36.8$

$r = \frac{36.8}{6.284}$

$r = 5.86$ cm

### To determine area of a circle

**EXAMPLE 8**

Calculate the area of a circle with

(a) diameter 10 cm (Use $\pi = \frac{22}{7}$)

(b) radius 7 cm

**REMEMBER**

radius, $r = \frac{\text{diameter}}{2}$

diameter, $d = 2r$

*Solution:*

(a) Area $= \pi r^2$

$= \frac{22}{7} \times \left(\frac{10}{2}\right)^2$

$= 78.57$ cm$^2$

(b) Area $= \pi r^2$

$= \frac{22}{7} \times 7^2$

$= 154$ cm$^2$





**EXAMPLE 9**

Given the area of a circle is 616 cm$^2$, calculate the radius and diameter. (Use $\pi = \frac{22}{7}$)

*Solution:*

Area $= \pi r^2$

$\pi r^2 = 616$

$\frac{22}{7} \times r^2 = 616$

$\frac{1\cancel{22}}{\cancel{7}_1} \times \frac{1\cancel{7}}{\cancel{22}_1} \times r^2 = 616 \times \frac{7}{22}$

$r^2 = 616 \times \frac{7}{22}$

$r^2 = 196$

$r = \sqrt{196}$

$r = 14$ cm

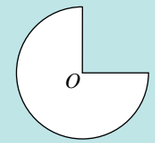
Diameter $= 2 \times 14$

$= 28$ cm

**THINK SMART**

(a) Calculate the area of quadrant if the radius is 7 cm.

(b) Calculate the area of the semi circle if the radius is 7 cm.



(c) Calculate the area of the three quadrant if the radius is 7 cm.

**EXAMPLE 10**

Given the circumference is 66 cm, calculate the area of the circle. (Use $\pi = \frac{22}{7}$)

*Solution:*

Circumference $= 66$ cm

$2\pi r = 66$

$2 \times \frac{22}{7} \times r = 66$

$r = 66 \times \frac{7}{44}$

$r = 10.5$ cm

Area $= \pi r^2$

$= \frac{22}{7} \times 10.5^2$

$= 346.5$ cm$^2$

**EXAMPLE 11**

Given area of circle is 75.46 cm$^2$. Calculate the circumference. (Use $\pi = \frac{22}{7}$)

*Solution:*

Area $= \pi r^2$

$\pi r^2 = 75.46$

$\frac{22}{7} \times r^2 = 75.46$

$r^2 = 75.46 \times \frac{7}{22}$

$r^2 = 24.01$

$r = \sqrt{24.01}$

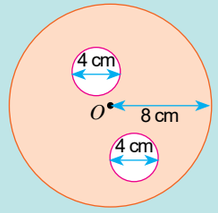
$r = 4.9$ cm

Circumference $= 2\pi r$

$= 2 \times \frac{22}{7} \times 4.9$

$= 30.8$ cm

**THINK SMART**

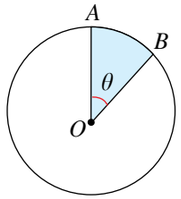



The diagram shows two circles in a bigger circle. Calculate the area of the shaded region.

## Determining length of arc in a circle

The arc $AB$ is part of the circumference of the circle. The length of arc is proportional to the angle at the centre of the circle.

$$\frac{\text{Length of arc}}{\text{Circumference}} = \frac{\text{Angle at centre}}{360°}$$




Therefore, $\dfrac{\text{Length of arc}}{2\pi r} = \dfrac{\theta}{360°}$

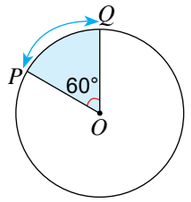
**TIPS**

The symbol $\theta$ is read as theta, a Greek letter used to represent angle.

**EXAMPLE 12**

The diagram below shows a circle with a radius of 14 cm and centred at $O$. Calculate the length of minor arc $PQ$ which encloses 60° at the centre. Write your answer to two decimal places.

*Solution:*



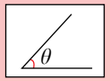

$$\frac{\text{Length of arc}}{2\pi r} = \frac{\theta}{360°}$$

$$\text{Length of arc} = \frac{\theta}{360°} \times 2\pi r$$

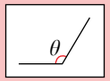
$$\text{Length of arc} = \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 14$$

$$= 14.67 \text{ cm}$$

**FLASHBACK**

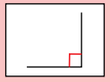



Acute angle
$0° < \theta < 90°$




Obtuse angle
$90° < \theta < 180°$


Reflex angle
$180° < \theta < 360°$

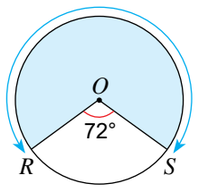


Right angle 90°

**EXAMPLE 13**

The diagram below shows a circle with a radius of 21 cm and centered at $O$. $\angle ROS$ is 72°. Calculate the length of major arc $RS$.

*Solution:*




$$\text{Angle at centre} = 360° - 72°$$
$$= 288°$$

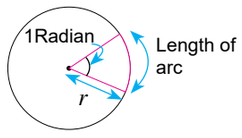
$$\frac{\text{Length of arc}}{2\pi r} = \frac{\theta}{360°}$$

$$\text{Length of arc} = \frac{\theta}{360°} \times 2\pi r$$

$$\text{Length of arc} = \frac{288°}{360°} \times 2 \times \frac{22}{7} \times 21$$

$$= 105.6 \text{ cm}$$

**DO YOU KNOW ?**




Angles can be measured using radians. 1 radian (1 rad) is the angle at the centre of the circle when the length of the arc is equal to the radius.





**EXAMPLE 14**

Given the length of the arc of a circle is 11 cm and the angle at the centre of the circle is 45°. Calculate in cm the radius of the circle.

*Solution:*

$$\frac{\theta}{360°} = \frac{\text{Length of arc}}{2\pi r}$$

$$2\pi r = \text{Length of arc} \times \frac{360°}{\theta}$$

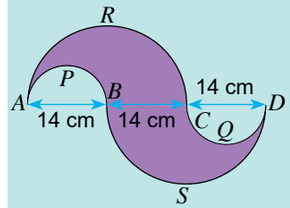
$$2 \times \frac{22}{7} \times r = 11 \times \frac{360°}{45°}$$

$$r = 11 \times \frac{360°}{45°} \times \frac{7}{22} \times \frac{1}{2}$$

$$r = \frac{27\,720}{1\,980}$$

$$r = 14 \text{ cm}$$

**THINK SMART**

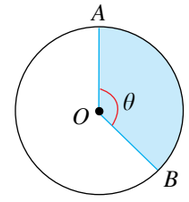



*ARC*, *APB*, *BSD* and *CQD* are arcs of the circles wheras *AB*, *AC*, *BD* and *CD* are the diameter of the circles. Calculate the area of the shaded region.

## To determine area of a sector

The area of a sector is a region bounded by an arc and two radii. The area of the sector is proportional to the area of the circle.

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Angle at centre}}{360°}$$




Therefore,

$$\frac{\text{Area of } AOB}{\pi r^2} = \frac{\theta}{360°}$$

**EXAMPLE 15**

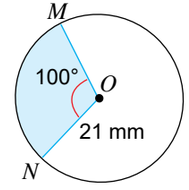
The diagram below shows a circle with centre $O$ and radius 21 mm. Calculate the area of the minor sector $MON$.

*Solution:*

$$\frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{360°}$$

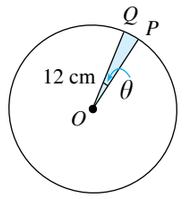
$$\text{Area of sector } MON = \frac{100°}{360°} \times \frac{22}{7} \times 21^2$$

$$= 385 \text{ mm}^2$$



**EXAMPLE 16**

Given the area of a sector $QOP$ is 18.48 cm$^2$ and the radius is 12 cm. Calculate the value of $\theta$.




***Solution:***

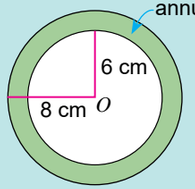
$$\frac{\text{Area of a sector}}{\pi r^2} = \frac{\theta}{360^\circ}$$

$$\frac{\theta}{360^\circ} = \frac{18.48}{\frac{22}{7} \times 12^2}$$

$$\theta = \frac{18.48}{\frac{22}{7} \times 12 \times 12} \times 360^\circ$$

$$\theta = 14.7^\circ$$

**THINK SMART**




The shaded area is an annulus.Determine the formula to calculate the area of the annulus.

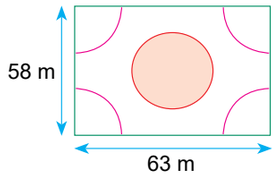
## 5.3.4 Solving problems

**LEARNING STANDARD**

Solve problems involving circles.

**EXAMPLE 17**

Majlis Bandaraya Melaka Bersejarah intends to build a rectangular recreational park with a length of 63 m and a width of 58 m. At every corner of the park, a quadrant with radius of 7 m will be planted with flowers. A circular shaped fish pond with a diameter of 28 m will be built in the middle of the park.The remaining areas will be planted with grass. Calculate the area covered with grass.(Use $\pi = \frac{22}{7}$)




***Solution:***

**Understanding the problem**

Radius of quadrant = 7 m

Garden is rectangular.
Length = 63 m
Width = 58 m

Diameter of fish pond = 28 m

Calculate the area covered with grass.

**Planning the strategy**

Recreational park area = length × width

Flower area = $4 \times \frac{1}{4} \pi r^2$

The fish pond area = $\pi r^2$

Area covered with grass
= Recreational area − flower area − fish pond area

**Implementing the strategy**

(i) Recreational park area = $58 \times 63$
$= 3\,654$ m$^2$

(ii) Flower area = $4 \times \frac{1}{4} \times \pi r^2$
$= \frac{22}{7} \times 7^2$
$= 154$ m$^2$

(iii) Fish pond area
$= \pi r^2$
$= \frac{22}{7} \times 14^2$
$= 616$ m$^2$

**Conclusion**

Thus, the area covered with grass is
$3\,654 \text{ m}^2 - 154 \text{ m}^2 - 616 \text{ m}^2$
$= 2\,884$ m$^2$







**SELF PRACTICE 5.3**

1. Calculate the circumference of a circle that has
   (a) a radius of 7 cm.
   (b) a radius of 56 cm.
   (c) a diameter of 9.2 cm.
   (d) a diameter of 98 mm.

   Give answers correct to two decimal places. (Use $\pi = \frac{22}{7}$)

2. Given circumference of a circle is 24.5 cm. Calculate
   (a) the diameter
   (b) the radius

   Give answers correct to two decimal places. (Use $\pi = 3.142$)

3. Calculate the area of the circle with the following radius.
   (a) 21 m
   (b) 56 mm
   (c) 7 cm
   (d) $1\frac{2}{5}$ cm

   Give answers correct to two decimal places. (Use $\pi = \frac{22}{7}$)

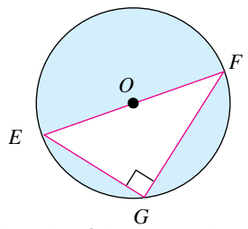
4. The area of a circle is 38.5 cm$^2$. Calculate.
   (a) the radius
   (b) the circumference of the circle

   Give answers correct to two decimal places. (Use $\pi = \frac{22}{7}$)

5. Calculate the area of a circle, if the circumference is 15.4 cm.

   Give answers correct to two decimal places. (Use $\pi = \frac{22}{7}$)

6. The diagram below shows a circle with centre $O$. Given $OF = 6.5$ cm and $EG = 5$ cm calculate the area of the shaded region, in cm$^2$. Give answers correct to two decimal places. (Use $\pi = 3.142$)



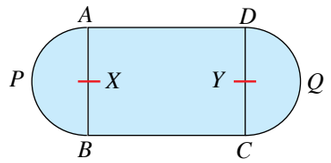

7. Calculate the radius when the length of the arc and angles at the centre of the circle are given. State the answer correct to two decimal places.

| | Length of arc(cm) | Angle at centre |
|---|---|---|
| (a) | 11 | 45° |
| (b) | 4.3 | 35° |
| (c) | 30.8 | 120° |
| (d) | 110 | 200° |

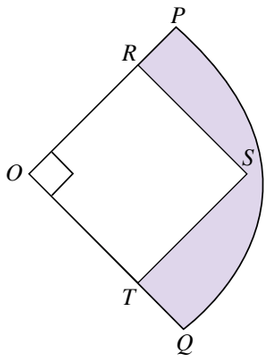
8. Given the radius and area of the circle, calculate the angle at the centre of the circle.

| | Radius | Area of sector |
|---|---|---|
| (a) | 14 cm | 18.48 $cm^2$ |
| (b) | 21 m | 27.72 $m^2$ |
| (c) | 8.4 cm | 15.4 $cm^2$ |

9. The diagram below shows a plan for a park. $ABCD$ is a rectangle. $APB$ and $DQC$ are semicircles centred at $X$ and $Y$. Given $AB = 7$ cm and $AC = 25$ cm. Calculate the perimeter of the park in cm.

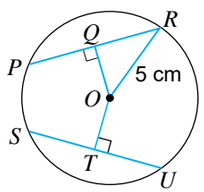



10. The diagram below shows the quadrant $OPQ$ centred at $O$. $ORST$ is a square. Given $OP = 10$ cm and $OR = 7$ cm. Calculate the area of the shaded region, in $cm^2$. State the answer in $\pi$.




## GENERATING EXCELLENCE

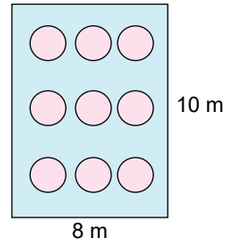
1. The diagram shows a circle with centre $O$. $PQR$ and $STU$ are straight lines. Given $PQR = STU = 6$ cm, calculate the length.
   (a) $PQ$
   (b) $ST$
   (c) $OT$



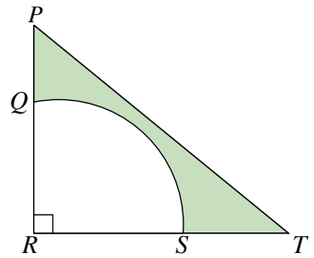




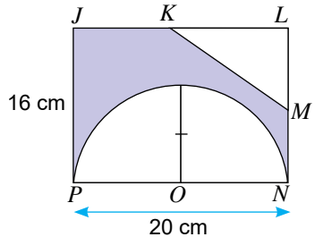

2. The diagram shows a dining hall measuring 10 m long and 8 m wide. It is laid with nine circular carpets, each has a diameter of 200 cm. Calculate the area that is not covered by the carpet in square metres.



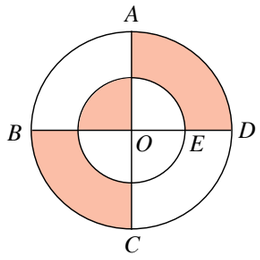

3. The diagram shows a right-angled triangle, $PRT$. $R$ is the centre for the quadrant. Given $RS = 14$ cm, $ST = 10$ cm and $PQ = 4$ cm. Calculate the perimeter of the shaded area in cm.
   (Use $\pi = \frac{22}{7}$)



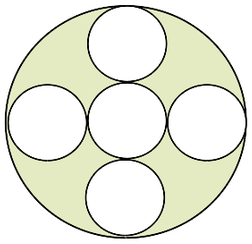

4. The diagram shows a rectangular piece of land owned by Encik Rashid. Encik Rashid divided his land into 3 parts. The first part is a triangle $KLM$. $K$ is the midpoint of $JL$ and $M$ is the midpoint of $LN$. The second part is a semicircle. Encik Rashid intends to plant vegetables in the first and second part. Calculate the area that is not planted with vegetables. (Use $\pi = 3.142$)




5. Kevin wants to build a dartboard. The dartboard consists of two circles centred at $O$ and three shaded regions as in the diagram. The diameters $BOD$ and $AOC$ are perpendicular to each other. Given $OE = ED = 10$ cm. Calculate the area of the shaded regions in $cm^2$. (Use $\pi = \frac{22}{7}$)



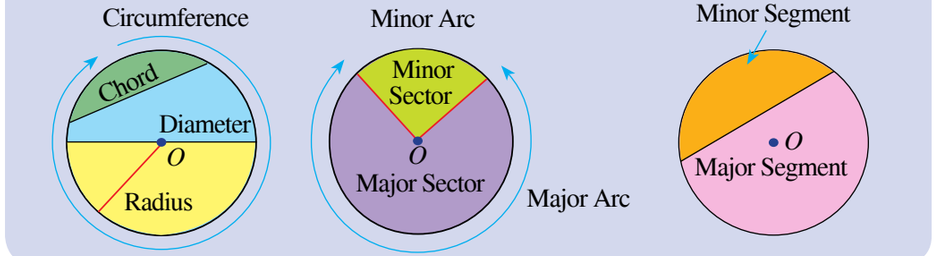

6. In a museum there is a round window decorated with circular rings of the same size as in the diagram. The radius of the window is 45 cm. Calculate the area that is not covered by the decoration. (Use $\pi = 3.142$)



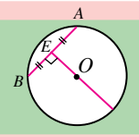
## CHAPTER SUMMARY

### Circles

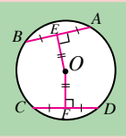
### Parts of a Circle



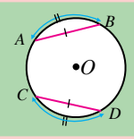

The radius perpendicular to the chord bisects the two chords and vice versa $AE = BE$.




Chords, which are equidistant from a centre of a circle are equal in lengths and vice versa.




Equal chords form arcs with equal lengths and vice versa
$AB = CD$.




### Formulae for a circle

Circumference $= \pi d$
$= 2\pi r$
$\rightarrow \quad \frac{\text{Length of Arc}}{2\pi r} = \frac{\theta}{360°}$

Area of a circle $= \pi r^2$
$\rightarrow \quad \frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{360°}$







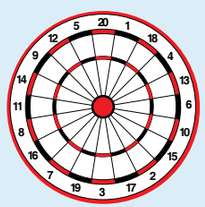
## SELF REFLECTION

| At the end of the chapter, I am able to: | |
|---|---|
| 1. Recognise parts of a circle and explain the properties of a circle. | |
| 2. Construct a circle and parts of the circle based on the conditions given. | |
| 3. Verify and explain that: | |
| (a) Diameter of a circle is an axis of symmetry of the circle. | |
| (b) A radius that is perpendicular to the chord bisects the chord and vice versa. | |
| (c) Perpendicular bisectors of two chords intersect at the centre. | |
| (d) Chords that are equal in length produce arcs of the same length and vice versa. | |
| (e) Chords that are equal in length are equidistant from the centre of the circle and vice versa. | |
| 4. Determine the centre and radius of a circle by using geometrical construction. | |
| 5. Solve problems involving symmetrical properties of chords. | |
| 6. Determine the relationship between circumference and diameter of a circle, and hence define the $\pi$ and derive the circumference formula. | |
| 7. Derive the formula for the area of a circle. | |
| 8. Determine the circumference, area of a circle, length of arc, area of a sector and other related measurements. | |
| 9. Solve problems involving circles. | |

## MINI PROJECT

**Title:** Number board game

You are required to build a number board like the one on the right. The number board consists of four circles with the radius of 5 cm, 15 cm, 20 cm and 25 cm respectively. All four circles share the same centre. The circles should be divided into 20 sectors. Each sector should be labelled with scores/points. This number board can be built using manila card, poster paper or polystyrene board. Arrows can be made from small sticks that are attached to adhesive tape. You can start the games by throwing the arrows towards the board to score points.

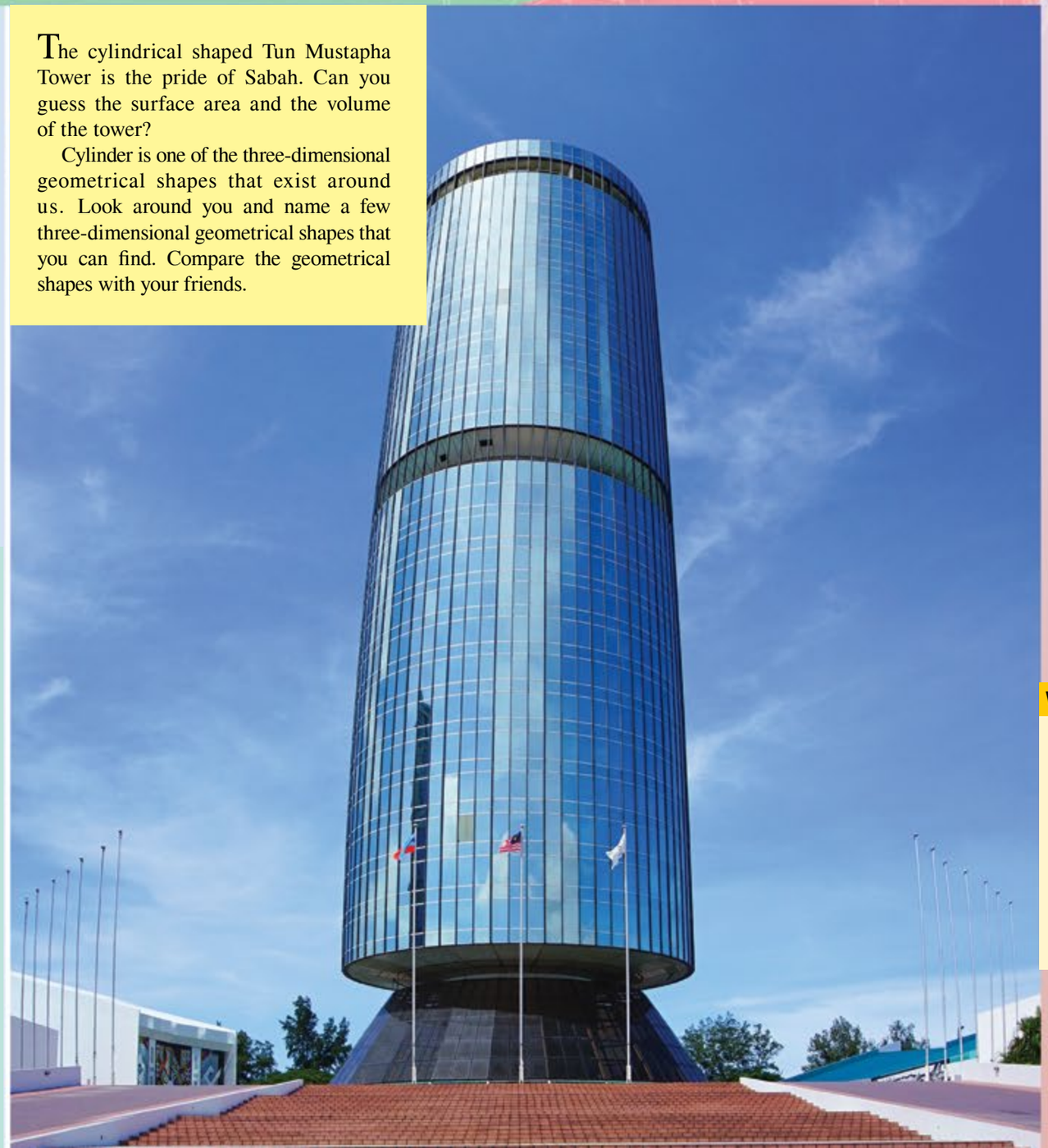


# CHAPTER 6

# Three-Dimensional Geometrical Shapes

The cylindrical shaped Tun Mustapha Tower is the pride of Sabah. Can you guess the surface area and the volume of the tower?

Cylinder is one of the three-dimensional geometrical shapes that exist around us. Look around you and name a few three-dimensional geometrical shapes that you can find. Compare the geometrical shapes with your friends.



## WHAT WILL YOU LEARN?



## WORD LINK

- Two-dimensional shape
- Three-dimensional shape
- Geometrical characteristic
- Net
- Surface area
- Volume
- Subject of formula
- Cross section

- *Bentuk dua dimensi*
- *Bentuk tiga dimensi*
- *Sifat geometri*
- *Bentangan*
- *Luas permukaan*
- *Isi padu*
- *Perkara rumus*
- *Keratan rentas*

## WALKING THROUGH TIME

The word geometry originated from two Greek words 'geo' meaning earth and 'metria' that means measurement. Euclid who revolutionised the geometrical research is often referred to as the 'Father of Geometry'. His book entitled 'Elements' is used as the main reference in the field of Mathematics, especially geometry in the early 20th Century.

For more information:

http://rimbunanilmu.my/mat_t2e/ms099

## WHY STUDY THIS CHAPTER?

- The knowledge and skills in this chapter will help an architect and an engineer in designing and drawing blueprints of a building.
- Interior designers also use knowledge of geometry to create attractive landscape and interior design that optimises the area allocated.

**CREATIVE ACTIVITY**

**Aim:** Identifying three-dimensional shapes
**Materials:**

**Steps:**
1. Name the geometrical shapes of the objects above.
2. Compare and list the differences between the objects above in terms of:
   (i) Surface properties
   (ii) Shape
3. Discuss your opinions with your friends.

Each of the objects above has its own geometrical characteristics. Two-dimensional geometrical shapes like squares and triangles have width and length, while three-dimensional shapes have width, length and height. However, in a circle, radius is used. We will be discussing, on the geometrical characteristics of three-dimensional shapes in this topic.

## 6.1 Geometric Properties of Three-Dimensional Shapes

### 6.1.1 Three-dimensional shapes

**LEARNING STANDARD**

Compare, contrast and classify three-dimensional shapes including prisms, pyramids, cylinders, cones and spheres, and hence describe the geometric properties of prisms, pyramids, cylinders, cones and spheres.

**COGNITIVE STIMULATION** Individual

**Aim:** Exploring the concept of two-dimensional and three-dimensional shapes
**Material:** Dynamic geometry software
**Steps:**
1. Open the file MS100.
2. Drag the red slider from *Open* to *Close* indicator. Take note of the differences between the two-dimensional and three-dimensional shapes in the diagram.
3. Repeat step 2 until the blue slider reaches $n = 11$.

**Discussion:**
The difference between a two-dimensional shape and a three-dimensional shape.

**QR CODE**

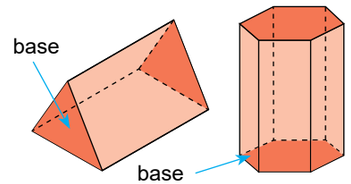
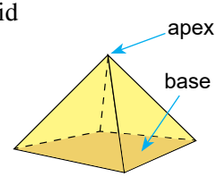
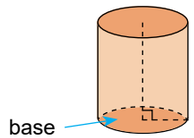
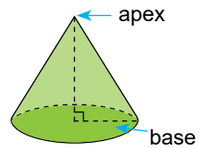
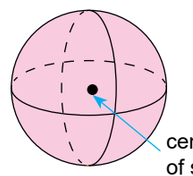
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms100 to explore three-dimensional shapes.

From the activity above, it can be concluded that three-dimensional shapes are formed out of two-dimensional shapes.





The table below shows three-dimensional shapes and their characteristics.

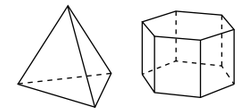
| Geometrical shape | Geometrical characteristics |
|---|---|
| Prism   | • Two flat bases that are polygons which are congruent and parallel.<br>• Flat rectangular shaped side.<br>• Uniform cross section. |
| Pyramid   | • One flat base that is polygon shaped.<br>• The other sides are triangular shaped that meet at the apex. |
| Cylinder   | • Two circular bases which are congruent and parallel.<br>• One curved surface. |
| Cone   | • One circular base.<br>• One apex.<br>• One curved surface that merges the base and the apex. |
| Sphere   | • All points on the surface are equidistant from the centre of the sphere.<br>• One curved surface. |

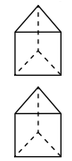
**FLASHBACK**

Congruent means an object that has the same size and shape.

**DO YOU KNOW?**

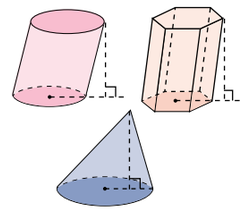
The pyramid and prism are named according to the shape of their base.



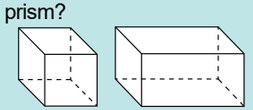

**DO YOU KNOW?**

Oblique shapes.



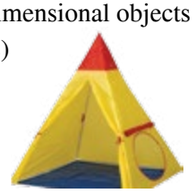
**THINK SMART**

Is a cube and a cuboid a prism?



**SELF PRACTICE 6.1**

1. List the geometrical characteristics for the three-dimensional objects below:

(a) 
(b) 
(c) 
(d) 




CHAPTER 6

2. List the three-dimensional shape that has geometrical characteristics as stated below.
   (a) One vertex with one curved surface.
   (b) One vertex with polygonal base.
   (c) Every point on the surface has the same length from the centre of the object.

## 6.2 Nets of Three-Dimensional Shapes

### 6.2.1 Nets

Net of a three-dimensional shape is obtained by opening and laying out each surface of a three-dimensional object to become two-dimensional.

**LEARNING STANDARD**

Analyse various nets including pyramids, prisms, cylinders and cones, and hence draw nets and build models.

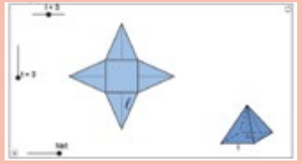
**COGNITIVE STIMULATION** Group

**Aim:** Analysing nets of cone, cylinder, prism and pyramid
**Materials:** Dynamic geometry software, scissors and adhesive tape
**Steps:**

Nets of cone

Nets of a cylinder

Nets of prism



Nets of pyramid

1. Open the file MS102A.
2. Drag the slider for each layout and observe the nets.
3. Open the file MS102B and print it.
4. Students are required to cut the net.
5. Fold the nets along the dotted lines.
6. Use the adhesive tape to form the three-dimensional shape.
   **Example:**

Step 4

Step 5

Step 6

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms102a to view the nets.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms102b to print the layout.

**DO YOU KNOW?**

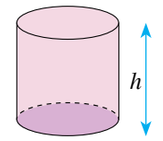
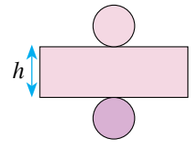
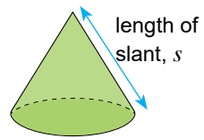
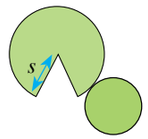
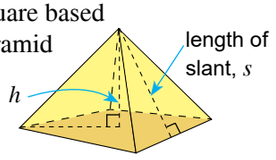
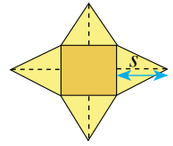
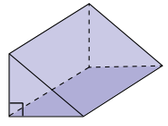
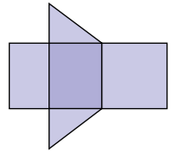
A cube can be filled up with six pyramids with the same square base.





**Discussion:**
(i) Can the net of a three-dimensional object be customised?
(ii) Sketch the various nets of a cube.

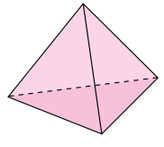
From the activity it can be concluded that the net of three-dimensional object can be vary. The table below shows three-dimensional geometrical shapes and net.

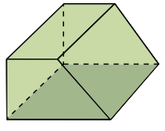
| Geometrical shape | Net |
|---|---|
| Cylinder   |   |
| Cone   |   |
| Square based pyramid   |   |
| Triangular based prism  |  |

**EXAMPLE 1**

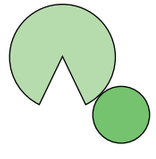
Draw the net for the three-dimensional shapes below.

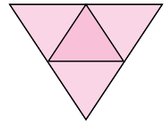
(a) 

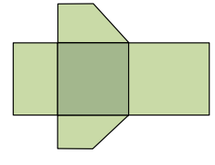
(b) 

(c) 

*Solution:*

(a) 

(b) 

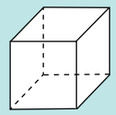
(c) 

**THINK SMART**

What is the net of a sphere?

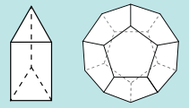
**THINK SMART**

How many nets are there for a cube?



**THINK SMART**

What are the nets of these prisms?

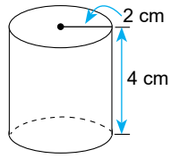


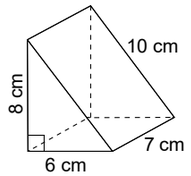
**DO YOU KNOW?**

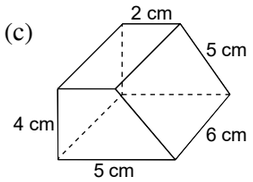
A cone is generated by the rotation of a right-angled triangle.

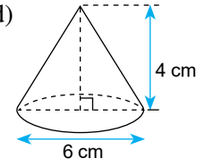
## SELF PRACTICE 6.2

1. Using 1 cm grid paper, draw the net and build a model for each of the three-dimensional shapes below.

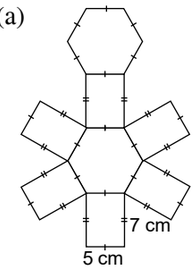
(a) 

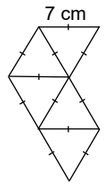

(b) 

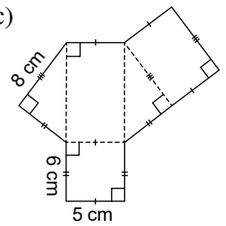

(c) 

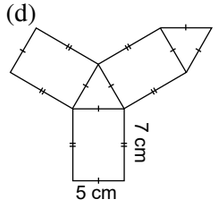

(d) 


2. State the three-dimensional shapes that can be built with the following net. Build an actual model.

(a) 


(b) 


(c) 


(d) 


# 6.3 Surface Area Three-Dimensional Shapes

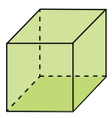
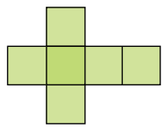
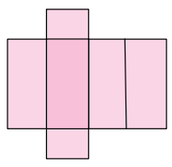
## 6.3.1 Surface area of cube, cuboid, pyramid, prism, cylinder and cone

**LEARNING STANDARD**

Derive the formulae of the surface areas of cubes, cuboids, pyramids, prisms, cylinders and cones, and hence determine the surface areas of the shapes.

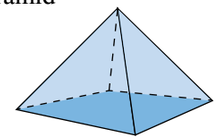
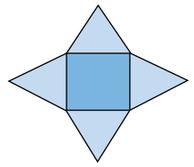
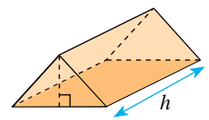
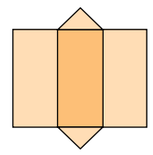
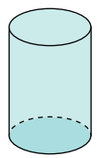
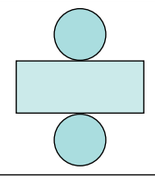
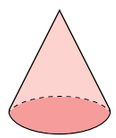
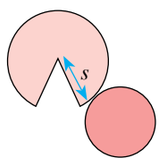
**COGNITIVE STIMULATION** Individual

**Aim:** Deriving the surface area of three-dimensional geometrical shapes
**Material:** Worksheet
**Steps:**
Fill in the box with the number of surfaces for each of the three-dimensional shapes below.

| Shape | Net | Surface area |
|---|---|---|
| Cube  |  | ☐ × area of a square |
| Cuboid  |  | ☐ × area of a rectangle + ☐ × area of a square |



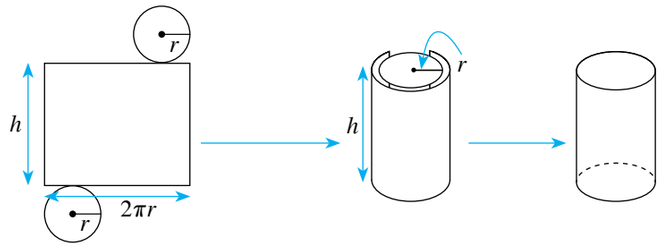


| | | |
|---|---|---|
| Pyramid  |  | ☐ × area of a square + ☐ × area of a triangle |
| Prism   |  | ☐ × area of a triangle + ☐ × area of a rectangle |
| Cylinder  |  | ☐ × area of a circle + ☐ × area of a rectangle |
| Cone  |   | ☐ × area of a circle + ☐ × area of a curved surface |

**Discussion:**
Determine the surface area for each of the three-dimensional shapes above.

The surface area of the three-dimensional geometrical shapes can be calculated by adding all the surface area of the net.

### The surface area of a closed cylinder




**DO YOU KNOW?**

Cube is also known as hexahedron because a cube has six surfaces.

From the net of a cylinder, the length of the rectangle is the circumference of circle and the width of the rectangle is the height of the cylinder.

Surface area of a closed cylinder = (2 × area of circle) + area of rectangle
$= (2 \times \pi r^2) + (2\pi r \times h)$
$= 2\pi r^2 + 2\pi rh$

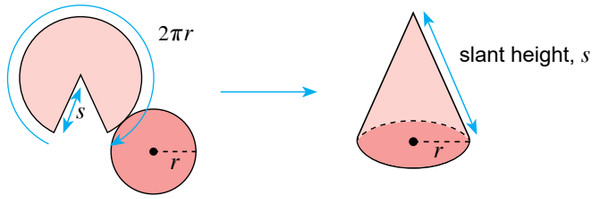
**FLASHBACK**

Area of circle = $\pi r^2$
Circumference of circle = $2\pi r$



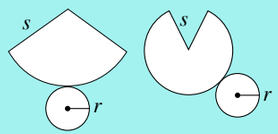

CHAPTER 6

## Surface area of a cone is calculated from the cone's net

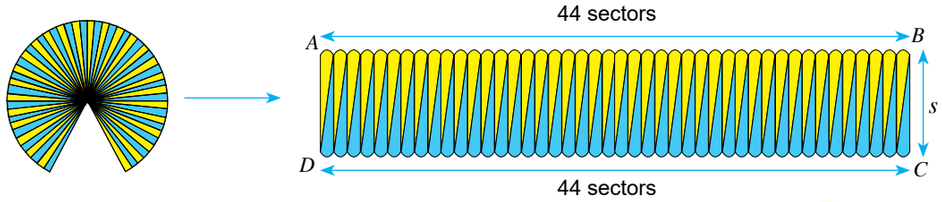



**THINK SMART**

What are the differences between the nets below?




Cut the curved surface into 88 equal sectors. Then arrange them accordingly as in the diagram below.




A rectangle $ABCD$ is formed. The circumference of the base of the cone is,

$$AB + CD = \text{circumference of circular base}$$
$$= 2\pi r$$

Therefore, length $AB$ = Length $CD$

$$= \frac{1}{2} \times 2\pi r$$
$$= \pi r$$

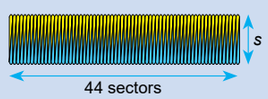
Curved surface area = Area of rectangle $ABCD$
$= \text{length} \times \text{width}$
$= AB \times BC$
$= \pi r \times s$
$= \pi rs$

Area of the circular base $= \pi r^2$

Cone surface area = area of circular base + curved surface area
$= \pi r^2 + \pi rs$

**TIPS**

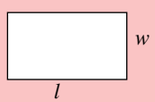
Cut the curved surfaces into 88 equal sectors:




The more sectors are cut, the greater the pieces will resemble a rectangle.

**FLASHBACK**

Area of rectangle
= Length × width



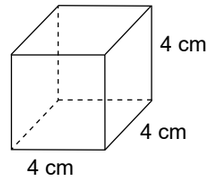




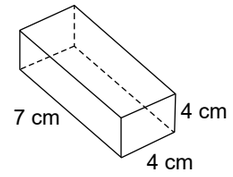

**EXAMPLE 2**

Calculate the surface area of the geometrical shapes below.

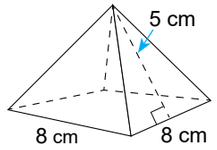
(a)



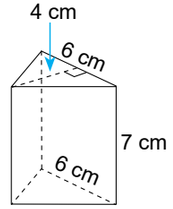

(b)




(c)




(d)




*Solution:*

(a) Surface area of a cube
$= 6 \times$ area of square
$= 6 \times (4 \text{ cm} \times 4 \text{ cm})$
$= 6 \times 16 \text{ cm}^2$
$= 96 \text{ cm}^2$

(b) Surface area of a cuboid
$= (4 \times \text{area of rectangle}) + (2 \times \text{area of square})$
$= (4 \times 4 \text{ cm} \times 7 \text{ cm}) + (2 \times 4 \text{ cm} \times 4 \text{ cm})$
$= (4 \times 28 \text{ cm}^2) + (2 \times 16 \text{ cm}^2)$
$= 144 \text{ cm}^2$

(c) Surface area of a pyramid
$= (4 \times \text{area of triangle}) + (\text{area of square})$
$= 4\left(\frac{1}{2} \times 8 \text{ cm} \times 5 \text{ cm}\right) + (8 \text{ cm} \times 8 \text{ cm})$
$= 80 \text{ cm}^2 + 64 \text{ cm}^2$
$= 144 \text{ cm}^2$

**DO YOU KNOW?**

The Autocad software can be used to calculate the surface area of a geometrical shape.

**TIPS**

**A two-dimensional shape** has two measurements, length and width which will give the surface area. Two-dimensional shapes do not have volume.

**A three-dimensional shape** has the measurements length, width and height. Three-dimensional shapes have volume.

**DO YOU KNOW?**

There are two types of solid, polyhedron and non-polyhedron. A solid polyhedron has flat surface with every side being a polygon. Non-polyhedron is a solid object with a curved surface like sphere, cylinder and cone.

(d) Surface area of a prism
= (3 × base area of rectangle) + (2 × area of triangle)

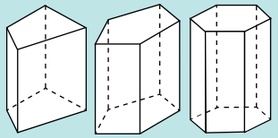
$= \left[(1 \times 6\text{ cm} \times 7\text{ cm}) + (2 \times 5\text{ cm} \times 7\text{ cm})\right] + 2\left(\frac{1}{2} \times 4\text{ cm} \times 6\text{ cm}\right)$

$= 42\text{ cm}^2 + 70\text{ cm}^2 + 24\text{ cm}^2$

$= 136\text{ cm}^2$

**THINK SMART**

How do you measure the surface area of the prisms below?



**EXAMPLE 3**

Calculate the surface area of a cylinder. The radius of the circle is 7 cm. Use $(\pi = \frac{22}{7})$

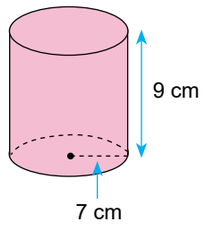
*Solution:*

Surface area of a cylinder $= 2\pi r^2 + 2\pi rh$

$= \left(2 \times \frac{22}{7} \times 7^2\right) + \left(2 \times \frac{22}{7} \times 7 \times 9\right)$

$= 308 + 396$

$= 704\text{ cm}^2$




**EXAMPLE 4**

The diagram below shows a cone. The circle radius is 3 cm. Calculate the surface area of the cone.
Use $(\pi = \frac{22}{7})$

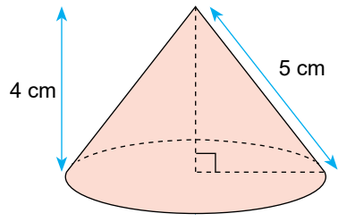
*Solution:*

Surface area of a cone $= \pi r^2 + \pi rs$

$= \left(\frac{22}{7} \times 3^2\right) + \left(\frac{22}{7} \times 3 \times 5\right)$

$= 28.29 + 47.14$

$= 75.43\text{ cm}^2$



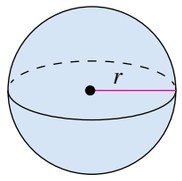

## 6.3.2 Surface area of a sphere

**LEARNING STANDARD**

Determine the surface area of spheres using formula.

The surface area of a sphere with radius $r$ cm can be determined by using the formula:

Surface area of a sphere $= 4\pi r^2$




**THINK SMART**

Many spherical shapes exist in our environment, for example, bubbles and water droplets. Can you think of another example?





**EXAMPLE 5**

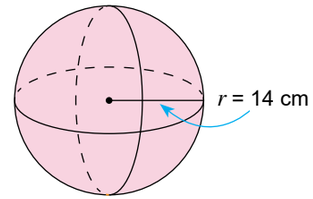
The diagram below shows a sphere with the radius, $r = 14$ cm.
Calculate the surface area of the sphere. (Use $\pi = \frac{22}{7}$)

*Solution:*

Surface area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 14^2$

$= 2\,464\text{ cm}^2$




**THINK SMART**

Sphere: $v = \frac{\pi d^3}{6}$ — Cube: $v = a^3$
Cylinder: $v = \frac{\pi d^2 h}{4}$ — Rectangle prism: $v = abh$

Can the formulae above be used to calculate volume?

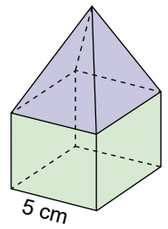
## 6.3.3 Solving problems

**LEARNING STANDARD**

Solve problems involving the surface area of three-dimensional shapes.

**EXAMPLE 6**

The diagram shows an object made up of a pyramid and a cube. The height of the object is 11 cm. Calculate the surface area of the object. State your answer in $\text{m}^2$.




*Solution:*

**Understanding the problem**

Calculating the surface area of a combined three-dimensional shape.

**Planning the strategy**

(i) Identifying the shapes.

(ii) Identifying the surface area formula for each shape.

**Conclusion**

$1\text{ m}^2 = 10\,000\text{ cm}^2$

$\therefore \frac{190\text{ cm}^2}{10\,000\text{ cm}^2} \times 1\text{ m}^2 = 0.019\text{ m}^2$

Combined surface area is $0.019\text{ m}^2$.

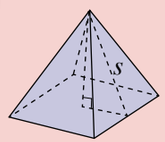
**Implementing the strategy**

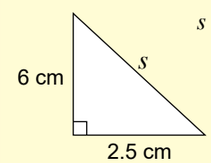
The shapes are pyramid and cube.
Surface area
= 5 × (surface area of cube) + 4 × (surface area of triangle)

$= 5(5 \times 5) + 4\left(\frac{1}{2} \times 5 \times 6.5\right)$

$= 125 + 65$

$= 190\text{ cm}^2$




$s$ = length of slant of the pyramid
$= \sqrt{6^2 + 2.5^2}$
$= 6.5$ cm

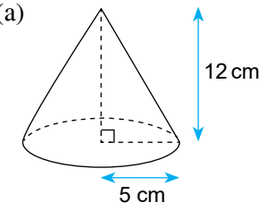
**TIPS**

1 m = 100 cm
$1\text{ m}^2 = 1 \times (100 \times 100)\text{ cm}^2$
$= 10\,000\text{ cm}^2$

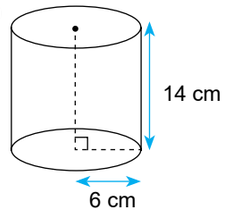
## SELF PRACTICE 6.3

1. Calculate the surface area of the three-dimensional objects below.

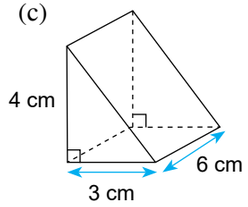
(a)




(b)

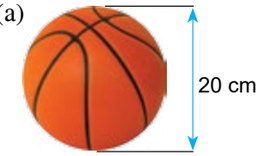



(c)

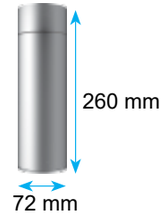



2. Calculate the surface area of the following objects.

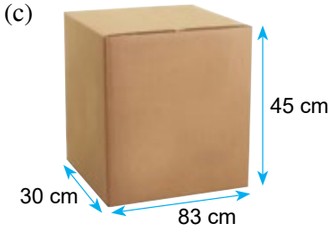
(a)




(b)

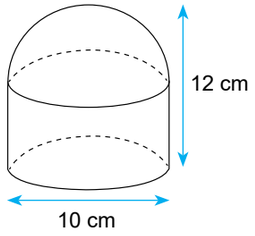



(c)

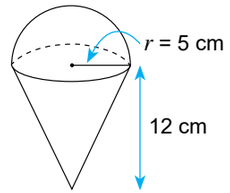



3. Calculate the combined surface area of the following three-dimensional objects.

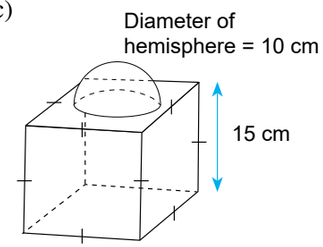
(a)




(b)




(c)




## 6.4 Volume of Three-Dimensional Shapes

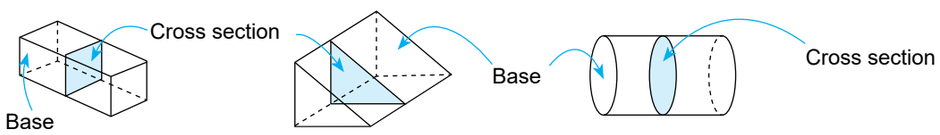
### 6.4.1 Deriving the formulae

#### Volume of prisms and cylinders

**LEARNING STANDARD**

Derive the formulae of the volumes of prisms and cylinders, and hence derive the formulae of pyramids and cones.

The volume of a three-dimensional shape is the measures of the amount of space it occupies. The shape is measured in cubic unit such as cubic millimetre ($mm^3$), cubic centimetre ($cm^3$) or cubic metre ($m^3$). Analyse the three-dimensional shapes below. What is the relationship between the cross-section and the base?








#### Volume of prism

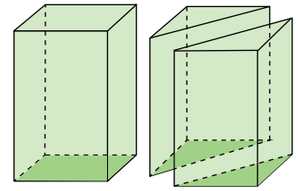
**DO YOU KNOW?**

Cuboid is a type of prism.

Analyse the cuboid below.

Volume of a cuboid = length × width × height
= area of base × height

The cuboid is divided into two equal parts. Two triangular prisms are formed. The relationship between the volume of cuboid and the volume of prism is



$$\text{Volume of a prism} = \frac{1}{2} \times \text{cuboid volume}$$

$$= \frac{1}{2} \times \text{area of base} \times \text{height}$$

area of triangle

$$= \boxed{\frac{1}{2} \times \text{length} \times \text{width}} \times \text{height}$$

Therefore, Volume of triangular prism = area of cross section × height

#### Volume of cylinder

The diagram above shows a coin in the shape of circle. If 10 coins are arranged upright it will produce a cylinder.

Therefore, volume of cylinder = area of base × height

$$= \pi r^2 \times h$$

Volume of a cylinder = $\pi r^2 h$

#### Volume of pyramid

Analyse a cube that has length ($l$), width (w) and height ($h$). Six pyramids of equal size can be fitted into the cube with the same base area as the pyramid, just like the base area of a cube and the height of the pyramid is half of the height of cuboid.

Area of base of the pyramid $= l \times w$

Height of pyramid $= \frac{h}{2}$

Height of cube, $h$ = 2 × height of pyramid

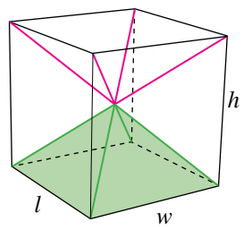
Volume of pyramid $= \frac{\text{Volume of pyramid}}{6}$

$$= \frac{l \times w \times h}{6}$$

$$= \frac{l \times w \times (2 \times \text{height of pyramid})}{6}$$

$$= \frac{l \times w \times \text{height of pyramid}}{3}$$

$$= \frac{\text{area of base of pyramid} \times \text{height of pyramid}}{3}$$




**THINK SMART**

Can the same activity be carried out using the rectangular-based pyramid and cuboid?

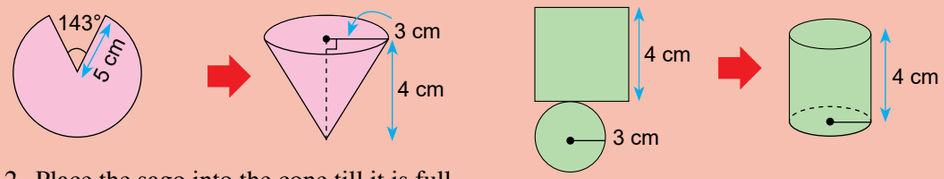
Therefore,

Volume of pyramid,
$= \frac{1}{3} \times$ base area × height

## Volume of cone

**COGNITIVE STIMULATION** Group

**Aim:** Producing the formula for the volume of cone
**Materials:** Manila card, scissors, glue and sago
**Steps:**
1. Using the net below, make a cone and cyclinder. Build an open cone and open cyclinder with the height upright and the base area according to the diagram below.




2. Place the sago into the cone till it is full.
3. Pour the sago from the cone into the cylinder.
4. Repeat steps 2 and 3 until the cylinder is full. How many cones of sago are needed to fill the cyclinder?

**Discussion:**
(i) Compare your results with your friends.
(ii) The relationship between the volume of cone and cylinder.

From the activity above, you would need 3 cones of sago to fill the cylinder.
Therefore, 3 × volume of cone = 1 × volume of cyclinder

$$\text{Volume of cone} = \frac{1}{3} \times \text{volume of cyclinder}$$

Therefore, $\text{Volume of cone} = \frac{1}{3}\pi r^2 h$

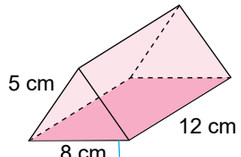
## 6.4.2 Calculation of volume

**LEARNING STANDARD**

Determine the volume of prisms, cylinders, cones, pyramids and spheres using formulae.

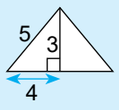
**EXAMPLE 7**

Calculate the volume of the prism shown.




*Solution:*

Volume of prism = Cross section area × Height
= Area of triangle × Height
$= (\frac{1}{2} \times 8 \times 3) \times 12$ cm
$= 144$ cm$^3$




Using Pythagoras theorem:
Height of triangle $= \sqrt{5^2 - 4^2}$
$= 3$ cm

**FLASHBACK**

SI unit for:
(i) Area is cm$^2$ (square centimetre)
(ii) Volume is cm$^3$ (cubic centimetre)



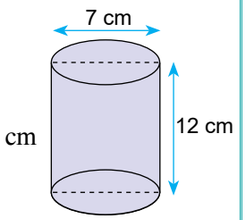


**EXAMPLE 8**

Calculate the volume of the cylinder. (Use $\pi = \frac{22}{7}$)

*Solution:*

Volume of cylinder = Cross section area × Height
$= \pi r^2 h$
$= (\frac{22}{7} \times 3.5 \text{ cm} \times 3.5 \text{ cm}) \times 12$ cm
$= 462$ cm$^3$

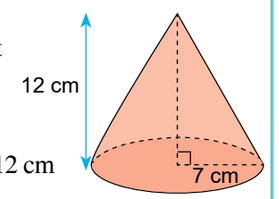



**EXAMPLE 9**

Calculate the volume of the cone on the right. (Use $\pi = \frac{22}{7}$)

*Solution:*

Volume of cone = $\frac{1}{3}$ × Area of base × Height
$= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times (\frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm}) \times 12$ cm
$= 616$ cm$^3$

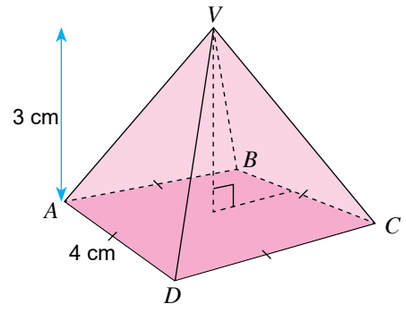



**EXAMPLE 10**

Calculate the volume of the pyramid.

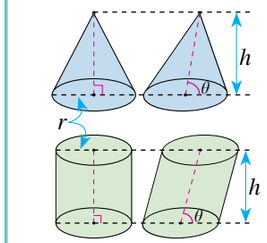
*Solution:*

Volume of pyramid = $\frac{1}{3}$ × Area of base × Height
$= \frac{1}{3} \times (4 \text{ cm} \times 4 \text{ cm}) \times 3$ cm
$= 16$ cm$^3$




**DO YOU KNOW?**

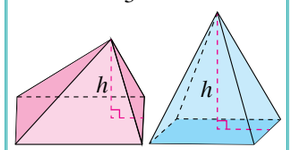
Volume of three-dimensional oblique-shaped objects.




$h$ = height of cone
$B$ = area of base

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}\pi r^2 h$

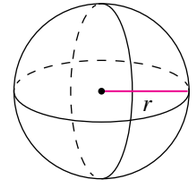



Volume = $\frac{1}{3}Bh$

## Volume of sphere

Sphere is a three-dimensional geometrical shape that has one point known as centre of the sphere. All the points are equidistant from the centre. Volume of the sphere with radius, $r$ is

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$





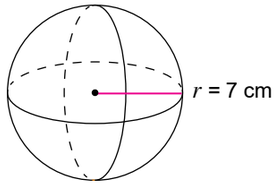


CHAPTER 6

**EXAMPLE 11**

Calculate the volume of the sphere with the radius 7 cm. (Use $\pi = \frac{22}{7}$)

*Solution:*




Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7\text{ cm} \times 7\text{ cm} \times 7\text{ cm}$

$= 1437.33\text{ cm}^3$

**EXAMPLE 12**

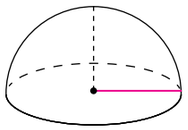
Calculate the volume of hemisphere on the right. (Use: $\pi = \frac{22}{7}$)

*Solution:*

Volume of hemisphere $= \frac{1}{2} \times$ Sphere volume

$= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

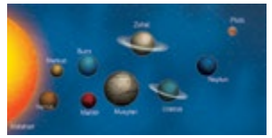



$= \frac{2}{3} \times \frac{22}{7} \times 5\text{ cm} \times 5\text{ cm} \times 5\text{ cm}$

$= 261.90\text{ cm}^3$

**DO YOU KNOW?**

The solar system consists of the sun and other planets that are spherical. This includes the planet Earth. Take note of the Earth's position in the solar system.



Radius of each planet,
Mercury = 2 423 km
Venus = 6 059 km
Earth = 6 378 km
Pluto = 1 180 km
Mars = 3 394 km

**THINK SMART**

A metal ball used in a competition has a radius of 4.9 cm. The density of the metal that is used to make ball is 7.8 g/cm³. Calculate the mass of the metal ball.

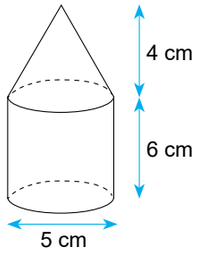
## 6.4.3 Solving problems

**LEARNING STANDARD**

Solve problems involving the volume of three-dimensional shapes.

**EXAMPLE 13**

Salim is an ice cream entrepreneur. He sells his ice creams in a container as shown in the diagram below. If he aims to sell 10 000 containers a month, how many litres of ice cream does he need in a month? Round off the answer to the nearest liters . (Use $\pi = \frac{22}{7}$)




**DO YOU KNOW?**

The Malaysian Health Ministry has organised a healthy eating campaign among Malaysians to consume the right amount of calorie according to the age and the daily needs of an individual. The calorie intake needed by a male aged 13-15 is 2 200 calories a day. Whereas, a female aged 13-15 needs 1 800 calories of food a day.





*Solution:*

**Understanding the problem**

To calculate the volume of ice cream needed to produce 10 000 containers of ice cream to the nearest litre.

**Planning the strategy**

(i) To determine the volume of the container

(ii) To determine the volume of 10 000 containers

**Conclusion**

1 litre = 1 000 cm³

$1\,440\,500\text{ cm}^3 = \frac{1\,440\,500\text{ cm}^3}{1\,000\text{ cm}^3} \times 1\text{ litre}$

$= 1\,440.5\text{ litre}$

Then, 1 440.5 litres of ice cream is needed.

**Implementing the strategy**

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 2.5 \times 2.5 \times 6$

$= 117.86\text{ cm}^3$

Volume of cone $= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 4$

$= 26.19\text{ cm}^3$

Therefore, volume of container = 117.86 + 26.19

$= 144.05\text{ cm}^3$

Total volume of 10 000 containers

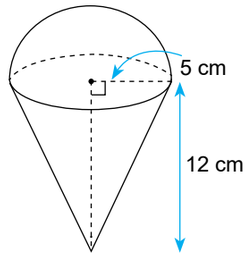
$= 10\,000 \times 144.05$

$= 1\,440\,500\text{ cm}^3$

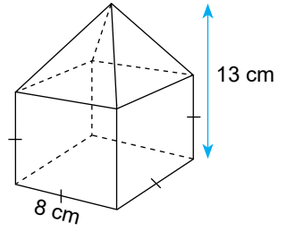
**SELF PRACTICE 6.4**

1. Calculate the volume of the following.

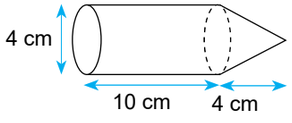
(a)




(b)

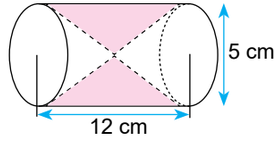



(c)

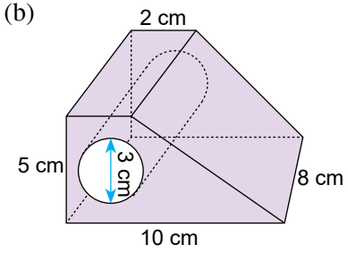



2. Calculate the volume of the shaded region.

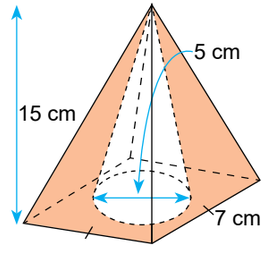
(a)



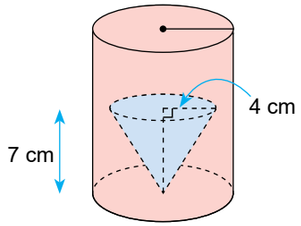

(b)




(c)



3. Ali poured water into a cylindrical container that has a radius of 7 cm and height of 15 cm until it is full. A solid shaped cone is inserted fully into the cylinder as shown in the diagram below. After a while, the solid cone is taken out from the cylinder. Calculate the volume of water that is left in the cylinder.

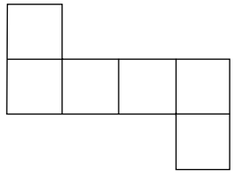



4. A block of metal pyramid with a square base, with side 15 cm and height 10 cm is melted down to form a few balls of spheres with a radius of 5 mm. How many pyramid blocks are needed to form 2 850 balls of spheres?

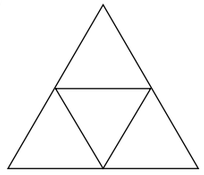
## GENERATING EXCELLENCE

1. State the three-dimensional shape of the nets.

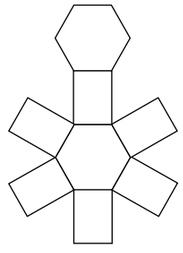
(a)



(b)



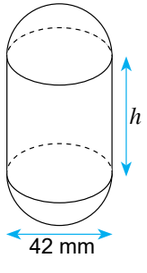
(c)



2. A cylindrical water bottle with a height of 20 cm and diameter of 5.5 cm is filled with water until it is full. Vincent wants to transfer the water in the bottle into a cubic container. State the minimum length of a side of the cube.

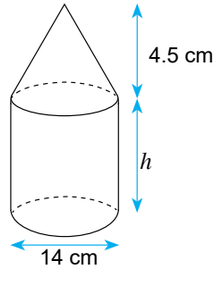
3. Given the volume of the block, calculate the value of $h$.

(a)



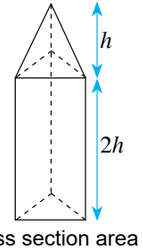

Volume = 122 000 mm$^3$

(b)




Volume = 1 540 cm$^3$

(c)



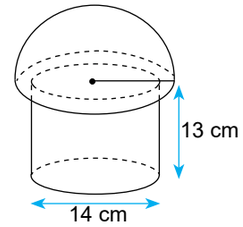

cross section area of prism = 325 cm$^2$

Volume = 6 825 cm$^3$



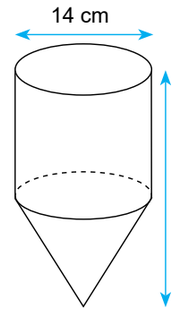


4. Study the diagram. The diameter of the hemisphere is 22 cm. Calculate
   (a) the volume of the combined shapes.
   (b) the total number of marbles with a volume of 343 mm$^3$ which can be filled into the container.




5. An artist wants to do a full painting on the surface of a pottery. The pottery in the shape of a cylinder has the height of 10 cm and a radius of 3.5 cm. If one tube of colour can paint 100 cm$^2$ of drawing, how many tubes are needed to paint 10 potteries of the same type?

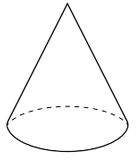
6. The diagram shows a solid made by combining a cylinder and a cone. $\frac{1}{2}$ kg of sugar can produce 1 litre of syrup to make candies shaped like the solid. If the height of the cylinder is twice the radius of the cylinder, how many candies can be produced using 100 kg of sugar?



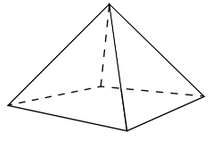

7. A cylinder open at the top with a height twice the radius of the base, is filled with water three quarter full. 539 m$l$ water is needed to fill up the cylinder. Calculate the surface area of the cylinder, in cm$^2$. (Use $\pi = \frac{22}{7}$)

8. The diagram shows a block of cone and pyramid. If the volume of the pyramid is three times the volume of the cone, and the surface area of the pyramid is twice the surface of the cone, calculate the height of the cone and the pyramid, if the height of the cone is 18 cm.

(Use $\pi = \frac{22}{7}$)

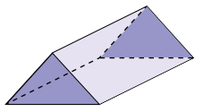
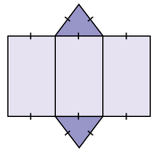
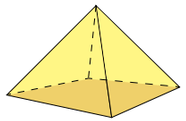
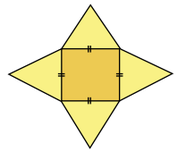
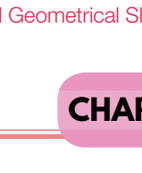
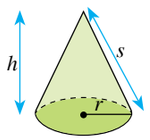
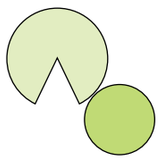
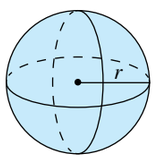


cone



pyramid

## CHAPTER SUMMARY

| Geometrical shape | Net | Surface area | Volume |
|---|---|---|---|
| Prism  |  | (2 × area of triangle) + (3 × area of rectangle) | Area of cross section × height |
| Pyramid  |  | Area of base + (4 × area of triangle) = (length × width) + $4(\frac{1}{2} \times \text{base} \times \text{height})$ | $\frac{1}{3}$ × area of base × height |
| Cylinder   |  | $2\pi r^2 + 2\pi rh$ | $\pi r^2 h$ |
| Cone   |  | $\pi r^2 + \pi rs$ | $\frac{1}{3}\pi r^2 h$ |
| Sphere   | | $4\pi r^2$ | $\frac{4}{3}\pi r^3$ |



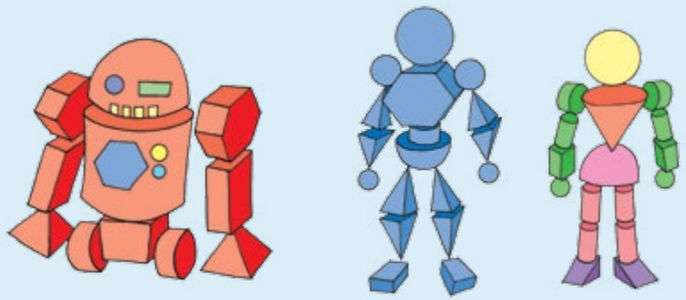


## SELF REFLECTION

| At the end of the chapter, I am able to: | |
|---|---|
| 1. Compare, contrast and classify three-dimensional shapes including prisms, pyramids, cylinders, cones and spheres, and hence describe the geometric properties of prisms, pyramids, cylinders, cones and spheres. | |
| 2. Analyse various nets including pyramids, prisms, cylinders and cones, and hence draw nets and build models. | |
| 3. Derive the formulae of the surface area of cubes, cuboids, pyramids, prisms, cylinders and cones, and hence determine the surface areas of the shapes. | |
| 4. Determine the surface area of spheres using formula. | |
| 5. Solve problems involving the surface area of three-dimensional shapes. | |
| 6. Derive the formulae of the volumes of a prisms and cylinders, and hence derive the formulae of pyramids and cones. | |
| 7. Determine the volume of prisms, cylinders, cones, pyramids and spheres using formulae. | |
| 8. Solve problems involving the volume of three-dimensional shapes. | |

## MINI PROJECT

Design a robot with the combination of shapes such as cube, cuboid, prism, pyramid, cylinder, cone and sphere. Students should create the shapes by themselves. You may combine the three-dimensional shapes.



Example of robots

# CHAPTER 7

# Coordinates

## WHAT WILL YOU LEARN?

**7.1** Distance in the Cartesian Coordinate System

**7.2** Midpoint in the Cartesian Coordinate System

**7.3** The Cartesian Coordinate System

## WORD LINK

| | |
|---|---|
| • Midpoint | • *Titik tengah* |
| • Distance | • *Jarak* |
| • Position | • *Kedudukan* |
| • Coordinate | • *Koordinat* |
| • *x*-axis | • *Paksi-x* |
| • *y*-axis | • *Paksi-y* |
| • Hypotenuse | • *Hipotenus* |
| • Origin | • *Asalan* |
| • Plots | • *Plot* |
| • Cartesian Plane | • *Satah Cartes* |
| • Scale | • *Skala* |

Cartesian Coordinate System is a method to determine the position of a point or object on a plane, or into two or three dimensions.

The position on a plane is determined by the position of the point on a straight line or number. The position of a point in two dimensions is determined by the coordinate system on a Cartesian plane. The position in three dimensions is determined by three numbers.


## WALKING THROUGH TIME

The Cartesian Coordinate System was introduced by René Descartes from France or better known as Cartesius. He introduced a coordinate plane which is formed by two perpendicular lines called 'axis'. Coordinates are a set of numbers that locate a point or a line.

For more information:

http://rimbunanilmu.my/mat_t2e/ms121

## WHY STUDY THIS CHAPTER?

- The coordinate system has contributed a lot in the field of archaeology and geography.
- Archaeologists begin their search according to coordinate points on a map digitally.
- Astronomers can determine the position of the stars through this coordinate system.
- A location is determined by a combination of coordinate points which help geographers to identify the area and position on Earth.

**CREATIVE ACTIVITY**

**Aim:** Identifying the position of a point
**Material:** Worksheet
**Steps:**

1. Open the file MS122A and print out the worksheet.
2. By joining the vertical and horizontal distances, determine the position of the following towns: Batu Pahat, Kluang and Segamat.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms122a to get the worksheet.

You have learnt about coordinates of a location on a certain Cartesian plane. A coordinate is a pair of numbers that is used to determine the position of a point on the Cartesian plane. The coordinate of a point is determined based on the distance from $x$-axis, the distance from $y$-axis and the origin. Were you able to determine the distance between two points from the activity above?

## 7.1 Distance in a Cartesian Coordinate System

### 7.1.1 Distance between two points on the Cartesian plane

**COGNITIVE STIMULATION** Group

**LEARNING STANDARD**

Explain the meaning of distance between two points on the Cartesian plane.

**Aim:** Identifying the distance between two points on a Cartesian plane
**Material:** Worksheet
**Step:**

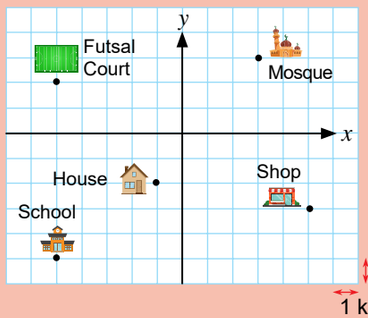



Diagram shows the plan of locations often passed by Azri.

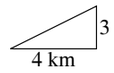
**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms122b to get the worksheet.

1. Open the file MS122B and print out the worksheet.
2. In pairs, identify Azri's movements to the destinations as in the table.
3. Azri's movement must be drawn in the form of a right angled triangle.
4. Measure the horizontal and vertical distances based on 1 grid box equal to 1 km and fill in the table as show in the example.
5. Add the total distance by completing the table.





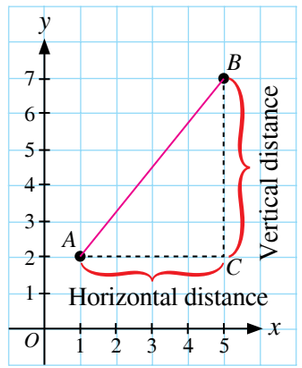
| Azri's Destination | Triangular representation | Horizontal Distance | Vertical Distance | Total distance travelled = Horizontal distance + Vertical distance |
|---|---|---|---|---|
| School to house |   | 4 km | 3 km | 4 km + 3 km = 7 km |
| House to futsal field | | | | |
| Mosque to shop | | | | |
| School to mosque | | | | |
| School to shop | | | | |

**Discussion:**

(i) From the representation of the right angled triangle, can you identify the nearest distance taken by Azri to a certain destination?
(ii) What is the easiest way to calculate the shortest distance?
(iii) What do you understand about distance on a Cartesian plane?

To determine distance between two points on a Cartesian plane, the right angled triangle representation method is used.
In this method you have to identify the horizontal distance and the vertical distance of two points on a Cartesian plane. This distance can be determined from the scale on the $x$-axis and the $y$-axis.

$AB$ is the shortest distance, taken without going through $C$

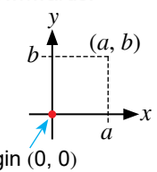



The Pythagoras theorem is used to calculate the distance $AB$, that is

$$AB^2 = AC^2 + CB^2$$
$$AB = \sqrt{AC^2 + CB^2}$$

**DO YOU KNOW?**

The Cartesian plane has two axes as in the diagram. The horizontal line is the $x$-axis and the vertical line is $y$-axis. Both lines will intersect perpendicularly. The intersection point is the origin which is the starting point for both $x$-axis and $y$-axis. The value of the numbers will increase when it moves to the right and upwards. However, the value of a number will decrease when it moves to the left and downwards.



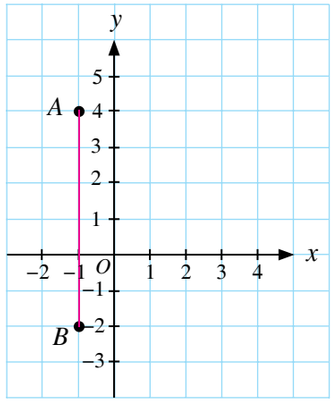

**TIPS**

In coordinates $(x, y)$, the value of $x$ is written first followed by the value of $y$.

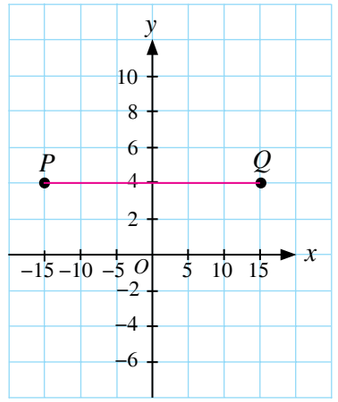
**EXAMPLE 1**

Determine the distance between two points in the following Cartesian plane.

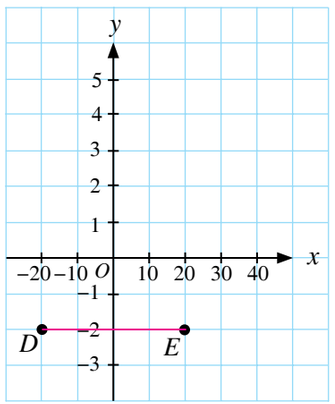
(a)



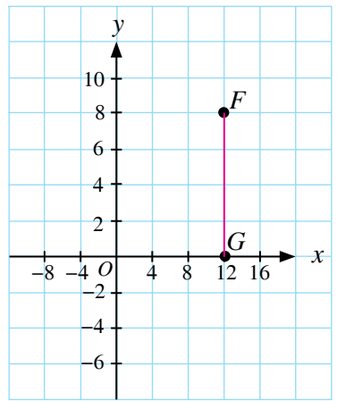

(b)




(c)

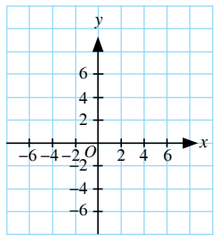



(d)




**DO YOU KNOW?**

What is a scale? Scales need to be determined in the Cartesian coordinate system. The units that can be written on the *x*-axis are 1, 2, 3, ... and on the text of the origin are −1, −2, −3, ... . The units that can be written on the *y*-axis are 1, 2, 3, … and the values below the origin are −1, −2, −3, ... . This is how each box is represented as one unit. Apart from that, scales can be written in the sequence of 2, 4, 6, 8, … or 5, 10, 15, … on both axes. These conditions depend on the suitability in certain situations.




Scale on *x*-axis is 2 units
Scale on *y*-axis is 2 units

*Solution:*

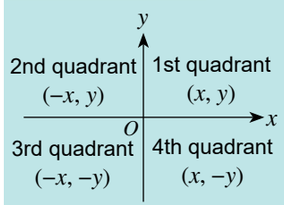
(a) The scale on *x*-axis and *y*-axis is 1 unit
Distance of *AB*
= 6 × 1
= 6 units

(b) The scale on *x*-axis is 5 units and *y*-axis is 2 units.
Distance of *PQ*
= 6 × 5
= 30 units

(c) The scale on *x*-axis is 10 units and *y*-axis is 1 unit.
Distance of *DE*
= 4 × 10
= 40 units

(d) The scale on *x*-axis is 4 units and *y*-axis is 2 units.
Distance of *FG*
= 4 × 2
= 8 units

**THINK SMART**




If (*x*, *y*) is (3, 4) in the 1st quadrant, state the coordinates of the point in the 2nd quardrant 3rd quardrant and 4th quardrant. What type of transformation is experienced by the point?





## 7.1.2 The formula if the distance between two points on the plane

**LEARNING STANDARD**

Derive the formula of the distance between two points on the Cartesian plane.

**COGNITIVE STIMULATION** Group

**Aim:** Determining the distance between two points with the same *x*-coordinate and *y*-coordinate.
**Material:** Printed Worksheet
**Steps:**

1. With a friend, identify the coordinates on the *x*-axis and the *y*-axis.
2. Complete the table by determining the coordinates with common axis.

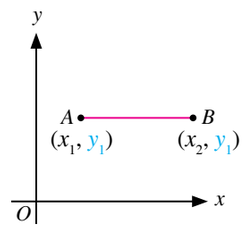
**Example:**

| Coordinate | | Same coordinate | Distance |
|---|---|---|---|
| *A* (2 , 1) | *B* (2 , 4) | *x*-coordinate | 4 – 1 = 3 unit |
| *C* (–1, 3) | *D* (7 , 3) | | |
| *E* (6 , 5) | *F* (6 , –5) | | |
| *G* (–7, 2) | *H* (1 , 2) | | |

**Discussion:**

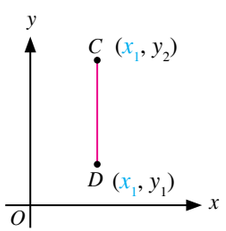
How can you create a simple formula for determining the distance between two points that has
(i) the same *x*-coordinate?
(ii) the same *y*-coordinate?

Distance can be determined if,
(i) Two points have the same *y*-coordinate




Distance for $AB = (x_2 - x_1)$ unit

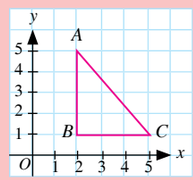
(ii) Two points have the same *x*-coordinate.




Distance for $CD = (y_2 - y_1)$ unit

**FLASHBACK**

Look at the triangle on the Cartesian plane below.




The base of the triangle *BC* is parallel to the *x*- axis. This makes the *y*- coordinates the same. This is called common *y*- axis. It is the same the other way around.

**EXAMPLE 2**

Calculate the distance between the points.
(a) (2, –3) and (4, –3)
(b) (0, 1) and (0, –2)

***Solution:***

(a) The distance between the two points is
$= 4 - 2$
$= 2$ units ← Horizontal distance = $x_2 - x_1$

(b) The distance between the two points is
$= 1 - (-2)$
$= 3$ units ← Vertical distance = $y_2 - y_1$

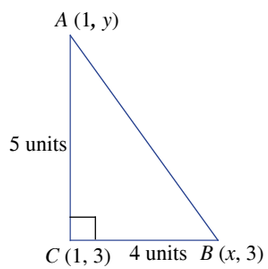
**EXAMPLE 3**

The diagram shows the distance between two points $A$ and $B$. Complete the coordinates of $A$ and $B$.

***Solution:***

$y - 3 = 5$ units
$y = 5 + 3$
$= 8$ units
Therefore, coordinate $A$ is (1,8).

$x - 1 = 4$ units
$x = 4 + 1$
$= 5$ units
Therefore, coordinate $B$ is (5, 3).




QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms126a for the Submarine Target game.

## 7.1.3 Distance between two points on a plane

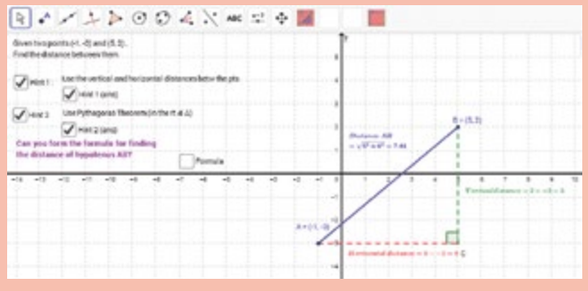
If the straight line that joins two points on a Cartesian plane is not parallel to the $x$-axis or $y$-axis, then the distance between the two can be determined using the Pythagoras theorem.

LEARNING STANDARD

Determine the distance between two points on a Cartesian plane.

**COGNITIVE STIMULATION** Group

**Aim:** Identifying the distance between two points
**Material:** Dynamic geometry software
**Steps:**
1. Open the file MS126B.



QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms126b to identify the distance between two points.





2. Move the coordinates $A$ and $B$ on the Cartesian plane based on the table below.
3. Identify the horizontal distance and the vertical distance for the line $AB$.
4. Compare the displayed answers with the answers using the formula.
5. Complete the table below with the answers by choosing *Hint*.

| | Points | | Difference in Distance | | Distance $AB$ |
|---|---|---|---|---|---|
| | $A$ | $B$ | Horizontal $y_2 - y_1$ | Vertical $x_2 - x_1$ | $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ |
| (a) | (1, 5) | (1, 7) | 1 – 1 = 0 | 7 – 5 = 2 | |
| (b) | (4, 1) | (1, 1) | | | |
| (c) | (8, 2) | (0, –4) | | | |
| (d) | (6, 7) | (2, 4) | | | |

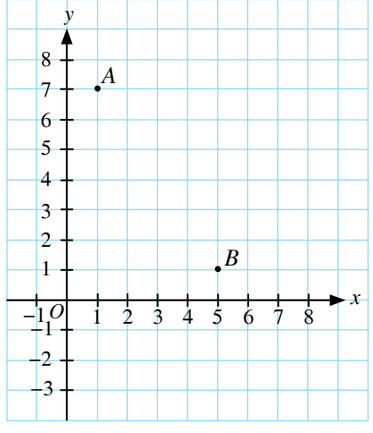
**Discussion:**
(i) What do you understand about the distance of $AB$?
(ii) What is the relevance of Pythagoras theorem?

The distance $AB$ is the hypotenuse. The Pythagoras theorem is used to determine the distance between two points on a Cartesian plane.

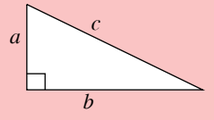
The distance between two points on a Cartesian plane $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**EXAMPLE 4**

Calculate the distance between point $A$ and point $B$ on the Cartesian plane in the diagram below.




**FLASHBACK**




$c = \sqrt{a^2 + b^2}$

What is this formula?

The theorem shows that for a right-angled triangle, the square of its hypotenuse is equal to the sum of the squares of the other two sides.

*Solution:*

**Method 1**

Based on the diagram, draw a right-angled triangle $ACB$.

$AC = 6$ units, $BC = 4$ units

Using Pythagoras theorem

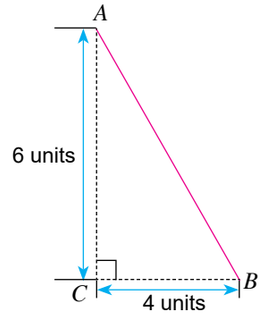
$AB^2 = BC^2 + AC^2$

$AB^2 = 4^2 + 6^2$

$AB^2 = 16 + 36$

$AB = \sqrt{52}$

$= 7.21$ units




**Method 2**

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

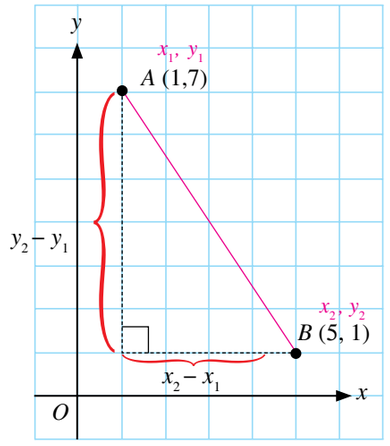
Distance $AB = \sqrt{(5-1)^2 + (1-7)^2}$

$= \sqrt{4^2 + (-6)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$

$= 7.21$ units

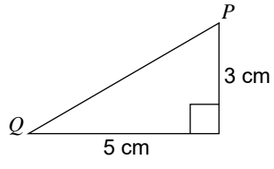



Therefore, the distance of $AB$ is 7.21 units.

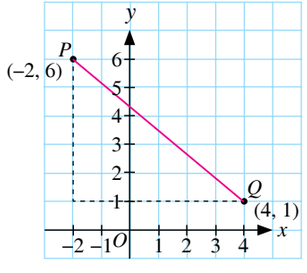
**EXAMPLE 5**

Calculate the distance between point $P$ and point $Q$.

(a)




(b)




*Solution:*

(a) $PQ^2 = 5^2 + 3^2$

$= 25 + 9$

$PQ = \sqrt{34}$

$= 5.83$ cm

Therefore, the distance of $PQ$ is 5.83 cm.

(b) $PQ^2 = \sqrt{[4-(-2)]^2 + (1-6)^2}$

$= \sqrt{6^2 + (-5)^2}$

$= \sqrt{36 + 25}$

$= \sqrt{61}$

$= 7.81$ cm

Therefore, the distance of $PQ$ is 7.81 cm.





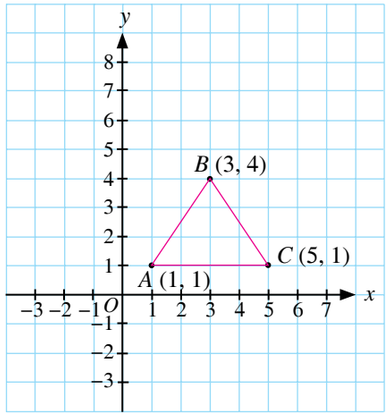
## 7.1.4 Solving problems

**LEARNING STANDARD**

Solve problems involving the distance between two points in the Cartesian coordinate system.

**EXAMPLE 6**

Calculate the perimeter of an isosceles triangle if the vertices for the triangle are $A$ (1, 1), $B$ (3, 4) and $C$ (5, 1).




*Solution:*

**Understanding the problem**

$ABC$ is an isosceles triangle with vertices $A$ (1, 1), $B$ (3, 4) and $C$ (5, 1).

**Planning the strategy**

- Draw and determine the points on a Cartesian plane.
- Perimeter $\Delta ABC = AB + BC + AC$
- The distance of $AC$ and $AB$.

**Implementing the strategy**

Distance $AB = \sqrt{3^2 + 2^2}$

$= \sqrt{9 + 4}$

$= \sqrt{13}$

$= 3.6$ units

$AB = BC$

**Conclusion**

Therefore, the perimeter of the triangle $ABC$ is $3.6 + 3.6 + 4 = 11.2$ units.

**TIPS**

The distance between two points

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance is the measurement of length between two points.

**EXAMPLE 7**

Given that the distance of $AB = 10$ units. Calculate the value of $v$.

*Solution:*

**Understanding the problem**

Calculate the value of $v$.

**Planning the strategy**

Distance $AB = 10$

Formula of distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Implementing the strategy**

$AB = \sqrt{(6-v)^2 + (9-3)^2}$

$10 = \sqrt{(6-v)^2 + 6^2}$

$10 = \sqrt{(6-v)^2 + 36}$

$10^2 = \left(\sqrt{(6-v)^2 + 36}\right)^2$

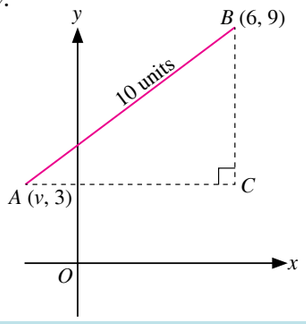
$10^2 - 36 = (6-v)^2$

$\sqrt{64} = 6 - v$

$8 = 6 - v$

$v = 6 - 8$

$v = -2$

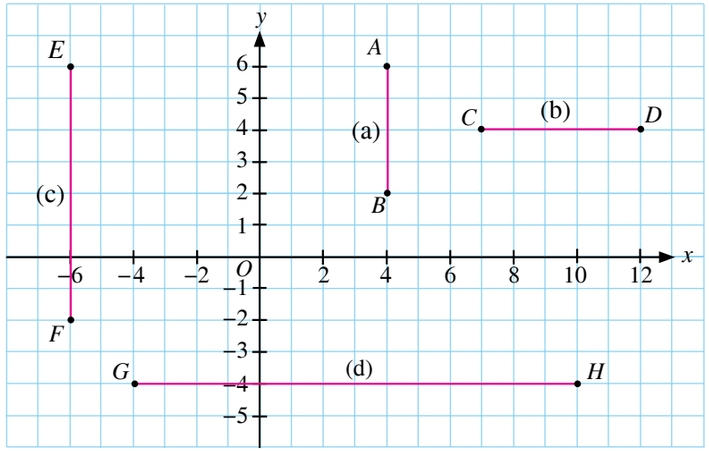



**Conclusion**

Therefore, the value of $v$ is –2.

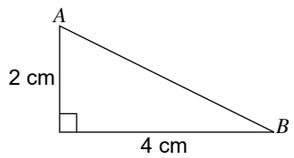
## SELF PRACTICE 7.1

1. Determine the distance between two points on the following Cartesian plane.

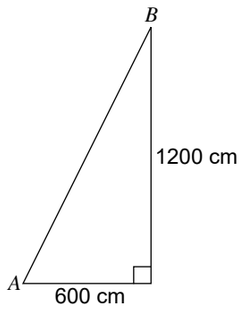



2. Calculate the distance of $AB$.

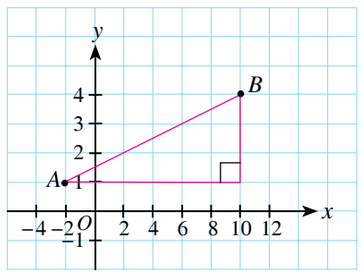
(a)



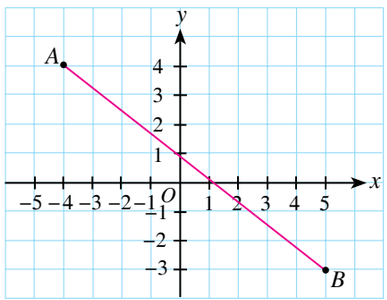

(b)




(c)




(d)



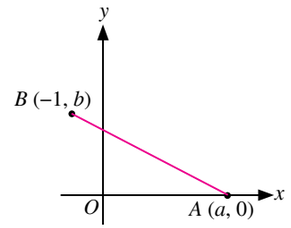

3. State the distance between each set of points below.
   (a) (1, 3) and (1, 7)
   (b) (0, −9) and (0, 9)
   (c) (5, −2) and (−2, −2)
   (d) (7, 4) and (8, 4)



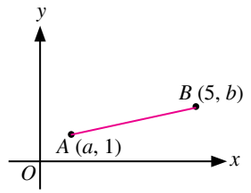


4. Given that the horizontal distance is 4 units and the vertical distance is 3 units for the points $A$ and $B$, calculate the values of $a$ and $b$.

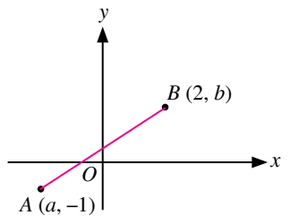
(a)



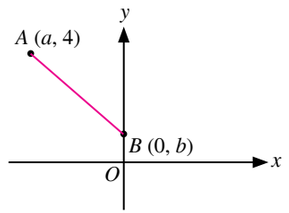

(b)



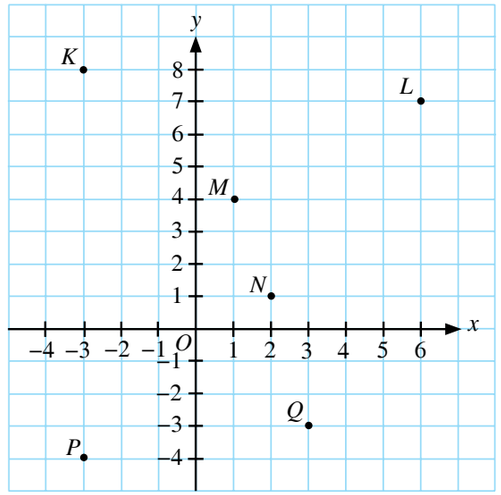

(c)




(d)




5. The diagram shows the points $K$, $L$, $M$, $N$, $P$ and $Q$ on the Cartesian plane.
   Calculate the distance between the points.
   (a) $KM$
   (b) $ML$
   (c) $PN$
   (d) $KQ$

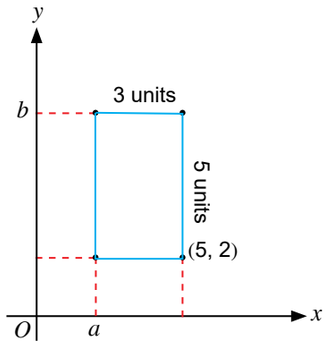



6. Determine the distance of the points $KL$ given $K$ (2, 2) and $L$ is on the $x$-axis with a distance of 7 units to the right from the $y$-axis.

7. Determine the distance of $AB$ if each of them is located on the $y$-axis with a distance of 5 units upwards and 2 units downwards from the $x$-axis.

8. Calculate the distance between the points $KL$ if $L$ is located on the origin and $K$ is 3 units to the left of $y$-axis and 5 units upwards from the $x$-axis.

9. Determine the values of $a$ and $b$ based on the information in the diagram below.

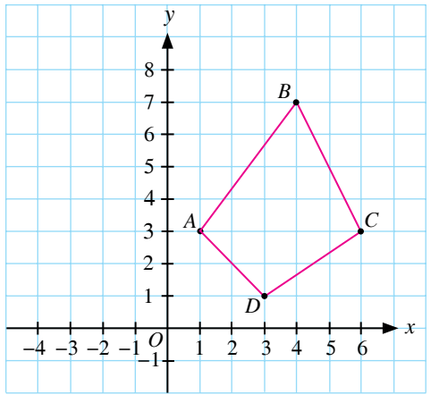



10. The vertical distance of point $V$ is 4 units to the north of point $W$. Determine the coordinates of $W$ if the coordinates of $V$ are

(a) (4, –3) (b) (2, –5)

(c) (5, –2) (d) (0, –4)

11. Based on the diagram, calculate the perimeter for $ABCD$.




12. The triangle $ABC$ has vertices $A$ (-2, –1), $B$ (–2, 5) and $C$ (1, –1). Calculate the perimeter for the triangle.

## 7.2 Midpoint in The Cartesian Coordinate System

### 7.2.1 Midpoint between two points

You have learned how to determine a radius for a certain diameter in a circle. Do you understand the concept of midpoint? Discuss this concept with your friends.

**LEARNING STANDARD**

Explain the meaning of midpoint between two points on the Cartesian plane.





**COGNITIVE STIMULATION** Group

**Aim:** Identifying the midpoint on a line

**Materials:** Grid paper, compasses and ruler

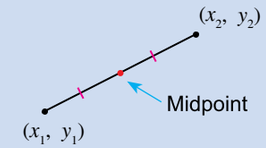
**Steps:**

1. Student $A$ constructs a Cartesian plane on grid paper.
2. Student $B$ chooses two coordinate points and draws a line that joins the points.
3. Student $C$ contructs a perpendicular bisector on the line.

**Discussion:**

What do you understand when you construct the perpendicular bisector on the line?

**TIPS**




**QR CODE**

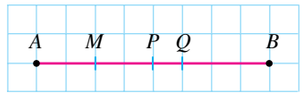
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms133 to watch an animated video on determining midpoints.

Midpoint is a point that divide a line segment equally.

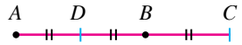
**EXAMPLE 8**

Determine the midpoint of the straight line $AB$.

(a)

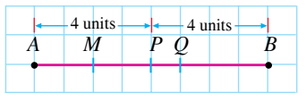



(b)




*Solution:*

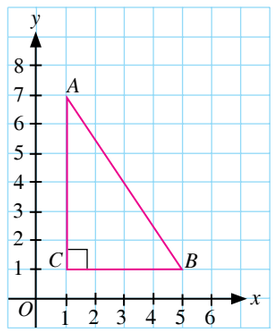
(a) Midpoint of the straight line $AB$ is $P$.




(b) Midpoint of the straight line $AB$ is $D$.

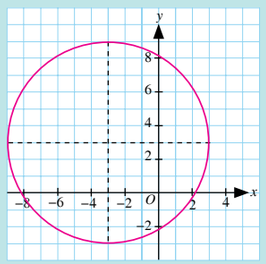

**EXAMPLE 9**

$P$ is the midpoint of the straight line $AB$. Determine coordinates $P$.




**THINK SMART**

State the coordinates of the centre of the circle for the diagram below. What is the connection between the centre of a circle and the midpoint?



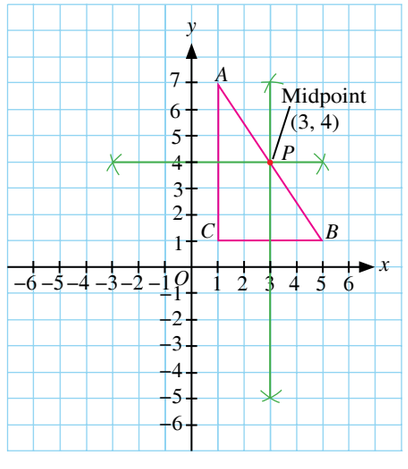
***Solution:***

Step 1: Determine the midpoint of $AC$ and $BC$.

Step 2: Construct a perpendicular bisector of $AC$ and $BC$.

Step 3: Intersection between the perpendicular bisector of $AC$ and $BC$ is the midpoint of the line $AB$.

Step 4: Therefore, point $P$ is (3, 4).

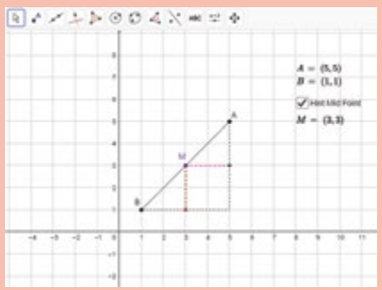



## 7.2.2 The midpoint formula

**COGNITIVE STIMULATION** Group

**Aim:** Deriving the midpoint formula
**Material:** Dynamic geometry software
**Steps:**

1. Open the file MS134.



2. Identify point $A$ and point $B$.
3. Change the location of the points as in the table provided.
4. Identify horizontal distance and vertical distance.
5. Open file MS135 and complete the table given.
6. Calculate the midpoint $M$.

**LEARNING STANDARD**

Derive the formula of the midpoint between two points on the Cartesian plane.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms134 to identify the midpoint.





| Point | | Midpoint of: | | Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ |
|---|---|---|---|---|
| $A$ | $B$ | Horizontal distance | Vertical distance | |
| (4, 5) | (2, 1) | | | |
| (–1, 5) | (3, 1) | | | |
| (1, 3) | (7, 1) | | | |
| (3, 4) | (–5, –1) | | | |
| (1, 2) | (–5, 2) | | | |

**Discussion:**

(i) Is the midpoint of the straight line $AB$ the result of the intersection for the midpoint of horizontal distance and vertical distance?

(ii) Derive the formula for midpoint.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms135 to get the worksheet.

Midpoint for a slanting line can be determined by identifying the horizontal distance and vertical distance which are both divided by two.

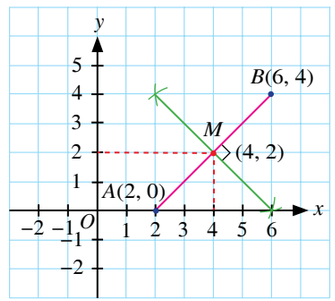
$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

## 7.2.3 Midpoint coordinates between two points

**LEARNING STANDARD**

Determine the coordinates of midpoint between two points on the Cartesian plane.

The location of a midpoint can be shown by constructing a perpendicular bisector. The intersection between the perpendicular bisector with the line segment will determine the coordinates of the midpoint on a Cartesian plane.



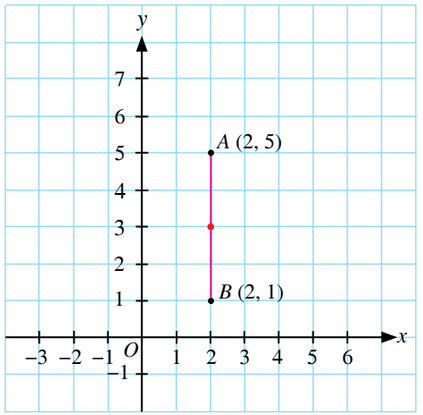

$$M = \left(\frac{6+2}{2}, \frac{4+0}{2}\right)$$

$$M = (4, 2)$$

**EXAMPLE 10**

Calculate the coordinate of the midpoint on the straight line $AB$ given $A$ (2,5) and $B$ (2,1).

***Solution:***




$A$ (2, 5) is $(x_1, y_1)$ and $B$ (2, 1) is $(x_2, y_2)$

$$\text{Midpoint } AB = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2+2}{2}, \frac{5+1}{2}\right)$$

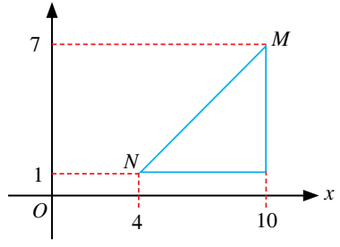
$$= \left(\frac{4}{2}, \frac{6}{2}\right)$$

$$= (2, 3)$$

Therefore, the midpoint of $AB$ is (2, 3).

**EXAMPLE 11**

Calculate the coordinate of the midpoint on the straight line $MN$.




***Solution:***

$M$ (10, 7) is $(x_1, y_1)$ and $N$ (4, 1) is $(x_2, y_2)$

$$\text{Midpoint } MN = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{10+4}{2}, \frac{7+1}{2}\right)$$

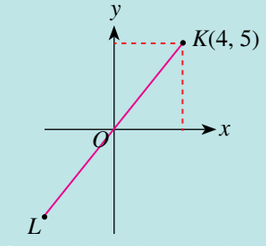
$$= \left(\frac{14}{2}, \frac{8}{2}\right)$$

$$= (7, 4)$$

Therefore, the midpoint of $MN$ is (7, 4).

**THINK SMART**

Midpoint is a point on a line segment that divides it into two equal parts.




If the origin is the midpoint for the line $KL$, can you determine the coordinates of $L$?





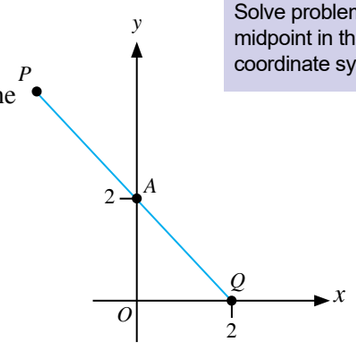
## 7.2.4 Solving problems

**LEARNING STANDARD**

Solve problems involving midpoint in the Cartesian coordinate system.

**EXAMPLE 12**

The diagram shows the line $PAQ$ on a Cartesian plane. $A$ is the midpoint for the line $PQ$. Determine the coordinates of $P$.



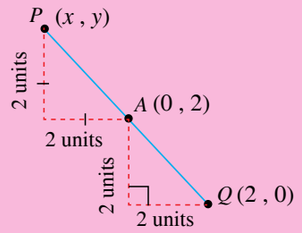

***Solution:***

**Understanding the problem**

Distance of $AP = AQ$.
Let $P = (x, y)$.

**Planning the strategy**

Distance of $AP = AQ$




**Implementing the strategy**

Calculate the horizontal and vertical distance starting from point $A$ which is 2 units respectively.

Midpoint, $A$ (0, 2)

$P(x, y)$ $\quad \frac{x+2}{2} = 0$ , $\frac{y+0}{2} = 2$

$Q(2, 0)$ $\quad x + 2 = 0$ , $y = 4$

$x = -2$

**Conclusion**

Therefore, coordinates of $P$ are (−2, 4).

**EXAMPLE 13**

Point $P$ is the midpoint on the straight line $KL$. Given coordinates of $K$ (−3, 12) and coordinates of $P$ (2, 9), calculate coordinates of $L$.

***Solution:***

$K$ (−3, 12) is $(x_1, y_1)$ and $L$ $(x_2, y_2)$

$$\text{Midpoint, } P = \left(\frac{-3 + x_2}{2}, \frac{12 + y_2}{2}\right)$$

$$(2, 9) = \left(\frac{-3 + x_2}{2}, \frac{12 + y_2}{2}\right)$$

$$\frac{-3 + x_2}{2} = 2 \text{ , } \frac{12 + y_2}{2} = 9$$

$$-3 + x_2 = 4 \text{ , } 12 + y_2 = 18$$

$$x_2 = 7 \text{ , } y_2 = 6$$

Therefore, coordinates of $L$ are (7, 6).

**THINK SMART**

The KLCC Tower has 88 floors. The distance which is the most suitable to build a skybridge is at the 42nd and the 43rd floors. Why?

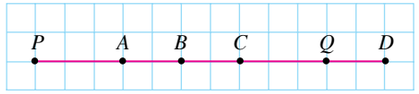



CHAPTER 7

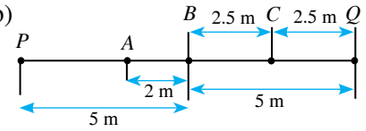
## SELF PRACTICE 7.2

1. In each of the diagrams below, determine the midpoint of the straight line $PQ$.

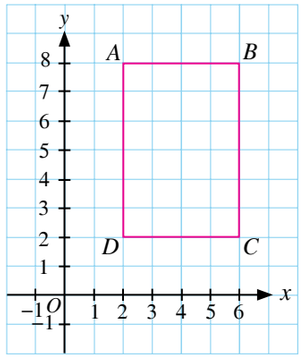
(a)



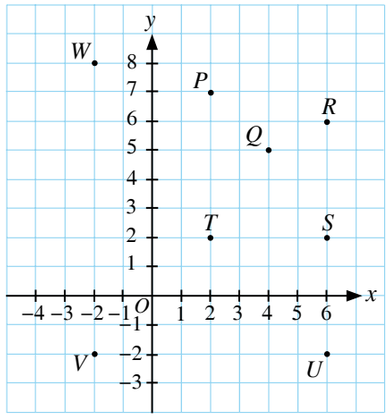

(b)




2. Based on the diagram below, state the coordinates of the midpoint of
   (a) $AB$
   (b) $CD$
   (c) $AD$




3. Determine the midpoint of the straight line
   (a) $PQ$
   (b) $RS$
   (c) $TU$
   (d) $WV$

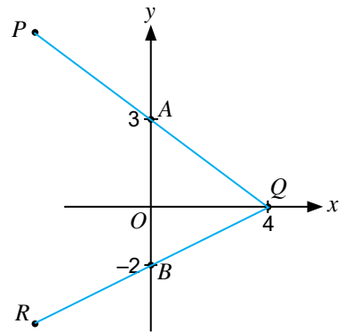



4. Determine the midpoint for the following coordinates
   (a) $P\,(-1, 7)$ and $Q\,(-1, 1)$.
   (b) $R\,(3, -6)$ and $S\,(3, 2)$.
   (c) $A\,(3, 1)$ and $B\,(5, 1)$.
   (d) $C\,(5, 0)$ and $D\,(1, 0)$.

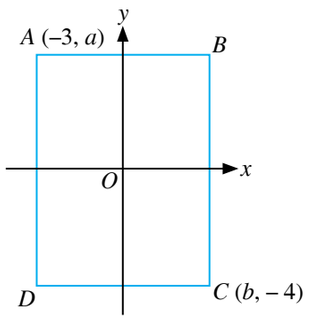




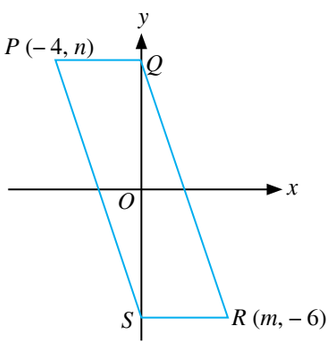
5. Referring to the diagram, $A$ is the midpoint of $PQ$ and $B$ is the midpoint of $RQ$. Determine the coordinates of $P$ and $R$.




6. Midpoint for the square in the diagram below is at the origin. Determine
   (a) the values of $a$ and $b$.
   (b) the distance of line $BC$.
   (c) the coordinates of $B$.




7. The origin is the midpoint for the height of the parallelogram. Calculate
   (a) the values of $m$ and $n$.
   (b) the midpoint of $PQ$.
   (c) the midpoint of $SR$.



8. Given the straight line $AB = BD$ with $D(-1, 3)$ and $B(1, 1)$, calculate the coordinates of point $A$.

9. A straight line that joins points $(-8, 3)$ and $(s, 3)$ has the midpoint $(0, u)$. Calculate the values of $s$ and $u$.

10. The line $AB$ is parallel to $x$-axis with point $A(3, a)$ and midpoint of $AB$ is $(5, 1)$. Calculate
    (a) the value of $a$.
    (b) the coordinates of $B$.

## 7.3 The Cartesian Coordinate System

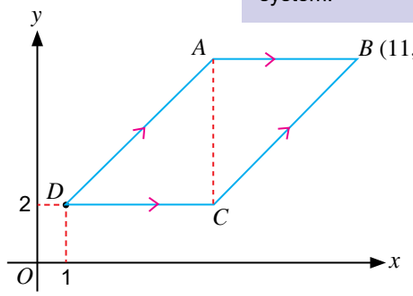
### 7.3.1 Solving problems

**LEARNING STANDARD**

Solve problems involving the Cartesian coordinate system.

**EXAMPLE 14**

The diagram shows a rhombus. Given the distance between the point $A$ and $B$ is 5 units. Calculate
(a) the coordinates of $A$.
(b) the midpoint of the straight line $AC$.




*Solution:*

(a)

**Understanding the problem**

Determine point $A$ where $AB$ is parallel to $DC$.

**Planning the strategy**

Straight line $AB$ is parallel to the $x$-axis. $y$-coordinate for point $A$ is 6.

**Implementing the strategy**

Distance of $AB = 5$ units.

$x$-coordinate $= 11 - 5$
$= 6$

**Conclusion**

Therefore, coordinates of $A$ are $(6, 6)$.

(b)

**Understanding the problem**

The line $AC$ is parallel to the $y$-axis, with a common $x$-coordinate, which is 6.

**Planning the strategy**

Formula for midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

**Implementing the strategy**

A $(6, 6)$ C $(6, 2)$
$x_1, y_1$ $x_2, y_2$

$\left(\frac{6+6}{2}, \frac{6+2}{2}\right) = (6, 4)$

**Conclusion**

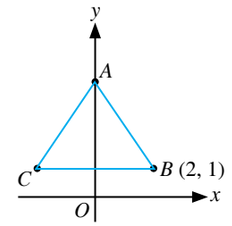
Therefore, midpoint of $AC$ is $(6, 4)$.



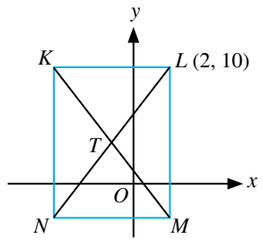


## SELF PRACTICE 7.3

1. The diagram on the right is an isosceles triangle where the height is 4 units. Calculate
   (a) the coordinates of $C$.
   (b) the coordinates of $A$.
   (c) the coordinates of midpoint of the line $AB$.
   (d) the distance of the line $AC$.




2. The diagram on the right is a rectangle. The distance of $KL$ is 8 units and $KN$ is 12 units. Calculate
   (a) the distance of $LN$.
   (b) the coordinates of midpoint of line $MN$.
   (c) the coordinates of $T$.



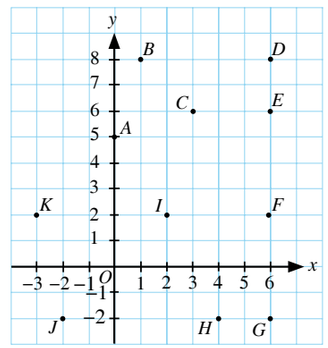

3. If the line $PQ$ is parallel to the $y$-axis and the midpoint $M(4, 0)$ with a distance of $MP$ is 3 units, calculate
   (a) the coordinates of $P$. (b) the coordinates of $Q$. (c) the distance of $PQ$.

4. The distance $AB = KL$, that is 8 units and each is parallel with the $y$-axis. If midpoint $AB$ is $(0, 3)$ and the distance from midpoint of $AB$ to midpoint of $KL$ is 2 units downwards, calculate
   (a) the coordinates of $K$ and $L$.
   (b) the coordinates of the midpoint of $KL$.

5. Given that $P(4, 0)$ and $Q$ is located on the $y$-axis with 6 units upwards from the $x$-axis, calculate
   (a) the midpoint of $PQ$.
   (b) the distance between the point $P$ and midpoint of $PQ$.

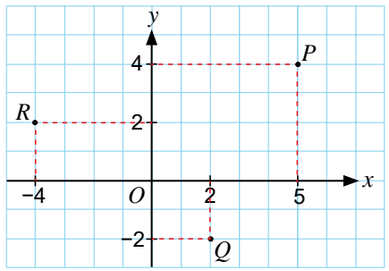
## GENERATING EXCELLENCE

1. Which of the following points represent
   (a) $(-3, 2)$
   (b) $(0, 5)$
   (c) $(4, -2)$
   (d) $(6, 8)$



2. If point $K$ is located on the $x$-axis and 4 units to the left of the $y$-axis, determine coordinates of $L$ which is 5 units upwards from point $K$.

3. If points $P$, $Q$ and $R$ each move 2 units to the south and 1 unit to the left respectively, state the new locations for the points. Calculate the distance for each new location from $PQ$ to $RQ$.




4. $ABCD$ is a square with $A$ as the origin and $B$ $(-5, 0)$. Calculate the perimeter of the square.

5. $KLM$ is a right angled triangle with points $K$ $(1, 0)$ and $L$ $(5, 0)$ as the base and $ML$ is the height for the triangle. If the distance from $M$ to $L$ is 5 units, calculate the area of the triangle.

6. The midpoint of the diagonal of a square is 2 units from the vertex of the square. Calculate the area of the square.

CHAPTER 7

## CHAPTER SUMMARY

**Coordinates**

**$x$-axis**
Axis which is horizontal and perpendicular to the $y$-axis in the Cartesian coordinate system.

**$y$-axis**
Axis which is vertical and perpendicular to the $x$-axis in the Cartesian coordinate system.

**Origin**
Point of intersection between horizontal and vertical axes. Coordinates of the origin are (0, 0).

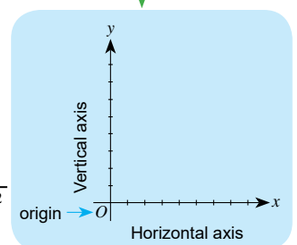
**Cartesian Plane**
Cartesian plane comprises one horizontal line and one vertical line that intersect at a right angle.

**Distance between two points**
Measurement of distance or length between two points.

Common axes

$(y_2 - y_1)$ and $(x_2 - x_1)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$




**Midpoint**
Point that bisects a line segment into two equal parts.

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$





## SELF REFLECTION

**At the end of this chapter, I am able to:**

| | |
|---|---|
| 1. Explain the meaning of distance between two points on the Cartesian plane. | |
| 2. Derive the formula of the distance between two points on a Cartesian plane. | |
| 3. Determine the distance between two points on the Cartesian plane. | |
| 4. Solve problems involving the distance between two points in the Cartesian coordinate system. | |
| 5. Explain the meaning of midpoint between two points on the Cartesian plane. | |
| 6. Derive the formula of the midpoint between two points on the Cartesian plane. | |
| 7. Determine the coordinates for midpoint between two points on the Cartesian plane. | |
| 8. Solve problems involving midpoint in the Cartesian coordinate system. | |
| 9. Solve problems involving the Cartesian coordinate system. | |

## MINI PROJECT

Draw a plan of your classroom seating position on a grid paper with a scale of 1 cm to 2 metres on the horizontal axis and 1 cm to 2 metres on the vertical axis. You may change the scale. Determine the coordinates of your friends' seats. Paste the plan in front of your class for reference.

CHAPTER 7

# CHAPTER 8

# Graphs of Functions

**WHAT WILL YOU LEARN?**

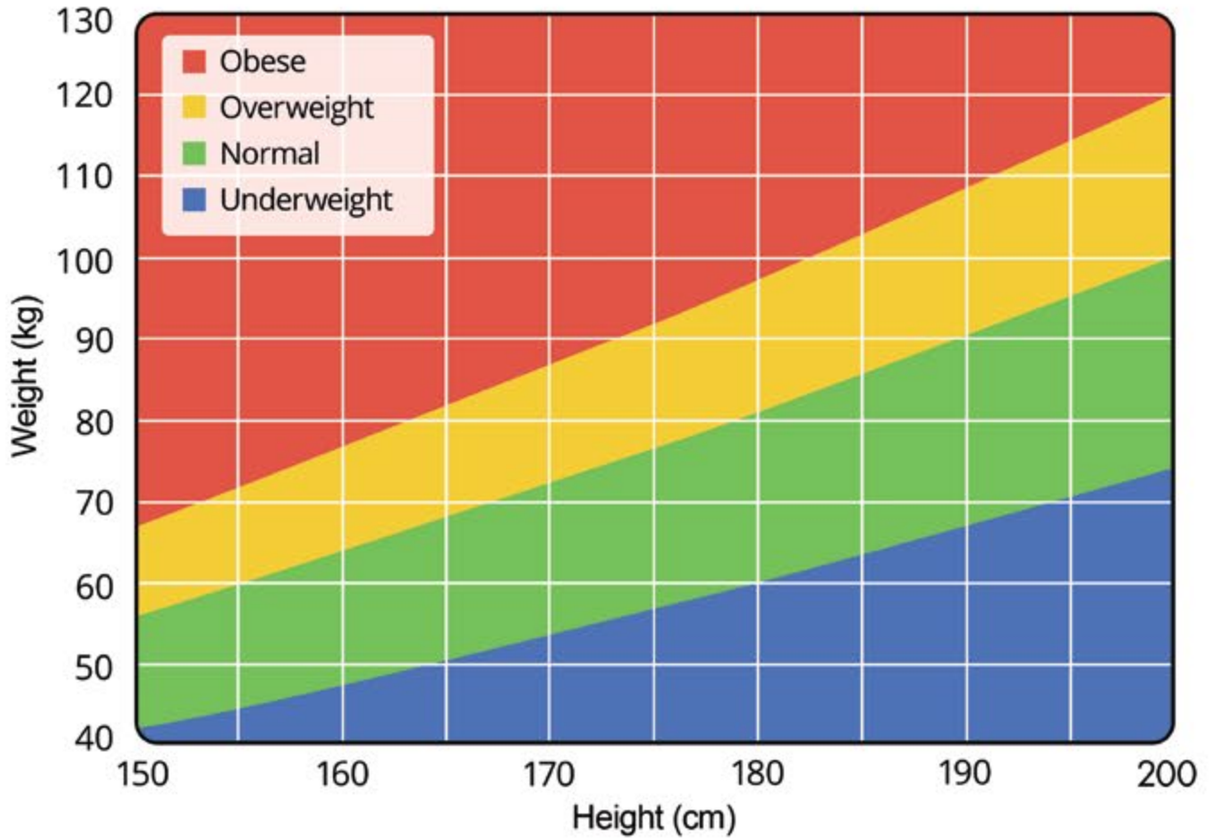


**WORD LINK**

| | |
|---|---|
| • Graph of function | • *Graf fungsi* |
| • Function | • *Fungsi* |
| • Variable | • *Pemboleh ubah* |
| • Relation | • *Hubungan* |
| • Linear equation | • *Persamaan linear* |
| • Table of value | • *Jadual nilai* |
| • Linear function | • *Fungsi linear* |
| • Non-linear function | • *Fungsi bukan linear* |
| • Scale | • *Skala* |
| • Reciprocal function | • *Fungsi salingan* |
| • Cubic function | • *Fungsi kubik* |
| • Quadratic function | • *Fungsi kuadratik* |



**B**ody mass index (BMI) is a measurement of body fat based on height and weight. A higher measurement of BMI indicates a lot of fat content.


**WALKING THROUGH TIME**

René Descartes (1596-1650), stated that function is a mathematical relationship between two variables. The term 'function' was introduced by Gottfried Wilhelm Leibniz (1646- 1716) in his book. The concept of function was further studied by Leonhard Euler (1707-1783) and he introduced the notation of function, that is $y = f(x)$.

For more information:

http://rimbunanilmu.my/mat_t2e/ms145

**WHY STUDY THIS CHAPTER?**

- Function is applied in the fields of economy, technology, science, engineering, banking and mathematics. Among the careers that need knowledge on functions are engineers, economist, auditors, lecturers and bankers.
- The concept of function helps in predicting the best time to trade shares in the stock market.

## CREATIVE ACTIVITY

**Aim:** Knowing relationship between two quantities

**Material:** Worksheet

**Steps:**

1. The advertisement below shows the entry rates to a water theme park according to categories. Based on the advertisement, complete the table.


| Family | Category | | |
|---|---|---|---|
| | Adults | Children | Elderly/Disabled |
| 1 | | | |
| 2 | | | |
| 3 | | | |
| **Total** | | | |

2.

| Category | Number | Cost | Total |
|---|---|---|---|
| Adults | 2 | $2 \times 30$ | 60 |
| Children | | | |
| Elderly/ Disabled | | | |
| **Total** | | | |

3. From the table above, what is the relationship between the total cost of the tickets for each family with the category of family members?

From the table above, we know that the total cost of the tickets depends on the number and category of family members.

# 8.1 Functions

## 8.1.1 Definition of functions

**LEARNING STANDARD**

Explain the meaning of functions.

## COGNITIVE STIMULATION

Group

**Aim:** Identifying functions

**Materials:** Worksheets and calculator

**Steps:**

1. Use the symbol $\sqrt[3]{\ }$ (cube root) on your calculator to determine the output number of some input numbers and complete Table *A*.
2. Use $x^3$ (power of cube) on your calculator to determine the output number of some input numbers and complete Table *B*.







| Input | $\sqrt[3]{\ }$ | Output |
|---|---|---|
| 64 | $\sqrt[3]{64}$ | 4 |
| 27 | $\sqrt[3]{27}$ | 3 |
| 0 | | |
| $\frac{1}{8}$ | | |
| $\frac{1}{125}$ | | |

Table *A*

| Input | $x^3$ | Output |
|---|---|---|
| 2 | $2^3$ | 8 |
| 3 | $3^3$ | 27 |
| 5 | | |
| 7 | | |
| 10 | | |

Table *B*

**Discussion:**

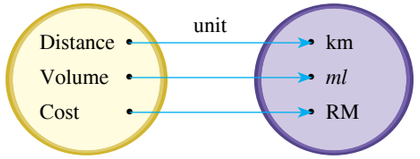
If the input is a domain while the output is a range, specify the range for set $A = \{64, 27, 0, \frac{1}{8}, \frac{1}{125}\}$ and set $B = \{2, 3, 5, 7, 10\}$.

From the activity above, function is a relation where each input has only one output.

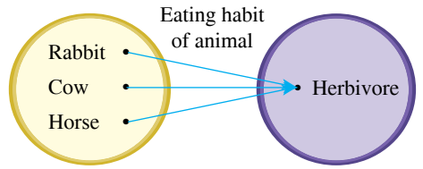
### Identifying functions

Relations that are functions

(a) One-to-one relation

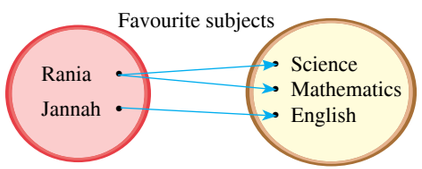



(b) Many-to-one relation

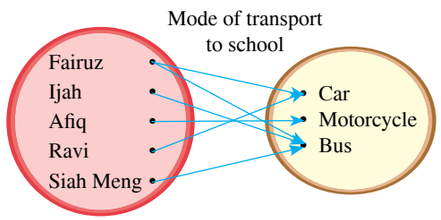



Relations that are not functions

(a) One-to-many relation




(b) Many-to-many relation




Relation is the matching of items from set *A* to set *B*. Relations can be represented by using the

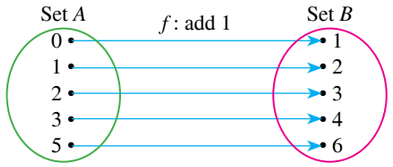
(a) arrow diagram

(b) graph

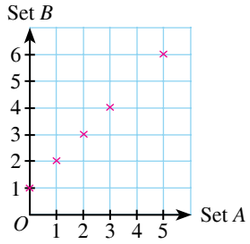
(c) ordered pair

**EXAMPLE 1**

(a) Arrow diagram




(b) Graph

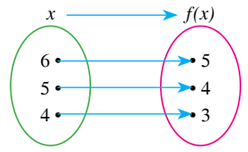



(c) Ordered pair

$P = \{(0, 1), (1, 2), (2, 3), (3, 4), (5, 6)\}$

**DO YOU KNOW ?**

$x$ is mapped to $f(x)$




The function above can be written with notations as follows:

$f : x \to x - 1$ or

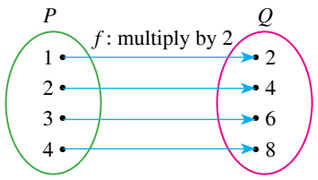
$f(x) = x - 1$

**EXAMPLE 2**

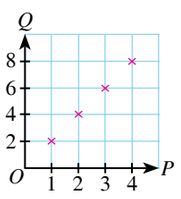
(a) **One-to-one functions**

Relation where the object in the domain has only one image.

(i)




(ii)

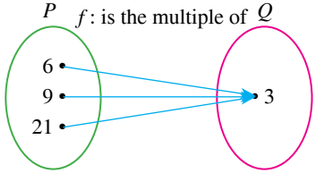



(iii) Ordered pair, $A = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$

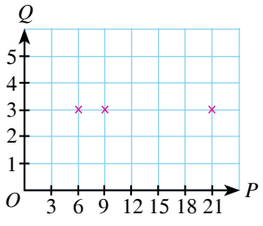
(b) **Many-to-one functions**

Relation where more than one object is matched to the same image.

(i)




(ii)




(iii) Ordered pair, $B = \{(6, 3), (9, 3), (21, 3)\}$



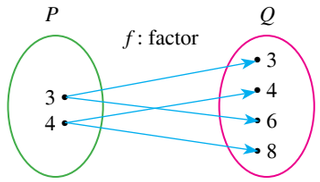




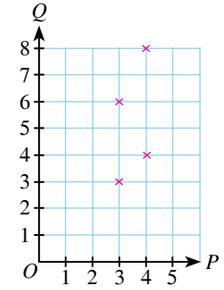
(c) **One-to-many relations**

Relation where the object in the domain has more than one image.

(i)




(ii)

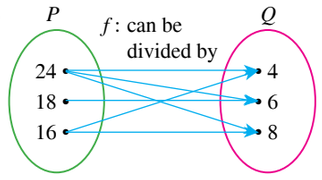



(iii) Ordered pair, $R = \{(3, 3), (3, 6), (4, 4), (4, 8)\}$

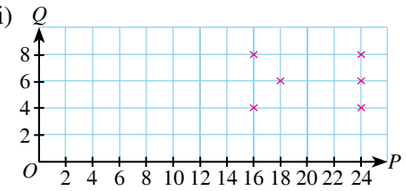
(d) **Many-to-many relations**

Relation where at least one object has more than one image, and more than one object has the same image.

(i)




(ii)




(iii) Ordered pair, $S = \{(24, 4), (24, 6), (24, 8), (18, 6), (16, 4), (16, 8)\}$

Provide justification based on the observation of the relation represented by a graph in the example above.

**FLASHBACK**

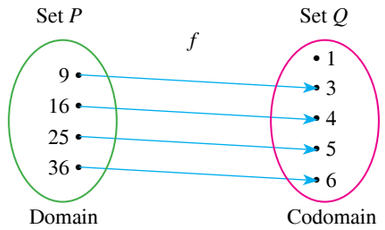
A straight line graph is obtained when all ordered pairs for linear equations are plotted and connected.

## 8.1.2 Function representation

**LEARNING STANDARD**

Identify functions and provide justifications based on function representations in the form of ordered pairs, tables, graphs and equations.

The diagram below shows the function $f$ that maps $x$ to $\sqrt{x}$ which is represented by $f(x) = \sqrt{x}$.




Set $P = \{9, 16, 25, 36\}$ is the domain and the element is the object. Set $Q = \{1, 3, 4, 5, 6\}$ is the codomain. The elements in set $Q$ that is matched to the object in set $P$ is the image. Set $\{3, 4, 5, 6\}$ is the range of the function.

**EXAMPLE 3**

Given set $P = \{1, 2, 3\}$ and set $Q = \{4, 5, 6\}$, the function $f$ maps $P$ to $Q$ by adding 3. Represent this function using

(a) ordered pair (b) table (c) graph (d) equation

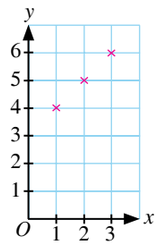
***Solution:***

(a) $\{(1, 4), (2, 5), (3, 6)\}$

(b)

| ***P*** | 1 | 2 | 3 |
|---|---|---|---|
| ***Q*** | 4 | 5 | 6 |

(c)




(d) $4 = 1 + 3$

$5 = 2 + 3$

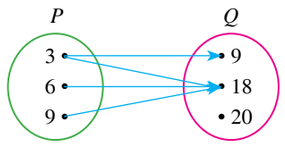
$6 = 3 + 3$

$y = x + 3$ or $f(x) = x + 3$

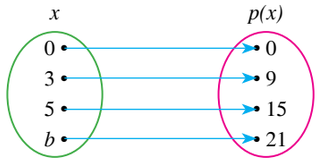
The function that maps $x$ to $y$ can be written using $f(x)$. Therefore, this function can be written as $f(x) = x + 3$.

**SELF PRACTICE 8.1**

1. The diagram shows the relation between set $P$ and set $Q$.
   State
   (a) the type of relation.
   (b) the range of the relation.

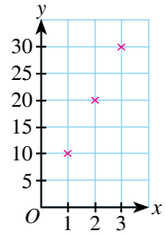



2. The diagram shows a function.
   State the value $b$.

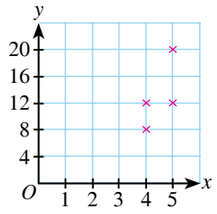



3. Determine whether the set of ordered pair is a function.
   (a) $P = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$
   (b) $Q = \{(1, 3), (0, 3), (2, 1), (4, 2)\}$
   (c) $R = \{(1, 6), (2, 5), (1, 9), (4, 3)\}$

4. Determine whether the relation in the following graph is a function or not a function.
   (a)




   (b)

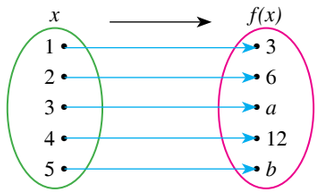




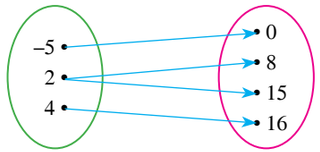



5. Given set $S = \{10, 12, 18, 20\}$ and set $R = \{2, 4, 10, 12\}$, set $S$ is mapped to set $R$ by subtracting 8. Represent the function using
   (a) ordered pair (b) table (c) graph (d) equation

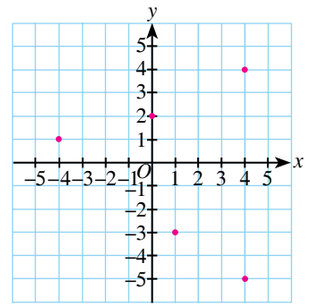
6. The following diagram shows the function $f(x) = 3x$ for the domain $1 \leqslant x \leqslant 5$.
   Determine the values $a$ and $b$.



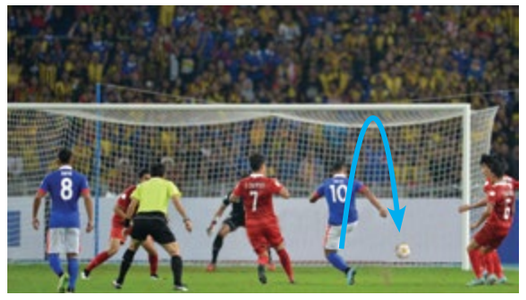

7. State the domain and range the following relation.
   (a)




   (b)




## 8.2 Graphs of Functions



We have learned that the representation of a function can be done in the form of a graph. A graph of function is the representation of a function on a Cartesian plane. By drawing a graph, we can explain the relationship between variables in the function. This graph also helps us identify information to solve problems.

The diagram shows a player kicking a ball, making it bounce into the goalmouth. The bouncing action forms a curve.

If the curve represents the function $s = 25t - 2.5t^2$, $t$ is the time in seconds and $s$ is the height in metre. The relationship between $s$ and $t$ can be represented in the form of a graph. Some information can be obtained from the graph, such as the maximum height of the ball, the time the ball takes to hit the ground again and the distance from where it was kicked.

**DO YOU KNOW?**

Malaysian football star from Penang Mohd Faiz Subri received the FIFA Puskas Award for the best goal in 2016.

### 8.2.1 Constructing a table of values

**LEARNING STANDARD**

Construct tables of values for linear and non-linear functions, and hence draw the graphs using the scale given.

From the given function, a **table of values** can be constructed to determine the corresponding value of the **ordered pair** $(x, y)$ before the graph is drawn.

## EXAMPLE 4

(a) Construct a table of values for the function $y = 5 - x$, given $x = -2, -1, 0, 1$.

(b) Construct a table of values for the function $y = 2x^2 - 1$, given $x = -1, 0, 1, 2$.

***Solution:***

(a) When $x = -2$
$y = 5 - x$
$y = 5 - (-2)$
$y = 5 + 2$
$y = 7$

When $x = -1$
$y = 5 - x$
$y = 5 - (-1)$
$y = 5 + 1$
$y = 6$

When $x = 0$
$y = 5 - x$
$y = 5 - 0$
$y = 5$

When $x = 1$
$y = 5 - x$
$y = 5 - 1$
$y = 4$

Therefore, the table of values for the function $y = 5 - x$ is

| $x$ | –2 | –1 | 0 | 1 |
|---|---|---|---|---|
| $y$ | 7 | 6 | 5 | 4 |

(b) When $x = -1$
$y = 2x^2 - 1$
$y = 2(-1)^2 - 1$
$y = 2 - 1$
$y = 1$

When $x = 0$
$y = 2x^2 - 1$
$y = 2(0)^2 - 1$
$y = 0 - 1$
$y = -1$

When $x = 1$
$y = 2x^2 - 1$
$y = 2(1)^2 - 1$
$y = 2 - 1$
$y = 1$

When $x = 2$
$y = 2x^2 - 1$
$y = 2(2)^2 - 1$
$y = 8 - 1$
$y = 7$

Therefore, the table of values for the function $y = 2x^2 - 1$ is

| $x$ | –1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | 1 | –1 | 1 | 7 |

**TIPS**

Use a calculator to calculate the values of $y = 2x^2 - 1$
Press

2 ALPHA ) ^ 2 - 1

For

$x = -1$,
press CALC -1 =
answer = **1**

$x = 0$,
press CALC 0 =
answer = **–1**

$x = 1$,
press CALC 1 =
answer = **1**

$x = 2$,
press CALC 2 =
answer = **7**

## Drawing a graph

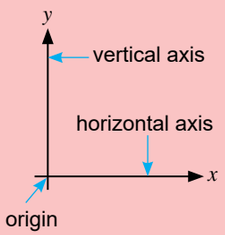
The ordered pairs *(x, y)* can be plotted on a Cartesian plane using the scale given. Next, the points are joined to form a graph. To make it easier in constructing the graph, we can use the following steps.

**Steps to draw a graph:**

1. Construct a table of values for the given function.
2. Draw and label each axis with the given scale or any suitable scale.
3. Plot the point $(x, y)$ for the ordered pairs from the table.
4. Join the points to form a straight line or a smooth curve.

**FLASHBACK**

- (0,0) is also known as origin.
- $x$-axis is known as horizontal axis
- $y$-axis is known as vertical axis.








## EXAMPLE 5

(a) Complete the table of values below for the function $y = 2x + 4$.

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 0 | | | 6 | | 10 |

(b) Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 2 units on the $y$-axis, draw a graph of the function for values of $x$ from $-2$ to 3.

This set of data can be written in the following form $-2 \leqslant x \leqslant 3$.

***Solution:***

(a) $y = 2x + 4$

When $x = -1$
$y = 2(-1) + 4$
$= -2 + 4$
$= 2$

When $x = 0$
$y = 2(0) + 4$
$= 0 + 4$
$= 4$

When $x = 2$
$y = 2(2) + 4$
$= 4 + 4$
$= 8$

Therefore, the table is

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 0 | 2 | 4 | 6 | 8 | 10 |

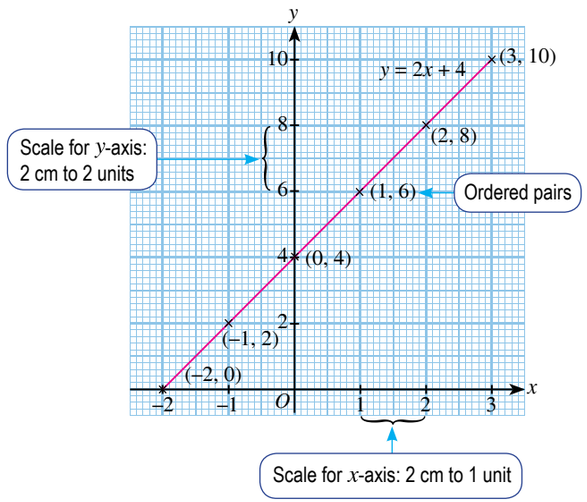
(b) Draw axes using the scales given.

Scale for $x$-axis : 2 cm to 1 unit.

Scale for $y$-axis : 2 cm to 2 units.

Plot the points according to ordered pairs from the table of values, (–2, 0), (–1, 2), (0, 4), (1, 6), (2, 8) and (3, 10).

Join the points with a straight line.




**TIPS**

This graph is also known as linear function graph, the highest power of a variable $x$ is 1.

**DO YOU KNOW ?**

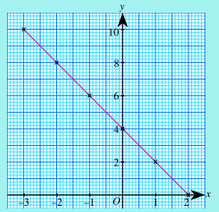
Equation for straight line $y = mx + c$, with $m$ as the gradient and $c$ is the $y$-intercept.

**TIPS**

Use a ruler to draw a straight line graph.

**THINK SMART**

What type of graph is shown below? State the function.



## EXAMPLE 6

(a) Complete the table of values below for the function $y = x^2 - 2x - 3$.

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | 5 | | –3 | | | 0 | 5 |

(b) Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 1 unit on the $y$-axis, draw a graph of the function for values of $x$ from $-2 \leqslant x \leqslant 4$.

***Solution:***

(a) $y = x^2 - 2x - 3$.

When $x = -1$
$y = (-1)^2 - 2(-1) - 3$
$= 1 + 2 - 3$
$= 0$

When $x = 1$
$y = 1^2 - 2(1) - 3$
$= 1 - 2 - 3$
$= -4$

When $x = 2$
$y = 2^2 - 2(2) - 3$
$= 4 - 4 - 3$
$= -3$

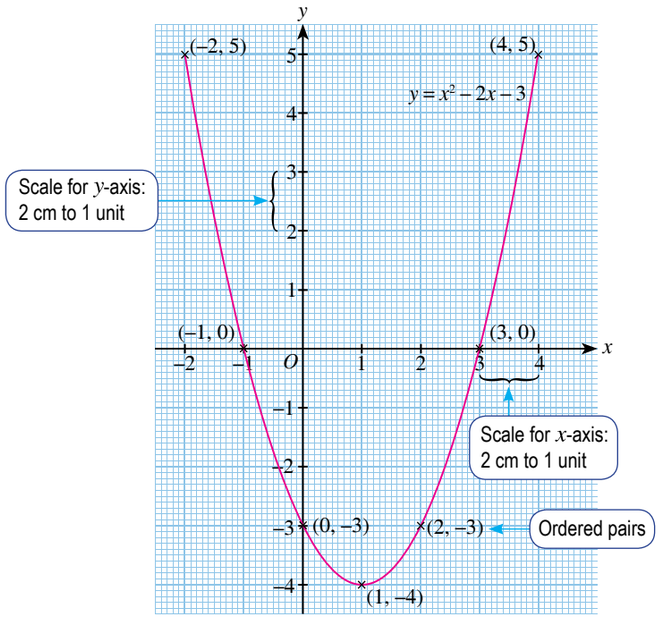
Therefore, the table is

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | 5 | 0 | –3 | –4 | –3 | 0 | 5 |

(b) Draw axes using the scale given. Plot the points using the table of values and join the points.

Scale for $x$-axis : 2 cm to 1 unit.

Scale for $y$-axis : 2 cm to 1 unit.




**TIPS**

Quadratic function
$f(x) = ax^2 + bx + c$,
The highest power for the variable in a quadratic function is 2, and $a \neq 0$.

**TIPS**

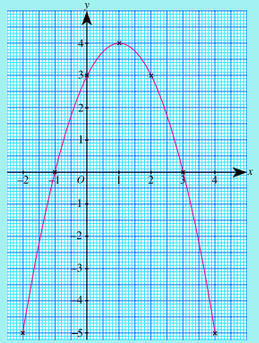
- A sharp pencil can help a student draw a line or curve smoothly.
- Students are allowed to use a flexible ruler to draw a curve.

**DO YOU KNOW ?**

This shape of graph is called parabola.

**THINK SMART**

What type of graph is shown below?
State the function.







## EXAMPLE 7

(a) Complete the table of values below for the function $y = 12 - x^3$.

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 39 | | 13 | 12 | | | –15 |

(b) Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 5 units on the $y$-axis, draw a graph of the function for $-3 \leqslant x \leqslant 3$.

***Solution:***

(a) $y = 12 - x^3$

When $x = -2$
$y = 12 - (-2)^3$
$= 12 + 8$
$= 20$

When $x = 1$
$y = 12 - (1)^3$
$= 12 - 1$
$= 11$

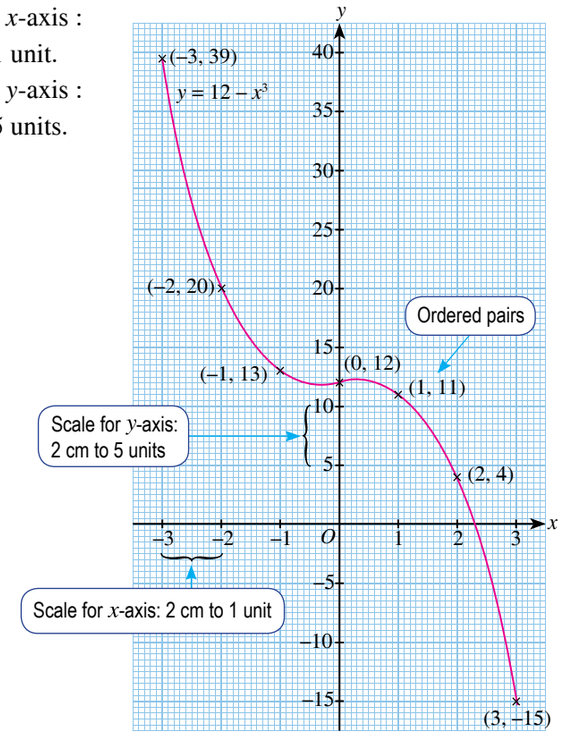
When = 2
$y = 12 - (2)^3$
$= 12 - 8$
$= 4$

Therefore, the table is

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 39 | 20 | 13 | 12 | 11 | 4 | –15 |

(b) Draw the axes using the scale given. Plot the points using the above table of values and join the points

Scale for $x$-axis :
2 cm to 1 unit.
Scale for $y$-axis :
2 cm to 5 units.

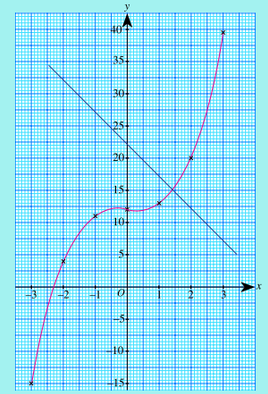



**TIPS**

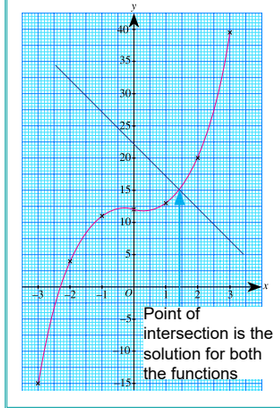
For the cubic function $ax^3 + c$, the highest power for the variable is 3.

**THINK SMART**

Determine the type of graph. State the function.



**DO YOU KNOW ?**



## EXAMPLE 8

(a) Complete the table of values for the function $y = \frac{24}{x}$.

| $x$ | −4 | −3 | −2 | −1 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $y$ | −6 | | −12 | −24 | | 12 | 8 | |

(b) Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 5 units on the $y$-axis, draw a graph of the function for $-4 \leqslant x \leqslant 4$.

***Solution:***

(a) $y = \frac{24}{x}$

When $x = -3$: $y = \frac{24}{-3} = -8$

When $x = 1$: $y = \frac{24}{1} = 24$

When $x = 4$: $y = \frac{24}{4} = 6$

Therefore, the table is

| $x$ | −4 | −3 | −2 | −1 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $y$ | −6 | −8 | −12 | −24 | 24 | 12 | 8 | 6 |

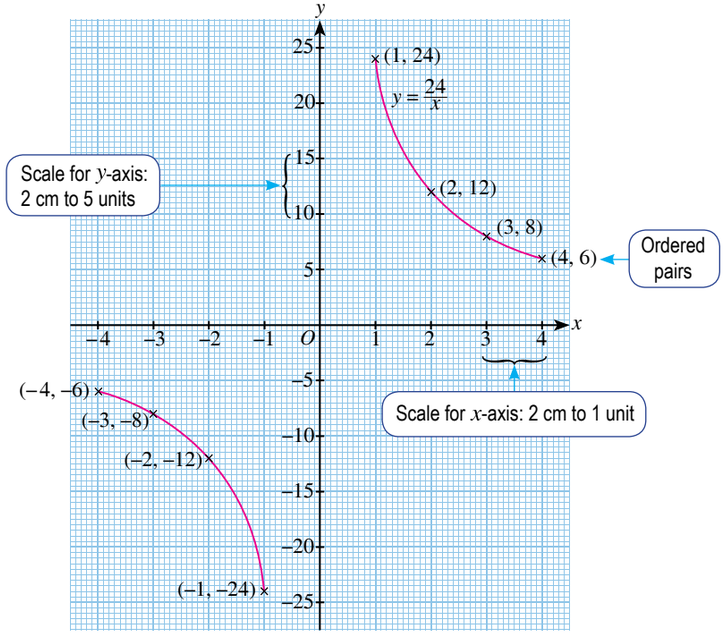
**DO YOU KNOW ?**

The reciprocal function $y = \frac{a}{x}$ is undefined if $x = 0$. The reciprocal function can also be written as $y = ax^{-1}$. This shape of graph is called hyperbola.

(b) Draw the axes using the scale given. Plot the points using the above table of values and join the points.

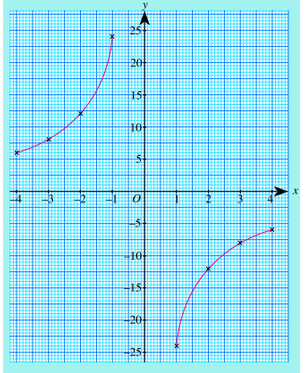
Scale for $x$-axis : 2 cm to 1 unit.

Scale for $y$-axis : 2 cm to 5 units.




**THINK SMART**

Determine the type of graph. State the function.







## EXAMPLE 9

(a) Complete the table of values for the function $y = x^{-2}$

| $x$ | −4 | −3 | −2 | −1 | −0.5 | 0.5 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 0.06 | | 0.25 | | 4 | | 1 | 0.25 | 0.11 | 0.06 |

(b) Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 0.5 unit on the $y$-axis, draw a graph of the function for $-4 \leqslant x \leqslant 4$.

***Solution:***

(a) $y = x^{-2}$

When $x = -3$: $y = (-3)^{-2} = 0.11$

When $x = -1$: $y = (-1)^{-2} = 1$

When $x = 0.5$: $y = (0.5)^{-2} = 4$

Therefore, the table is

| $x$ | −4 | −3 | −2 | −1 | −0.5 | 0.5 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 0.06 | 0.11 | 0.25 | 1 | 4 | 4 | 1 | 0.25 | 0.11 | 0.06 |

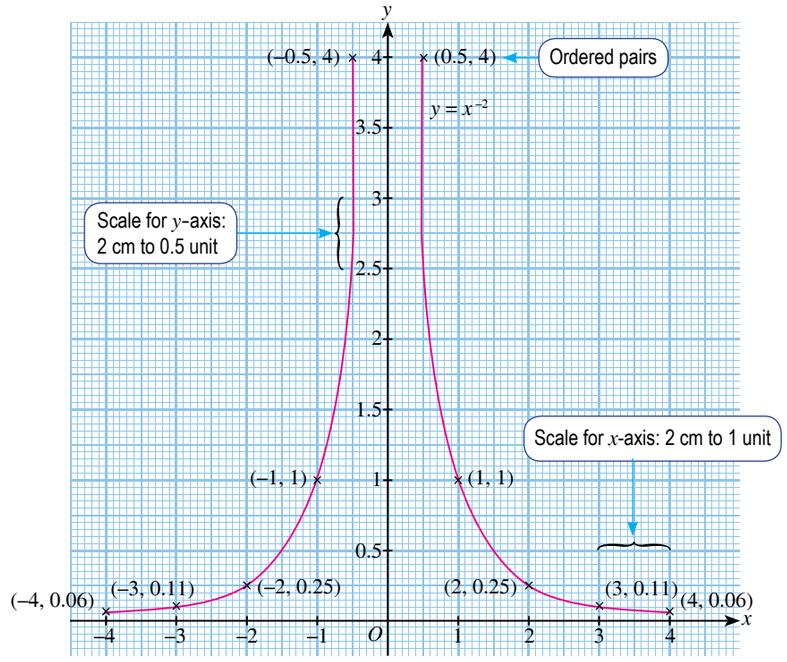
**TIPS**

$y = ax^n$ when $n = -1, -2$ is a reciprocal function.

(b) Draw the axes using the scale given. Plot the points using the above table of values and join the points.

Scale for $x$-axis: 2 cm to 1 unit

Scale for $y$-axis: 2 cm to 0.5 unit




This shape of graph is called hyperbola.

## 8.2.2 Interpreting graphs of functions

**LEARNING STANDARD**

Interpret graphs of functions.

Interpreting graphs of functions is like studying trends and making predictions according to the relations derived from the variables.

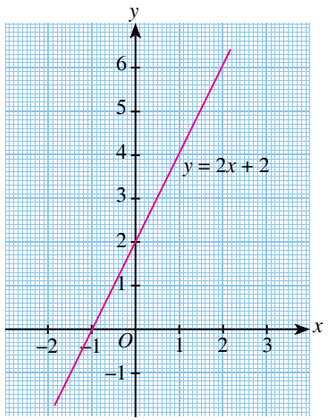
**EXAMPLE 10**

The diagram shows a graph of the function for $y = 2x + 2$.

From the graph, determine

(a) value of $y$ when $x = 2$

(b) value of $x$ when $y = 4$

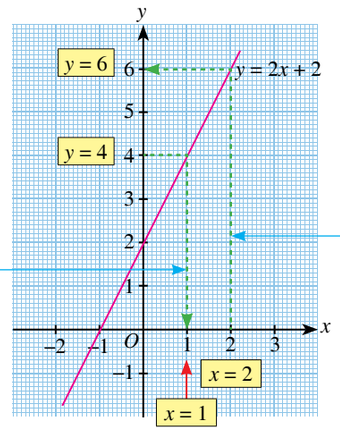



***Solution:***

From the graph:

(a) when $x = 2$, then $y = 6$

(b) when $y = 4$, then $x = 1$




When $y = 4$, draw a horizontal line to the right until it meets the function $y = 2x + 2$. Then, construct a vertical line downwards until it meets $x$-axis. State value $x$.

When $x = 2$, draw a vertical line upwards until it meets the function $y = 2x + 2$. Then, construct a horizontal line to the left until it meets $y$-axis. State value $y$.

**EXAMPLE 11**

The diagram shows a graph of the function for $y = x^2 - 2$. Based on the graph, determine

(a) value of $y$ when $x = 2$

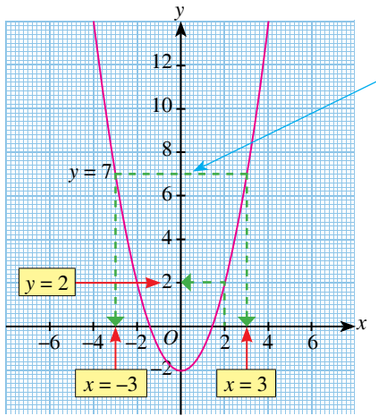
(b) value of $x$ when $y = 7$

***Solution:***

From the graph:

(a) when $x = 2$, then $y = 2$

(b) when $y = 7$, then $x = 3$ or $-3$




Construct a horizontal line $y = 7$, until it meets both sides of the function $y = x^2 - 2$. Then, construct a vertical line downwards towards $x$-axis.



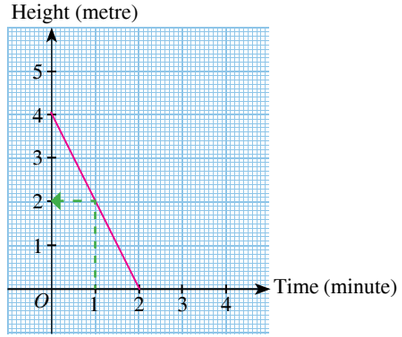


**EXAMPLE 12**

The graph of function shows the movement of a ball that was dropped from a height of 4 metres. Based on the graph, determine

(a) the distance of the ball from the ground at the first minute.

(b) time the ball touches the ground.




***Solution:***

From the graph:

(a) when $x = 1$, $y = 2$

Therefore, the distance of the ball from the ground is 2 metres.

(b) when the ball touches the ground, the height is zero.

When $y = 0$, $x = 2$

Therefore, ball touches the ground at the second minute.

**EXAMPLE 13**

During the entrepreneurial expo, Anis sold fried rice at the Consumer Club's stall. The graph shows the number of fried rice packets sold with the profit that Anis gained.

From the graph,

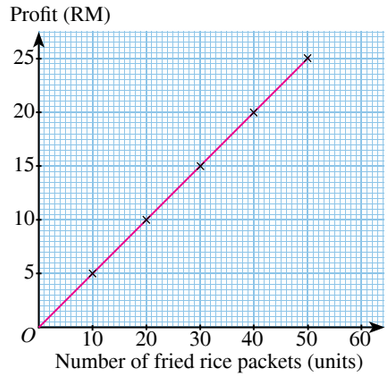
(a) what is the profit earned by Anis if she sold 20 packets of fried rice?

(b) if Anis earned a profit of RM20, how many packets of fried rice did she sell?

(c) state the profit made by Anis if she sold 26 packets of fried rice.

(d) state a suitable inference.

(e) predict the profit Anis would have made if 60 packets of fried rice were sold.




***Solution:***

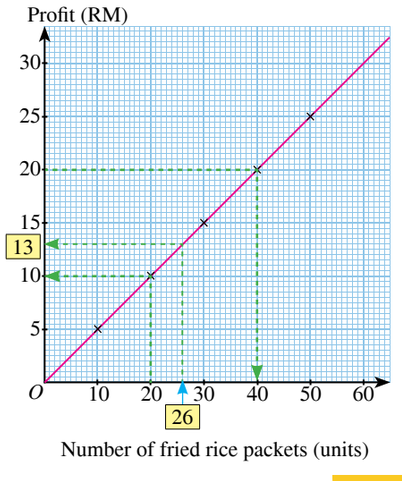
(a) RM10

(b) 40 packets

(c) Profit = RM13

(d) The more number of fried rice packets sold, the higher the profit gained.

(e) RM30



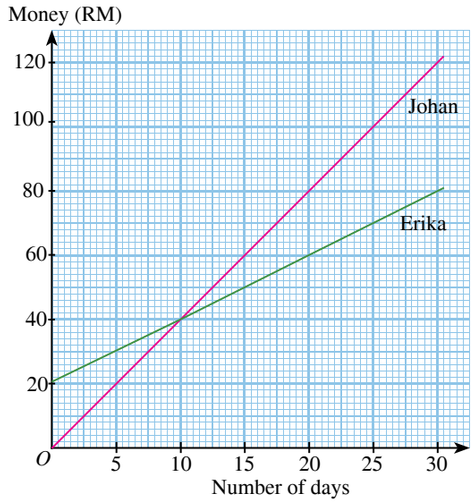
## 8.2.3 Solving problems

**LEARNING STANDARD**

Solve problems involving graphs of functions.

**EXAMPLE 14**

Every day Johan and Erika receive pocket money from their father. They keep part of the money in their saving boxes. The graph below shows the total amount of money saved (RM) according to the number of days.




(a) How much money is saved at the end of the 20th day by
   (i) Johan (ii) Erika

(b) When will the amount of money saved by Johan and Erika become equal?

(c) When will the amount of money between Johan and Erika have a difference of RM30?

(d) Their father promised to give a present to the person who saves the most at the end of one month. Who will get the present? Justify your answer.

*Solution:*

**Understanding the problem**

(a) Determine how much money is saved at the end of the 20th day by Johan and Erika.
(b) Determine when the amount of money saved by Johan and Erika become equal.
(c) Calculate when the amount of money between Johan and Erika will differ by RM30.
(d) Determine the person who would saves the most at the end of one month and justify it.

**Planning the strategy**

(a) State the value of $y$ when $x = 20$ for Johan and Erika's graph.
(b) State the value of $x$ at the point of intersection of the two lines.
(c) Calculate the difference of RM30 of the two lines.
(d) State the value of $y$ when the number of days is 30.

**Implementing the strategy**

(a) From the graph, on the 20th day
   (i) Johan's savings = RM80 (ii) Erika's savings = RM60
(b) Value of $x$ at the point of intersection in Johan and Erika's graph. Point of intersection (10, 40). Therefore on the 10th day the total savings for Johan and Erika is the same, which is RM40.
(c) RM100 – RM70 = RM30, on the 25th day.
(d) When $x = 30$; in Erika's graph, $y$ = RM80; in Johan's graph, $y$ = RM120. Hence Johan has more savings. Therefore, Johan will receive the present from his father.





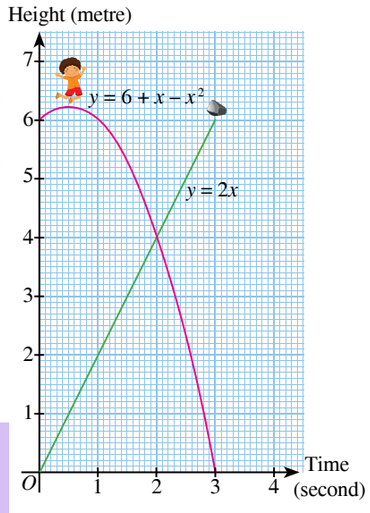


**Conclusion**

(a) Total amount of money saved on the 20th day by
   (i) Johan = RM80 (ii) Erika = RM60
(b) Johan and Erika have the same amount of money saved on the 10th day.
(c) The difference between their savings is RM30 on the 25th day.
(d) Johan will get the present as on the 30th day, he will have RM120, whereas Erika's savings will be only RM80.

**EXAMPLE 15**

In an animated game while the cartoon character Jibam jumps from a rock, a pebble is thrown at him to make him fall. Graph of the function $y = 6 + x - x^2$, represents Jibam's movements and graph of the function $y = 2x$ represents the movement of the pebble. $Y$ represents height in metre and $x$ represents time in seconds.




(a) What is the maximum height of Jibam's jump?

(b) When does the pebble touch Jibam?

(c) When does Jibam touch the ground?

*Solution:*

**Understanding the problem**

- The function $y = 6 + x - x^2$ represents Jibam's jump.
  The function $y = 2x$ represents the movement of the pebble.
- Identify the maximum height of Jibam's jump, the time the pebble touches Jibam and the time Jibam touches the ground.

**Planning the strategy**

(a) Determine maximum height from the graph $y = 6 + x - x^2$
(b) Determine the value of $x$ at the intersection of both graphs.
(c) Determine the value of $x$ when $y = 0$

**Implementing the strategy**

From the graph,
(a) the highest point is (0.5, 6.25), therefore the maximum height is 6.25 m.
(b) the intersection point of both graphs is (2, 4), the value of $x$ is 2. Therefore, the pebble touches Jibam at the 2nd second.
(c) when $y = 0$, $x = 3$. Therefore, Jibam touches the ground at the 3rd second.

**Conclusion**

Therefore,
(a) the maximum height of Jibam's jump is 6.25 metres.
(b) the pebble touches Jibam at the 2nd second.
(c) Jibam touches the ground at the 3rd second.

## SELF PRACTICE 8.2

1. Copy and complete the following table of values for the functions given.

(a) $y = 3x + 2$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | 5 | | | 14 |

(b) $y = 2x^2$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | | 8 | 18 | |

(c) $y = x^3 + 2$

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | –6 | | 2 | | | |

2. Construct a table of values for each of the following using the given value of $x$.

(a) $y = 2x - 2$ for $-3 \leqslant x \leqslant 3$.

(b) $y = 2x^2 + x - 5$ for $-1 \leqslant x \leqslant 3$.

(c) $y = 3x^3 - 6$ for $-2 \leqslant x \leqslant 4$.

3. Copy and complete the following tables of values for the functions given, and draw the graph using the given scale.

(a) $y = 5 + x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 2 | | 4 | 5 | | | 8 | 9 |

Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 1 unit on the $y$-axis, draw the graph of function $y = 5 + x$ for $-3 \leqslant x \leqslant 4$.

(b) $y = 4 - x^2$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | 0 | | | 3 | 0 | –5 |

Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 1 unit on the $y$-axis, draw the graph of function $y = 4 - x^2$ for $-3 \leqslant x \leqslant 3$.

(c) $y = 8 - x^3$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 35 | | 9 | 8 | | | –19 |

Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 10 units on the $y$-axis, draw the graph of function $y = 8 - x^3$ for $-3 \leqslant x \leqslant 3$.

(d) $y = \frac{4}{x}$

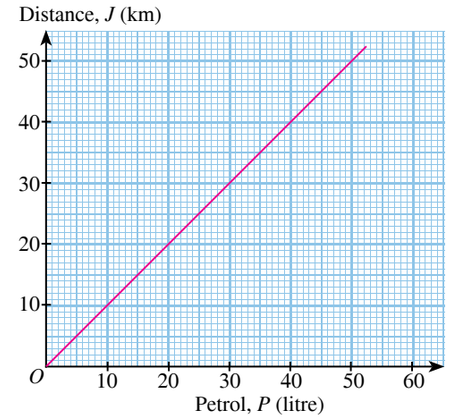
| $x$ | –4 | –3 | –2 | –1 | –0.5 | 0.5 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | –1 | –1.33 | | –4 | –8 | | 4 | | 1.33 | |

Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 2 units on the $y$-axis, draw the graph of function $y = \frac{4}{x}$ for $-4 \leqslant x \leqslant 4$.





4. The graph shows petrol $P$ (litre) used by a taxi for a distance of $J$ km.




From the graph,

(a) calculate how far will the taxi travel if its tank is filled with

(i) 30 litres of petrol

(ii) 42 litres of petrol

(b) calculate the cost of petrol for the taxi to travel 36 km if 1 litre of petrol costs RM 2.30.

5. Given a function $y = 5x^2 - 9x - 5$.

(a) Complete the table of values for the function above for $-2 \leqslant x \leqslant 3$.

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 33 | | –5 | –9 | | |

(b) Using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 5 units on the $y$-axis, plot all the points.

(c) Construct the graph of function.

(d) From the graph, determine the value of $x$ when $y = 0$.

## GENERATING EXCELLENCE

1. Determine if each of the relation is a function.

(a) {(0, 0), (1, 4), (2, 8), (3, 12)} (b) {(25, 5), (25, –5), (9, 3), (9, –3)}

2. Represent the relation of the set given in the form of ordered pairs, tables, graphs and equations.

(a) Set for integers, $B = \{1, 2, 3, 4, 5\}$
Set for multiples 11, $A = \{11, 22, 33, 44, 55\}$

(b) Set for integers, $I = \{1, 2, 3, 4, 5\}$
Set for perfect squares, $S = \{1, 4, 9, 16, 25\}$

3. The surface area of a ball $L$ in the shape of a sphere is the product of $4\pi$ with the square of its radius, $r$.

(a) State

(i) the dependent variable.

(ii) the independent variable.

(b) Write the relation between $L$ and $r$.

4. Given $T = \{1, 2, 3 ,4\}$ and $U = \{1, 8, 27, 64\}$. The relation from set $T$ to set $U$ is to the power of three. Represent the following functions in the form of

(a) ordered pair (b) table (c) graph (d) equation

5. Amira's father gave her RM100 as her spending money.

(a) If she spends RM2 every day, calculate the balance after

(i) 2 days (ii) 5 days (iii) 10 days

(b) Given RM$y$ represents the balance after $x$ days, complete the table of values below.

| $x$ | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | 70 | 60 | | 40 | | 20 | 10 | |

(c) Draw the graph for the function $y = 100 - 2x$ for $5 \leqslant x \leqslant 50$. Use the scale 2 cm to 10 units on the $x$-axis and 2 cm to 10 units on the $y$-axis.

(d) From the graph, calculate

(i) when Amira will spend all her money.

(ii) when Amira will have a balance of RM44.

6. Wilson wants to build a rectangular rabbit cage with the width $p$ metres and the length $3p$ metres. Given $A$ is the area of the cage, then $A = 3p^2$.

(a) Complete the table of values below for $0 \leqslant p \leqslant 6$

| $p$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $A$ | | | | 27 | | 75 | 108 |

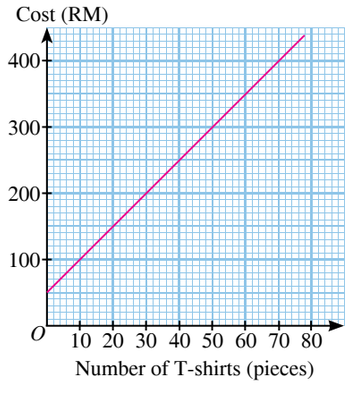
(b) Draw the graph of the function $A$ for $0 \leqslant p \leqslant 6$. Use the scale 2 cm to 1 unit on the $x$-axis and 2 cm to 10 units on the $y$-axis.

(c) Based on the graph, calculate

(i) the area of the rabbit cage when the width is 5.2 metres.

(ii) the area of the rabbit cage if Wilson has 40 metres of wire mesh.

7. Raj is the chairman of the Computer Club. He wants to order T-shirts for his club members from Puan Aini, the school cooperative teacher. Puan Aini has prepared a graph to show the cost in RM, with the number of T-shirts.




(a) Complete the table below based on the graph.

| **Number of T-shirts (pieces)** | 10 | 30 | 50 | 70 |
|---|---|---|---|---|
| **Cost (RM)** | | | | |

(b) After Raj studied the graph, he was surprised that 0 pieces of T-shirts cost RM50. If you were Puan Aini, what would your explanation be?

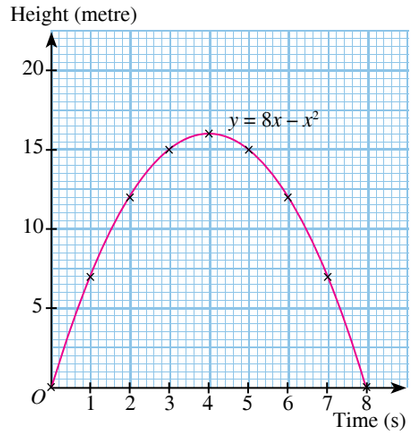
(c) Calculate the total cost Raj needs to pay for 68 pieces of T-shirts.

(d) If Raj has a budget of RM410, state the total number of T-shirts that he can order.





8. Nizam hits a golf ball. The height of the ball, $y$ metre from the surface of the ground after $x$ seconds is $y = 8x - x^2$. The graph shows the movement of the golf ball after being hit.




From the graph,

(a) what is the height of the ball at the third second?

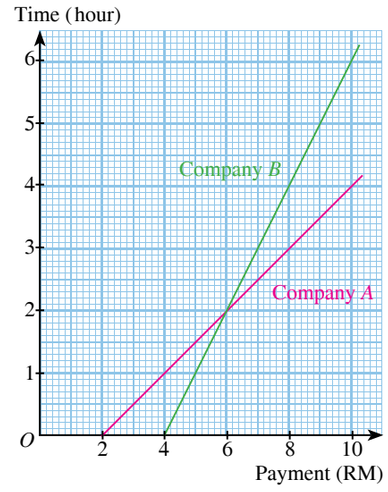
(b) calculate the time when the ball is at the height of 10 m.

(c) at which second will the ball fall on the ground?

(d) what is the maximum height achieved by the ball?

(e) what is the trend in the movement of the ball?

9. Zarul wants to rent a bicycle to go sightseeing at the recreational park. There are two shops offering bicycle rental services, Company $A$ and Company $B$. The graph shows the hourly rate charged by each company.




(a) How much does Company $A$ charge to rent a bicycle for 3 hours?

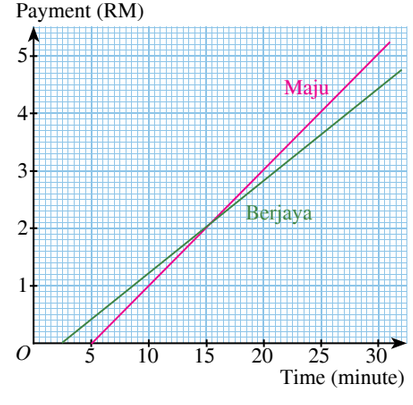
(b) If Zarul wants to rent a bicycle for only one hour, which company offers a cheaper rate? Explain.

(c) Zarul has RM7. From which company should he rent the bicycle? Explain.

(d) After which hour the payment charged at both bicycle companies will be the same.

(e) If Zarul rents a bicycle for 6 hours from Company $B$, how much must he pay?

10. Maju and Berjaya are two telecommunication companies offering packages for a mobile phone prepaid plan. The graph shows the payment rate and talk time offered by both companies.




(a) How much is the payment for 20 minutes talk time charged by

(i) Maju

(ii) Berjaya

(b) If Erin uses more than 30 minutes of talk time in a month, which company offers a cheaper rate? Explain.

(c) Umai wants to spend only RM4 for the service. Which company should Umai choose? Explain.

## CHAPTER SUMMARY

**Functions**

It is the relation in which each object in the domain has only one image in the codomain.

**One-to-one functions**
**Many-to-one functions**

**Representation of functions**

- Ordered Pairs
- Tables
- Graphs
- Equations

**Graphs of functions**

Representation of a function with a straight line or curve on a Cartesian plane.

Table of values is a table that shows values for independent and dependent variables and it is constructed before a graph is drawn.

Steps to draw a graph.

1. Construct a table of values for the selected range.
2. Draw and label axes with the scale given.
3. Plot points from the table of values.
4. Join the points to construct the graph.





## SELF REFLECTION

**At the end of the chapter, I am able to:**

| | |
|---|---|
| 1. Explain the meaning of functions. | ☺ 😐 ☹ |
| 2. Identify function and provide justifications based on function representations in the form of ordered pairs, tables, graphs and equations. | ☺ 😐 ☹ |
| 3. Construct tables of values for linear and non-linear functions and draw the graphs using the scale given. | ☺ 😐 ☹ |
| 4. Interpret graphs of functions. | ☺ 😐 ☹ |

## MINI PROJECT

You are required to design a greeting card by using the function given. Complete the table of values. Draw seven graphs of the functions below by using a scale of 2 cm to 1 unit on the $x$-axis and 2 cm to 2 units on the $y$-axis. Label each graph and colour each region with your favourite colours. Then, cut out the graph paper according to your chosen size and make sure that the coloured design covers the whole surface area. Paste it on a manila card and decorate it creatively into a greeting card.

Example of design from several graphs of functions

| Number | Function ($y$) | $x$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
| 1 | $y = x$ | | | | | | | |
| 2 | $y = -x + 9$ | | | | | | | |
| 3 | $y = 2x + 8$ | | | | | | | |
| 4 | $y = -2x + 4$ | | | | | | | |
| 5 | $y = x^2 - 3$ | | | | | | | |
| 6 | $y = -x^2 + 13$ | | | | | | | |
| 7 | $y = -x^3 + 5$ | | | | | | | |
| 8 | $y = 2x^{-2}$ | | | | | | | |

# CHAPTER 9

# Speed and Acceleration

## WHAT WILL YOU LEARN?

**9.1** Speed

**9.2** Acceleration

## WORD LINK

| | |
|---|---|
| • Speed | • *Laju* |
| • Distance | • *Jarak* |
| • Time | • *Masa* |
| • Unit | • *Unit* |
| • Acceleration | • *Pecutan* |
| • Deceleration | • *Nyahpecutan* |
| • Average speed | • *Laju purata* |
| • Uniform speed | • *Laju seragam* |
| • Non-uniform speed | • *Laju tak seragam* |
| • Stationary | • *Pegun* |



All our daily activities involve movement and speed. Speed change occurs when there is an activity that causes movement.

Azizulhasni Awang, our national cyclist has an illustrious career with stunning display of speed. He won a gold medal in the Track Cycling World Championship in Hong Kong for Men's keirin event.

## WALKING THROUGH TIME

Galileo Galilei is the first scientist who measured speed as distance per time.

For more information:

http://rimbunanilmu.my/mat_t2e/ms169

## WHY STUDY THIS CHAPTER?

- The knowledge in this chapter can be applied in the field of careers such as automotive engineering, astronauts, physics studies, sports and astronomy.

**CREATIVE ACTIVITY**

**Aim:** Introducing speed and acceleration.
**Materials:** Three sets of remote control cars, stopwatch, racing track and whistle.
**Steps:**
1. Choose three students.
2. Each person is given the same type of remote control car.
3. Each person uses the remote control to move the car on the track provided when the whistle is blown.
4. Record the time and winner for this activity.
5. What is the relationship between time, speed and acceleration in winning the race?

## 9.1 Speed

### 9.1.1 Speed as a rate

**LEARNING STANDARD**

Explain the meaning of speed as a rate involving distance and time.

Take a look at the back of a truck or bus. There is a speed limit symbol for each of the vehicles. What does the symbol mean? What will the consequences be if the driver does not comply with the speed limit stated on the symbol?


**COGNITIVE STIMULATION** Group

**Aim:** Explaining the meaning of speed as a rate involving distance and time.
**Materials:** Stop watch, manila card (50 m sprint results sheet)
**Steps:**
1. Students form groups of four.
2. Select three students as runners from each group.
3. The students will run 50 m on the track provided.
4. Records the time taken by each student.
5. Complete the table in the manila card.
   Next, calculate the value of distance divided by time taken for each runner.

| Name of runner | Distance (m) | Time (s) | $\frac{\text{Distance (m)}}{\text{Time (s)}}$ |
|---|---|---|---|
| | 50 | | |
| | 50 | | |
| | 50 | | |

6. Display your group's result.





**Discussion:**
(i) List the names of the first, second and third place winners in your group.
(ii) What conclusions can the group make based on the results?

In the activity above, the first student has completed the run in the shortest possible time and the last student recorded the longest time.

The students ran an equal distance, so the student's speed was the ratio of the distance over their running time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

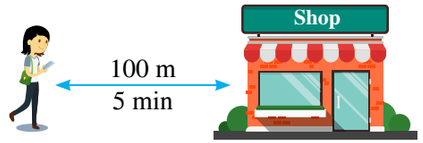
**TIPS**

If I walk 10 km in an hour, then my speed is 10 km/h. If a particle moves 1 metre in one second, then the speed of the particle is 1 m/s.

**EXAMPLE 1**

Aida walks to a shop over a distance of 100 m in 5 minutes.
Calculate the speed.

*Solution:*




$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{100 \text{ m}}{5 \text{ min}}$$

$$= 20 \text{ m/min}$$

Therefore, Aida walks 20 metres every minute.

**TIPS**

m/min is read as "metre per minute".

**EXAMPLE 2**

Khairul Hafiz and Badrul Hisham are young state sprinters. During a 100 m event, Khairul Hafiz finished in 10.18 seconds while Badrul Hisham finished in 10.25 seconds. Calculate their speed respectively.

*Solution:*

$$\text{Speed of Khairul Hafiz} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{100 \text{ m}}{10.18 \text{ s}}$$

$$= 9.82 \text{ m/s}$$

$$\text{Speed of Badrul Hisyam} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{100 \text{ m}}{10.25 \text{ s}}$$

$$= 9.76 \text{ m/s}$$

**TIPS**


$D$ : Distance
$S$ : Speed
$M$ : Time

$$S = \frac{D}{M}$$

$$M = \frac{D}{S}$$

$$D = S \times M$$

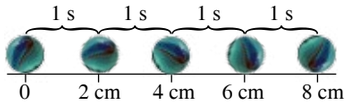
## 9.1.2 Uniform and non-uniform speed

**LEARNING STANDARD**

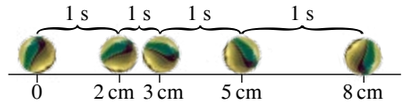
Describe the differences between uniform and non-uniform speed.

Look at the movement of the marbles in the diagram below. Marble *A* and marble *B* are rolled on the table. The movement of the marbles is described as follows.

**Marble *A***

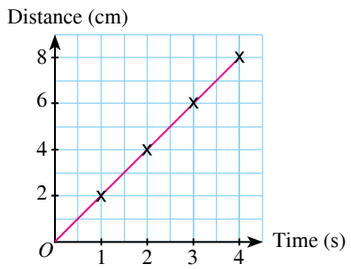



**Marble *B***

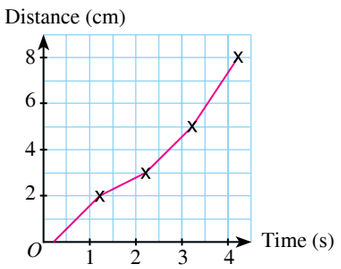



Compare the distance travelled by marbles *A* and *B* within 4 seconds. Which marble has a uniform speed and which marble has a non-uniform speed? Explain.

**Distance-Time Graph for Marble *A***




**Distance-Time Graph for Marble *B***




**Marble *A***

Marble *A* moves at equal distance in equal interval of time. Thus, marble *A* moves at **a uniform speed**.

Therefore, **uniform speed** is the speed that covers equal distance in equal interval of time.

**Marble *B***

Marble *B* moves at unequal distance in equal interval of time. Thus, marble *B* moves at **a non-uniform speed**.

Therefore, **non-uniform speed** is the speed that covers unequal distance in equal interval of time.

**EXAMPLE 3**

Farid drove a trailer for 170 km within the first 2 hours and 190 km within the next 4 hours. Did Farid drive the trailer at a uniform speed? Explain.

***Solution:***

$$\text{Speed for the first 2 hours} = \frac{170\text{ km}}{2\text{ h}}$$
$$= 85\text{ km/h}$$

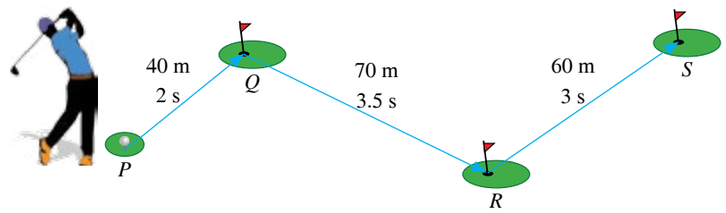
$$\text{Speed for next 4 hours} = \frac{190\text{ km}}{4\text{ h}}$$
$$= 47.5\text{ km/h}$$

Therefore, the speed of the trailer is **non-uniform**.

**TIPS**

km/h can also be written as $\frac{\text{km}}{\text{h}}$ or $\text{kmh}^{-1}$.

**EXAMPLE 4**




Encik Mahesh hit the golf ball from pole *P* to pole *S* passing through pole *Q* and pole *R*. Did the golf ball have a uniform speed? Explain.

***Solution:***

$$\text{Speed of ball from } P \text{ to } Q = \frac{40\text{ m}}{2\text{ s}}$$
$$= 20\text{ m/s}$$

$$\text{Speed of ball from } Q \text{ to } R = \frac{70\text{ m}}{3.5\text{ s}}$$
$$= 20\text{ m/s}$$

$$\text{Speed of ball from } R \text{ to } S = \frac{60\text{ m}}{3\text{ s}}$$
$$= 20\text{ m/s}$$

Therefore, the speed of the golf ball is **uniform**.

**THINK SMART**

| Time (s) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Distance (m) | 0 | 10 | 20 | 30 | 40 |

| Time (s) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Distance (m) | 0 | 3 | 7 | 16 | 30 |

Draw the distance-time graph. From the graph conclude on uniform and non-uniform speed.

## 9.1.3 Average speed

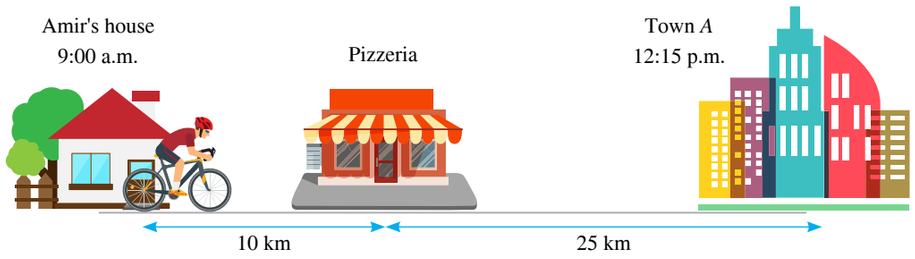
**LEARNING STANDARD**

Perform calculation involving speed and average speed including unit conversion.

The Electric Train Service (ETS) from Kuala Lumpur to Butterworth moves at a non-uniform speed. In this situation, the average speed is used to give an idea of the speed of the train.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

## EXAMPLE 5




Amir started cycling from his house to town *A* at 9:00 a.m.. Along the way, he stopped at a pizzeria to rest and eat before he resumed his journey. He arrived in town *A* at 12:15 p.m.. Calculate the average speed of the journey in km/h.

***Solution:***

Total distance = 10 km + 25 km
= 35 km

Total time taken = 12:15 – 9:00
= 3 h 15 min

$$\text{Average speed} = \frac{35 \text{ km}}{3.25 \text{ h}}$$

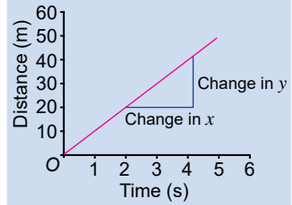
$$= 10.77 \text{ km/h}$$

3 h 15 min
$= 3 \text{ h} + \left(\frac{15}{60}\right) \text{h}$
= 3 h + 0.25 h
= 3.25 h

**TIPS**

DISTANCE-TIME GRAPH

Gradient of distance-time = Speed graph




$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x}$$

$$= \frac{20 \text{ m}}{2 \text{ s}}$$

Speed = 10 m/s

## EXAMPLE 6

A bus departs from Puchong at 0825 and arrives in Perai at 1345. If the total distance travelled is 354 km, calculate the average speed in km/h.

***Solution:***

Total distance = 354 km

Total time taken = 1345 – 0825
= 5 hours 20 min

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$= \frac{354 \text{ km}}{5.33 \text{ h}}$$

$$= 66.42 \text{ km/h}$$

$5 \text{ h} + \left(\frac{20}{60}\right) \text{h}$
= 5 h + 0.33 h
= 5.33 h





## EXAMPLE 7

The speed limit on Jalan Persekutuan is 90 km/h. Convert this speed to
(a) m/s (b) km/min

***Solution:***

(a) $90 \text{ km/h} = \frac{90 \text{ km}}{1 \text{ h}}$

$$= \frac{90 \times 1\,000 \text{ m}}{1 \times 3\,600 \text{ s}}$$

$$= \frac{90\,000 \text{ m}}{3\,600 \text{ s}}$$

= 25 m/s

(b) $90 \text{ km/h} = \frac{90 \text{ km}}{1 \text{ h}}$

$$= \frac{90 \text{ km}}{1 \times 60 \text{ min}}$$

$$= \frac{90 \text{ km}}{60 \text{ min}}$$

= 1.5 km/min

## EXAMPLE 8

Convert 120 m/s to km/min.

***Solution:***

$$120 \text{ m/s} = \frac{120 \text{ m}}{1 \text{ s}}$$

$$= \frac{120 \div 1\,000 \text{ km}}{1 \div 60 \text{ min}}$$

$$= \frac{0.12 \text{ km}}{0.017 \text{ min}}$$

= 7.06 km/min

**FLASHBACK**

1 km = 1 000 m
1 m = 100 cm

×1 000 ×100
km m cm
÷1 000 ÷100

1 hour = 60 minutes
1 minute = 60 seconds

×60 ×60
hour minute second
÷60 ÷60

## EXAMPLE 9

Karmila has won a gold medal in the women's triathlon event. She began the event with a 10 km run, followed by a 1 500 m swim and ended with a 40 km cycling. She completed the event in 1 hour 56 minutes. Calculate the average speed in km/h for the entire event.

***Solution:***

$$\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$= \frac{10 \text{ km} + 1\,500 \text{ m} + 40 \text{ km}}{1 \text{ hour } 56 \text{ min}}$$

$$= \frac{10 \text{ km} + 1.5 \text{ km} + 40 \text{ km}}{1.93 \text{ h}}$$

$$= \frac{51.5 \text{ km}}{1.93 \text{ h}}$$

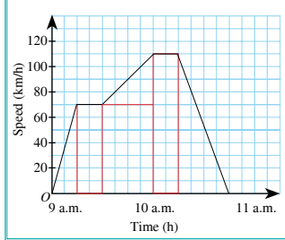
= 26.68 km/h

$\frac{1\,500}{1\,000}$ m
= 1.5 km

1 h + 56 min
$= 1 \text{ h} + \left(\frac{56}{60}\right) \text{h}$
= 1 h + 0.93 h
= 1.93 h

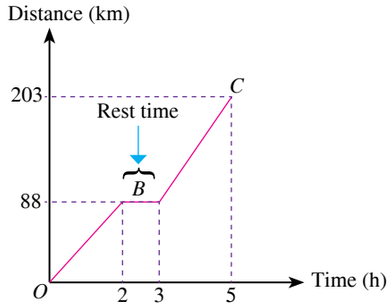
**DO YOU KNOW?**

The area under the graph is equal to distance.



**EXAMPLE 10**

Siti joined a school trip to Kuala Lumpur. The school bus departed from school ($O$) at 7:00 a.m.. On their way, they stopped at Ulu Bernam rest area $B$ for a short break. They continued their journey to Kuala Lumpur ($C$). The graph shows the movement of the bus from the school to Kuala Lumpur. Calculate the average speed of the trip in km/h.




*Solution:*

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{203 \text{ km}}{5 \text{ h}}$$

$$= 40.6 \text{ km/h}$$

## 9.1.4 Solving problems

**LEARNING STANDARD**

Solve problems involving speed.

**EXAMPLE 11**

Khairul Idham Pawi has made the country proud in the World Motorcycle Championship (MotoGP) when he won the Moto3 category at the German Grand Prix. He took 47 minutes 8 seconds to finish the race on the 40.38 km track. Calculate

(a) Khairul's motorcycle speed in km/h.

(b) the difference in time between Khairul's speed and second place winner's speed if the latter speed was 0.85 km/min.

*Solution:*

**Understanding the problem**

- Race distance = 40.38 km.
- Race time taken = 47 minutes 8 seconds.
- Calculate the speed of Khairul's motorcycle and difference in time with the second place winner.

**Planning the strategy**

- Time = $\frac{\text{Distance}}{\text{Speed}}$
- Time difference = Second place winner – Winner's time





**Implementing the strategy**

(a) Khairul's motorcycle speed = $\frac{\text{Distance}}{\text{Time}} = \frac{40.38 \text{ km}}{47 + \left(\frac{8}{60}\right) \text{ min}}$ ← Change unit: seconds to minute.

$$= \frac{40.38 \text{ km}}{47.13 \text{ min}}$$

$$= \frac{40.38 \text{ km}}{\left(\frac{47.13}{60}\right) \text{hour}}$$ ← Change unit: minutes to hour.

$$= \frac{40.38 \text{ km}}{0.79 \text{ hour}}$$

$$= 51.11 \text{ km/h}$$

(b) The second place winner's motorcycle time = $\frac{\text{Distance}}{\text{Speed}}$

$$= \frac{40.38 \text{ km}}{0.85 \text{ km/min}}$$

$0.5 \times 60 = 30$ seconds to change from minute to second.

$$= 47.5 \text{ mins}$$

$$= 47 \text{ mins } 30 \text{ s}$$

Time difference between Khairul and the second place winner = 47 mins 30 s – 47 mins 8 s

= 22 s

**Conclusion**

(a) The speed of Khairul's motorcycle is 51.11 km/h.

(b) The time difference between the second place winner and Khairul is 22 seconds.

**EXAMPLE 12**

Mr Tan took 3 hours 7 minutes to drive from Kuala Lumpur to Skudai at an average speed of 103 km/h. However he took $\frac{3}{4}$ of an hour longer on his return trip from Skudai to Kuala Lumpur. Calculate the average speed of Mr Tan's return journey in km/h.

*Solution:*

**Understanding the problem**

- Average speed = 103 km/h.
- Duration of the trip = 3 hours 7 minutes.
- Return period = $\frac{3}{4}$ of an hour more than the trip to Skudai.
- Calculate the average speed of Mr Tan's return journey.

**Planning the strategy**

- Distance = Speed × Time
- Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

**Implementing the strategy**

Distance from Kuala Lumpur to Skudai

$= 103 \text{ km/h} \times \left(3 + \frac{7}{60}\right) \text{h}$

Change unit from minute to hour

$= 103 \text{ km/h} \times 3.12 \text{ h}$

$= 321.36 \text{ km}$

Average speed of Mr Tan's return journey

$= \frac{321.36 \text{ km}}{3 \text{ h} + \left(\frac{7 + 45}{60}\right) \text{h}}$ ← $\frac{3}{4}$ h = 45 minutes

Change unit from minute to hour

$= \frac{321.36 \text{ km}}{3 \text{ h} + 0.87 \text{ h}}$

$= \frac{321.36 \text{ km}}{3.87 \text{ h}}$

$= 83.04 \text{ km/h}$

**Conclusion**

The average speed of Mr Tan's return journey is 83.04 km/h.

## SELF PRACTICE 9.1

1. Match the correct time for the given distance and speed.

| | | |
|---|---|---|
| Speed = 44.1 km/h<br>Distance = 150 km | Speed = 120 km/h<br>Distance = 90 km | Speed = 125 m/s<br>Distance = 500 m |
| 4 seconds | 3 hour 24 minutes | 45 minutes |

2. Based on the following diagram, calculate the distance travelled for each given situation.

(a) 

Speed = 80 km/h, Time = $1\frac{1}{2}$ hours

(b) 

Speed = 343 km/min, Time= 4.5 minutes

(c) 

Speed = 3 m/min, Time = 5.5 minutes

(d) 

Speed = 250 km/h, Time = 2 hours 40 minutes





3. Malaysia's Paralympic athlete, Mohamad Ridzuan Mohamad Puzi clocked the fastest time of 12.07 seconds in the 100 metre event at the 2016 Paralympic Games in Rio de Janeiro, Brazil. Calculate the speed in the m/s.

4. The distance from Tanjung Malim to Muar is 272 km. A bus departs from Tanjung Malim at 0830. The average speed of the bus is 80 km/h. At what time will the bus arrive in Muar? State your answer in the 24-hour system.

5. Convert the following speed unit to the specified unit.
(a) 50 km/h to m/min. (b) 0.8 m/s to km/h. (c) 110 km/h to km/min.

6. Umar drove his taxi from Ipoh to Kuala Lumpur via the highway. He stopped at Tapah to pick up some goods before proceeding to Kuala Lumpur. He drove from Ipoh to Tapah at an average speed of 100 km/h over a distance of 60 km. If the average speed of his taxi from Ipoh to Kuala Lumpur is 110 km/h over a distance of 220 km, calculate the average speed of Umar's taxi from Tapah to Kuala Lumpur.

7. A leopard can reach a speed of 25.9 m/s, especially when chasing its prey. State the speed in km/h.

# 9.2 Acceleration

## 9.2.1 Acceleration and deceleration

LEARNING STANDARD

Explain the meaning of acceleration and deceleration as a rate involving speed and time.

Sprinters begin a race at the starting line. Once the race begins, their speed will increase as they sprint towards the finishing line. The increase in speed results in acceleration.

After they have passed the finishing line, they will slow down. The decrease in speed results in deceleration.

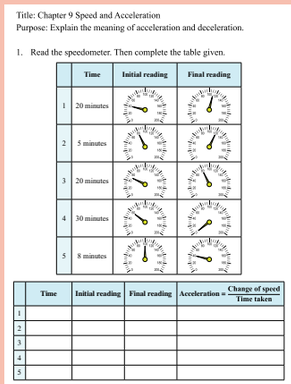
**COGNITIVE STIMULATION** Individual

**Aim:** Explaining the meaning of acceleration and deceleration

**Material:** Worksheets

**Steps:**

1. Open the file MS179.
2. Complete the table.
3. State whether it is an acceleration or a deceleration.



Title: Chapter 9 Speed and Acceleration
Purpose: Explain the meaning of acceleration and deceleration.

1. Read the speedometer. Then complete the table given.

| | Time | Initial reading | Final reading |
|---|---|---|---|
| 1 | 20 minutes | | |
| 2 | 5 minutes | | |
| 3 | 20 minutes | | |
| 4 | 30 minutes | | |
| 5 | 8 minutes | | |

| | Time | Initial reading | Final reading | Acceleration = $\frac{\text{Change of speed}}{\text{Time taken}}$ |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |

**DO YOU KNOW?**

The speed of a vehicle is usually expressed in rotation per minute (rpm).

**QR CODE**

Scan the QR Code or file http://rimbunanilmu.my/mat_t2e/ms179 to get the worksheet.

CHAPTER 9

The change of speed can be calculated by finding the difference between the final speed and the initial speed of a linear moving object. From the activity, the increase in speed results in acceleration and the decrease in speed results in deceleration. Therefore, acceleration and deceleration is a rate involving speed and time.

**EXAMPLE 13**

A racing car accelerates from a stationary state and reaches a speed of 120 km/h in 6 seconds. Calculate the acceleration.

***Solution:***

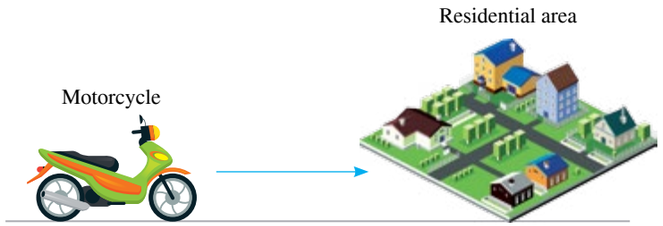
$$\text{Change of speed} = 120 \text{ km/h} - 0 \text{ km/h}$$
$$= 120 \text{ km/h}$$
$$\frac{120 \text{ km}}{1 \text{ h}} = \frac{120 \text{ km}}{60 \times 60 \text{ s}}$$ ← 60 x 1 min
$$= 0.033 \text{ km/s}$$
$$\text{Acceleration} = \frac{0.033 \text{ km/s}}{6 \text{ s}}$$ ← $\frac{120 \text{ km}}{1 \text{ hour}} = \frac{120 \text{ km}}{3\,600 \text{ s}}$
$$= 0.0056 \text{ km/s per second or km/s}^2$$

**ATTENTION**

The stationary state is the fixed state of an object.

Speed = 0

**EXAMPLE 14**




A motorcycle moves from a stationary state and accelerates uniformly to reach a speed of 20 m/s in 5 seconds. What is the speed of the motorcycle?

***Solution:***

$$\text{Acceleration} = \frac{(20 - 0)\text{m/s}}{5\text{s}}$$
$$= \frac{20 \text{ m/s}}{5\text{s}}$$
$$= 4 \text{ m/s}^2$$

**TIPS**

Uniform acceleration means the speed increases at a similar rate.

| Time (s) | Speed (m/s) |
|---|---|
| 0 | 0 |
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| 4 | 16 |
| 5 | 20 |

**DO YOU KNOW ?**

If an object moves at a uniform speed, the acceleration is zero.





After the motorcyclist applies the brake, the motorcycle moves slower at a uniform rate until it stops within 4 seconds. What is the speed of the motorcycle?

***Solution:***

$$\text{Acceleration} = \frac{(0 - 20)\text{m/s}}{4\text{s}}$$
$$= \frac{-20 \text{ m/s}}{4\text{s}}$$
$$= -5 \text{ m/s}^2$$

Thus, deceleration is 5 m/s$^2$.

**ATTENTION**

For deceleration, the negative sign does not need to be written
Example:
Acceleration = –5 m/s$^2$
or
Deceleration = 5 m/s$^2$

**DO YOU KNOW ?**

When any object falls from a high point due to gravitational pull, the acceleration is 9.81 ms$^{-2}$.

Units commonly used to measure acceleration

| Speed unit | Time unit | Acceleration unit |
|---|---|---|
| km/h | hour | km/h$^2$ or kmh$^{-2}$ or km/h per hour |
| m/s | second | m/s$^2$ or ms$^{-2}$ or m/s per second |

## 9.2.2 Unit conversion

**LEARNING STANDARD**

Perform calculations involving acceleration including unit conversion.

**EXAMPLE 15**

Convert 420 m/min per min to km/min per minute.

***Solution:***

$$\frac{420 \text{ m/min}}{\text{min}} = \frac{420 \text{ m}}{\text{min}} \div \text{min}$$
$$= \frac{420 \times 1 \text{ m}}{\text{min}} \div \text{min}$$
$$= \left(\frac{420 \times \frac{1}{1\,000} \text{ km}}{\text{min}}\right) \div \text{min}$$
$$= \frac{\frac{420}{1\,000} \text{km}}{\text{min}} \div \text{min}$$
$$= \frac{0.42 \text{ km}}{\text{min}} \times \frac{1}{\text{min}}$$
$$= 0.42 \text{ km/min}^2$$

**EXAMPLE 16**

Rani cycles for $\frac{3}{4}$ of an hour from her home to the Cultural Festival held in the city with a speed change of 18 km/h. Traveling to the Cultural Festival takes 40% less time compared to the time for the return trip at the same speed change. Calculate the difference in acceleration between the trips to and from the Cultural Festival.

***Solution:***

$$\text{Acceleration when going} = \frac{18 \text{ km/hour}}{\frac{3}{4} \text{ hour}}$$
$$= 24 \text{ km/hour}^2$$

**TIPS**

Unit km/hour$^2$ can be written as kmh$^{-2}$ or km/h$^2$.

$$\text{Time reduced} = \left(\frac{40}{100}\right) \times \frac{3}{4} \text{ hour}$$
$$= 0.3 \text{ hour}$$

$\frac{3}{4}$ hour

$$\text{Time taken for return trip} = 0.75 \text{ hour} - 0.3 \text{ hour}$$
$$= 0.45 \text{ hour}$$

$$\text{Acceleration during return trip} = \frac{18 \text{ km/hour}}{0.45 \text{ hour}}$$
$$= 40 \text{ km/hour}^2$$

$$\text{Difference in acceleration} = 40 \text{ km/hour}^2 - 24 \text{ km/hour}^2$$
$$= 16 \text{ km/hour}^2$$

**EXAMPLE 17**

Samy drove at a speed of 70 km/h. He increased his speed to 100 km/h in 30 minutes. Calculate the acceleration in

(a) km/h per hour (b) km/h per second

***Solution:***

(a) Change of speed = 100 km/h – 70 km/h
= 30 km/h

$$\text{Acceleration} = \frac{30 \text{ km/h}}{30 \text{ min}}$$
$$= \frac{30 \text{ km/h}}{\left(\frac{1}{2}\right) \text{h}}$$

30 minutes = $\frac{1}{2}$ hour

$$= 60 \text{ km/h per hour}$$

(b) Time = 30 minutes ← 30 x 1 min
= 30 × 60 s
= 1 800 seconds

$$\text{Acceleration} = \frac{30 \text{ km/h}}{1\,800 \text{ s}}$$
$$= 0.0167 \text{ km/h per second}$$





## 9.2.3 Solving problems

**LEARNING STANDARD**

Solve problems involving acceleration.

**EXAMPLE 18**

Lisnah accelerates her car 4 km/h per second while overtaking a car. If she has been driving at 100 km/h, calculate her speeds after 5 seconds.

***Solution:***

**Understanding the problem**
- Acceleration = 4 km/h per second
- Began driving at 100 km/h
- Time = 5 seconds
- Calculate her speeds after 5 seconds

**Planning the strategy**

Acceleration is the result of increase in speed.

**Implementing the strategy**

$$\frac{4 \text{ km/h}}{\text{s}} = \frac{\text{Speed after 5 seconds} - 100 \text{ km/h}}{5 \text{ s}}$$

$$\frac{4 \text{ km/h}}{\text{s}} \times 5 \text{ s} = \text{Speed after 5 seconds}$$

20 km/h = Speed after 5 seconds – 100 km/h

$$\text{Speed after 5 seconds} = 20 \text{ km/h} + 100 \text{ km/h}$$
$$= 120 \text{ km/h}$$

**Conclusion**

Speed after 5 seconds is 120 km/h.

**EXAMPLE 19**

A motorcycle that moves at a speed of 40 km/h decreases its speed by 20% of it beginning speed within 40 seconds. Calculate the acceleration.

***Solution:***

**Understanding the problem**
- Began riding at 40 km/h
- Time = 40 seconds
- Calculate the acceleration

**Planning a strategy**

Acceleration is the result of increase in speed.

**Implementing the strategy**

Speed reduction: 100% – 20% = 80%

$$\text{End speed} = \frac{80}{100} \times 40 \text{ km/h}$$
$$= 32 \text{ km/h}$$

$$\text{Acceleration} = \frac{(32 - 40) \text{ km/h}}{40 \text{ s}}$$
$$= -0.2 \text{ km/h per second}$$

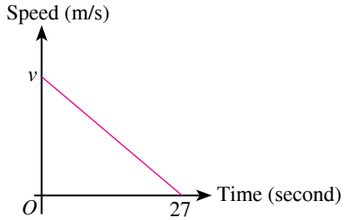
Deceleration = 0.2 km/h per second

**Conclusion**

The motorcycle acceleration is – 0.2 km/h per second.

## EXAMPLE 20

The diagram shows a speed-time graph for the movement of a toy lorry in 27 seconds. The deceleration of the toy lorry is 0.741 m/s$^2$.




(a) Calculate the speed, $v$, in m/second.

(b) Calculate the distance covered by the toy lorry after 2.2 seconds.

***Solution:***

**Understanding the problem**
- Acceleration = $-0.741$ m/s$^2$
- Duration = 27 seconds
- Calculate the speed, $v$.
- Calculate distance after 2.2 seconds

**Planning the strategy**

Distance = Speed × Time
Speed = Deceleration × Time

**Implementing the strategy**

(a) $-0.741 \text{ m/s}^2 = \dfrac{0 - v}{27 \text{ s}}$

$-0.741 \text{ m/s}^2 \times 27 \text{ s} = 0 - v$

$v = 20 \text{ m/s}$

(b) Distance = Speed × Time
= 20 m/s × 2.2s
= 44 m

**Conclusion**

(a) Speed of toy lorry movement is 20 m/s.
(b) Distance covered by the toy lorry is 44 m.

## SELF PRACTICE 9.2

1. State whether each of the following statements is True or False.

| Situation | Acceleration | True/False |
|---|---|---|
| (a) The speed of a ball rolling on the floor is reduced from 12 cm/s to 2 cm/s in 4 seconds. | –2.5 cms$^{-2}$ | |
| (b) A trailer accelerates from 90.5 km/h to 123 km/h in $\frac{3}{4}$ hours. | 40 kmh$^{-2}$ | |
| (c) A coconut fell from a tree at a speed of 7 m/s in 0.71 s. | 9.86 ms$^2$ | |
| (d) Puan Mages reduced the speed of her car from 80 km/h to 60 km/h in 0.5 hours | 54 km/h$^2$ | |

2. Calculate the acceleration for the following situations.
   (a) A car accelerates from 60 km/h to 110 km/h in 30 minutes.
   (b) The speed of a boat decreases from 70 km/h to 40 km/h in 5 minutes.





3. Vinod cycled to his aunt's house at a speed of 8 m/s. Within 4 seconds, he increased his speed to 10 m/s. Calculate the acceleration in ms$^{-2}$.

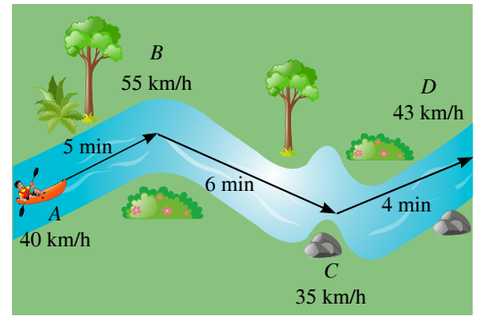
4. Based on an experiment, the speed of an object decreases from 145 cm/s to 75 cm/s in 8 seconds. Calculate the deceleration.

## GENERATING EXCELLENCE

1. Categorise the object in the box whether it has uniform or non-uniform speed.

| lift | hour | waves | fan | wind | mini bus |
|---|---|---|---|---|---|

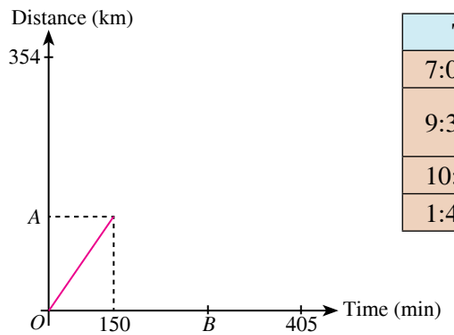
2. Wafi participated in a kayaking competition at Sungai Lembing. He started the competition from station $A$ and proceeded to $B$, $C$ and $D$.




Based on the given information, calculate the kayak acceleration from
(a) station $A$ to station $B$.
(b) station $B$ to station $C$.
(c) station $C$ to station $D$.

3. Every morning Shu Mei cycles to school from her home via the post office. The distance from her home to the post office is 4 km, while the distance from the post office to the school is 5 km. Given the average speed of the bicycle is 18 km/h, calculate
   (a) the time for the entire trip for Shu Mei to reach school in minutes.
   (b) the time Shu Mei reaches her school if she starts riding her bicycle at 6:40 a.m.

4. Syahmi drove 354 km from Kuala Lumpur to his hometown in Terengganu. The table below shows his travel notes.



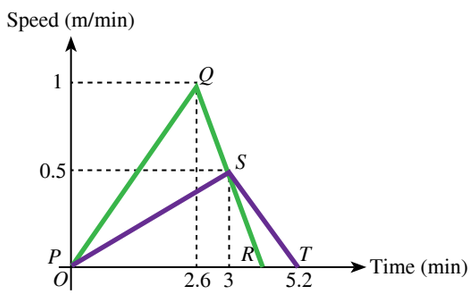

| Time | October 21, 2017 / Saturday |
|---|---|
| 7:00 a.m. | Started the trip. |
| 9:30 a.m. | Stopped at Temerloh Rest and Service Area for breakfast after driving 185 km. |
| 10:05 a.m. | Continued journey to Terengganu. |
| 1:45 p.m. | Arrived at the village. |

(a) State the value of $A$ and $B$.

(b) Complete the graph for Syahmi's whole journey.

(c) Calculate the average speed, in km/h, for the whole trip.

5. The diagram below shows the speed-time graph for the movement of two marbles from the opposite directions. The $PQR$ graph represents the movement of the green marbles and the $PST$ graphs represent the movement of the purple marbles. Both marbles are on the same path.




(a) Calculate the acceleration of the green marbles within the first 2.6 minutes.

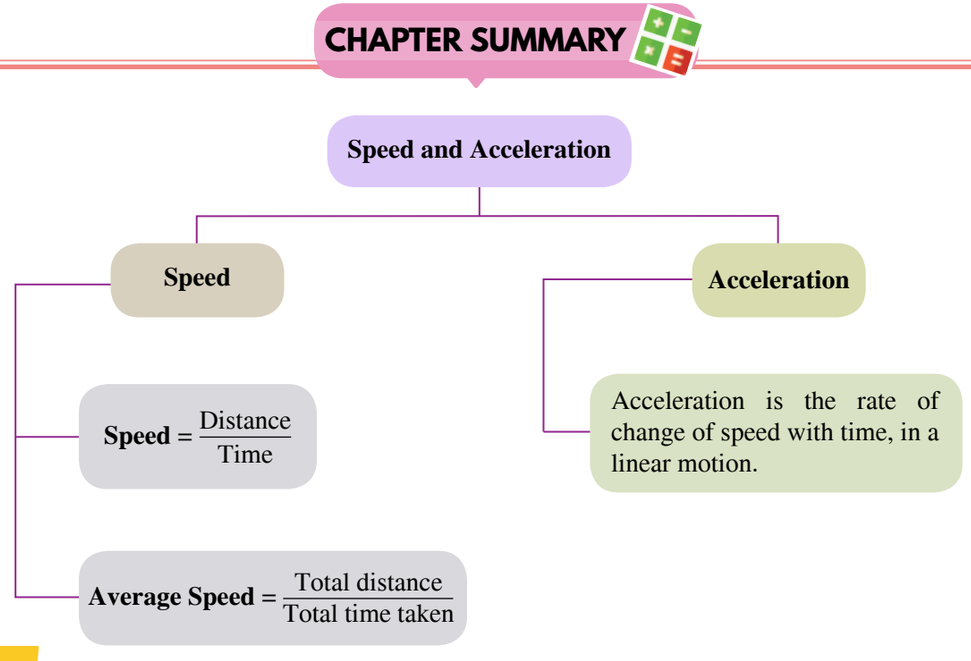
(b) When will the purple marble stop moving?

(c) What is the maximum speed of the green marbles?

(d) Calculate the time in which the two marbles collide.

6. The distance between Tanjung Malim and Sungai Petani is $x$ km. A car was driven from Tanjung Malim to Sungai Petani at an average speed of 90 km/h. The return journey when the car was driven at an average speed of 105 km/h took 30 minutes. Calculate the value of $x$.

## CHAPTER SUMMARY








## SELF REFLECTION

**At the end of the chapter, I will be able to:**

| | |
|---|---|
| 1. Explain the meaning of speed as a rate involving distance and time. | ☺ ☹ |
| 2. Describe the differences between uniform and non-uniform speed. | ☺ ☹ |
| 3. Perform calculation involving speed and average speed including unit conversion. | ☺ ☹ |
| 4. Solve problems involving speed. | ☺ ☹ |
| 5. Explain the meaning of acceleration and deceleration as a rate involving speed and time. | ☺ ☹ |
| 6. Perform calculations involving acceleration including unit conversions. | ☺ ☹ |
| 7. Solve problems involving acceleration. | ☺ ☹ |

## MINI PROJECT

Heading the speed limit is one of the regulations when travelling on the road. Maximum speed limit is determined according to certain areas. Compliance to the speed limit is very important to ensure the safety of road users.

You are required to make a report of the speed limit in the following areas.

(a) School

(b) Hospital / clinic

(c) Highway

(d) Hilly areas

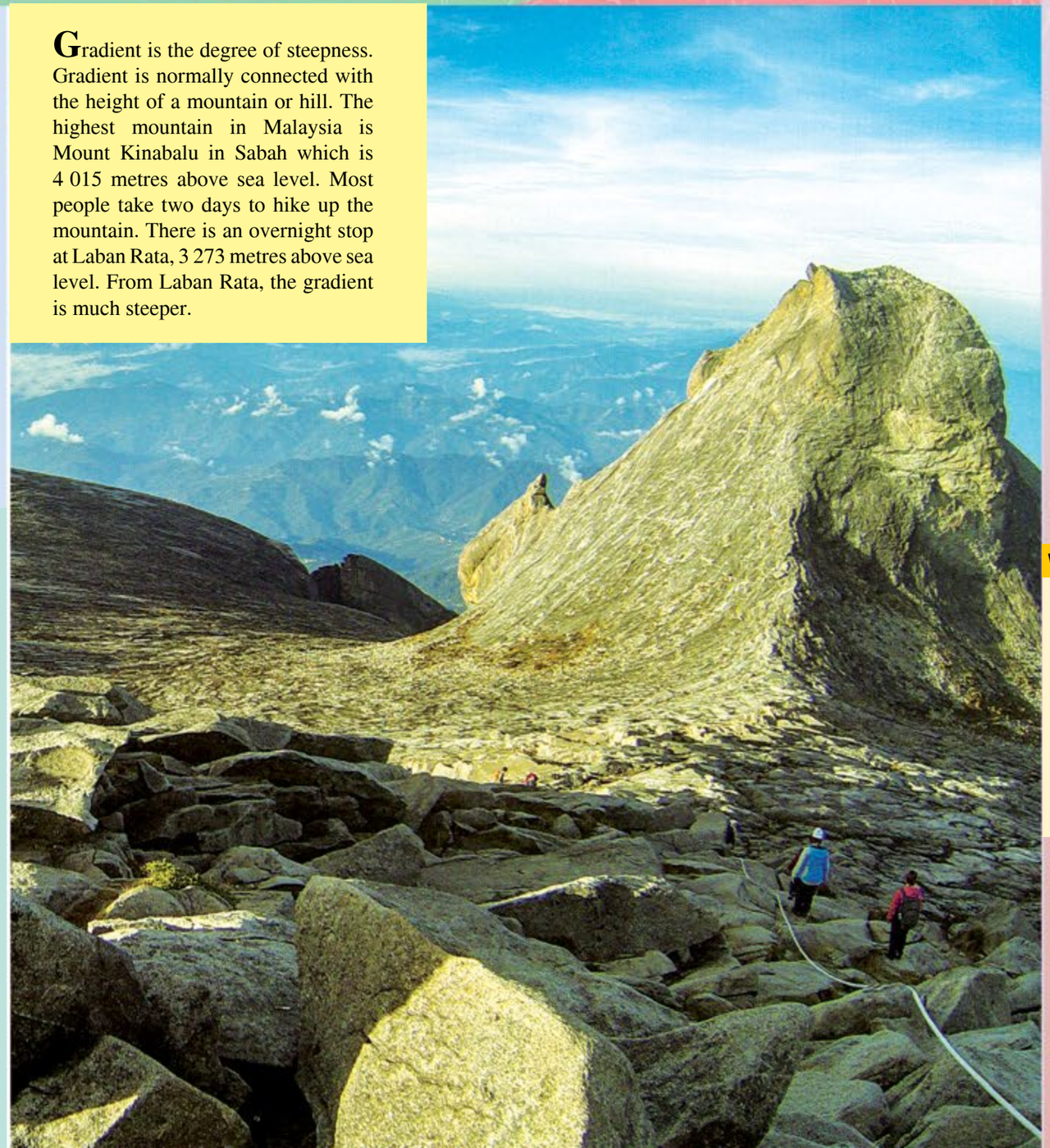
Attach images of speed limit signage taken at relevant areas to support your report.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____

CHAPTER 10

# Gradient of a Straight Line

Gradient is the degree of steepness. Gradient is normally connected with the height of a mountain or hill. The highest mountain in Malaysia is Mount Kinabalu in Sabah which is 4 015 metres above sea level. Most people take two days to hike up the mountain. There is an overnight stop at Laban Rata, 3 273 metres above sea level. From Laban Rata, the gradient is much steeper.



## WHAT WILL YOU LEARN?

**10.1** Gradient

## WORD LINK

- Steepness — *Kecuraman*
- Straight line — *Garis lurus*
- Intercept — *Pintasan*
- Inclination — *Kecondongan*
- Ratio — *Nisbah*
- Vertical distance — *Jarak mencancang*
- Horizontal distance — *Jarak mengufuk*
- Gradient — *Kecerunan*

## WALKING THROUGH TIME

Edwin Bedwell Wilson (1879-1964) was a mathematician who applied the concept of straight line to gradient. He was a vector analysis expert who published his famous book entitled 'Vector Analysis' in 1901. The gradient concept is applied in vector calculation where the vector's gradient will explain the change.

For more information:

http://rimbunanilmu.my/mat_t2e/ms189

## WHY STUDY THIS CHAPTER?

- Learning about gradient will open doors to the careers in mathematics and physics. The formulae used can provide exact calculations to problems in product design.
- In addition, construction engineers, especially those involved in land surveying, use gradient to determine the stability or elevation of a building area.

**CREATIVE ACTIVITY**

**Aim:** Understanding the concept of gradient

**Materials:** Manila card measuring 20 cm x 9 cm, five to six erasers and one marble

**Steps:**

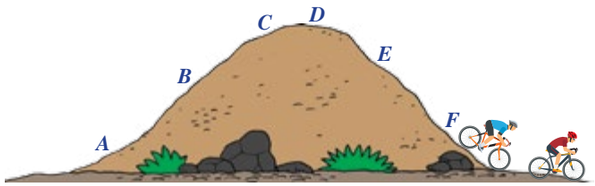
1. Form groups of 3 or 4.
2. Fold the manila card measuring 9 cm as shown in the diagram.
3. Stack three erasers on top of each other and place them at one end under the folded manila card and another eraser under the other end of the manila card.
4. Place the marble at the higher end of the manila card and let it roll along the route.
5. Increase the height of the manila card by adding more erasers to the higher end. Repeat this at the lower end.
6. Take note of the movement of the marble moving along the route.
7. Your friends and you can explore the movement of the marble when the height of both ends of the manila card is of the same level.

The activity above shows the movement of the marble at different speeds. The marble's speed depends on the height of its launch site. When the height is increased the speed increases.

## 10.1 Gradient

### 10.1.1 Steepness and inclination area

View the picture. Which area is steeper? Why?




**LEARNING STANDARD**

Describe gradient and direction of inclination based on real life situations, and then explain the meaning of gradient as a ratio of vertical distance to horizontal distance.

The diagram shows a hilly area travelled by Farid and Afif when they went mountain biking. They found it difficult to continue their ride when they were going up route *C*. However, when they were going down route *E*, their bicycles moved faster. Why did this situation occur?

**COGNITIVE STIMULATION** Group

**Aim:** Identifying the area of steepness and inclination

**Material:** Dynamic geometry software

**Steps:**

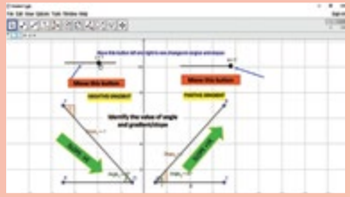
1. Open the file MS190.
2. Move the round button *j* and *n* to the left and right.
3. Observe the value of angle and value of gradient.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms190 to watch the animated video on gradient.







**TIPS**

The value of an angle is always measured from the *x*-axis.

**Discussion:**

(i) Does the value of angle *D* and *A* affect the gradient value?

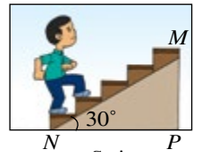
(ii) What is the relationship between the steepness of the line and the direction of the slant.

(iii) Does the negative value of gradient indicates that the slope is decreasing?

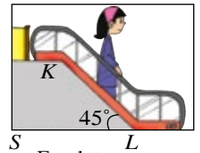
The steepness of a straight line is determined by its gradient value. The greater the gradient value, the steeper the slope of the straight line. The negative or positive gradient value determines the direction of the slope of the straight line.

**EXAMPLE 1**

Look at the diagram on the side. Compare the direction of the tilt and the steepness of the *MN* line with *KL*. Draw conclusions from both diagrams.



Stairs



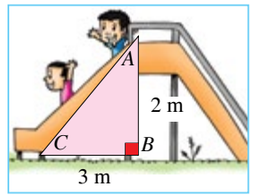
Escalator

*Solution:*

The line *KL*, has a higher inclination than the line *MN*. The larger the angle value, the higher the gradient value. Thus, line *KL* is steeper than *MN*.

### Gradient is the ratio of the vertical distance to the horizontal distance

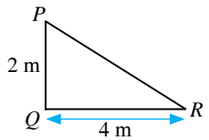
The diagram on the right shows children going down a slide at a playground. The length of the straight line which connects point *A* and point *B* is 2 m. The length of the straight line that connects point *C* and point *B* is 3 m. *CB* is the **horizontal distance** and *AB* is the **vertical distance**.




**EXAMPLE 2**

In each of the diagrams below, state the horizontal distance and the vertical distance between point *P* and point *R*.

(a)

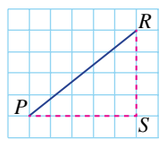



*Solution:*

Vertical distance, $PQ = 2$ m

Horizontal distance, $QR = 4$ m

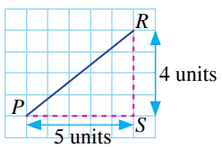
(b)




*Solution:*

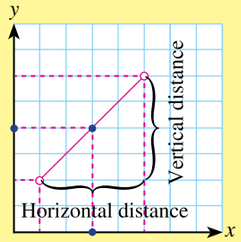
Vertical distance, $RS = 4$ units

Horizontal distance, $PS = 5$ units




**THINK SMART**

Drivers of heavy lorry will step on the accelerator when going up a steep hill. They will also accelerate from the base of the hill before the climb. Why?




Change in $y$ = Vertical distance
Change in $x$ = Horizontal distance

Gradient, $m = \dfrac{\text{Vertical distance}}{\text{Horizontal distance}}$

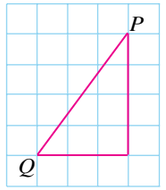
**TIPS**

$m$ represents gradient of a straight line.

**EXAMPLE 3**

From the following diagram, specify the gradient of the straight line $PQ$ and $BC$. Describe the steepness of line $PQ$ and $BC$.

(a)



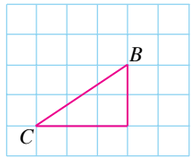

*Solution:*

Vertical distance is 4 units. Horizontal distance is 3 units.

$$\frac{\text{Vertical distance}}{\text{Horizontal distance}} = \frac{4}{3}$$

Thus, the gradient of $PQ$ is $\frac{4}{3}$.

(b)




*Solution:*

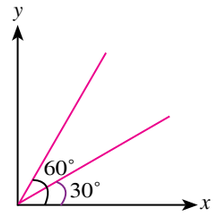
Vertical distance is 2 units.
Horizontal distance is 3 units

$$\frac{\text{Vertical distance}}{\text{Horizontal distance}} = \frac{2}{3}$$

Thus, the gradient of $BC$ is $\frac{2}{3}$.

**DO YOU KNOW ?**

Line that has a steeper gradient has a greater angle.




## 10.1.2 Formula of gradient of a straight line on a Cartesian plane

In the Cartesian coordinate system, the gradient of a straight line that passes through two points $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ can be calculated using the ratio between the vertical distance to the horizontal distance.

**LEARNING STANDARD**

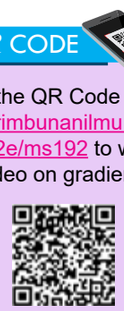
Derive the formulae for gradient of a straight line in the Cartesian plane.

**COGNITIVE STIMULATION** Group

**Aim:** Identifying straight lines on the Cartesian plane
**Material:** Worksheets
**Step:**
1. Open the file MS192.
2. Change the corresponding coordinate point values as in the table given in the coordinate spaces.
3. Take note of the changes that occur on each of the straight line.
4. Specify the gradient value for both points.
5. State whether the straight line passes through the origin, parallel to the $x$-axis or parallel to the $y$-axis.

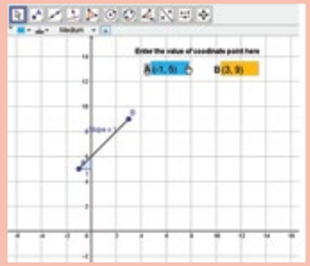
**QR CODE**

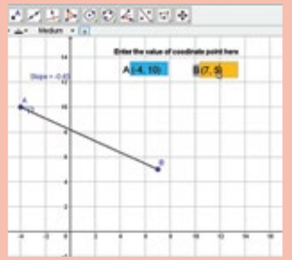
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms192 to watch the video on gradient.











| Coordinate | | Value of gradient | Position of straight line |
|---|---|---|---|
| A | B | | |
| (3,1) | (3, 9) | | |
| (3, –3) | (–2, 2) | | |
| (–1, 5) | (7, 5) | | |
| (4, 4) | (0, 0) | | |
| (0, 6) | (–2, 0) | | |
| (0, 2) | (3, 0) | | |
| $(x_1, y_1)$ | $(x_2, y_2)$ | | |

**Discussion:**

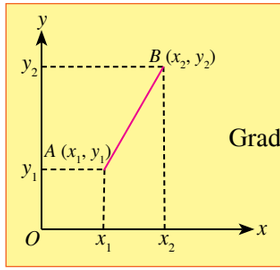
(i) Identify the straight line intersecting the $x$-axis and the $y$-axis.

(ii) Prove by using the formula:

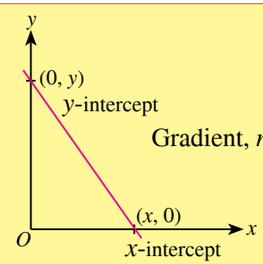
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{and} \quad m = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

that your gradient value is the same as displayed.

The intersection point between the straight line and the $x$-axis is called $x$-intercept, while the intersection point between the straight line and the $y$-axis is called $y$-intercept.




Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$




Gradient, $m = -\dfrac{y\text{-intercept}}{x\text{-intercept}}$

**EXAMPLE 4**

Determine the gradient of the following coordinate points.

(a) $A$ (3, 1) and $B$ (6, 7)

*Solution:*

$x_1\ y_1 \qquad x_2\ y_2$

(a) $A$ (3, 1) and $B$ (6, 7)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7 - 1}{6 - 3}$$
$$= \frac{6}{3}$$
$$= 2$$

(b) $P$ (4, −1) and $Q$ (3, 5)

*Solution:*

$x_1\ y_1 \qquad x_2\ y_2$

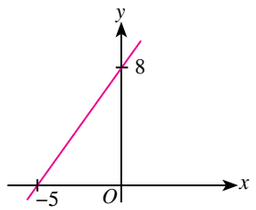
(b) $P$ (4, −1) and $Q$ ( 3, 5)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - (-1)}{3 - 4}$$
$$= \frac{6}{-1}$$
$$= -6$$

**EXAMPLE 5**

Determine the gradient of the following points of coordinate.

(a)



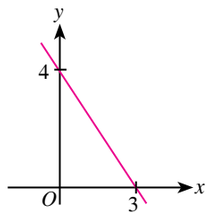

*Solution:*

$y$-intercept = 8

$x$-intercept = −5

$$\text{Gradient} = -\frac{8}{(-5)}$$
$$= \frac{8}{5}$$

(b)




*Solution:*

$y$-intercept = 4

$x$-intercept = 3

$$\text{Gradient} = -\frac{4}{3}$$

**EXAMPLE 6**

Determine the gradient of the coordinates given.

(a) $L$ (4, 0) and $M$ (0, 8)

*Solution:*

(a) $y$-intercept = 8

$x$-intercept = 4

$$\text{Gradient} = -\frac{8}{4} = -2$$

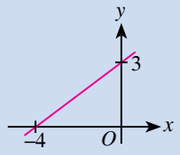
(b) $G$ (−3, 0) and $K$ (0, 9)

(b) $y$-intercept = 9

$x$-intercept = −3

$$\text{Gradient} = -\frac{9}{(-3)} = 3$$

**TIPS**

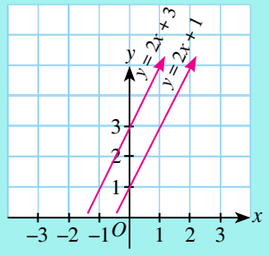



$y$-intercept coordinates are (0, 3).

$x$-intercept coordinates are (−4, 0).

**THINK SMART**

A straight line is represented by $y = mx + c$, $m$ is the gradient while $c$ is the $y$-intercept. State the gradient and the $y$-intercept of the line below and the relationship between the two straight lines.

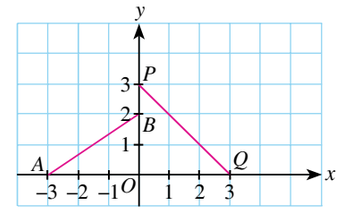







**EXAMPLE 7**

Calculate the gradient of a straight line $AB$ and $PQ$ based on the diagram on the right.




*Solution:*

$$\text{Gradient, } m = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

(i) Gradient $AB = -\dfrac{2}{(-3)}$

$$= \frac{2}{3}$$

Then, gradient $AB$ is $\frac{2}{3}$.

(ii) Gradient $PQ = -\dfrac{3}{3}$

$$= -1$$

Then, gradient $PQ$ is −1.

## 10.1.3 Gradient for a straight line

**LEARNING STANDARD**

Make generalisation for the gradient of a straight line.

**COGNITIVE STIMULATION** Group

**Aim:** Identifying slant

**Materials:** Graph paper and card with coordinates

| | | | |
|---|---|---|---|
| $P$ (1, 1)<br>$Q$ (3, 5) | $R$ (−2,−2)<br>$S$ (−2, 8) | $W$ (−4, 1)<br>$V$ (−7, 8) | $T$ (−4, 3)<br>$U$ (6, 3) |

**Steps:**

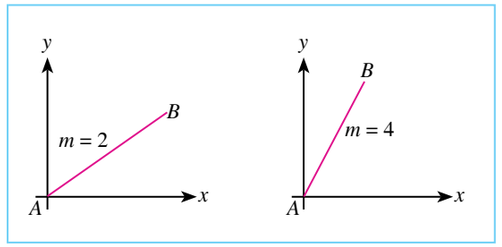
1. Student $A$ constructs a graph with a scale of 1 cm to 1 unit on the $x$-axis, and 1 cm to 1 unit on the $y$-axis.
2. Student $B$ matches the value of the point on the card by plotting the coordinates on the Cartesian plane.
3. Student $C$ draws a straight line and determines the gradient of each pair of coordinate points given.
4. Student $D$ will complete the table below. Other friends will discuss and review.

| Straight line | Gradient | Direction of inclination right or left | Value of gradient positive or negative |
|---|---|---|---|
| $PQ$ | | | |
| $RS$ | | | |
| $WV$ | | | |
| $TU$ | | | |

**Discussion:**

(i) The relationship between the gradient value and the direction of inclination.

(ii) Arrange the gradient of the straight line in descending order.

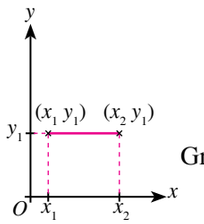



**TIPS**

Relationship of gradient and straight line.

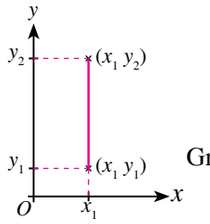

The more the straight line $AB$ approaches the $y$-axis, the greater the gradient value and vice-versa. Thus, the greater the absolute value of the gradient, the steeper the straight line.

Any point in a straight line that is parallel to the $x$-axis has the common $y$-coordinate. Thus, the gradient is zero.




Gradient = 0

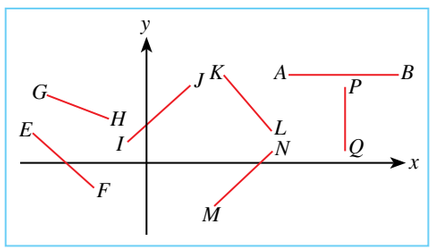
The $x$–coordinate for any two points in a straight line that is parallel to the $y$–axis is the same. This will give an undefined gradient.




Gradient is undefined ($\infty$)

**EXAMPLE 8**

Identify the straight line that has the gradient value of positive, negative, zero or undefined in the diagram below. Justify.




*Solution:*

Gradient of straight lines $IJ$ and $MN$ are positive because the lines are inclined to the right.

Gradient of straight lines $EF$, $GH$ and $KL$ are negative because the lines are inclined to the left.

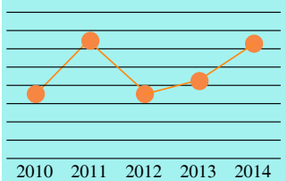
Gradient of line $AB$ is zero because the line is parallel to the $x$-axis.

Gradient of line $PQ$ is undefined because the line is parallel to the $y$-axis.

**THINK SMART**

Based on the graph below. Observe on which year does the inflation rate shows a negative gradient? Discuss.

**Inflation Rate (%) in Malaysia (2010-2014)**



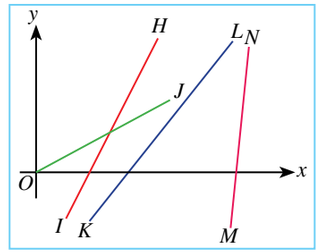

Source: World Bank
https://www.imoney.my/articles/realiti-tentang-inflasi





**EXAMPLE 9**

Determine the gradient of the four straight line in the diagram on the right. State the line with the highest and lowest gradient. Give reasons.




*Solution:*

Straight line $MN$ is the straight line that has the highest gradient and it is more vertically inclined.

Straight line $OJ$ is the straight line that has the smallest gradient because it is more horizontally inclined.

## 10.1.4 Determining gradient

**LEARNING STANDARD**

Determining the gradient of a straight line.

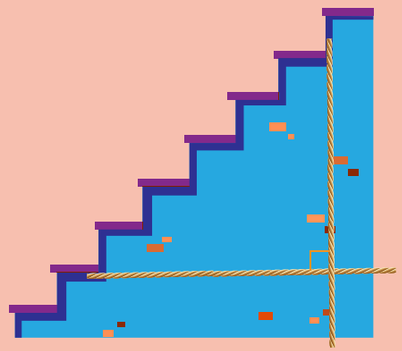
**COGNITIVE STIMULATION** Group

**Aim:** Determining gradient

**Materials:** Stairs, ropes, measuring tape

**Steps:**

1. In group, determine the gradient of the stairs in your school.
2. Choose any two suitable stairs.
3. Choose two suitable points as shown in the diagram.
4. Use ropes to determine the vertical and horizontal length. Make sure the angle where the two strings meet is at 90°.
5. Repeat steps 3 and 4 for the second stairs.



**Discussion:**

(i) State the vertical and the horizontal distance of the stairs.

(ii) Calculate the gradient of the two stairs.

(iii) What is the relationship between the ratio of 'the vertical to the horizontal distance' of the two stairs?

(iv) The ratio of 'the horizontal distance to the vertical distance' is not used to determine gradient. Discuss.

The ratio of 'the vertical distance to the horizontal distance' is used to determine the gradient of a straight line. The greater the gradient, the steeper the straight line.

**QR CODE**

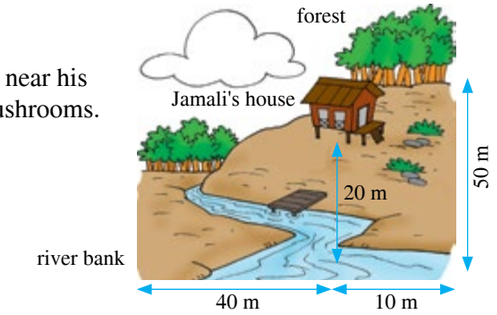
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms197 to view the activity on gradient.

**EXAMPLE 10**

Every day Jamali fetches water from the river bank near his house and later goes into the forest to search for mushrooms. Calculate the gradient of

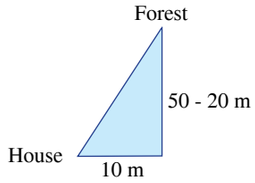
(i) his house to the forest

(ii) the river bank to his house




***Solution:***

(a)

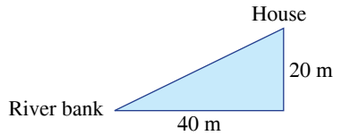



$$\text{Gradient} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

$$= \frac{30}{10} = 3$$

Thus, the gradient from Jamali's house to the forest is 3.

(b)




$$\text{Gradient} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

$$= \frac{20}{40} = \frac{1}{2}$$

Thus, the gradient from the river bank to his house is $\frac{1}{2}$.

## 10.1.5 Solving problems

**LEARNING STANDARD**

Solve problems involving the gradient of a straight line.

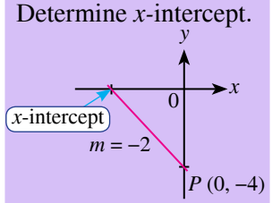
**EXAMPLE 11**

(a) Determine the $x$-intercept of a straight line that passes through the point $P$ (0, –4) with the gradient –2.

(b) Determine the coordinate of $y$-intercept of a line that passes through the point $Q$ (6, 0) and the gradient $\frac{1}{3}$.

***Solution:***

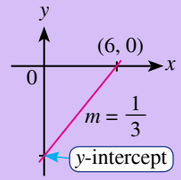
(a)

| **Understanding the problem** | **Planning the strategy** | **Implementing the strategy** | **Conclusion** |
|---|---|---|---|
| Determine $x$-intercept.<br>  <br> | Use the formula:<br>$x\text{-intercept} = -\frac{(y\text{-intercept})}{m}$<br>$y\text{-intercept} = -4$ | Substitute the value and calculate.<br>$x\text{-intercept} = -\frac{\cancel{-4}^{2}}{\cancel{-2}_{1}}$<br>$= -2$ | Thus, $x$-intercept is –2 with the coordinates (–2, 0). |



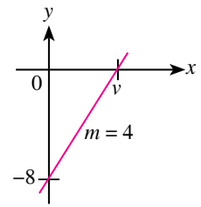


(b)

| **Understanding the problem** | **Planning the strategy** | **Implementing the strategy** | **Conclusion** |
|---|---|---|---|
| Determine the position of $y$-intercept.<br>  <br> | Using the formula:<br>$y\text{-intercept} = -m \times x\text{-intercept}$ | Substitute the value and calculate.<br>$y\text{-intercept} = -\frac{1}{\cancel{3}_{1}} \times \cancel{6}^{2}$<br>$= -2$ | Thus $y$-intercept is –2 with the coordinates (0, –2). |

**EXAMPLE 12**

Calculate the value of $v$ in the diagram on the right.




***Solution:***

| **Understanding the problem** | **Planning the strategy** | **Implementing the strategy** | **Conclusion** |
|---|---|---|---|
| Determine the position of $v$.<br>$v = x\text{-intercept}$ | Use the formula:<br>$x\text{-intercept} = -\frac{(y\text{-intercept})}{m}$ | $v = -\left(\frac{-8}{4}\right)$<br>$= 2$ | Thus, $x$-intercept is 2. |

**EXAMPLE 13**

Given $A$ (–9, 2), $B$ (–7, 2), $C$ (– 4, 3) and $D$ (–6, –1) are the vertices of a quadrilateral. Determine the type of the quadrilateral.

***Solution:***

**Understanding the problem**

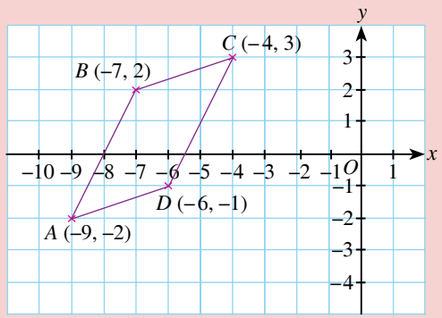
Determine the type of the quadrilateral.

**Planning the strategy**

- Determine the gradient of the straight line $AD$, $BC$, $AB$ and $DC$ by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Draw the quadrilateral.

**Implementing the strategy**

- Draw the graph.




Gradient of the straight line $AD$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-2)}{-6-(-9)} = \frac{1}{3}$$

Gradient of the straight line $DC$

$$m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{-4-(-6)} = 2$$

Gradient of the straight line $BC$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{-4-(-7)} = \frac{1}{3}$$

Gradient of the straight line $AB$

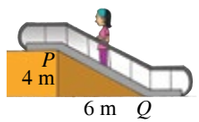
$$m_4 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-2)}{-7-(-9)} = 2$$

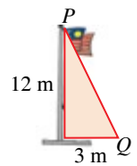
**Conclusion**

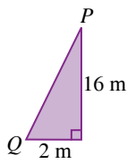
- Straight line $AD$ is parallel to the straight line $BC$, $m_1 = m_2$.
- Straight line $AB$ is parallel to the straight line $DC$, $m_3 = m_4$.
- $ABCD$ is a parallelogram.

## SELF PRACTICE 10.1

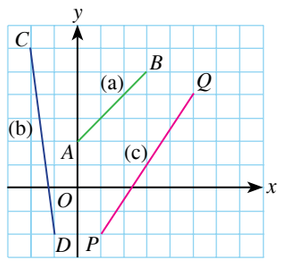
1. Determine the vertical distance and the horizontal distance of point $P$ and point $Q$.

(a) 

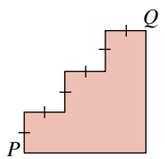

(b) 


(c) 


2. Determine the vertical distance and the horizontal distance of line $AB$, $CD$ and $PQ$ in the following Cartesian plane.




3. Calculate the vertical distance and horizontal distance in metres between $P$ and $Q$ in the diagram if the distance of each stair is 12 cm.





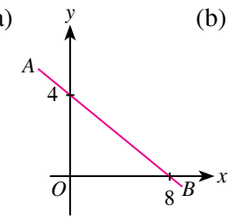


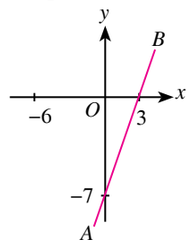

4. State the vertical distance and horizontal distance of the two pairs of points given.

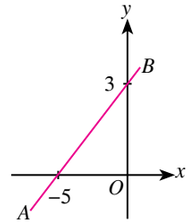
(a) (3, 0) and (–2, 6) (b) (1, 1) and (6, 5) (c) (3, 1) and (1, 5)

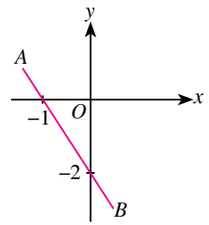
(d) (0, 0) and (4, 4) (e) (1, –2) and (2, 4) (f) (3, 6) and (6, –3)

5. State the value of the $x$-intercept and the value of the $y$-intercept of the straight line $AB$.

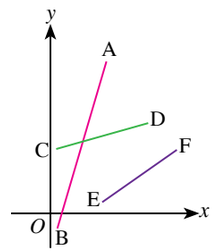
(a) 

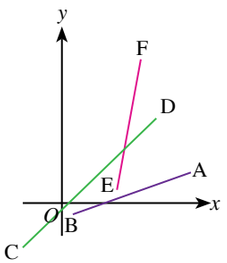

(b) 


(c) 


(d) 

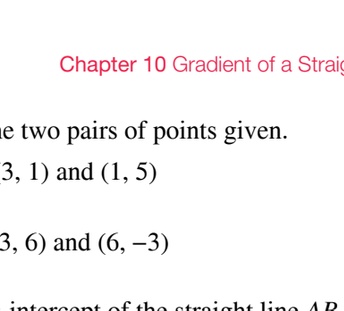

6. Identify the straight line that has the highest gradient in each of the diagrams below.

(a) 

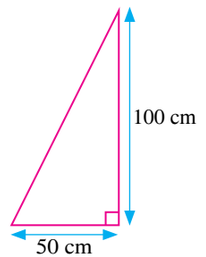

(b) 

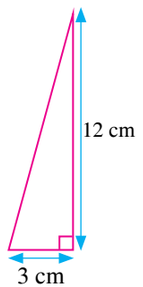

7. Based on the diagram, state whether the gradient is positive or negative.

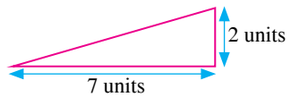
(a) $LM$
(b) $MN$
(c) $NO$
(d) $OQ$



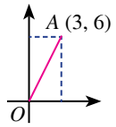

8. State the gradient of the straight line for each of the following.

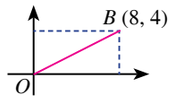
(a) 

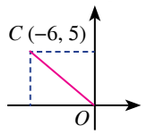

(b) 

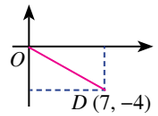

(c) 

9. Calculate the gradient of the straight line for each of the following.

(a) 


(b) 


(c) 


(d) 


10. Calculate the gradient of a straight line that passes through the following pairs of points.

(a) $A$ (4, 5) and $B$ (3, 2)
(b) $E$ (−1, −2) and $F$ (0, 7)
(c) $C$ (6, 6) and $D$ (3,1)
(d) $G$ (2, 4) and $H$ (6, 1)

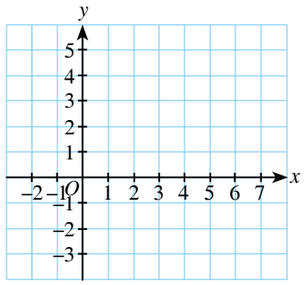
11. Calculate the gradient of the following straight line,

(a) $x$-intercept = 4, $y$-intercept = 1
(b) $x$-intercept = 9, $y$-intercept = 10
(c) $x$-intercept = −3, $y$-intercept = 8
(d) $x$-intercept = −5, $y$-intercept = −5

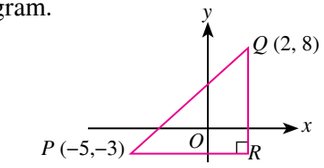
## GENERATING EXCELLENCE

1. Draw the straight line that passes through the given pairs of coordinates on the Cartesian plane. Determine whether the gradient of the straight line is positive or negative.

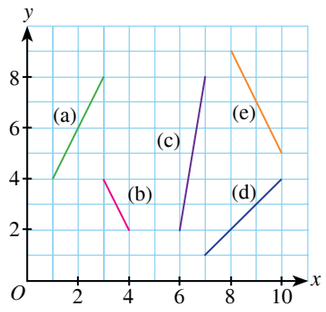
(a) (−1, 0) and (−2, 5)
(b) (0, 1) and (3, 5)
(c) (1, −3) and (2, 4)
(d) (7, −2) and (2, 2)
(e) (0, 1) and (5, 3)
(f) (0, 3) and (5, 0)
(g) (0, 0) and (6, 5)




2. Determine the gradient of the straight line $PQ$ in the diagram.




3. Calculate the gradient of all the straight lines in the following diagram. Compare and determine the line with the steepest gradient.








4. Complete the table below.

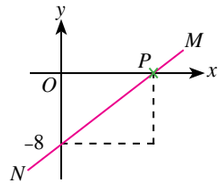
| | $x$-intercept | $y$-intercept | Gradient |
|---|---|---|---|
| (a) | 4 | | 2 |
| (b) | −2 | 4 | 2 |
| (c) | −4 | | −3 |
| (d) | −1 | 4 | 4 |
| (e) | −1 | 2 | |
| (f) | 5 | | 1 |

5. Given the gradient of the straight line that passes through $M$ (1, $k$) and $N$ (−2, 3) is −2, determine the value of $k$.

6. The gradient of straight line $PQ$ is −1 with point $P$ (2, −1) and the vertical distance of point $Q$ is 3 units to the left of $y$-axis. State the coordinate $Q$.

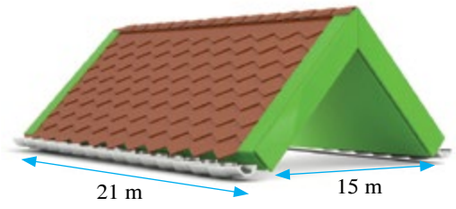
7. If the gradient of a straight line is 2 and the $y$-intercept is −18, determine $x$-intercept of the straight line.

8. Calculate the gradient of the straight line $MN$, if the horizontal distance of $P$ from the $y$-axis is 6 units.



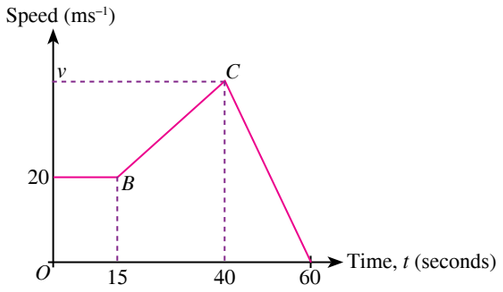

9. If points $A$ and $B$ are on the same straight line with the gradient $\frac{4}{3}$ and point $A$ (0, 8), determine the coordinate of $B$ if $B$ is $x$-intercept.

10.

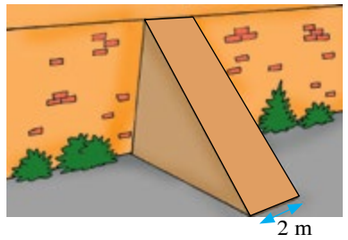



The diagram above is the roof of a terrace house. If the height of the roof is 5 m, calculate
(a) the gradient of the roof
(b) the length of the slant of the roof

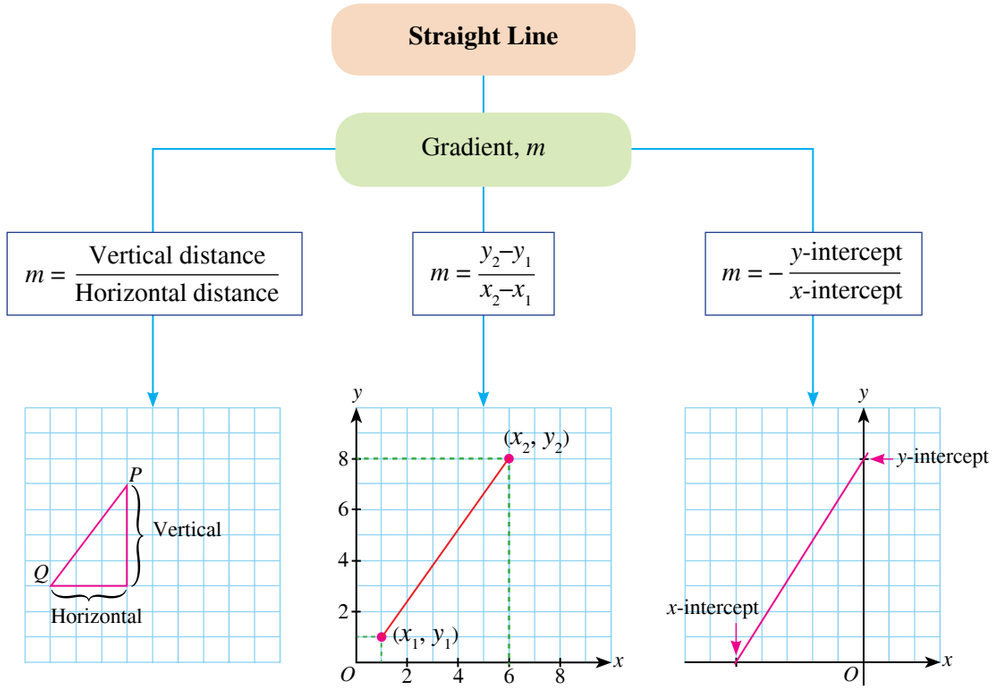
11. The diagram shows the journey of a motorcycle in 60 seconds.
    (a) State the speed of the motorcycle at the constant phase.
    (b) Calculate the value of $v$ if the motorcycle moves at 0.88m/s when $t = 15$ s.




12. The cross section area of a brick wall that is shaped as right-angled triangle is $12m^2$ and the height is 6 metre. Calculate the gradient and the area of the slanted surface of the stone wall.




## CHAPTER SUMMARY








## SELF REFLECTION

**At the end of the chapter, I am able to:**

| | |
|---|---|
| 1. Describe the steepness and inclination directions based on daily situations, and further explaining the meaning of gradient as a ratio for vertical distance and horizontal distance. | |
| 2. Derive the formulae for gradient of a straight line in the Cartesian plane. | |
| 3. Make generalisation for the gradient of a straight line. | |
| 4. Determine the gradient of a straight line. | |
| 5. Solve problems involving the gradient of a straight line. | |

## MINI PROJECT

Students are required to search for information on mountains in Malaysia. The information should be on

(i) the height above sea level

(ii) the horizontal distance

You can calculate the gradient of each mountain and arrange the value of gradients from the highest value to the lowest value. Compare your information with your friends. You can also research on the mountains of South East Asia.


Gunung Tahan, Pahang


Gunung Korbu, Perak


Gunung Mulu, Sarawak

CHAPTER 11

# Isometric Transformations

## WHAT WILL YOU LEARN?



## WORD LINK

| | |
|---|---|
| • Object | • *Objek* |
| • Image | • *Imej* |
| • Clockwise | • *Ikut arah jam* |
| • Anticlockwise | • *Lawan arah jam* |
| • Isometry | • *Isometri* |
| • Congruency | • *Kekongruenan* |
| • Orientation | • *Orientasi* |
| • Axis | • *Paksi* |
| • Centre of rotation | • *Pusat putaran* |
| • Reflection | • *Pantulan* |
| • Transformation | • *Penjelmaan* |
| • Translation | • *Translasi* |
| • Symmetry | • *Simetri* |
| • Vector | • *Vektor* |
| • Rotation | • *Putaran* |
| • Rotational symmetry | • *Simetri putaran* |
| • Order of rotational symmetry | • *Peringkat simetri putaran* |



Tuanku Mizan Zainal Abidin Mosque was built on 5 April 2004. The mosque is adjacent to the beautiful Putrajaya lakes and clear water. The image of the picture below shows a transformation on the lake. How does this phenomenon occur?

## WALKING THROUGH TIME

The mathematicians Felix Klein (1849-1925) argued that isometry is the balance produced by the movement of a common form or movement by a group of similar forms. Isometry in a pattern is a movement of the same shape. There are four types of isometry; translation, rotation, reflection and glide reflection.

For more information:

http://rimbunanilmu.my/mat_t2e/ms207

## WHY STUDY THIS CHAPTER?

- Knowledge in the field of transformation can be applied in the manufacturing and fashion design industries. Vehicle designs such as motorcycles, cars and planes require the design of symmetrical objects. Meanwhile, fashion designers will produce different patterns in each of their designs.

**CREATIVE ACTIVITY**

**Aim:** Identifying characteristics of transformation

**Material:** Extracts of stories

Every day before going to school, Akmal brushes his hair in front of the mirror and keeps it neat. While enjoying breakfast, he will sit under a ceiling fan to avoid sweating. After completing breakfast, Akmal walks from his home to the bus stop to go to school.

**Steps:**

1. In groups, discuss the actions of:
   (i) Akmal in front of the mirror.
   (ii) the rotating ceiling fan.
   (iii) Akmal's walk from his home to a bus stop.
2. Does the actions change the appearance of Akmal and the fan blades? From these situations, what do you understand about the meaning of transformation in Akmal's daily life?

Transformation is the process of changing the direction, orientation or size of an object's image through translation, reflection and rotation. The image produced by isometric transformation is congruent.

## 11.1 Transformations

### 11.1.1 Transformation in the shapes, sizes, directions and orientations of an object

**LEARNING STANDARD**

Describe the changes of shapes, sizes, directions and orientations of an object under a transformation, and hence explain the idea of one-to-one correspondence between points in a transformation.

Transformation involves the transfer of a position on a plane.

**COGNITIVE STIMULATION** Group

**Aim:** Identifying the transformation through position, size and appearance

**Material:** Manila card, water colour and flashlight

**Steps:**

1. Dip both sides of your palms into the water colour and press the palms onto the manila card side by side as shown in Diagram *A*.
2. When the right palm is coloured, press your right palm twice side by side with one side a little lower as shown in Diagram *B*.

Diagram *A*

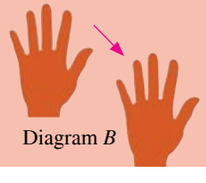


Diagram *B*





3. Repeat step 2 but the second impression is in a twist as if the hands are waving as shown in Diagram *C*.
4. Direct the flashlight to the palm of your hand and notice the resulting shade on the board. Move the flashlight forward and backwards to see the size of the shadow.

Diagram *C*

**Discussion:**

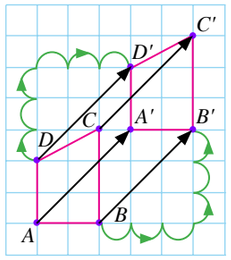
Based on the activities above, what is the conclusion from the orientation of the movement, when

(i) the position of palms are side-by-side.
(ii) one palm is positioned lower than the other.
(iii) the position of the palms are in a waving pose.
(iv) the size of the shadow of your palm varies.

**THINK SMART**

Is the shadow an image?

Before transformation takes place, the original figure is called the **object**. After the transformation, the shape is called the **image**. Transformation is a match of a point on a plane. When the object moves in a transformation, each point of the object follows the same pattern of movement.




Transformation is a movement with a certain orientation and match without changing the shape.

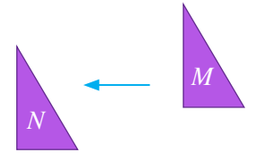
The diagram shows is the movement of an object *ABCD* into the image $A'B'C'D'$ with a movement of three units to the right and three units upwards.

**EXAMPLE 1**

Which of the following shows a transformation and why?

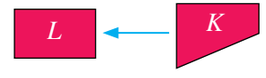
(a)


(b)




(c)


(d)



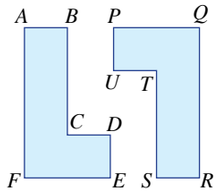

***Solution:***

(a) Transformation because it does not change the shape.
(b) Transformation because it only changes position and does not change shape.
(c) Not a transformation due to a change in appearance.
(d) Not a transformation due to a change in appearance.

**EXAMPLE 2**

The diagram shows *ABCDEF* as an object, while *PQRSTU* is an image. State the image of

(a) point *C* (b) the line *AB* (c) ∠ *BCD*




*Solution:*

(a) The image of point C is T because of the same shape, but the position is different.

(b) The image of the line *AB* is *RS*. This is because the image has the same length.

(c) The image for ∠ *BCD* is ∠ *STU* because ∠ *BCD* is of the same size as ∠ *STU*.

## 11.1.2 Congruence

**COGNITIVE STIMULATION** Group

**Aim:** Identifying congruency

**Materials:** Blue and red coloured paper, ruler, protractor and scissors

**Steps:**

1. In small groups of 4 to 5, students are required to cut out two triangles.
2. Students *A* and *B* will form triangles using blue coloured paper with measurements of 5 cm, 8 cm and 11 cm.
3. Students *C* and *D* will cut out triangles of the same size using red coloured paper.
4. Student *E* will combine both triangles.
5. Students will measure angles of the triangle using a protractor.

**Discussion:**

(i) Provide the properties that can be obtained from both forms of the triangle.

(ii) If congruence is the similarity of shape and size, do the triangles meet the criteria of being congruent?

Two objects are congruent if both have the same shape and size, regardless of the orientation of the movement.

**LEARNING STANDARD**

Explain the idea of congruency in transformations.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms210 to view video on congruency.

**THINK SMART**

Look at the object above. Are the two objects congruent? If the weight of the two objects is equal, does the number of coins stored in the savings box have the same value? Is that what is meant by congruence?



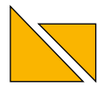


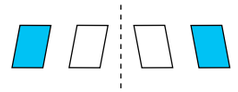
Regular tools used to determine congruency are rulers, protractors, compasses and tracing paper. You can use these tools to explore the nature of congruency.

Study the 20 sen and 10 sen coins. Can you distinguish them from the shape of the coin? Coins that are round are similar. Do you agree with this statement? All 10 sen coins are congruent, while 20 sen and 10 sen coins are similar but not congruent.

**EXAMPLE 3**

Which of these pairs are congruent? State why.

(a) 

(b) 

*Solution:*

(a) Not congruent because the size is not the same.

(b) Congruent as they are of the same size, although the position and orientation are not the same.

**DO YOU KNOW?**

Orientation is something to do. For example, clockwise, counter-clockwise, left and right.

**TIPS**

Congruents are the same form and size.

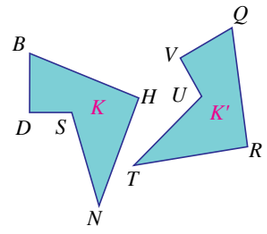
**SELF PRACTICE 11.1**

1. Which of the following diagram does not show a transformation?

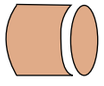
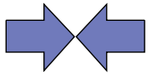
(a)  (b)  (c)  (d) 

2. $K'$ is the image of $K$ under a transformation. Identify

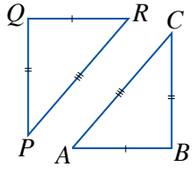
(a) the image of vertex $N$ (b) the image of $BH$ (c) the image of ∠$SDB$




3. Identify congruent pairs and state why.

(a)  (b)  (c)  (d) 

4. The diagram below are two congruent triangles. Complete the table below with matching lines and equal angles.




| Triangle | Side | Side | Angle | Angle |
|---|---|---|---|---|
| *PQR* | *QP* | | ∠*PQR* | |
| *CBA* | | *AB* | | ∠*CAB* |

# 11.2 Translation

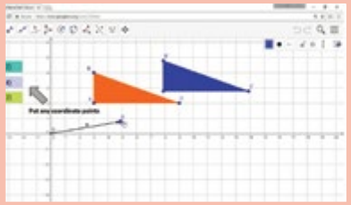
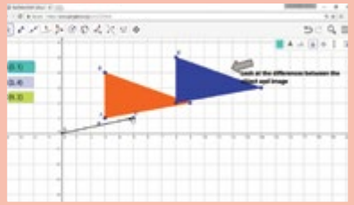
## 11.2.1 Translation

**LEARNING STANDARD**

Recognise translation.

**COGNITIVE STIMULATION** Individual

**Aim:** Identifying features of translation
**Material:** Dynamic geometry software
**Steps:**

1. Open the file MS212.
2. You can explore any coordinates for *A, B* and *C*.
3. Observe the resulting blue image after the point is changed.
4. Movement of the image depends on the assigned arrow *E*. You can also move the blue slider to see the movement of the image.

**Discussion:**

(i) What conclusions can be made from the exploration activities above?

(ii) How does the features of the image change when the coordinate value at the point of the object changes?

**QR CODE**

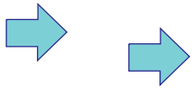
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms212 to view video on translation.

Translation is the transfer of all points on a plane in the same direction and through the same distance.

With a translation, objects and images have the same shape, size and orientation.

**EXAMPLE 4**

Identify which diagram shows translation. Justify.

(a) 

(b) 

***Solution:***

(a) Translation because the shape, size and orientation are the same.

(b) Not a translation because the orientation is not the same.





## 11.2.2 Representation of translation in the form of vector

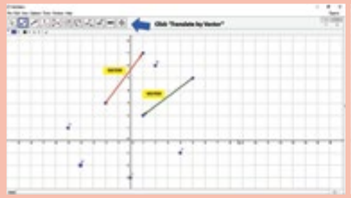
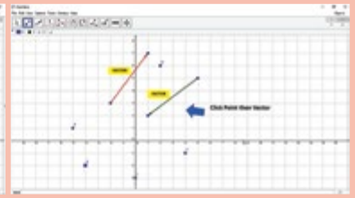
**LEARNING STANDARD**

Describe translation by using various representations including vector form.

Vector of translation is a movement that has direction and magnitude. This vector is also represented by an arrow. Determining translation is based on the value and direction of a vector.

**COGNITIVE STIMULATION** Group

**Aim:** Exploring vectors of translations
**Material:** Dynamic geometry software

**Steps:**

1. Open the file MS213. Follow the instruction of the video.
2. Given Vector 1 and Vector 2, determine the coordinate images of *A, B, C, D* and *E*.
3. You can select any vector to specify the image for those points.
4. Complete the table below.

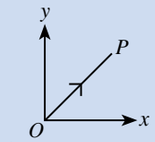
| Coordinate of Object | Total unit of movements Left/Right a | Total unit of movements Upwards/ Downwards b | Form of vector $\begin{pmatrix} a \\ b \end{pmatrix}$ | Coordinate of image |
|---|---|---|---|---|
| $A$ (  ) | | | | $A'$ (  ) |
| $B$ (  ) | | | | $B'$ (  ) |
| $C$ (  ) | | | | $C'$ (  ) |
| $D$ (  ) | | | | $D'$ (  ) |
| $E$ (  ) | | | | $E'$ (  ) |

**Discussion:**

(i) Is the direction of motion of the object equal to the direction of movement of the arrow?

(ii) How do you write unit of movement for vector of translation $\begin{pmatrix} \text{Movement right/left} \\ \text{Movement upwards/downwards} \end{pmatrix}$?

**TIPS**




$\overrightarrow{OP}$ is known as a vector.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms213 to view a demonstration on vector of translation.

**FLASHBACK**

Coordinate $(x, y)$. The value of $x$ is written first followed by the value of $y$.

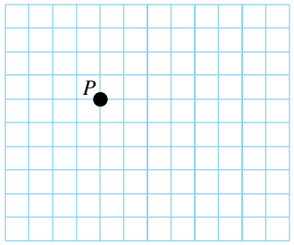
Translation can be described by specifying the direction and distance of the movement. Translation can be described using two methods,

(a) Direction of movement: to the right, left, upwards, downwards.
Distance of movement: number of units.

(b) Write in vector form $\begin{pmatrix} a \\ b \end{pmatrix}$

**EXAMPLE 5**

Determine the position of point $P$ in the given diagram for the translation description and draw the vector translation.



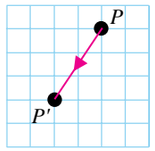

(a) Point $P$ moves 2 units to the left and 3 units downwards.

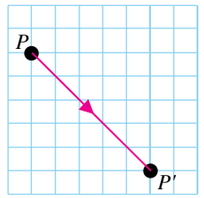
(b) Point $P$ moves 5 units to the right and 5 units downwards.

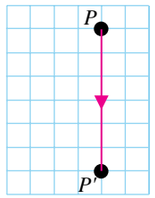
(c) Point $P$ moves 6 units downwards.

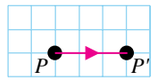
(d) Point $P$ moves 3 units to the right.

***Solution:***

(a)



(b)



(c)



(d)



**DO YOU KNOW?**

Vector translation is written as $\begin{pmatrix} a \\ b \end{pmatrix}$.

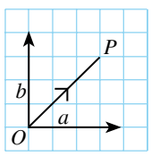
$a$ represents a movement parallel to the $x$-axis which is positive if the object moves to the right and is negative if the object moves to the left.

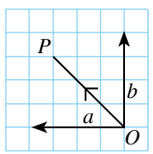
$b$ represents a movement parallel to the $y$-axis which is positive if the object moves upwards and is negative if the object moves downwards. Therefore, the translation of the image generated by the object in the diagram below is $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

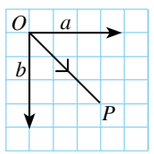



**EXAMPLE 6**

Determine the vector translation of $OP$ in the diagram below.

(a)



(b)



(c)



***Solution:***

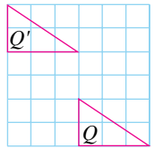
(a) $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ (b) $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ (c) $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

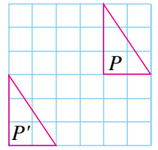


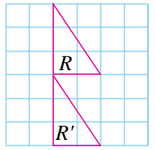


**EXAMPLE 7**

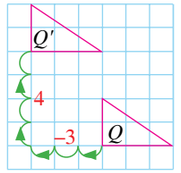
Determine the translation for the diagrams below.

(a)


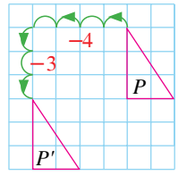

(b)



(c)


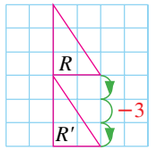

***Solution:***

(a)


$\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

(b)


$\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

(c)


$\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

## 11.2.3 Image and object under a translation

**LEARNING STANDARD**

Determine the image and object under a translation.

**COGNITIVE STIMULATION** Group

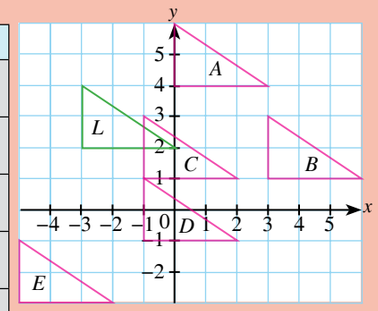
**Aim:** Identifying the image of an object in a translation

**Material:** Worksheet

**Steps:**

1. Study the diagram on the right. Identify the image of the object $L$ for the translation provided.
2. Complete the table.

| Translation | Image |
|---|---|
| $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ | |
| $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ | |
| $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ | |
| $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ | |
| $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$ | |




**Discussion:**

(i) Compare the length of the sides and the value of angle for the object as well as the image.

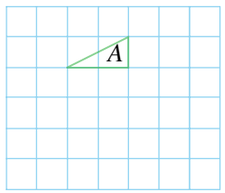
(ii) Conclusion on features of translation.

The image of an object in a translation will always be the same in terms of shape, size and orientation.

**EXAMPLE 8**

Draw the image of object A in the diagram with the translation

(a) $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$







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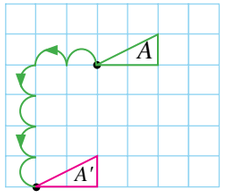
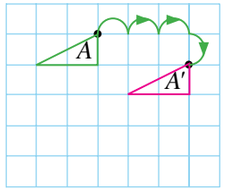
*Solution:*

(a)

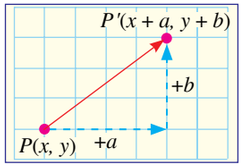



(b)




## Determining the coordinates of image when coordinates of the object is given

To locate the **image** with translation $\begin{pmatrix} a \\ b \end{pmatrix}$, the coordinates of the object $P(x, y)$ will be mapped to $P'(x + a, y + b) = P'(x' y')$

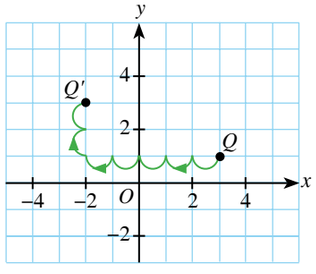



**EXAMPLE 9**

Determine the coordinates of the image for point $Q$ (3, 1) with translation $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$.

*Solution:*

**Method 1:** Draw on a Cartesian plane




**Method 2:** Calculate

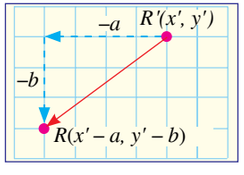
i. $Q\,(3, 1) \longrightarrow Q'\,(3 + (-5), 1 + 2)$
$= (-2, 3)$

ii. $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Therefore, image for $Q$ (3, 1) is (–2, 3).

## Specifying coordinates of object when coordinates of the image is given

To locate an **object** with translation $\begin{pmatrix} a \\ b \end{pmatrix}$, coordinates of object $R'(x', y')$ will be mapped to $R(x' - a, y' - b) = R(x\ y)$




**EXAMPLE 10**

Object $A$ undergoes the translation $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Determine the coordinates of object $A$ if the image $A'$ is as follows.

(a) (–6, 1) (b) (9, 0)

*Solution:*

(a) Coordinate $A = [-6 - 3\ ,\ 1 - (-2)]$
$= (-9\ ,\ 3)$

(b) Coordinate $A = [9 - 3\ ,\ 0 - (-2)]$
$= (6, 2)$

**TIPS**

Alternative methods

$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a + x \\ b + y \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a - x \\ b - y \end{pmatrix}$

To change from vector form to ordered pairs.

$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow (x, y)$

(a) $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -9 \\ 3 \end{pmatrix}$

(b) $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 2 \end{pmatrix}$





## Defining vector translation when position of image and object is given

**EXAMPLE 11**

Given $P'$ (3, 6) is the image of $P$ (2, 9), determine the translation.

*Solution:*

Vector translation $= \begin{pmatrix} x' - x \\ y' - y \end{pmatrix}$

$= \begin{pmatrix} 3 - 2 \\ 6 - 9 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

**TIPS**

Given object $(x, y)$ and the image $(x', y')$. Vector translation is $\begin{pmatrix} x' - x \\ y' - y \end{pmatrix}$

## 11.2.4 Solving problems

**LEARNING STANDARD**

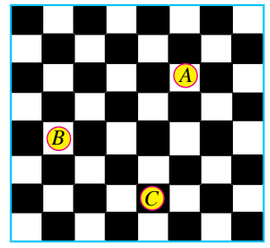
Solve problems involving translation.

**EXAMPLE 12**

Agnes moves her checker piece from $A$ to $B$ and then to $C$. Indicate her movement in the form of translation of the checker piece marked

(a) $A$ to $B$.

(b) $B$ to $C$.




*Solution:*

**Understanding the problem**

Translation movement to the left or right, upwards or downwards.

**Planning the strategy**

(a) 4 units to the left, 2 units downwards.

(b) 3 units to the right, 2 units downwards.

**Conclusion**

(a) Therefore, vector translation $A$ to $B$ is $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$.

(b) Therefore, vector translation $B$ to $C$ is $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

**Implementing the strategy**

Using $\begin{pmatrix} a \\ b \end{pmatrix}$

(a) $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

(b) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

**THINK SMART**

A shot putt can be thrown in two different styles. Does the force affect the direction of the throw? Discuss the relationship with the concept of translation.

**TIPS**

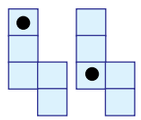
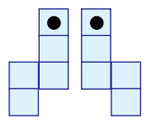
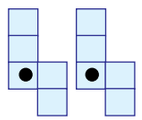
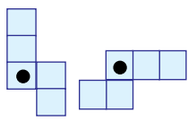
Translation movements always begin from the left or the right, then upwards or downwards.

**THINK SMART**

| Object | Translation | Image |
|---|---|---|
| $A$ (–3, 4) | $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ | |
| $B$ (7, 9) | $\begin{pmatrix} -4 \\ -5 \end{pmatrix}$ | |
| | $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ | $P'$(–5, 2) |
| | $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ | $Q'$(4, 1) |

## SELF PRACTICE 11.2

1. Which of the following pairs is a translation?

(a)  (b)  (c)  (d) 

2. Determine the coordinates of the image for the object (5, −3) under translation

(a) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ (b) $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ (c) $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$

3. Determine the coordinates of the object for the image (−1, −4) under translation

(a) $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ (c) $\begin{pmatrix} -8 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$

4. State the vector translation for the following points.

(a) $A$ (1, 2), $A'$ (3, 6) (b) $B$ (5, 7), $B'$ (−1, −1)

(c) $C$ (4, 4), $C'$ (8, 0) (d) $D$ (6, 4), $D'$ (3, −3)

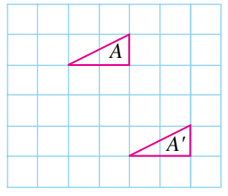
5. The object $L$ (1, 4) is mapped to a position $L'$ (3, −5) after a translation. Determine the position of an image or object with the same translation for the coordinate points below.

(a) $A$ (3, 1) (b) $S'$ (4, −2)

(c) $J'$ (5, −6) (d) $D$ (−7, −8)

6. Using the same orientation with the diagram on the right, determine the coordinate of the image for the following.

(a) (−1, −4) (b) (5, −5)




CHAPTER 11

## 11.3 Reflection

### 11.3.1 Reflection

LEARNING STANDARD

Recognise a reflection.

When Preveena looks at the mirror while brushing her hair, she will be able to see her appearance in the mirror. The image of Preveena in the mirror is the result of reflection. Reflection is a transformation that occurs when all the points on the plane are reversed in the same plane on a line. The line is called the **axis of reflection**.

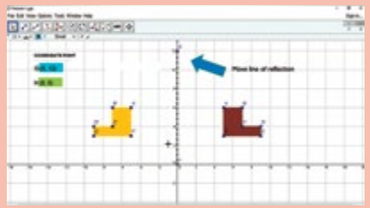




## COGNITIVE STIMULATION Group

**Aim:** Identifying the features of reflection

**Material:** Dynamic geometry software



QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms219 to view a video on features of reflection.

**Steps:**

1. Open the file MS219.
2. Observe the line changes when point $G$ and $H$ change.
3. See the changes that occur to the image.

**Discussion:**

(i) What do you understand about the axis of reflection?

(ii) What will happen to the image in yellow when the axis for line $GH$ is moved?

(iii) From the activity, what do you understand about characteristic of symmetry?

Reflections have certain features which are

(i) the object and the image are on the opposite sides of the axis of reflection.

(ii) the object and its image have the same perpendicular distance from the axis of reflection.

(iii) the shape and size of the image is the same as the object, but the orientation is reversed.

(iv) the image of a point on the axis of reflection is the point itself.

TIPS

All the points located on the axis of reflection do not change their position during a transformation.

Symmetry is a type of match in terms of size and shape between one side or one part in the direction of an object. The line of symmetry is a line that divides a form into two congruent parts. This line also devides the perpendicular lines that connect all points which join the object and the image. The line of symmetry is the axis of reflection of the image and object.

The properties of an image of a reflection are

(a) the image is the same shape and the same size as the object.

(b) the image has different orientations, inverted sides and forms mirror images with one another.

**EXAMPLE 13**

Which of the following patterns show the orientation of a reflection?

(a)  (b)  (c) 

*Solution:*

(a) Yes (b) No (c) Yes

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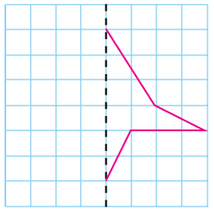
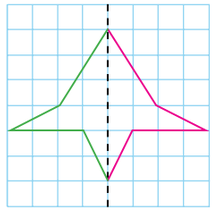
**EXAMPLE 14**

Complete the sketch below.



*Solution:*



**THINK SMART**

| Object | Transformation | Image |
|---|---|---|
| (3, 4) | Reflection in $x$-axis | |
| (–3, –5) | Reflection in $x$-axis | |
| (3, 4) | Reflection in $y$-axis | |
| (–3, –5) | Reflection in $y$-axis | |

## 11.3.2 Describing reflection

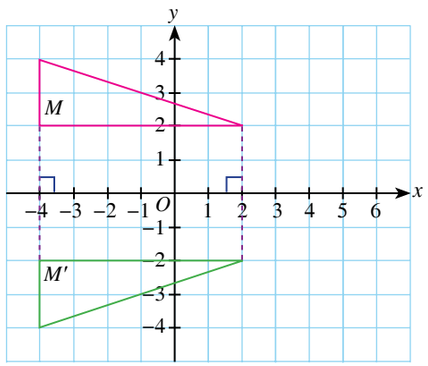
**LEARNING STANDARD**

Describe reflection using various representations.

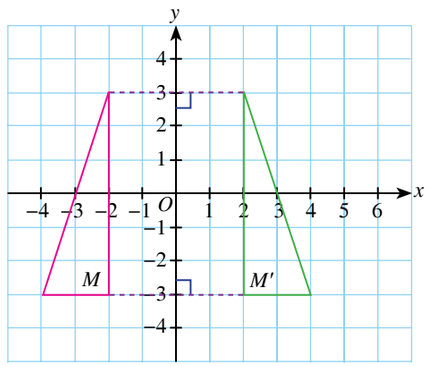
**EXAMPLE 15**

In the following Cartesian diagram, the triangle $M'$ is the image of the triangle $M$ under a reflection. Describe the reflection.

(a)




(b)




*Solution:*

(a) The object $M$ is reflected in the $x$-axis.

(b) The object $M$ is reflected in the $y$-axis.

## 11.3.3 Image of an object

**LEARNING STANDARD**

Determine the image and object under a reflection.

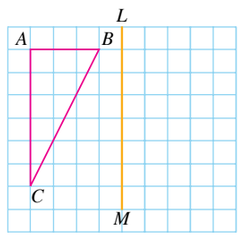
**EXAMPLE 16**

Draw the image of the triangle $ABC$ with a reflection on line $LM$.

*Solution:*

Step 1: Select any vertex and construct a perpendicular line from the vertex to the line $LM$ and extend beyond the axis of reflection.

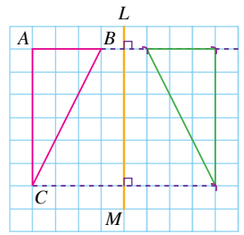
Step 2: Draw parallel lines to all other vertices.








Step 3: Determine the distance of each vertex from the axis of reflection and mark an equal distance from the axis to the same line. Do the same for all vertices.



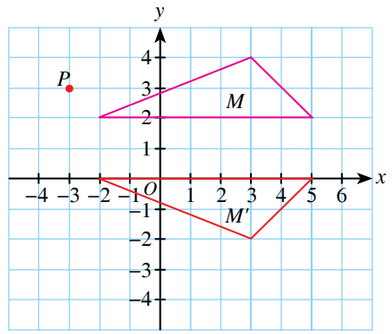

## 11.3.4 Solving problems

**LEARNING STANDARD**

Solve problems involving reflection.

**EXAMPLE 17**

In the diagram on the right, $M'$ is the image of $M$ in an axis of reflection. Determine the coordinates of $P'$ under the same axis of reflection.




*Solution:*

**Understanding the problem**

$M'$ is an image of $M$. Determine the axis of reflection.

**Planning the strategy**

Determine the pair of vertices for the images and objects.
Draw a perpendicular line for both pairs of vertices.
Construct or determine the bisectors for the perpendicular line.

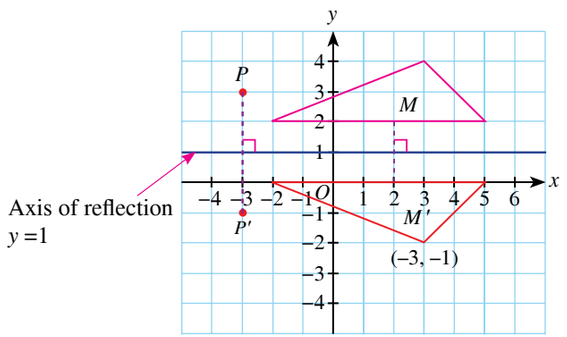
**Conclusion**

Axis of reflection $y = 1$, coordinate of $P'$ are $(–3, –1)$.

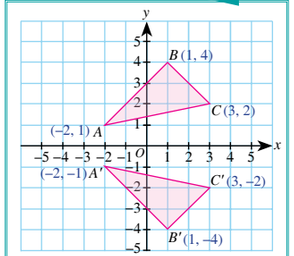
**Implementing the strategy**

Axis of reflection is at $y = 1$.

Use this axis to locate $P'$.

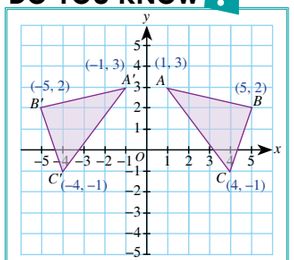



**DO YOU KNOW?**




Reflection of the point $(x, y)$ in $x$-axis is the point $(x, –y)$

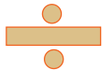
**DO YOU KNOW?**



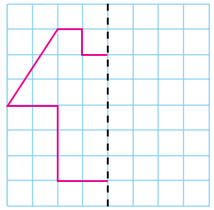

Reflection of the point $(x, y)$ in $y$-axis is the point $(–x, y)$

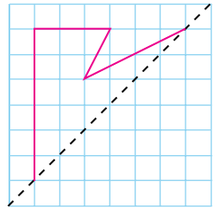
## SELF PRACTICE 11.3

1. Which of the following is a reflection?

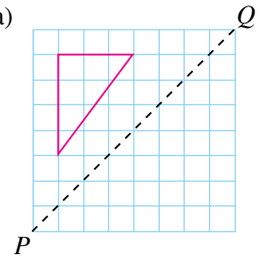
(a)  (b)  (c)  (d) 

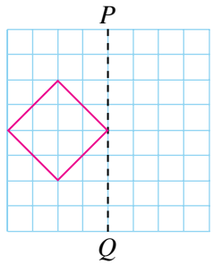
2. Complete the diagram below.

(a) 

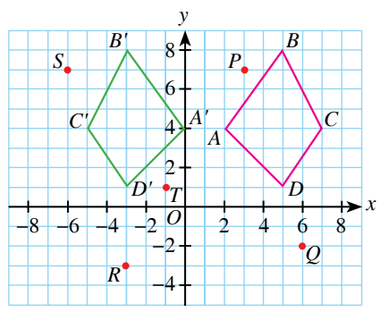
(b) 

3. Construct an image for the object below under a reflection in the line $PQ$.

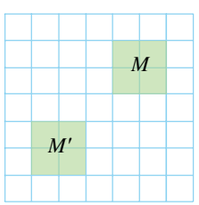
(a) 

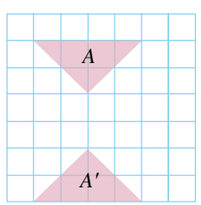

(b) 


4. $A'B'C'D'$ is the image for the object $ABCD$ in an axis of reflection. Determine the coordinate of the image for the points $P$, $Q$, $R$ and $S$ under the same axis of reflection.




5. Draw the axis of reflection for the following diagrams.

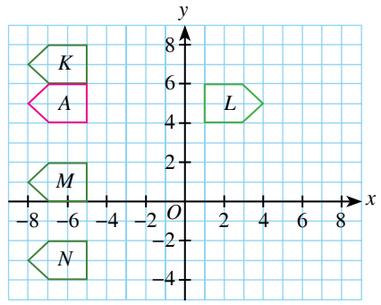
(a) 


(b) 






6. Based on the Cartesian plane on the right, describe the mapping of reflection of polygon $A$ to the polygon
(a) $K$ b) $L$ (c) $M$ (d) $N$




7. Identify the axis of reflection and describe the representation of reflection for these pair of coordinate points.
(a) $A$ (3, 1) and $A'$ (−3, 1) (b) $B$ (−4, 2) and $B'$ (−4, −2)
(c) $C$ (5, 6) and $C'$ (−5, 6) (d) $D$ (2, 2) and $D'$ (4, 2)

8. If $L$ (4, 1) is mapped to $L'$ (4, 5) under a reflection, determine
(a) coordinate of the image for (−3, −1) in the same axis of reflection.
(b) coordinate of the object for (7, 2) under the same reflection.

## 11.4 Rotation

### 11.4.1 Rotation

**LEARNING STANDARD**
Recognise a rotation.

Take note of the rotating objects around you, such as clocks, ceiling fans and tyres. The hands of a clock makes a full rotation every twelve hours. However the tyre depends on the forward or backward movement. All these movements have a centre of rotation.

**COGNITIVE STIMULATION** Individual

**Aim:** Recognising rotation

**Material:** Dynamic geometry software

**Steps:**

1. Open the file MS223.
2. Drag the green button and take note of the rotation.
3. Adjust the button to see the object being rotated.


**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms223 to view a video on rotation.

**Discussion:**

(i) Can you identify the image of the triangle that moves when the angle of rotation is adjusted? What conclusions can you make of the triangle image?

(ii) What are the properties of the image in the activity above?

The properties of the image of a rotation:

(a) The resulting image has the same shape, size and orientation as the object.

(b) The centre of rotation is a stationary point.

(c) The distance of all the points of the image to the centre of the rotation is equal to the distance of the object to the centre of the rotation.

## 11.4.2 Rotation in various representations

**LEARNING STANDARD**

Describe a rotation using various representations.

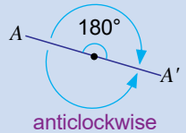
When we describe a rotation, we need to state the **centre**, **the angle** and **the direction of the rotation** that maps the object to the image.

Clockwise

Anticlockwise

**TIPS**

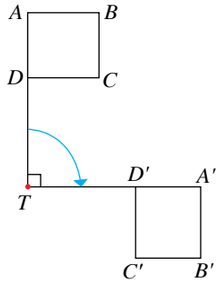
The image generated by a rotation of 180° clockwise is equal to the rotation of 180° anticlockwise.



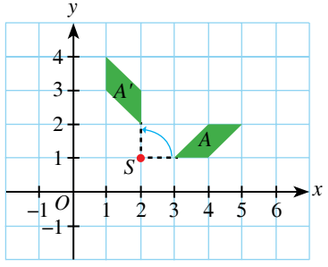

**EXAMPLE 18**

Describe the rotation for the diagram below.

(a)




(b)




*Solution:*

(a) Clockwise rotation of 90° at point $T$.

(b) Anticlockwise rotation of 90° at point $S$.

## 11.4.3 Determining the image and object under a rotation

**LEARNING STANDARD**

Determine the image and object under a rotation.

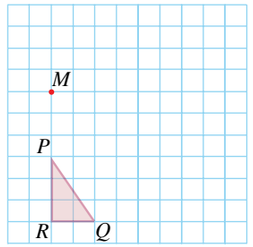
We can use tracing paper, protractor and compasses to determine the image or object under a rotation.





**EXAMPLE 19**

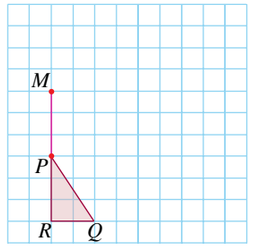
Determine the image of $\Delta PQR$ when rotated 90° anticlockwise at point $M$.



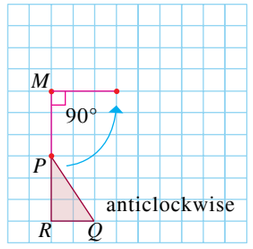

*Solution:*

**Method 1 (Using tracing paper)**

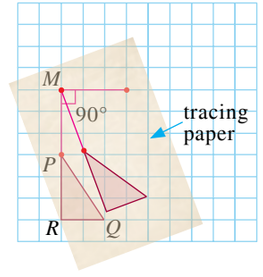
Step 1: Draw the line from point $M$ to point $P$.



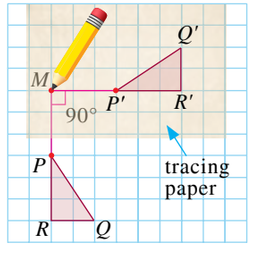

Step 2: Determine the angle of 90° anticlockwise.



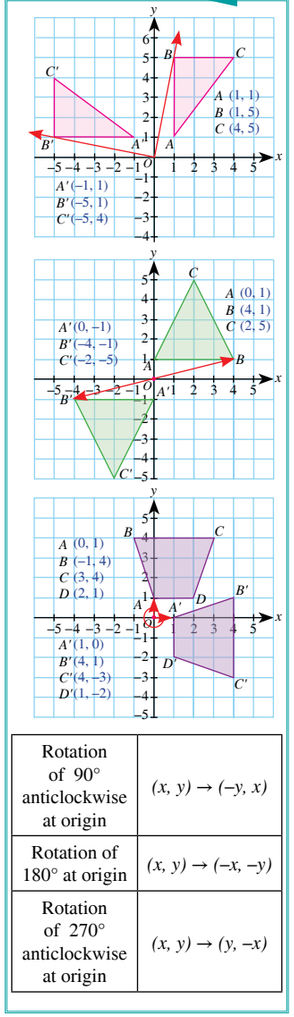

Step 3: Redraw the triangle $PQR$ on tracing paper.




Step 4: Press the tip of a pencil at the point $M$, turn the tracing paper 90° anticlockwise.




**DO YOU KNOW?**




| | |
|---|---|
| Rotation of 90° anticlockwise at origin | $(x, y) \rightarrow (-y, x)$ |
| Rotation of 180° at origin | $(x, y) \rightarrow (-x, -y)$ |
| Rotation of 270° anticlockwise at origin | $(x, y) \rightarrow (y, -x)$ |

**THINK SMART**

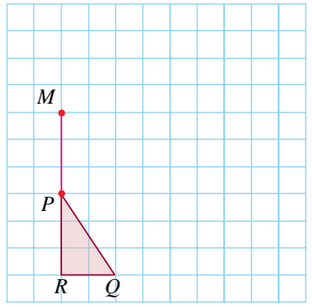
| Object | Transformation | Image |
|---|---|---|
| (5, 2) | Clockwise rotation of 90° at the point (0, 0) | |
| (–3, 4) | Anticlockwise rotation of 90° at the point (2, 1) | |
| (–4, 7) | Rotation of 180° at the point (–1, 3 ) | |



CHAPTER 11

**Method 2 (Using protractor)**

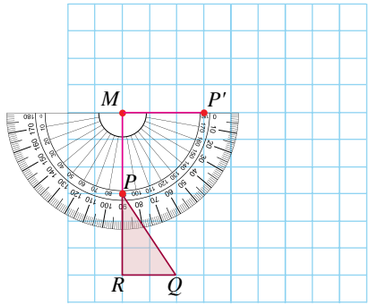
Step 1: Construct the line $MP$.



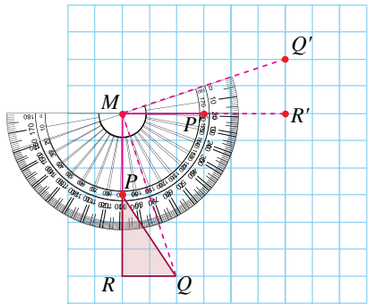

**THINK SMART**

| Object | Transformation | Image |
|---|---|---|
| | Clockwise rotation of 90° at the point (–2, 3) | (–3, 1) |
| | Anticlockwise rotation of 90° at the point (1, 3 ) | (3, 2) |
| | Rotation of 180° at the point (–3, 4) | (2, 1) |

Step 2: Using a protractor, draw a line $MP$ in an anticlockwise rotation of 90° with an equal distance to $MP'$.



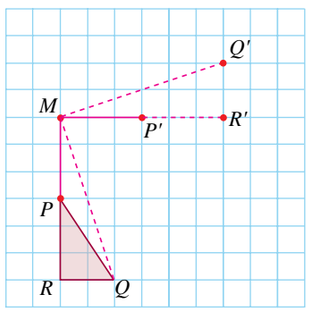

Step 3: Repeat step 2 with lines $MR$ and $MQ$.




**TIPS**

If the question uses a square grid, then you do not have to use protractor for rotations of 90°, 180° and 270°.

Step 4: Join all the points $P'$, $R'$ and $Q'$ to form a triangle similar to $PRQ$.

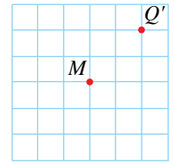






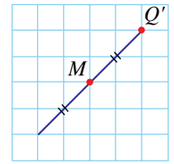

**EXAMPLE 20**

Determine the object for the image point $Q'$ when rotated 180° clockwise at point $M$.

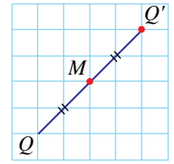



*Solution:*

Step 1: Draw a line to join the points $M$ and $Q'$ and extend it to an equal distance to $MQ'$ in the opposite direction.

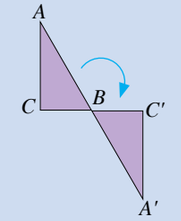



Step 2: Mark point $Q$ on the extended line with $MQ = MQ'$.




**TIPS**

If $B$ is the centre of rotation, then the position of image $B$ will not change.




## 11.4.4 Solving problems

**LEARNING STANDARD**

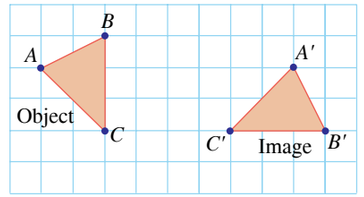
Solve problems involving rotation.

Do you know that if the object and image of a rotation is given, centre, angle and rotational direction can be determined using the method of geometric construction?

### Determining centre, angle and direction of rotation

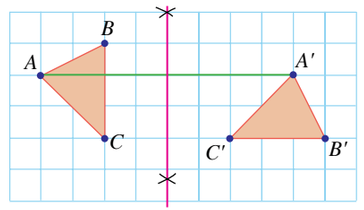
**EXAMPLE 21**

$A'B'C'$ is the image for $ABC$ after a rotation. Determine the angle, direction and centre under a rotation.

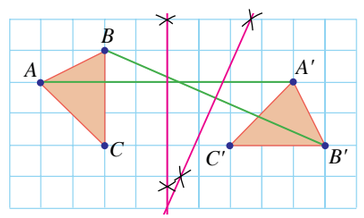



*Solution:*

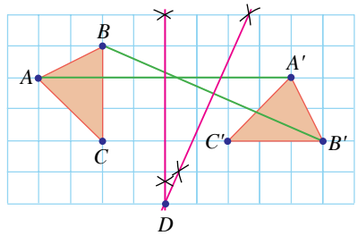
Step 1: Join point $A$ to $A'$. Construct a perpendicular bisector for the line segment $AA'$.




Step 2: Repeat step 1 for the lines $BB'$ or $CC'$.

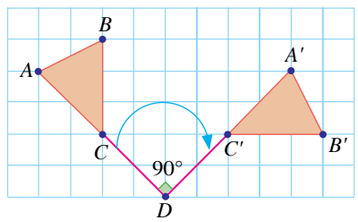


Step 3: The intersection point of two perpendicular lines is the centre of the rotation. Mark the centre of the rotation as $D$.




Step 4: Measure the angle $CDC'$ using a protractor.

Therefore, the image is under rotation of 90° clockwise at point $D$.



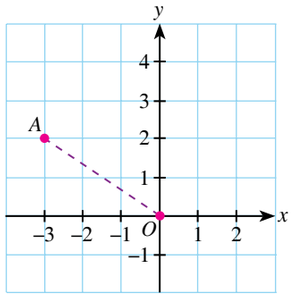

## Defining coordinates of the image when coordinates of the object are given

**EXAMPLE 22**

Determine the coordinates of the image of point $A$ (–3, 2) under a rotation of 90° clockwise at $O$ origin.

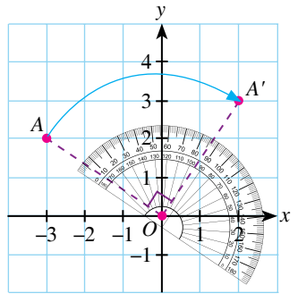
*Solution:*

**Step 1:**
Join the line $OA$.




**Step 2:**
Rotate the line $OA$ 90° clockwise at origin $O$ using a protractor.

From the diagram, the coordinates of the image $A'$ are (2, 3).




## Determining the coordinates of objects when the coordinates of the image is given

**EXAMPLE 23**

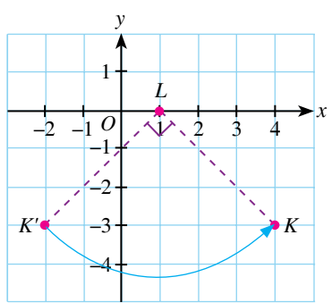
If $K'$ (–2, –3) is the image of $K$ with a rotation of 90° clockwise at point $L$ (1, 0), state coordinates $K$.

*Solution:*

**Step 1:** Reverse the rotation direction to locate the coordinates of the object, that is, the point $K$.

**Step 2:** By using a protractor, rotate the line $K'L$ at point L, at 90° anticlockwise.

From the diagram, the coordinates of $K$ are (4,–3).



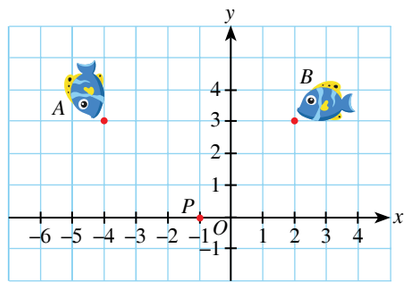




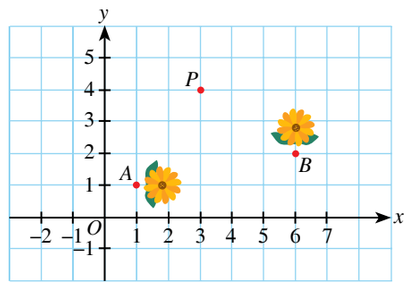

## SELF PRACTICE 11.4

1. Describe the rotations at centre $P$ when $A$ is the object and $B$ is the image.

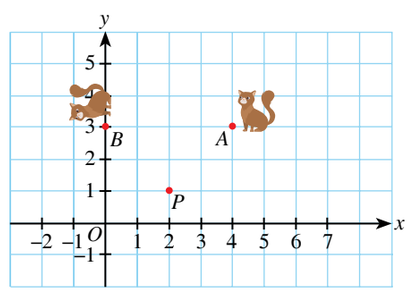
(a)



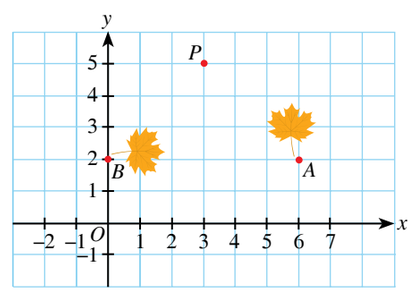

(b)




(c)

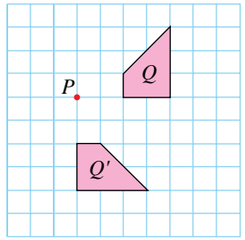



(d)

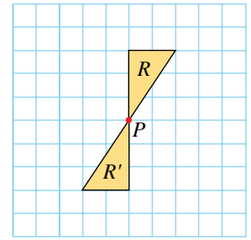



2. Describe the rotations that map the object to its image.

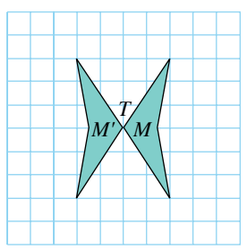
(a)



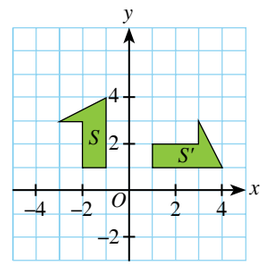

(b)




(c)




(d)



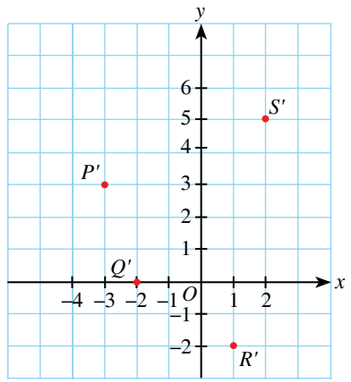

3. Draw the image for $R$ under a rotation for each of the following.

Anticlockwise rotation of 90° at centre $O$.


Rotation of 180° at centre $O$.

4. Determine the coordinates of objects for the following points at the given rotations below.

| Point | Rotation | | | Coordinate |
|---|---|---|---|---|
| | Centre | Angle | Direction | |
| $P$ | $(-2, 1)$ | 90° | clockwise | |
| $Q$ | $(0, 0)$ | 90° | anticlockwise | |
| $R$ | $(0, -1)$ | 90° | anticlockwise | |
| $S$ | $(0, 4)$ | 90° | clockwise | |

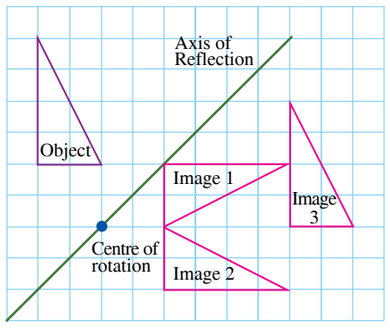



## 11.5 Translation, Reflection and Rotation as an Isometry

### 11.5.1 Relationship between translation, reflection and rotation with isometry

**LEARNING STANDARD**

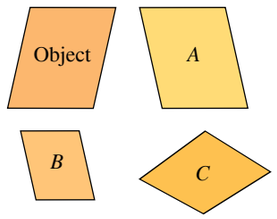
Investigate the relationship between the effects of translation, reflection and rotation and the distance between two points on an object and image, and hence explain isometry.




You have learned about the transformation for translation, reflection and rotation for an object. Each one has certain properties. Study the diagram on the left. Are you able to recognise the transformation of Image 1, Image 2 and Image 3? What is the relationship between the distance of the object and the image? If an object is mapped to a congruent image, then it is an isometry. Isometry is a transformation that maintains the distance between any two points on the original object. The isometric transformation will retain the original shape and size of the object.

**EXAMPLE 24**

Which of these diagrams $A$, $B$ and $C$ are the isometric images of the object under an isometry?




*Solution:*

Figure $A$: an isometric image as it has the same shape and size.
Figure $B$: non-isometric image because it is not of the same size.
Figure $C$: non-isometric as the shape and size are not the same.

You are able to recognise that translation, reflection and rotation are isometric transformations.





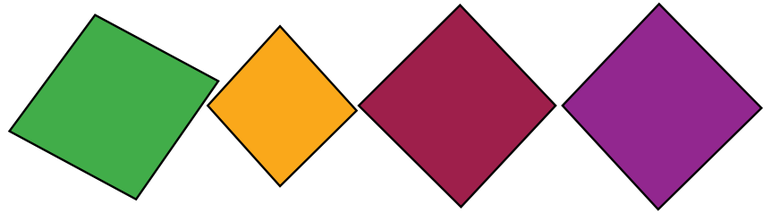
### 11.5.2 Relationship between isometry and congruence

**LEARNING STANDARD**

Explain the relationship between isometry and congruency.

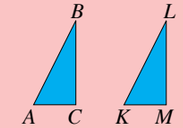
Look at the purple object. Can you state the image which is congruent under a transformation of reflection?

Can you determine the axis of reflection for this isometric transformation?



**FLASHBACK**

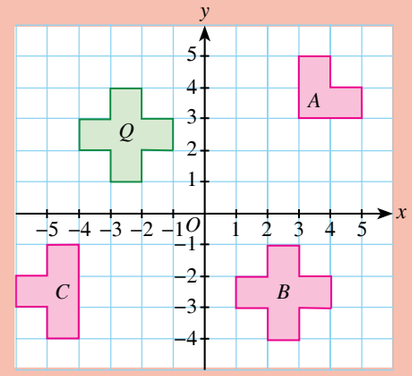
Two objects are congruent if the shape and size are the same.




$ABC$ and $KLM$ are congruent under a translation.

**COGNITIVE STIMULATION** Group

**Aim:** Identifying the relationship between isometry and congruence
**Materials:** Tracing paper and ruler



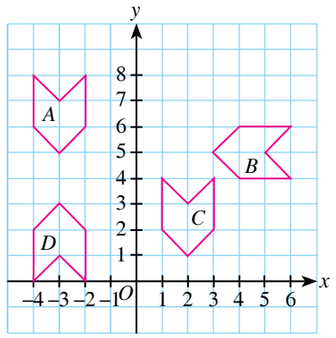

**Steps:**
1. Study the diagram above. $Q$ is the object of an image.
2. Work together with your friends, to identify congruent images.
3. Identify other possible isometry that produces images that are congruent.

**Discussion:**
(i) If the images $A$ and $C$ are non-congruent, are the images an isometry?
(ii) What is the relationship between isometry and congruence?

Under an isometry, objects and images are of equal shape and size. Therefore, objects and images are **congruent**. Isometry is a transformation where the image is congruent with the object.

**EXAMPLE 25**

Objects $A$, $B$, $C$ and $D$ are congruent. State the isometry of
(a) object $A$ to object $B$.
(b) object $A$ to object $C$.
(c) object $A$ to object $D$.

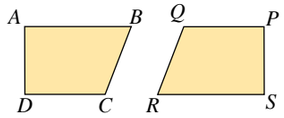



***Solution:***

(a) Rotation
(b) Translation
(c) Reflection

## 11.5.3 Solving problems

When naming a congruent polygon, the order must be based on the vertex or the corresponding angle.




The rectangles $ABCD$ and $SRQP$ are congruent.

**LEARNING STANDARD**

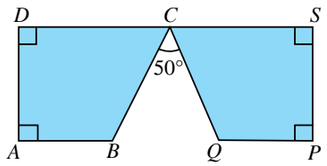
Solve problems involving isometry and congruency.

**FLASHBACK**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms232 to view an animation on congruence.

**EXAMPLE 26**

In the diagram, $ABCD$ is the image for $PQCS$ under an isometry.
Given $DCS$ is a straight line, determine $\angle PQC$.




***Solution:***

**Understanding the problem**

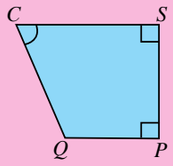
$ABCD$ is the image of $PQCS$.

$DCS$ is a straight line.

$\angle PQC$ is a part of the rectangle $PQCS$.

**Planning the strategy**

Determine




$$\angle QCS = \frac{180° - 50°}{2}$$
$$= \frac{130°}{2}$$
$$= 65°$$

**Conclusion**

Therefore, $\angle PQC$ is 115°.

**Implementing the strategy**

$$\angle PQC = 360° - 90° - 90° - 65°$$
$$= 115°$$

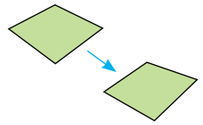




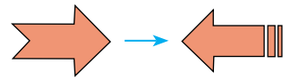
## SELF PRACTICE 11.5

1. Determine whether the following transformation is an isometry.

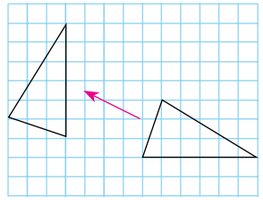
(a)



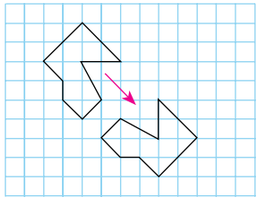
(b)



(c)

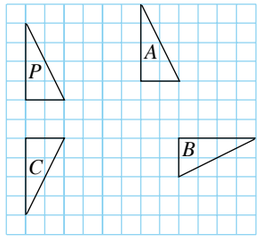


(d)

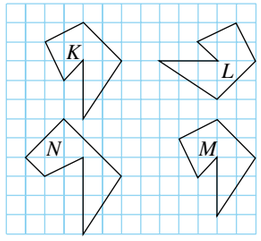


2. Determine whether each of the following transformations is an isometry.
(a) A reflection followed by another reflection.
(b) A translation.
(c) A repeated rotation.

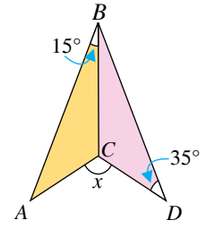
3. In the diagram, $A$, $B$ and $C$ are images for the object $P$.
State the type of transformation.




4. The diagram shows several shapes. State the shapes that are congruent.




5. In the diagram, $\Delta ABC$ is the image for $\Delta BCD$, under an isometric transformation. Calculate the value $x$.



# 11.6 Rotational Symmetry

## 11.6.1 Rotational symmetry

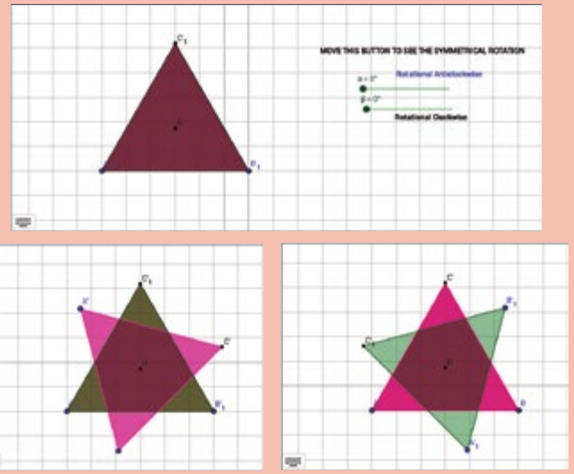
A shape has rotational symmetry if the shape does not change after rotation even though it has less than one rotation.

**LEARNING STANDARD**

Explain rotational symmetry.

**COGNITIVE STIMULATION** Group

**Aim:** Identifying rotational symmetry
**Material:** Dynamic geometry software
**Steps:**




1. Open the file MS234.
2. Move the green button in an anticlockwise rotation at angles120°, 240° and 360°. Note the changes in the green triangle. Move the button back to its original position.
3. Move the green button in clockwise rotation at 120°, 240° and 360°. Note the changes in the pink triangle.

**Discussion:**

(i) Can you identify the rotational symmetry of the hexagon?

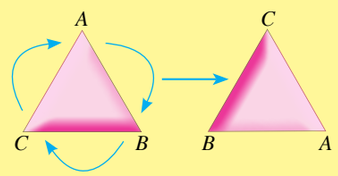
(ii) If *D* is the centre of the rotation, what do you understand by rotational symmetry.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms234 to view an animation on rotational symmetry.

**DO YOU KNOW?**

The recycle symbol is an example of rotational symmetry.




Symmetry is an exact match in terms of size and shape of one part or the side of a direction or object. For rotational symmetry, when the shape or image is rotated less than 360 ° at a fixed point, the shape will still look the same.





**EXAMPLE 27**

Identify the objects which has rotational symmetry.

(a) 

(b) 

(c) 

*Solution:*

(a) No (b) Yes (c) No

## 11.6.2 Order of rotational symmetry of an object

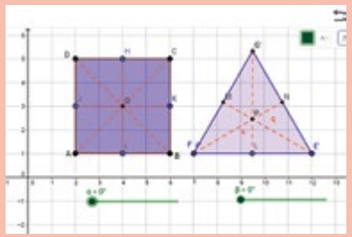
**LEARNING STANDARD**

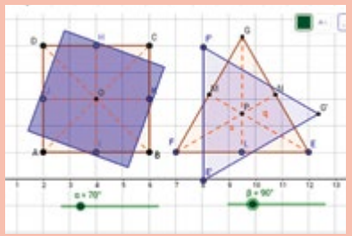
Determine the order of rotational symmetry of an object.

The number of images that can be generated in the same rotational centre and becomes the original object is called the order of rotational symmetry. The order of rotational symmetry is equal to the number of axes of symmetry of an object.

**COGNITIVE STIMULATION** Group

**Aim:** Identifying order of rotational symmetry
**Material:** Dynamic geometry software
**Steps:**





**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms235 to view this activity.

1. Open the file MS235.
2. There are two forms of geometry.
3. Move the angle buttons to get the original shape.
4. Count the number of rotational movements to get the original shape of the object.

**Discussion:**

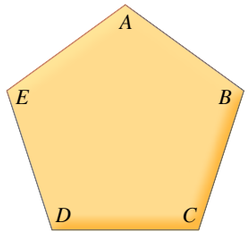
(i) Is the number of order of rotational symmetry equals to the number of axis of symmetry?

(ii) Can you determine the number of order of rotational symmetry?

The order of rotational symmetry is the number of times a shape moves to return to its original shape in a complete rotation. The number of axis of symmetry is equals to the number of order of rotational symmetry.

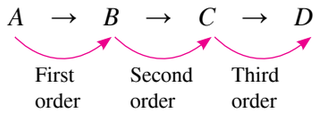
**EXAMPLE 28**

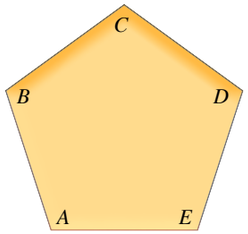
Determine the order of rotational symmetry when the position of $A$ changes to position $D$ in the diagram on the right.




*Solution:*

Using tracing paper, draw and determine the rotation order $A$ to $D$.

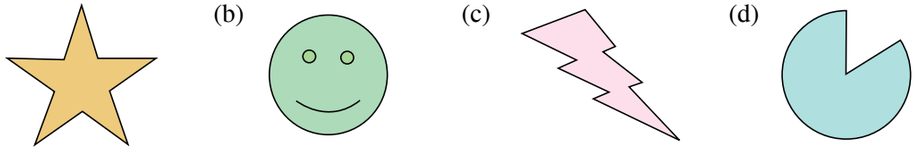



**SELF PRACTICE 11.6**

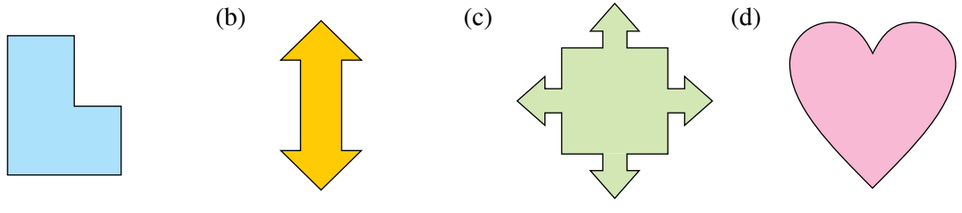
1. Which of the following objects has rotational symmetry?

(a) (b) (c) (d)

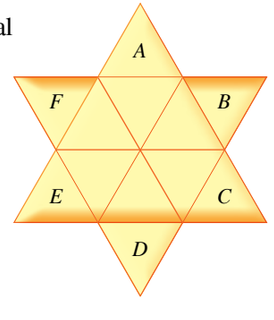


2. Determine the order of rotational symmetry for the following objects.

(a) (b) (c) (d)



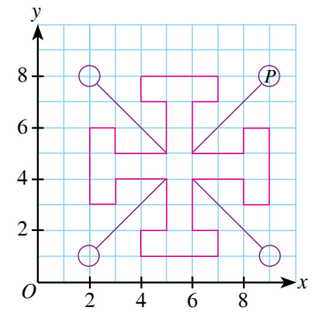
3. The symmetrical object is rotated at a point. State the order of rotational symmetry if
   (i) position $A$ is at position $C$.
   (ii) position $B$ is at position $D$.
   (iii) position $C$ is at position $B$.





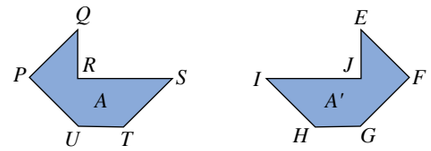



4. The symmetrical object lies on the Cartesian plane. State the coordinates of $P$ under the third order of symmetry.



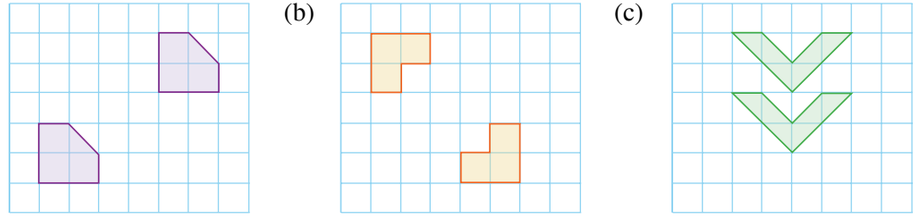

## GENERATING EXCELLENCE

1. The diagram on the right shows polygon $A$ mapped to polygon $A'$ under a reflection. Identify the point that matches
   (a) the image of point $P$.
   (b) the object of point $G$.



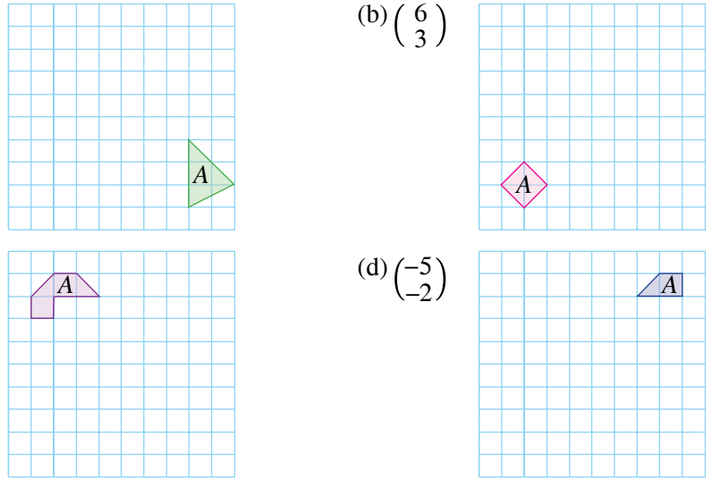

2. Which of the following is a translation and state the reason for your answer.

(a) (b) (c)



3. Draw the image for object $A$ under the given translation.

(a) $\begin{pmatrix} -7 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

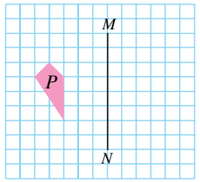


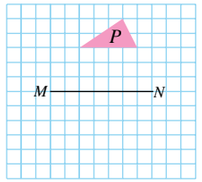

(c) $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$ (d) $\begin{pmatrix} -5 \\ -2 \end{pmatrix}$

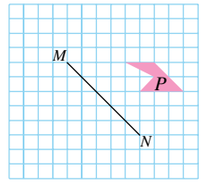
4. If point $K$ $(-2, -2)$ is the object, identify the image under the following vector translation.

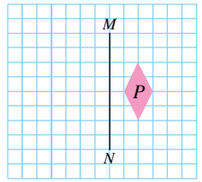
(a) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ (d) $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ (e) $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ (f) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

5. Draw the image $P'$ of the object $P$ under the reflection in the line $MN$.

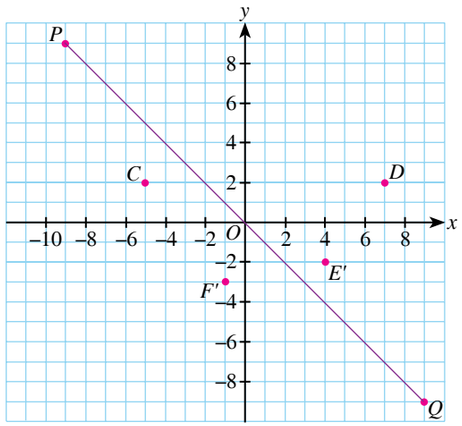
(a) 


(b) 


(c) 

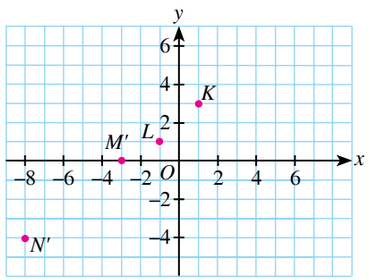

(d) 


6. Determine the coordinates of the image or object of the following points, under the given axis of reflection.



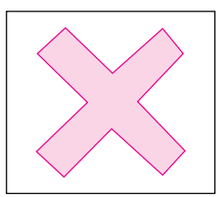

| Point | Axis of reflection | Coordinate |
|---|---|---|
| $C$ | $y$-axis | $C'$ (  ) |
| $D$ | $x$-axis | $D'$ (  ) |
| $E'$ | Line $PQ$ | $E$ (  ) |
| $F'$ | Line $PQ$ | $F$ (  ) |

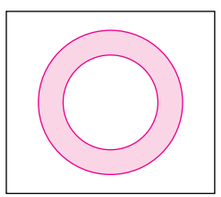
7. Determine the coordinates of the image or object of the following points, under the given rotation.




| Point | Rotation | | | Coordinate |
|---|---|---|---|---|
| | Centre | Angle | Direction | |
| $K$ | (0, 0) | 90° | clockwise | $K'$ (  ) |
| $L$ | (0, 2) | 180° | clockwise | $L'$ (  ) |
| $M'$ | (0, 0) | 90° | anticlockwise | $M$ (  ) |
| $N'$ | (−3, −4) | 180° | clockwise | $N$ (  ) |

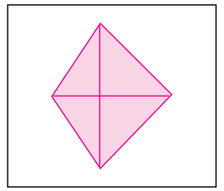
8. Which of the following has rotational symmetry?

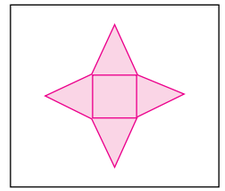
(a) 

(b) 



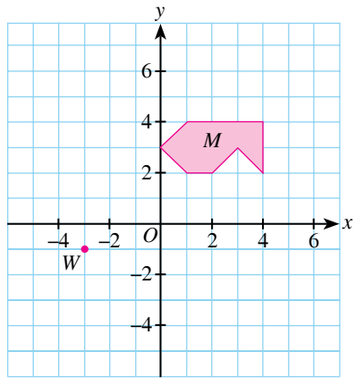


(c) 

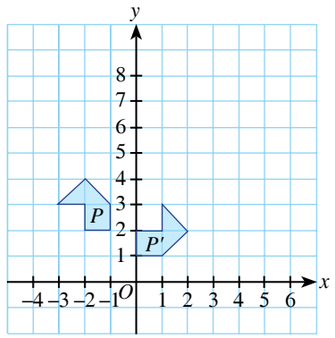
(d) 

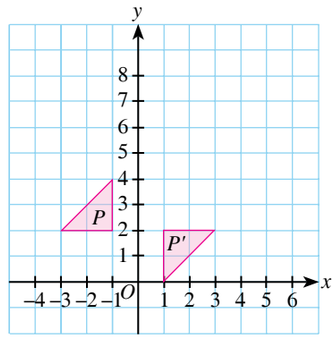
9. (i) In the diagram on the right, draw the image of $M$ under the reflection in the $x$-axis.

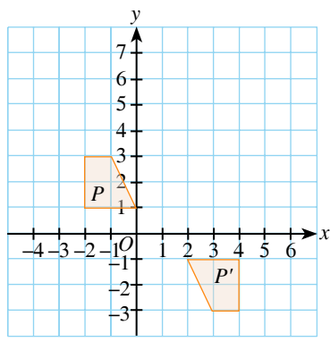
(ii) State the coordinates of the image $W'$ under the same reflection.

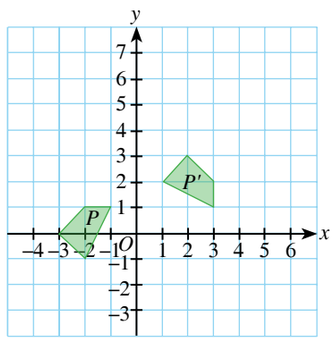



10. Given $P'$ is the image of $P$ under rotation, provide a complete description about the rotation.

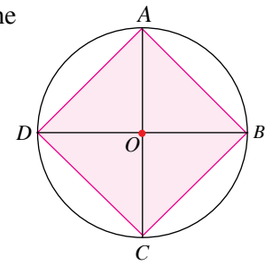
(a) 


(b) 

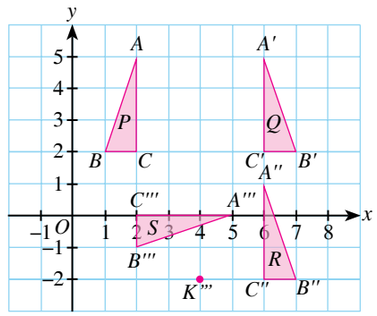

(c) 


(d) 

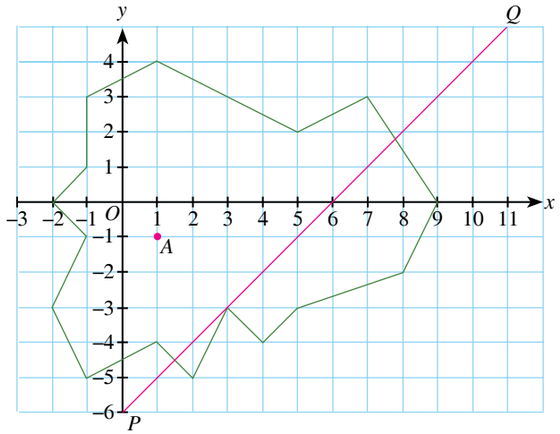
11. In the diagram on the right, $ABCD$ is a square. State the image of the triangle $OAB$ under the following rotations.
    (i) Clockwise rotation of 90° at point $O$.
    (ii) Rotation of 180° at point $O$.
    (iii) Anticlockwise rotation of 270° at point $O$.



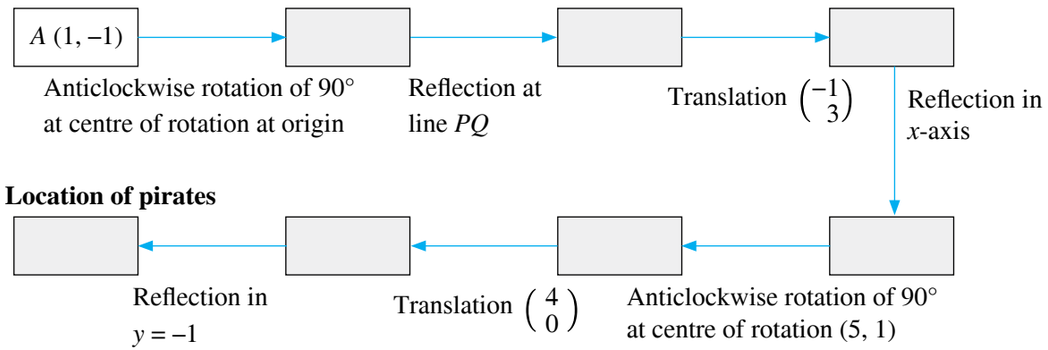

12. The diagram on the right shows triangle $ABC$ which has three transformations, $P \rightarrow Q \rightarrow R \rightarrow S$.
    (a) Describe the transformations.
    (b) If the point $K'''$ is the image of point $K$, state the coordinates of the object $K$ under the same transformation.




13. The diagram below shows a map of the sea of Kejora.



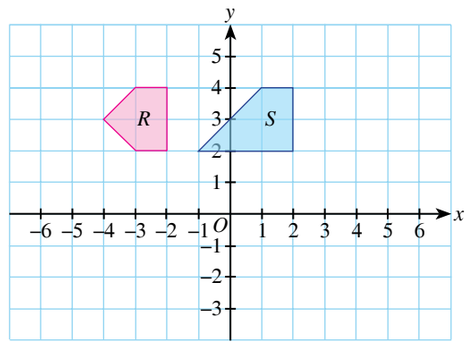

Point $A$ is the position of Makdis military vessel. Help Makdis soldiers track pirates in the following order of transformations.





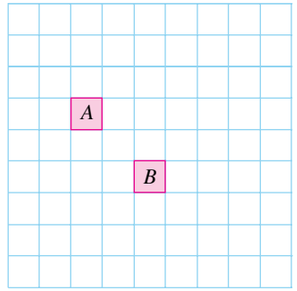



14. The diagram on the right shows the object $R$ and $S$.
    (a) Draw the image of $R$ under
        (i) anticlockwise rotation of 90° at the point of origin followed by the translation $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$.
        (ii) the reflection on the straight line $x = 1$.
    (b) Draw the image of $S$ under the reflection in the $x$-axis followed by the translation $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ followed by the rotation of 180° at the point $(0, -1)$.
    Next, name the polygon.

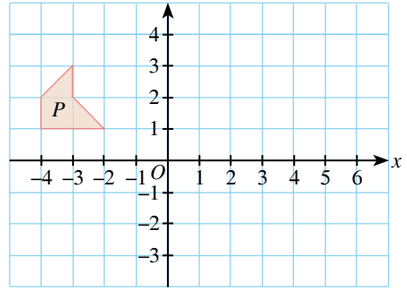



15. The diagram on the right shows rectangles $A$ and $B$ drawn on a square grid.

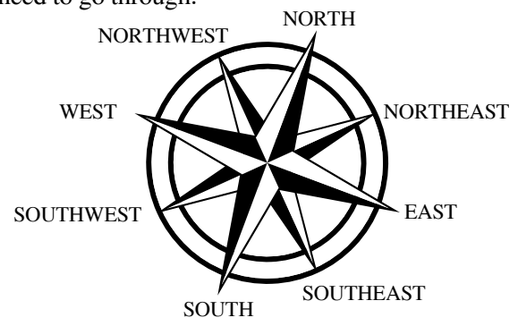
    The rectangle $A$ is the image of rectangle $B$ under one transformation. Describe the five possible transformations.




16. Based on the diagram on the right,
    (a) rotate object $P$ at 180° at point (1, 1). Label the rotational image as $Q$.
    (b) make a translation $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$ of $Q$ and label it as $R$.
    (c) describe the other transformations that map the object $P$ to the image $R$.




17. Based on the diagram below, Fauzah and Zainun are located in Southeast and Southwest respectively. If they plan to meet in an area located in the Northeast, state the order of rotational symmetry that they need to go through.







CHAPTER 11

## CHAPTER SUMMARY

**Isometric Transformations**

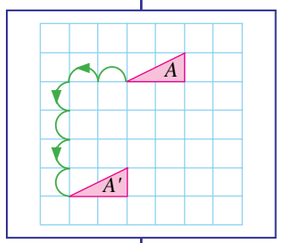
Transfer of points on a plane

**Translation**

The transfer of all points on a plane in the same direction and magnitude of a vector. The properties of translation are:

(i) the image does not change.

(ii) the image lies in a particular vector of the object.




**Reflection**

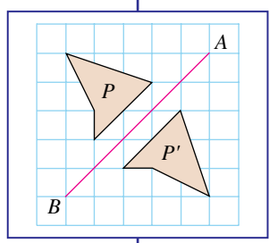
The transformation that reverses the points on a plane based on a line known as the axis of reflection. The properties of reflection are:

(i) the object and the image are on the opposite sides of the axis of reflection.

(ii) the object and its image have the same perpendicular distance from the axis of reflection.

(iii) the shape and size of the image are the same as the object, but the orientation is reversed.

(iv) the image of a point on the axis of reflection is the point itself.



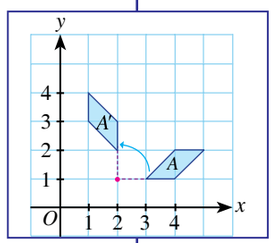

**Rotation**

The process of transformation that occurs when each point is rotated at a fixed point through a certain angle and in a certain direction. The properties of rotation are:

(i) rotates at a specific centre of rotation.

(ii) has a rotational angle.

(iii) the image retains the original form but the position changes.




**Isometry**

The transformation that shows the original object and its image is congruent. In isometry, the distance between two points on the original object equals the distance between the two corresponding points in the image. Reflection, rotation and translation are isometric.

**Congruence**

Object and image has the same size and shape.

**Rotational Symmetry**

The shape or image is rotated in less than 360° at a fixed point. Its shape remains the same.





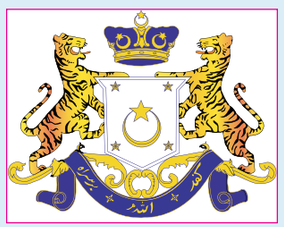
## SELF REFLECTION

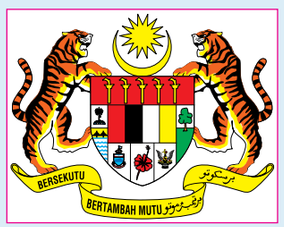
**At the end of this chapter, I will be able to:**

| | |
|---|---|
| 1. Describe translation, reflection and rotation using various representations. | |
| 2. Determine the image and object under a translation, reflection or rotation. | |
| 3. Solve problems involving translation, rotation and reflection. | |
| 4. Investigate the relationship between the effects of translation, reflection and rotation and the distance between two points on an object and image, and hence explain isometry. | |
| 5. Explain the relationship between isometry and congruency. | |
| 6. Solve problems involving isometry and congruency. | |
| 7. Explain rotational symmetry. | |
| 8. Determine the order of rotational symmetry order of an object. | |

## MINI PROJECT

You are asked to design a class logo that symbolises cooperation, solidarity, tolerance, respect and determination. These features should be translated in the form of isometric transformation pattern that are diversify yet moderate. Then, explain the meaning of each item you have chosen.






CHAPTER 12

# Measures of Central Tendencies

## WHAT WILL YOU LEARN?

**12.1** Measures of Central Tendencies

## WORD LINK

- Measure of central tendency
- Mode
- Median
- Mean
- Extreme value
- Data
- Table
- Data representation
- Pie chart
- Bar chart
- Dot plot
- Stem and leaf plot
- Frequency table

- *Sukatan kecenderungan memusat*
- *Mod*
- *Median*
- *Min*
- *Nilai ekstrem*
- *Data*
- *Jadual*
- *Perwakilan data*
- *Carta pai*
- *Carta palang*
- *Plot titik*
- *Plot batang dan daun*
- *Jadual kekerapan*

Statistics is a branch of mathematics that uses data.

This is because statistics involves the collection, compilation, description and analysis of data and conclusions from the results of data analysis.

One example of application of statistical knowledge is the stock market. In the stock market, statistics are applied in various ways by using data representations. In this way they are able to study various information and make various inferences from profit data sets, economic development, business, inflation and national finances.

## WALKING THROUGH TIME

John Graunt was a well-known statistician. He used statistical knowledge to make some conclusions and predictions about population and mortality rates in his initial study.

For more information:

http://rimbunanilmu.my/mat_t2e/ms245

## WHY STUDY THIS CHAPTER?

- Measures of central tendencies is often used in areas related to data.
- Career fields that use this knowledge are economics, statistics, business, entrepreneur and education.

**CREATIVE ACTIVITY**

**Aim**: Identifying information from data representation

**Materials**: Booklet and calculator

**Steps:**

1. Survey the number of siblings for each student in your class.
2. Organize the data by constructing the frequency table below.

| Number of siblings | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |

3. List the information obtained from the frequency table above.

(i) The number of siblings with the highest frequency.

(ii) The number of siblings with the lowest frequency.

**FLASHBACK**

Frequency is the number of times an item appears in a data.

**FLASHBACK**

Frequency table is a table that consists of data and frequency of the item.

**FLASHBACK**

**Data Representation**
- Pie chart
- Bar chart
- Line graph
- Dot plot
- Stem-and-leaf plot

## 12.1 Measures of Central Tendencies

Measures of central tendencies are measures that show the position of a group of data and describe the information of that data with only one value.

Mohd Azizulhasni Awang, also known as 'Pocket Rocket Man' is a professional Malaysian Track Cyclist. In 2017, he became the champion for the men's keirin competition during the Track Cycling World Championship in Hong Kong.

Based on his outstanding achievements, are we able to predict that he will improve or maintain his record of achievement in the upcoming Olympic Games? This prediction can be made based on Mohd Azizulhasni's achievement data through certain justifications. From this justification, analysis and interpretation can be done.

This process is known as measures of central tendencies. The three types of **measures of central tendencies** are mean, median and mode.


Source: http://www.astroawani.com/berita-sukan/fakta-tentang-jaguh-pelumba-negara-azizulhasni-awang-139401





### 12.1.1 Mode, mean and median for a set of ungrouped data

**LEARNING STANDARD**

Determine the mode, mean and median of a set of ungrouped data.

#### Mode

**COGNITIVE STIMULATION** Individual

**Aim:** Determining the mode

**Material:** Worksheet

**Steps:**

1. Open the file MS247.
2. Look at the lyrics of Negaraku as attached.
3. Complete the table.

**Discussion:**

Which vowel has the highest frequency?

Title: Worksheets 12.1
Aim: Determine the mode

1. Study the lyrics of Negaraku. Tally the vowel and calculate the frequency.

Negaraku
Tanah tumpahnya darahku,
Rakyat hidup
bersatu dan maju,

Rahmat bahagia
Tuhan kurniakan,
Raja kita
selamat bertakhta

| Vowel | Tally | Frequency |
|---|---|---|
| a | | |
| e | | |
| i | | |
| o | | |
| u | | |

Vowel that have the highest frequency = _____

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms247 to retrieve the worksheet.

From the activity above, the most frequent vowel is known as **mode**.

The **mode** of a set of data is **the highest value of its frequency**.

Sometimes there are two modes in a set of data where the highest frequency is equal. However, when the frequency of a set of data is the same then the set of data is to be said as no mode.

**EXAMPLE 1**

State the mode for each set of data.

(a) 4, 5, 2, 3, 4, 4, 5

(b) *M*, *N*, *L*, *M*, *L*, *P*, *L*, *L*, *P*

(c) Coffee, Tea, Coffee, Coffee, Milk, Tea, Milk, Tea

(d) 2, 4, 6, 8, 10

***Solution:***

(a) 4, 5, 2, 3, 4, 4, 5 ← 4 has the highest frequency, which is 3
Mode = 4

(b) *M*, *N*, *L*, *M*, *L*, *P*, *L*, *L*, *P* ← *L* has the highest frequency, which is 4
Mode = *L*

(c) Coffee, Tea, Coffee, Coffee, Milk, Tea, Milk, Tea ← Coffee and tea have the highest frequency, which is 3
Mode = Coffee and Tea

(d) 2, 4, 6, 8, 10 ← None of the numbers is repeated.
No mode.

## Median

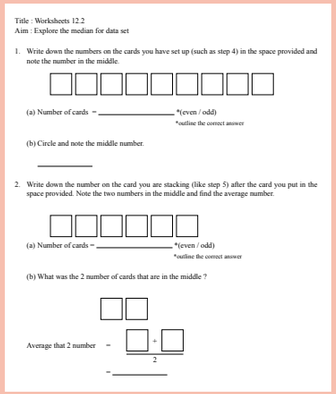
**COGNITIVE STIMULATION** Group

**Aim:** Exploring the median for a set of data

**Material:** Worksheets

**Steps:**

1. Open the file MS348 and print the worksheet.
2. Cut out all the cards one by one.
3. Arrange the number cards in ascending order.
4. Identify the card in the middle. Record the number in the provided worksheet.
5. Now, remove any 3 cards at random.
6. Rearrange the cards in ascending order.
7. Identify the two numbers in the middle. Calculate the average of the two numbers. Record on the worksheet.


Title : Worksheets 12.2
Aim : Explore the median for data set

1. Write down the numbers on the cards you have set up (such as step 4) in the space provided and note the number in the middle.

(a) Number of cards = _____ *(even / odd)
*outline the correct answer

(b) Circle and note the middle number.

_____

2. Write down the number on the card you are stacking (like step 5) after the card you put in the space provided. Note the two numbers in the middle and find the average number.

(a) Number of cards = _____ *(even / odd)
*outline the correct answer

(b) What was the 2 number of cards that are in the middle ?

Average that 2 number = $\frac{\square + \square}{2}$

= _____

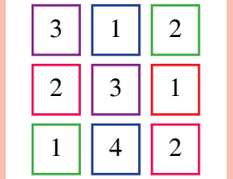
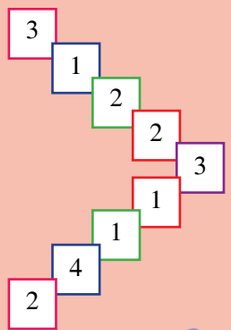


Figure *A*

QR CODE

Scan the QR Code or browse http://rimbunanilmu.my/mat_t2e/ms248 to retrieve the worksheet.




THINK SMART

Try to repeat this activity by organising the card in descending order. Did you get the same results?

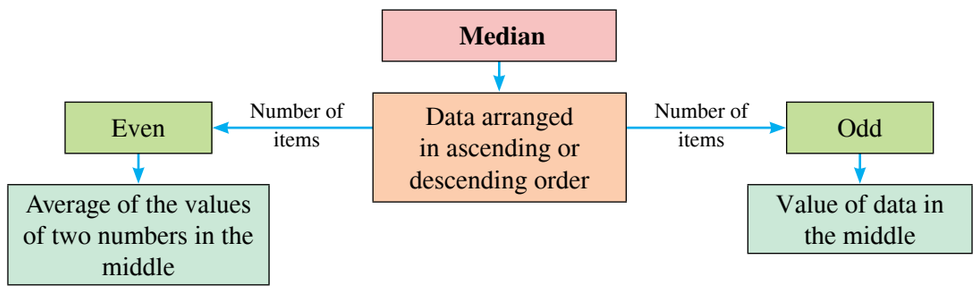
**Discussion:**

How can you locate the middle value for a set of data if the number of items are

(i) odd (ii) even

In the activity above, you have defined the **median** for odd and even data. Note in Step 3, the number of all the cards you have arranged is 9 pieces (odd) and in Step 6 the number of cards arranged is 6 pieces (even). Therefore,

The **median** for a set of data with an odd number of items is the value in the **middle**, while the median for a set of data with an even number of items is the average value of two numbers in the middle arranged in ascending or descending order.




CHAPTER 12





**EXAMPLE 2**

The data below is the pocket money to for five students each day. Determine the median.

RM5 RM8 RM3 RM7 RM5

*Solution:*

3 5 5 7 8 ← Arrange data in ascending order

3 5 5 7 8 ← Mark the data in the middle

Median = 5

**EXAMPLE 3**

The data below shows the total number of goals scored by the Seladang team in 10 games. Determine the median.

1 5 1 1 4 2 5 1 4 4

*Solution:*

1 1 1 1 2 4 4 4 5 5 ← Arrange data in ascending order

1 1 1 1 2 4 4 4 5 5 ← Circle the data in the middle

$\frac{2+4}{2} = \frac{6}{2} = 3$ ← Calculate the average of the two numbers

Median = 3

Another way to find a median is by deleting left and right data in pairs (ascending or descending).

**EXAMPLE 4**

Determine median for each set of the following data.

(a) 4, 7, 2, 3, 4, 9, 6, 2, 1

(b) 28, 27, 21, 23, 24, 21, 25, 24

*Solution:*

(a) (a) Arrange data in ascending order.

1, 2, 2, 3, 4, 4, 6, 7, 9

Value in the middle

Median = 4

(b) Arrange data in ascending order.

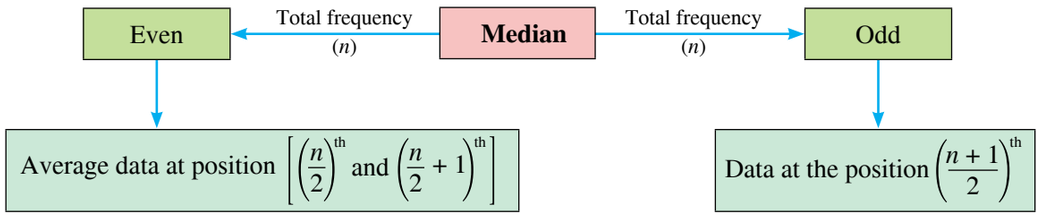
21, 21, 23, 24, 24, 25, 27, 28

Two values in the middle

Median = $\frac{24+24}{2}$ = 24

CHAPTER 12

## Determine the median for even or odd numbers of data in frequency table and data representation




with $n$ as the total frequency.

**EXAMPLE 5**

1. The table shows time taken to build a model of a rocket in a science activity for 11 groups of students.

| Time (minutes) | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Frequency | 1 | 6 | 3 | 1 |

Determine the median using the frequency table

*Solution:*

Total Frequency = 11

Median = data at $\left(\frac{n+1}{2}\right)$

= data at $\left(\frac{11+1}{2}\right)$

= data at $\frac{12}{2}$

= data at $6^{th}$

| Time (minutes) | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Frequency | 1 | 6 | 3 | 1 |
| Position of data | 1 | 2 - 7 | 8 - 10 | 11 |

1st data is 10 — 2nd data till the 7th data is 20

The $6^{th}$ data is 20, therefore the median is 20.

2. The table shows time taken to solve the number of crossword puzzles in the Malay Language Society activity for 12 groups of students.

| Time (minutes) | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Frequency | 2 | 4 | 5 | 1 |

Determine the median for the frequency table.





*Solution:*

Number of frequencies = 12

Median = Average data at $\left[\left(\frac{n}{2}\right)^{th} \text{ and } \left(\frac{n}{2}+1\right)^{th}\right]$

= Average data at $\left[\left(\frac{12}{2}\right)^{th} \text{ and } \left(\frac{12}{2}+1\right)^{th}\right]$

= Average data at ($6^{th}$ and $7^{th}$)

= $\frac{\text{Data at } 6^{th} + \text{Data at } 7^{th}}{2}$

| Time (minutes) | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Frequency | 2 | 4 | 5 | 1 |
| Position of data | 1 - 2 | 3 - 6 | 7 - 11 | 12 |

The 3rd till the 6th data is 20 — The 7th till the 11th data is 30

Median = $\frac{6^{th} \text{ data} + 7^{th} \text{ data}}{2}$

= $\frac{20+30}{2}$

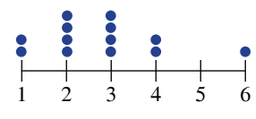
= 25

Therefore, median is 25.

**EXAMPLE 6**

Calculate the median for the situation on the right.

1. The dot plot shows the total students at the library in a six days.




*Solution:*

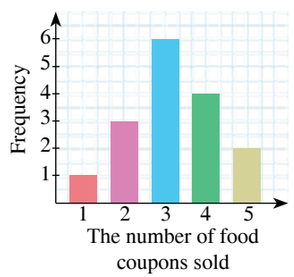
Number of frequency = 13 ← Total frequency, $n$ is odd

Median = data at $\left(\frac{13+1}{2}\right)^{th}$

= data at $7^{th}$

= 3

2. The bar chart shows the number of food coupons sold by Form 2S teachers during Co-curricular Day.




*Solution:*

Number of frequency = 16 ← Total frequency, $n$ is even

Median = Average data at $\left[\left(\frac{16}{2}\right)^{th} \text{ and } \left(\frac{16}{2}+1\right)^{th}\right]$

= Average data at ($8^{th}$ and $9^{th}$)

= $\frac{\text{Data at } 8^{th} + \text{data at } 9^{th}}{2}$

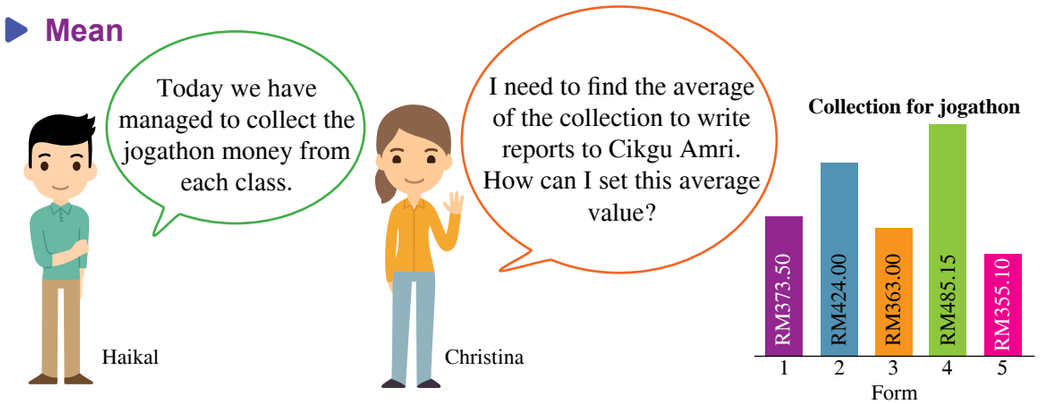
= $\frac{3+3}{2}$

= 3




CHAPTER 12

## Mean




In the situation above, we can find an average value of the collection of the jogathon money. The average value is also called **mean**.

**Mean** for a set of data is the value obtained when **the sum of the data values is divided by the number of data**.

$$\text{Mean} = \frac{\text{Total value of data}}{\text{Number of data}}$$

**EXAMPLE 7**

Calculate the average jogathon money that Haikal collected from each class.

*Solution:*

$$\text{Mean} = \frac{\text{RM373.50} + \text{RM424.00} + \text{RM363.00} + \text{RM485.15} + \text{RM355.10}}{5}$$
$$= \frac{\text{RM2 000.75}}{5}$$
$$= \text{RM400.15}$$

**FLASHBACK**

The data below is called ungrouped data: 2,3,1,1,2,2,4,4

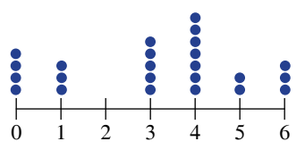
This data can also be compiled in the frequency table as follows.

| Number | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Frequency | 2 | 3 | 1 | 2 |

**EXAMPLE 8**

This dot plot shows the results of a survey on the consumption of canned carbonated drinks of 26 students per day.

Calculate the mean of the number of cans of carbonated drinks consumed by them in a day.




*Solution:*

$$\text{Mean of number of cans of carbonated drinks} = \frac{(4 \times 0) + (3 \times 1) + (0 \times 2) + (5 \times 3) + (7 \times 4) + (2 \times 5) + (3 \times 6)}{4 + 3 + 2 + 5 + 7 + 2 + 3}$$
$$= \frac{78}{26}$$
$$= 3$$

Therefore, the number of cans of carbonated drinks consumed by them in a day is 3 cans.





**EXAMPLE 9**

The table shows the hours of Internet usage of Form 2 students in a day.

| Internet usage (hours) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of students | 2 | 6 | 11 | 7 | 9 |

Calculate the mean for the frequency table above.

*Solution:*

| Internet usage (hours) | Number of students | Internet usage × Number of students |
|---|---|---|
| 1 | 2 | $1 \times 2 = 2$ |
| 2 | 6 | $2 \times 6 = 12$ |
| 3 | 11 | $3 \times 11 = 33$ |
| 4 | 7 | $4 \times 7 = 28$ |
| 5 | 9 | $5 \times 9 = 45$ |
| **Total** | 35 | 120 |

Number of frequency

Sum of data (frequency x data)

$$\text{Mean} = \frac{\text{Sum of data (frequency} \times \text{data)}}{\text{Number of frequency}}$$
$$= \frac{120 \text{ hours}}{35}$$
$$= 3.43 \text{ hours}$$

Therefore, mean is 3.43 hours.

Mean for the data in the frequency table can be calculated by finding the sum of the data from the corresponding frequency divided by the number of frequencies.

$$\text{Mean} = \frac{\text{Sum (data} \times \text{frequency)}}{\text{Number of frequencies}}$$

## Existence of extreme value

**Extreme value** is a value that is **too small** or **too large** in a set of data. It means the value is too far from the value of the other data in the set.

**EXAMPLE 10**

The time in minutes taken by 7 students to complete a three-dimensional polygon model using the toy blocks is

5, 6, 7, 7, 8, 9, 20

Which of these data is one of the extreme value? Explain.

*Solution:*

20 is extreme value because its value is much larger than the other data.

**EXAMPLE 11**

Determine the extreme data in the data below. Explain your answer.

–5, 0, 1, 3, 3, 5, 6

*Solution:*

–5 is extreme value because its value is much smaller than other data.

### Impact of extreme value

**EXAMPLE 12**

1. A set of data of pocket money of 5 students to the school.

   RM3, RM4, RM4, RM6, RM8

   Calculate the mode, median and mean of the data above.

2. You are required to replace RM8 to RM32, then calculate new mode, median and mean .

*Solution:*

RM3, RM4, RM4, RM6, RM8

1. Mode = RM4

   median = RM4

   $\text{mean} = \frac{\text{RM3} + \text{RM4} + \text{RM4} + \text{RM6} + \text{RM8}}{5}$

   $= \frac{\text{RM25}}{5}$

   = RM5

RM3, RM4, RM4, RM6, RM32 ← Extreme value

2. Mode = RM4

   median = RM4

   $\text{mean} = \frac{\text{RM3} + \text{RM4} + \text{RM4} + \text{RM6} + \text{RM32}}{5}$

   $= \frac{\text{RM49}}{5}$

   = RM9.80

When an extreme value exists in a set of data, it will affect the value of mean. As in the example, the value of mean shows an increase of RM4.80, while the value of median and mode do not change with extreme values.

## 12.1.2 The effect of changing a set of data to the mode, mean and median

LEARNING STANDARD

Make conclusion about the effect of changes in a set of data to value of mode, mean and median.

### Data is changed uniformly

Carry out the given activity to identify the effect on mode, median, and mean when each data is changed uniformly or non-uniformly.

**COGNITIVE STIMULATION** Group

**Aim**: Investigating the effects of changes in mean, median and mode if any data is changed in a uniform manner

**Material:** Worksheets

**Steps:** Five students are given Mathematics Quiz questions with a minimum score of 20. The table below shows their results.





| Students | Amin | Ben | Chia | Don | Eva |
|---|---|---|---|---|---|
| Score | 3 | 4 | 4 | 6 | 8 |

1. Copy and complete the following table.

| | Score | Students | | | | | Mean | Median | Mode |
|---|---|---|---|---|---|---|---|---|---|
| | | Amin | Ben | Chia | Don | Eva | | | |
| Row 1 → | $n$ | 3 | 4 | 4 | 6 | 8 | | | |
| Row 2 → | $n + 1$ | | | | | | | | |
| Row 3 → | $n \times 2$ | | | | | | | | |

Table 1

2. Copy and complete the following table.

| Score | Students | | | | | Mean | Median | Mode |
|---|---|---|---|---|---|---|---|---|
| | Amin | Ben | Chia | Don | Eva | | | |
| Original Score | 3 | 4 | 4 | 6 | 8 | | | |
| Addition to the Score | +1 | +2 | +3 | +4 | +5 | | | |
| New Score | 4 | | | | | | | |

Table 2

**Discussion:**

(i) Compare the answers obtained in row 1, row 2, and row 3 in Table 1 when the data is uniformly changed?

(ii) Compare the mean, median and mode between the original scores and new scores in Table 2 when each data is changed in a non-uniform manner?

From the activity above, when 1 is added to each original data (row 2) or multiplied by 2 (row 3), we find that the values of mean, median and mode will also be added with 1 or multiplied by 2.

This means that **a uniform change in data** will result in a uniform change in **values for mean**, **median** and **mode**.

However, if the data is changed in a non-uniform manner, the values of mean, median and mode will also change in a **non-uniform** manner.

**EXAMPLE 13**

Kanang bought 5 types of stationery at the school cooperative which cost RM1, RM2, RM3, RM3 and RM6 respectively.

(a) Calculate the mean, median and mode for the data.

(b) Calculate the new mean, median and mode if the price of each stationery

(i) is increase by RM 2 (ii) is multiplied by 3

*Solution:*

(a) RM1, RM2, RM3, RM3, RM6

$$\text{Mean} = \frac{\text{RM1} + \text{RM2} + \text{RM3} + \text{RM3} + \text{RM6}}{5} \qquad \text{median} = \text{RM3} \qquad \text{mode} = \text{RM3}$$
$$= \frac{\text{RM15}}{5}$$
$$= \text{RM3}$$

(b) (i) New data (price increased by RM 2) = RM3, RM4, RM5, RM5, RM8

$$\text{Mean} = \frac{\text{RM3} + \text{RM4} + \text{RM5} + \text{RM5} + \text{RM8}}{5} \qquad \text{median} = \text{RM5} \qquad \text{mode} = \text{RM5}$$
$$= \frac{\text{RM25}}{5}$$
$$= \text{RM5}$$

← Value of original mean also increases by 2

Value of original median also added by 2

Value of original mode also added by 2

(ii) New data (price multiplied by 3) = RM3, RM6, RM9, RM9 dan RM18

$$\text{Mean} = \frac{\text{RM3} + \text{RM6} + \text{RM9} + \text{RM9} + \text{RM18}}{5} \qquad \text{median} = \text{RM9} \qquad \text{mode} = \text{RM9}$$
$$= \frac{\text{RM45}}{5}$$
$$= \text{RM9}$$

← Value of original mean also multiplied by 3

Value of original median also multiplied by 3

Value of original mode also multiplied by 3

**EXAMPLE 14**

Raju's scores in a Japanese quiz are 3, 6 and 6.

(a) Calculate the mean, median, and mode for the data.

(b) Add the first data by 1, add the second data by 2 and add the third data by 3. Next, determine the new values of mean, median and mode.

*Solution:*

(a) $\text{Mean} = \frac{3 + 6 + 6}{3}$ median = 6 mode = 6

$= \frac{15}{3}$

$= 5$

In the example above, the data is not uniformly changed. Similarly the new values of mean, median and mode **do not change in a uniform manner**.

(b) New data are (3 + 1), (6 + 2), (6 + 3), hence, they are 4, 8, 9.

$\text{Min} = \frac{4 + 8 + 9}{3}$

$= \frac{21}{3}$

$= 7$

median = 8, No mode

## 12.1.3 Organise data in frequency tables for grouped data

### Frequency table for grouped data

**LEARNING STANDARD**

Collect data, construct and interpret the frequency table for grouped data.





**COGNITIVE STIMULATION** Group

**Aim:** Organising data by group or class

**Materials:** Worksheets and weighing scales

**Steps:**

1. Students in the class are required to weigh themselves and record the weight on the whiteboard.
2. Organise the weight in kg, in the table on the right according to the following classes.
   30 - 39, 40 - 49, 50 - 59, 60 - 69, 70 - 79
3. Tally and complete the frequency table on the right.

| Weight (kg) | Tally | Frequency |
|---|---|---|
| 30 - 39 | | |
| 40 - 49 | | |
| 50 - 59 | | |
| 60 - 69 | | |
| 70 - 79 | | |

**Discussion:**

What is the difference between the grouped data frequency table and the ungrouped data frequency table that you have learned earlier?

From the activities above, for a table of a grouped data, data is classified with a uniform class interval to prevent the data from overlapping.

This class can be used to categorise those data into appropriate groups such as grades, passes, failures and achievement levels. The information will help us make a conclusion.

This situation is very important if we want to organise a large set of data.

**EXAMPLE 15**

The data on the right shows the mathematics test score for 30 Form Two Saturn students in the mid-year examination. Organise the data in the frequency table according to classes.

| Marks | Tally | Frequency |
|---|---|---|
| 0 - 19 | | |
| 20 - 39 | | |
| 40 - 59 | | |
| 60 - 79 | | |
| 80 - 99 | | |

**Mathematics Marks Form 2 Saturn**

| | | | | |
|---|---|---|---|---|
| 85 | 58 | 75 | 41 | 53 |
| 12 | 61 | 63 | 45 | 72 |
| 37 | 55 | 29 | 42 | 95 |
| 31 | 22 | 18 | 25 | 19 |
| 47 | 38 | 50 | 78 | 58 |
| 90 | 57 | 63 | 49 | 88 |

*Solution:*

| Marks | Tally | Frequency |
|---|---|---|
| 0 - 19 | /// | 3 |
| 20 - 39 | 𝍸 / | 6 |
| 40 - 59 | 𝍸 𝍸 / | 11 |
| 60 - 79 | 𝍸 / | 6 |
| 80 - 99 | //// | 4 |

Data in class 80 - 99 are 85, 88, 90 and 95

In the example above, the marks have been classified into five categories according to the same interval.

**REMEMBER**

Tally

𝍸 = 5

**TIPS**

How to tally the data into classes:

Example: 85 is located between 80-99 class interval then tally it in the 80-99 row.

**EXAMPLE 16**

Silvia interviewed 20 of her friends about the time they wake up in the morning during the school holidays. The findings from the interview are shown on the right.

Organise the data in a frequency table according to the following classes.

| Time (a.m.) | Tally | Frequency |
|---|---|---|
| 5:00 - 5:29 | | |
| 5:30 - 5:59 | | |
| 6:00 - 6:29 | | |
| 6:30 - 6:59 | | |
| 7:00 - 7:29 | | |

| Wake-up time (a.m.) | |
|---|---|
| 6:00 | 6:35 |
| 5:01 | 6:42 |
| 6:22 | 5:40 |
| 5:30 | 7:23 |
| 6:03 | 6:15 |
| 6:40 | 5:41 |
| 5:20 | 6:45 |
| 6:50 | 5:35 |
| 6:40 | 6:05 |
| 6:50 | 6:35 |

From the frequency table, answer the following questions.

(a) How many students wake up between 6:00 a.m. to 6:29 a.m.?

(b) Describe the highest and lowest frequencies of the time the students wake up.

***Solution:***

(a) Five students.

(b) From the frequency table, most of the students wake up between 6:30 a.m. to 6:59 a.m.. There are eight of them. Only one student wakes up between 7:00 a.m. to 7:29 a.m..

| Time (a.m.) | Tally | Frequency |
|---|---|---|
| 5:00 - 5:29 | // | 2 |
| 5:30 - 5:59 | //// | 4 |
| 6:00 - 6:29 | 𝍸 | 5 |
| 6:30 - 6:59 | 𝍸 /// | 8 |
| 7:00 - 7:29 | / | 1 |

## 12.1.4 Modal class and mean of a set of grouped data

**LEARNING STANDARD**

Determine the modal class and mean of a set of grouped data.

**EXAMPLE 17**

Below is a survey finding of the weekly pocket money, in RM brought by 30 students of SMK Tasek Damai.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 21 | 18 | 22 | 35 | 40 | 55 | 40 | 45 | 50 |
| 25 | 32 | 45 | 15 | 10 | 20 | 35 | 45 | 15 | 25 |
| 25 | 15 | 60 | 30 | 45 | 50 | 30 | 10 | 12 | 30 |

CHAPTER 12





1. Copy and complete the frequency distribution table below.

| Pocket money (RM) | Tally | Frequency |
|---|---|---|
| 1 - 10 | // | 2 |
| 11 - 20 | | |
| 21 - 30 | | |
| 31 - 40 | | |
| 41 - 50 | | |
| 51 - 60 | | |

2. From frequency distribution table, state the class with the highest frequency.

***Solution:***

1.

| Pocket money (RM) | Tally | Frequency |
|---|---|---|
| 1 - 10 | // | 2 |
| 11 - 20 | 𝍸 // | 7 |
| 21 - 30 | 𝍸 /// | 8 |
| 31 - 40 | 𝍸 | 5 |
| 41 - 50 | 𝍸 / | 6 |
| 51 - 60 | // | 2 |

Modal class → 21 - 30

8 ← Highest frequency

2. The class with the highest frequency is 21-30.

When the data is organised, we will know the highest frequency value and its class. In the Example 17, the highest frequency is 8 and the class is 21-30. Thus, class 21-30 are known as **modal class**.

**EXAMPLE 18**

The table of frequency below shows marks for an apptitude test for 30 students. Determine the modal class.

| Marks | 40 - 44 | 45 - 49 | 50 - 54 | 55 - 59 | 60 - 64 | 65 - 69 |
|---|---|---|---|---|---|---|
| Frequency | 7 | 4 | 1 | 4 | 9 | 5 |

***Solution:***

Modal class

| Marks | 40 - 44 | 45 - 49 | 50 - 54 | 55 - 59 | 60 - 64 | 65 - 69 |
|---|---|---|---|---|---|---|
| Frequency | 7 | 4 | 1 | 4 | 9 | 5 |

Highest frequency

Highest frequency = 9

Modal class = 60 - 64

CHAPTER 12

## Mean for grouped data

For grouped data, we have classified the data to a specific group, so before the mean for this data is determined, we must find the midpoint of this class to represent the class.

**EXAMPLE 19**

The table below recorded the number of newspapers sold by different stores in a week. Determine the midpoint for each class.

| Number of newspaper | Number of stores (Frequency) |
|---|---|
| 70 - 74 | 4 |
| 75 - 79 | 10 |
| 80 - 84 | 8 |
| 85 - 89 | 2 |

**DO YOU KNOW?**

| Number of newspaper | Number of stores (frequency) |
|---|---|
| 70 - 74 | 4 |

Lower limit, Upper limit

*Solution:*

| Number of newspaper | Midpoint of the class | Number of stores (Frequency) |
|---|---|---|
| 70 - 74 | $\frac{70+74}{2} = 72$ | 4 |
| 75 - 79 | $\frac{75+79}{2} = 77$ | 10 |
| 80 - 84 | $\frac{80+84}{2} = 82$ | 8 |
| 85 - 89 | $\frac{85+89}{2} = 87$ | 2 |

**TIPS**

Midpoint $= \frac{\text{Lower limit + upper limit}}{2}$

After obtaining the midpoint for each class, we calculate the mean with the following formula,

$$\text{Mean} = \frac{\text{The sum (frequency} \times \text{midpoint)}}{\text{Number of frequencies}}$$

**EXAMPLE 20**

The table below records the height of 30 saplings observed by Umeswary in a science experiment. Calculate the mean of the height of the tree.

| Height of tree (cm) | Frequency |
|---|---|
| 5 - 9 | 4 |
| 10 - 14 | 5 |
| 15 - 19 | 4 |
| 20 - 24 | 8 |
| 25 - 29 | 7 |
| 30 - 34 | 2 |





*Solution:*

1. Calculate the midpoint of each class. The following steps can be followed.

| Height of tree (cm) | Midpoint of the class | Frequency |
|---|---|---|
| 5 - 9 | $\frac{5+9}{2} = 7$ | 4 |
| 10 - 14 | $\frac{10+14}{2} = 12$ | 5 |
| 15 - 19 | $\frac{15+19}{2} = 17$ | 4 |
| 20 - 24 | $\frac{20+24}{2} = 22$ | 8 |
| 25 - 29 | $\frac{25+29}{2} = 27$ | 7 |
| 30 - 34 | $\frac{30+34}{2} = 32$ | 2 |

2. Multiply each midpoint with the frequency.

| Height of tree (cm) | Midpoint, $x$ | Frequency, $f$ | Frequency × midpoint, $fx$ |
|---|---|---|---|
| 5 - 9 | $\frac{5+9}{2} = 7$ | 4 | $4 \times 7 = 28$ |
| 10 - 14 | $\frac{10+14}{2} = 12$ | 5 | $5 \times 12 = 60$ |
| 15 - 19 | $\frac{15+19}{2} = 17$ | 4 | $4 \times 17 = 68$ |
| 20 - 24 | $\frac{20+24}{2} = 22$ | 8 | $8 \times 22 = 176$ |
| 25 - 29 | $\frac{25+29}{2} = 27$ | 7 | $7 \times 27 = 189$ |
| 30 - 34 | $\frac{30+34}{2} = 32$ | 2 | $2 \times 32 = 64$ |
| | | $\sum f = 30$ | $\sum fx = 585$ |

**TIPS**

Mean for grouped data can also be written in the form of a symbol.

$\sum$ is read as "sigma". $\sum$ is a notation for sum.

$fx$ represents frequency of multiplication of midpoint.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Symbol $\bar{x}$ for mean is read as "*x bar*".

$f$ represents frequency.

3. Calculate the mean of the height of sapling,

$$\text{mean} = \frac{\text{sum (frequency} \times \text{midpoint)}}{\text{Number of frequencies}}$$
$$= \frac{\sum fx}{\sum f}$$
$$= \frac{585}{30}$$
$$= 19.5$$

## 12.1.5 The most appropriate measure for central of tendencies

**LEARNING STANDARD**

Choose and justify the appropriate measures of central tendencies to describe the distribution of a set of data, including those with extreme values.

We can choose and justify any measure of central tendency to describe the distribution of a given set of data as seen appropriate.

The type of data is very important if we want to make the selection of the appropriate measure of central tendency. Justification for the choice should be clear and precise so that it can represent the entire data.

**Mean** is chosen as a measure of central tendency because it involves the entire data.When there is an extreme value, the mean cannot give an accurate interpretation of the data because the extreme value affects the mean.

**The median** is a more appropriate measure of central tendency to use when there is an extreme value because the median is not influenced by extreme values. It is the value at the midpoint of the set of data.

**Mode** is a more appropriate measure of central tendency when the set of data involves non-numerical values like category data. Additionally, mode is also suitable for favourite or popular items.

### EXAMPLE 21

Determine the type of measure of central tendencies that is suitable for the situations below.

1. The stems and leaf plot on the right shows the weight of marbles in 10 plastic jars.

**Weight of marbles**

| Stem | Leaf | | | |
|---|---|---|---|---|
| 5 | 0 | 6 | 8 | |
| 6 | 1 | 1 | 4 | 7 |
| 7 | 2 | 6 | 9 | |

Key: 5 | 0 refers to 50 g

*Solution:*

Mean because there is no extreme value in the set of data

2. The pictograph on the right shows the flavour of ice cream favoured by kindergarten students.

**Favourite ice cream flavour**

| Flavour | Frequency |
|---|---|
| Chocolate | |
| Pandan | |
| Yam | |
| Strawberry | |

represents 5 students

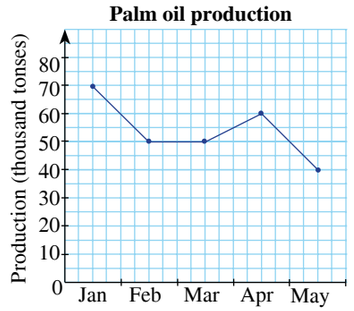
*Solution:*

Mode because this is a category data and it will determine the favourite item.





3. The line graph on the right shows the production of palm oil in a factory for 5 months.




*Solution:*

Mean - because there is no extreme value in the set of data.

4. The table below shows the number of hours spent browsing the Internet by Form 2 Melor students.

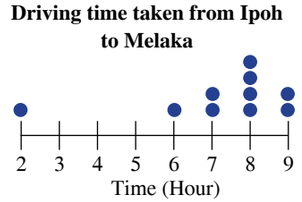
| Hours spent browsing the Internet | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of students | 2 | 5 | 5 | 7 | 6 | 4 | 3 |

**Hours spent browsing the Internet by Form 2 Melor students**

*Solution:*

Mean - because there is no extreme value in the set of data.

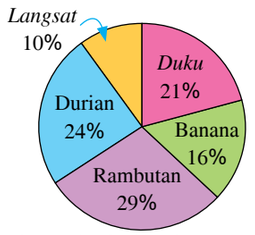
5. The dot plot on the right shows the time taken in hours when 10 drivers made a trip from Ipoh to Melaka by car.




*Solution:*

Median because there is an extreme value in the set of data.

6. The pie chart on the right shows the favourite fruits of the students of Form 2 Gemilang.

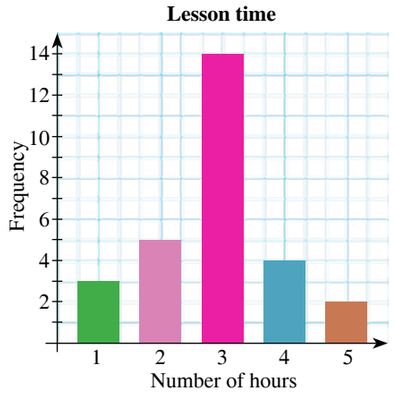



**The favourite fruits of the students of Form 2 Gemilang**

*Solution:*

Mode because this is a category data and it will determine the favourite item.

7. The bar chart on the right shows time taken in hours for some students to do revision in a day.




*Solution:*

Median because there is an extreme value in the set of data.

## 12.1.6 Mode, mean and median from data representation

**LEARNING STANDARD**

Determine mode, mean and median from data representations.

The use of measures of central tendencies in statistics or daily routine.

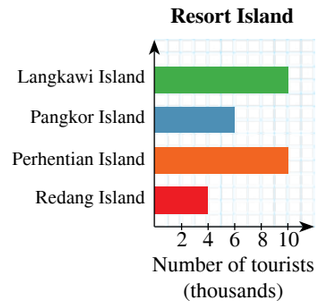
**EXAMPLE 22**

Determine the mode in the following data representations.

(a) The bar chart shows the number of tourists to resort islands.

*Solution:*

Modes are Perhentian Island and Langkawi Island.




(b) The pictograph shows the kind of fruits that are favoured by Form 2 Bestari students.

*Solution:*

No mode.

**Favoured fruits of Form 2 Bestari students.**

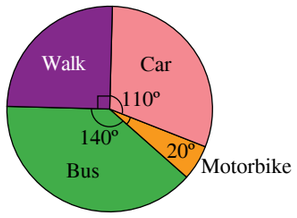
| | |
|---|---|
| Banana | |
| Watermelon | |
| Durian | |
| Mango | |

represents three students

(c) The pie chart shows how students go to school.

*Solution:*

Mode is bus.




**Transportation used by students to school**

(d) The table shows the profit of online sales in a year.

*Solution:*

Mode is women accessories.

| Item | Profit (%) |
|---|---|
| Books | 87 |
| Computer software | 54 |
| Cinema tickets | 72 |
| Women accessories | 130 |
| Travel package | 78 |

**Profit of online sales**



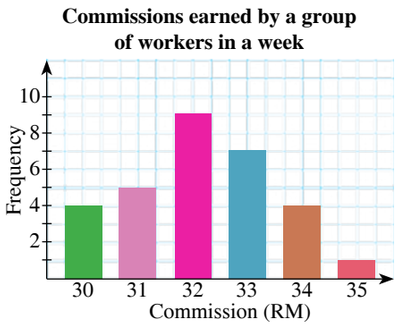


**EXAMPLE 23**

The bar chart on the right shows commissions earned by a group of workers at a restaurant in a week.

(a) Determine the mean, median and mode by the employee within a week.

(b) Determine the number of workers who receive commissions less or equal to RM32 compared to the total number of workers, in fraction form.




*Solution:*

(a) $\text{Mean} = \dfrac{4(30) + 5(31) + 9(32) + 7(33) + 4(34) + 1(35)}{4 + 5 + 9 + 7 + 4 + 1}$

$= \dfrac{965}{30}$

$= \text{RM}32.17$

$\text{Median} = \text{Average data at } \left[\left(\dfrac{30}{2}\right)^{\text{th}} \text{ and } \left(\dfrac{30}{2} + 1\right)^{\text{th}}\right]$

$= \text{Average data at } (15^{\text{th}} \text{ and } 16^{\text{th}})$

$= \dfrac{\text{Data at } 15^{\text{th}} + \text{data at } 16^{\text{th}}}{2}$

$= \dfrac{32 + 32}{2}$

$= \text{RM}32$

$\text{Mode} = \text{RM}32$

(b) The number of workers receiving commissions less or equal to RM32

$= \dfrac{4 + 5 + 9}{30}$

$= \dfrac{3}{5}$

**EXAMPLE 24**

The table shows the number of spelling errors made by Form 2 Amanah students when writing essay in Malay.

| Number of spelling errors | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Number of students | 4 | 8 | $x$ | 6 | 5 | 4 |

(a) If the mean of the students' spelling errors is 2.4, calculate the value of $x$.

(b) If the median for the frequency distribution is 3, calculate the maximum value of $x$.

(c) If the mode of students' spelling errors is 2, determine the minimum possible value of $x$.

*Solution:*

(a) $\text{Mean} = \dfrac{4(0) + 8(1) + x(2) + 6(3) + 5(4) + 4(5)}{4 + 8 + x + 6 + 5 + 4} = 2.4$

$\dfrac{2x + 66}{x + 27} = 2.4$

$2x + 66 = 2.4(x + 27)$

$2x + 66 = 2.4x + 64.8$

$2.4x - 2x = 66 - 64.8$

$0.4x = 1.2$

$x = 3$

(b) $\underbrace{0, 0, 0, 0}_{4}$ $\underbrace{1, 1, 1, 1, 1, 1, 1, 1}_{8}$ $\underbrace{2,\ldots,2}_{x}$ 3 $\underbrace{3, 3, 3, 3, 3}_{5}$ $\underbrace{4, 4, 4, 4, 4}_{5}$ $\underbrace{5, 5, 5, 5}_{4}$

The maximum value for $x$ if the median is located here

$4 + 8 + x = 5 + 5 + 4$

$12 + x = 14$

$x = 2$

Thus, the greatest value for $x$ is 2

Therefore, the maximum value of $x$ is 2.

(c) The minimum possible value of $x$ is 9.

## 12.1.7 Measures of central tendencies in making predictions, forming an argument and conclusion

**LEARNING STANDARD**

Apply the understanding of measures of central tendencies to make predictions, form convincing arguments and conclusions.

In making comparisons or selecting the most appropriate measure of central tendency, the importance of the range should be taken into consideration.

**EXAMPLE 25**

Encik Rahman would like to choose a school representative for the game of bowling at the zone level. Ramesh and Khairil are among those who have been shortlisted in this selection. In the last five exercises prior to the selection, Ramesh's score was 116, 118, 200, 207 and 209. Khairil's score was 240, 240, 75, 220 and 75. Which player will be selected as the school representative?

***Solution:***

$$\text{Mean score for Ramesh} = \frac{116 + 118 + 200 + 207 + 209}{5} = \frac{850}{5} = 170$$

$$\text{Mean score for Khairil} = \frac{240 + 240 + 75 + 220 + 75}{5} = \frac{850}{5} = 170$$

Both players have the same mean. Therefore, the mean cannot be used in the decision for the selection of the school representative.

Score range of Ramesh = 209 – 116 = 93

Score range of Khairil = 240 – 75 = 165

Ramesh's range of scores is lower than that of Khairil's because there is a very low score (extreme value) that caused the range to be large. Therefore, selecting Ramesh as a school representative is more suitable.

**FLASHBACK**

Range is the difference between the smallest value and the largest value.





**EXAMPLE 26**

Encik Johan who is a teacher formed three basketball teams. The table below shows the total number of goals scored by the teams in five competitions.

| Team | Competition | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Kijang | 65 | 95 | 32 | 96 | 88 |
| Harimau | 50 | 90 | 65 | 87 | 87 |
| Seladang | 90 | 85 | 46 | 44 | 80 |

(a) You want to join one of these teams.

(i) Given the mean, which team will you join? Justify your answer by showing the workings.

(ii) If you consider the median in making a decision, which group would you choose? Explain by showing your workings.

(b) If Encik Johan was asked to submit a report of the achievements by team Harimau to the school principal, which measures of central tendencies should by Encik Johan choose? Explain.

***Solution:***

(a) (i) $\text{Mean for team Kijang} = \frac{65 + 95 + 32 + 96 + 88}{5} = 75.2$

$$\text{Mean for team Harimau} = \frac{50 + 90 + 65 + 87 + 87}{5} = 75.8$$

$$\text{Mean for team Seladang} = \frac{90 + 85 + 46 + 44 + 80}{5} = 69$$

The Harimau team is selected because the mean is the highest; that is 75.8.

Data set for Kijang is 32, 65, 88, 95, 96. Thus, median is 88.

(ii) Team Kijang = 32, 65, (88), 95, 96. Median = 88

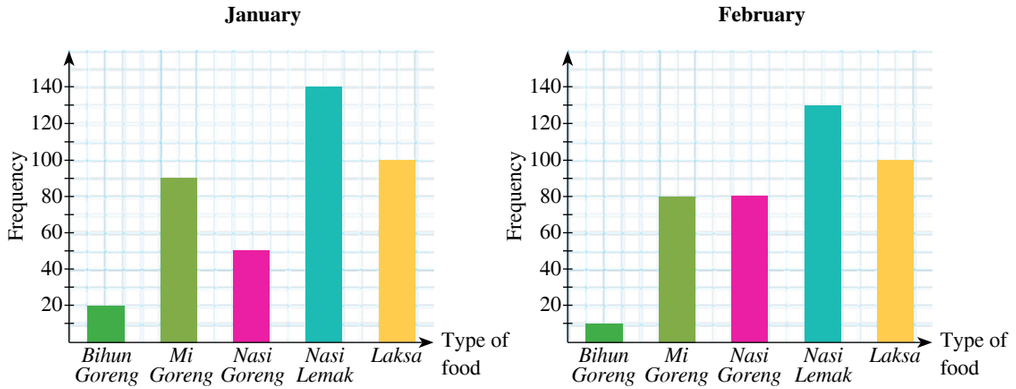
Team Harimau = 50, 65, (87), 87, 90. Median = 87

Team Seladang = 44, 46, (80), 85, 90. Median = 80

The Kijang team is selected because the value of the median is the highest; that is 88.

(b) Mean. This is because mean uses the entire set of data in the table. Therefore, mean is most appropriate because there is no extreme value in this set.

**EXAMPLE 27**




The bar chart above shows food selection at school canteens in January and February from a study conducted on 400 students.

(a) Which measures of central tendencies is appropriate for the above situation?
Explain.

Nasi lemak is the most favoured dish of the students.

(b) Do you agree with the above statement? Explain your reasons.

(c) You are a canteen committee member of the Consumer Association. You are asked to suggest a type of food that needs to be reduced. Give your reasons.

*Solution:*

(a) From the graphs above, mean and median are not suitable because the data provided are category data. So, mode is more appropriate.

(b) Yes because Nasi lemak is the mode for January and February.

(c) Bihun Goreng should be reduced due to its lowest frequency in January and February.

## SELF PRACTICE 12.1

1. Specify the mode for each of the following sets of data.
   (a) 3, 0, 1, 1, 4, 3, 2, 2, 1
   (b) RM10, RM8, RM7, RM7, RM8, RM9
   (c) 64, 60, 63, 60, 60, 67

2. The table below shows shirt size of 145 participants of the "Jom Sihat" run.

| Size | SS | S | M | L | XL | XXL |
|---|---|---|---|---|---|---|
| Frequency | 20 | 17 | 15 | 37 | 31 | 25 |

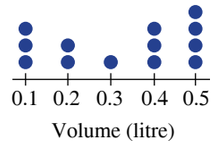
State the mode for the size of shirts.





3. State the mode for the data representation below.

(a) **Volume of oil in the bottle**

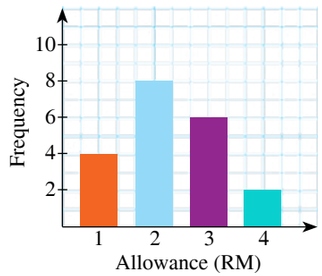



(b) **Marks for fitness test**

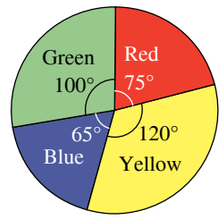
| Stem | Leaf |
|---|---|
| 2 | 6 6 7 8 |
| 3 | 1 1 2 2 2 3 6 7 7 7 |
| 4 | 0 2 5 |

Key: 2 | 6 refers to 26 km

(c) **The allowance paid to students to sell bookmarks**




(d) **Team Helang's favourite colour**

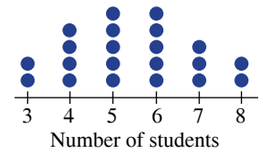



4. Determine the median for the following sets of data.
   (a) 7, 5, 7, 8, 3, 12.
   (b) 37, 38, 27, 28, 48, 47, 58, 68.
   (c) 3, 200, 4, 10, 50, 7, 90, 3, 50, 11, 3.

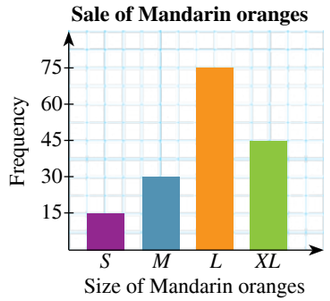
5. The table shows the number of ferry passengers at Pulau Pangkor jetty in January. Calculate the median.

| Number of passengers | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Frequency | 5 | 8 | 7 | 10 |

6. Calculate the median for the following representation of data.
   (a) Dot plot indicates the number of students visiting the access centre in a week.




   (b) Bar chart shows the size of mandarin oranges sold at a store during Chinese New Year.



7. Calculate the mean for the following set of data.
   (a) 9, 5, 2, 3, 11, 12 (b) 3.5, 2.4, 1.7, 3.2, 4.5

8. (a) Given the value of mean of 4, 7, $x$, 9, 8 is 6, calculate $x$.
   (b) Given the value of mean of 7 cm, 15 cm, 12 cm, 5 cm, h cm and 13 cm is 10 cm. calculate the value $h$.

9. The table below shows the trend of absenteeism among 40 students in January.

| Number of days absent | 0 | 1 | 2 | 3 | 4 | 5 | 8 |
|---|---|---|---|---|---|---|---|
| Frequency | 24 | 3 | 4 | 5 | 2 | 1 | 1 |

Calculate the mean. Round off your answer to the nearest whole number.

10. Complete the of frequency table below.

(a)

| | | | |
|---|---|---|---|
| 18 | 28 | 18 | 24 |
| 18 | 23 | 30 | 24 |
| 26 | 35 | 22 | 13 |
| 16 | 33 | 19 | 32 |
| 6 | 16 | 34 | 27 |

The data above shows the age for 20 visitors at the National Museum.

| Age (year) | Tally | Frequency |
|---|---|---|
| 6 - 10 | / | 1 |
| 11 - 15 | | |
| 16 - 20 | | |
| 21 - 25 | | |
| 26 - 30 | | |
| 31 - 35 | | |

(b)

| | | | |
|---|---|---|---|
| 47 | 34 | 23 | 23 |
| 47 | 48 | 54 | 42 |
| 42 | 65 | 43 | 15 |
| 31 | 32 | 48 | 58 |
| 35 | 39 | 42 | 31 |

The data above shows the number of table tennis balls contained inside 20 baskets.

| Number of table tennis balls | Tally | Frequency |
|---|---|---|
| 10 - 19 | / | 1 |
| | | |
| | | |
| | | |
| | | |
| | | |

11. 2, 2, 3, 5, 7, 10, 11, 16, 17, 40
   (a) Calculate the value of mean, median and mode.
   (b) Which measure of central tendencies is suitable? Explain.



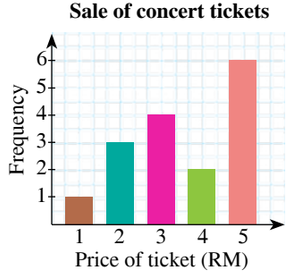


12. The table below shows the scores of an English spelling test for a group of Form 1 students.

| Score | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|
| Number of students | 4 | 16 | 12 | 7 | 6 | 5 |

   (a) Calculate the mean, median and mode.
   (b) Which measure of central tendencies is suitable? Explain.

13. Determine the measures of central tendencies suitable in the situation below. Justify your answer.
   (a) The bar chart shows the number and the price of concert tickets sold by the school Theatre Club.




   (b) The stem-and-leaf plot shows the volume of chemical solution, in m$l$, for 19 different bottles.

Volume of chemical solution

| Stem | Leaf |
|---|---|
| 2 | 0 1 3 5 6 |
| 3 | 6 7 1 1 0 1 1 2 3 5 |
| 4 | 1 1 1 |
| 13 | 7 |

Key: 2 | 0 refers to 20 m$l$

14. Which measures of central tendency is suitable to describe situation below?
   (a) The number of students for school societies and uniformed bodies.
   (b) Students' favourite television programmes in your class.
   (c) Number of pets owned by Form 2 Amanah students.

## GENERATING EXCELLENCE

1. The table below shows the number of children from 40 families in a motivational programme.

| Number of children | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 3 | 2 | 8 | 5 | 17 | 5 |

Determine the mode.

2. The mean for seven numbers is 10. Five of the numbers are 6, 5, 14, 10 and 11. Two other numbers are represented by $k$. Calculate
   (a) the sum of the seven numbers. (b) the value of $k$.

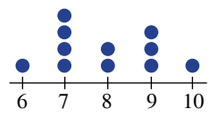
3. In each of the data representation below, calculate the mean.

(a) **Marks for Mathematics test**

| Stem | Leaf |
|---|---|
| 7 | 2 3 |
| 8 | 1 1 4 5 |
| 9 | 2 6 |

Key: 7 | 2 refers to 72 marks

(b) **Packets of noodles sold**




Number of packets

4. The table shows the qualifying marks for a History quiz obtained by a group of students. Calculate the median.

| **Marks** | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| **Frequency** | 2 | 7 | 5 | 11 | 9 | 7 |

5. Given the following numbers: 2, 4, 6, 6, 8 and 12,

(a) determine the mean, median and mode of the set of data above.

(b) calculate the new mean, median and mode if

(i) 2 is added to each number. (ii) each number is multiplied by 2.

(iii) 2 is substracted from each number. (iv) each number is divided by 2.

6. The mean of four numbers is 14. If two numbers, $x$ and $x + 2$ are added to the set of data, the new mean is 15. Calculate the value of $x$.

7. The mean of four numbers is 71. Two of the numbers are 56 and 48. The value of the other two numbers is $x$ respectively.

(a) Calculate,

(i) the sum of the four numbers. (ii) the value of $x$.

(b) Subtract 5 from each of the four numbers. Calculate the new value of mean.

8. The stem-and-leaf table below represent the distance, in km, by a group of runners during a cross-country event.

**Distance**

| Stem | Leaf |
|---|---|
| 2 | 3 4 6 9 |
| 3 | 0 1 2 2 2 4 4 5 8 |
| 4 | 2 2 |

Key: 2 | 3 refers to 23 km

(a) Determine

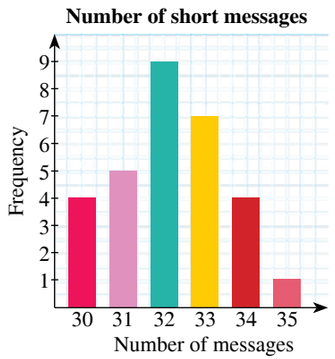
(i) mean (ii) mode (iii) median

(b) What is the percentage of participants who ran a distance of 32 km or more?





9. The bar chart shows the number of short messages sent by 30 students a week.




(a) Calculate

(i) mean (ii) mode (iii) median

(b) Calculate the number of students who sent less than 33 messages in a week compared to the total number of students, in fraction form.

10. Time taken for 40 students to finish crossword puzzles is recorded in the table below.

| **Time (minutes)** | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| **Number of students** | $x$ | 2 | $y$ | 6 | 14 |

(a) Show that $x + y = 18$.

(b) If $y = 6$, calculate the mean.

(c) Determine: (i) median (ii) mode

11. Malek, Rani and Yip have been selected to the final round in the long jump competition. They have made their respective jumps and their distance is recorded in metre.

| **Participants** | **Jump** | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| **Malek** | 3.2 | 4.5 | 6.1 |
| **Ravi** | 6.3 | 3.4 | 5.2 |
| **Yip** | 4.5 | 6.7 | 4.9 |

From the table above, which measures of central tendencies would you choose to determine the winners of the gold, silver and bronze medals? Explain your answers.

12. Joshua has scored 74, 95, 98, 84 and 74 in his History tests.

(a) How could Joshua convince his parents that he had worked hard to achieve the best results in History? Which measures of central tendencies should Joshua use for this purpose? Justify your answer.

(b) Encik Shamsudin is Joshua's History teacher. He encouraged Joshua to work harder because his marks for History were still inconsistent. Which of the marks did Encik Shamsudin refer to when he expressed his concern for Joshua's achievement?

## CHAPTER SUMMARY

**Measures of Central Tendencies**

The average value that represents a set of data

**Mode**
The most frequent data in a set of data.

**Modal class**
The class with the highest frequency

**Median**
The value or data in the middle after the data is arranged in ascending or descending order.

**Mean**

$$\text{Mean} = \frac{\text{Sum of values of data}}{\text{Number of data}}$$

$$\text{Mean} = \frac{\text{Sum (frequency x midpoint)}}{\text{Total number of frequency}}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Selection of measures of central tendencies

**Mean**
Selected to represent data when it involves the whole data when extreme value does not exist.

**Median**
Selected to represent data when extreme values exist.

**Mode**
Selected to represent data when we intend to determine the item with the highest frequency.





## SELF REFLECTION

**At the end of the chapter, I will be able to:**

| | |
|---|---|
| 1. Determine the mode, mean and median values of set of a non-grouped data. | ☺ 😐 ☹ |
| 2. Makes conclusion about the effect of changes in a set of data to the value of mode, median and mean. | ☺ 😐 ☹ |
| 3. Collect data, construct and interpret the frequency table for grouped data. | ☺ 😐 ☹ |
| 4. Determine the modal class and mean of a set of a grouped data. | ☺ 😐 ☹ |
| 5. Select and justify the appropriate measures of central tendencies to describe the distribution of a set of data, including those with extreme values. | ☺ 😐 ☹ |
| 6. Determine mode, mean and median from data representations. | ☺ 😐 ☹ |
| 7. Apply the understanding of the measures of central tendencies to make predictions, forming convincing arguments and make conclusions. | ☺ 😐 ☹ |

## MINI PROJECT

You are required to find information and write reports on students' height and weight from three different form 2 classes. Obtain the data through questionnaires containing gender, height and weight.

Then, organise your data using a suitable frequency table. You can use a computer software or do it manually in writing this report.

For the data from each class, analyse the data by using measures of central tendencies that is mode, mean, and median. State the measures of central tendencies you would choose to represent the data. Next, calculate BMI for each student and provide suggestions to stay healthy.

# CHAPTER 13

# Simple Probability

## WHAT WILL YOU LEARN?

**13.1** Experimental Probability

**13.2** Probability Theory involving Equally Likely Outcomes

**13.3** Probability of the Complement of an Event

**13.4** Simple Probability

A pink team and a blue team are playing against each other in a netball game. Based on the records, the pink team scored 12 goals out of 18 attempts. The blue team scored 18 goals out of 30 attempts. What is the ratio of the goals to the attempts for the pink team and blue team? In your opinion, which team will win the game?


## WORD LINK

- Probability — *Kebarangkalian*
- Sample space — *Ruang sampel*
- Event — *Peristiwa*
- Complement of an event — *Peristiwa pelengkap*
- Theoretical probability — *Kebarangkalian teori*
- Experimental probability — *Kebarangkalian eksperimen*
- Tree diagram — *Gambar rajah pokok*

## WALKING THROUGH TIME

Richard Carl Jeffrey was an innovative philosopher in the 20th century. He was also one of the members in the department of philosophy at Princeton University between 1974 to 1999. He contributed much in the field of logic and statistics. The book 'The Logic of Decision' written by him, discusses a new theory about making decisions in a state of uncertainty and the belief in possibilities. His writings are widely used in the field of logic including 'Formal Logic: Its Space and Limits' and 'Computability and Logic'. He also wrote the book 'Probability and the Art of Judgement' and 'Subjective Probability: The Real Thing'.

For more information:

http://rimbunanilmu.my/mat_t2e/ms277

## WHY STUDY THIS CHAPTER?

- Economists use the knowledge of probability in predicting the increase or decrease in stock value depending on the current economic situation and political stability of a country.
- Meteorologist use the science of probability in predicting weather and wind change for the days ahead.
- Businessmen also use knowledge of probability to review their business profit statistics and to forecast the future profits and earnings.

**CREATIVE ACTIVITY**

**Aim:** Identifying probability

**Materials:** Weather forecast results, blue and red marbles

**Steps:**

1. Consider the following situations:
   (a) Rain is expected the next day.
   (b) Choose a female student from the Girl Guides for a netball game.
   (c) The possibility of a black marble is taken from a box containing 3 blue marbles and 7 red marbles.
2. Discuss the possibility of each of the above situations and appropriate value to represent each possibility.

The situations above indicate that the events may occur, will occur and may not occur. The possibility of an event occurring is determined by the value between 0 to 1 and is known as probability.

> Probability is the measurement of possible occurrence of an event expressed either in the form of fractions or percentages.

## 13.1 Experimental Probability

In the Creative Activity, you are introduced to the probability concept. Now let's look at the relationship between frequency and the number of trials.

### 13.1.1 Experimental probability

**LEARNING STANDARD**

Perform simple probability experiments, and hence state the ratio $\frac{\text{frequency of an event}}{\text{number of trials}}$ as the experimental probability of an event.

**COGNITIVE STIMULATION** Group

**Aim:** Introducing simple probability

**Materials:** A coin

**Steps:**

1. Flip the coin 25 times.
2. Note down whether it is head or tail.
3. Then, repeat step one 50 times.
4. Next, repeat step one 100 times.
5. Write the result obtained in the table.

| | Number of flips | | | Ratio $\frac{\text{frequency of appearance}}{\text{number of flips}}$ | | |
|---|---|---|---|---|---|---|
| **The frequency of appearance** | 25 | 50 | 100 | 25 | 50 | 100 |
| head | | | | | | |
| tail | | | | | | |

**Discussion:**

The relationship between the ratio obtained and the experimental probability.





Experimental probability is the probability that is obtained from an experiment. Ratio 'frequency of appearance towards number of flips' that is obtained from flipping the coin is the experimental probability for an event.

In general,

$$\text{The experimental probability of an event} = \frac{\text{Frequency of an event}}{\text{Number of trials}}$$

### 13.1.2 Experimental probability of an event

**LEARNING STANDARD**

Make conclusions about the experimental probability of an event when the number of trials are large enough.

**COGNITIVE STIMULATION** Group

**Aim:** Making a conclusion on experimental probability

**Materials:** Dynamic geometry software

**Steps:**

1. Open the file MS279.
2. Click the *New experiment* button.
3. Click the *Start* button. Study the parallel marker and the reading on the graph.
4. Repeat steps 2 and 3 for 4 times.

**Discussion:**

(i) Differences in the graph that is formed in all five experiments.
(ii) The conclusion that can be obtained regarding experimental probability when the number of trials is higher.

**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms279 to view flipping coins experiment.

The file shows possible outcomes of obtaining head from flipping a coin. As much as 1 200 trials of flipping the coin has been done. From the graph that is shown, the experimental probability of obtained heads from 1 200 trials moves towards one value, that is 0.5.

It is observed that all the five graphs show the same shape. The conclusion that can be made is, experimental probability moves towards a certain value if the experiment is repeated with a higher number of trials.

**SELF PRACTICE 13.1**

1. Perform an experiment by throwing a fair dice. Write the ratio of the number of obtaining even numbers in 16 trials.

# 13.2 The Probability Theory Involving Equally Likely Outcomes

## 13.2.1 Sample space for an experiment

Before starting a football game, the referee usually flips the coin to decide the team that will start the game. Why does the referee use a coin and not dice or other objects? What is the sample space of the possible outcomes of flipping a coin?

**LEARNING STANDARD**

Determine the sample space and events of an experiment.

**COGNITIVE STIMULATION** Individual

**Aim:** Writing the possible end result for the fair dice throw

**Materials:** Fair dice

**Steps:**

1. Throw a dice and record the number that appears on the fair dice.
2. Complete the table below:

| **Number on the fair dice** | | | | | | |
|---|---|---|---|---|---|---|

3. Repeat step 1 a few times until you are sure that all the numbers on the fair dice have been obtained. (The number on the fair dice that has been obtained does not need to be recorded again).
4. List all numbers that appear using notation set { }.
5. State the relationship of the list in step 4 as sample space.

**Discussion:**

The possible outcomes of a fair dice throw.

When a fair dice is thrown, the number that is shown is either 1, 2, 3, 4, 5, 6. Although the number shown is repeated, it is still in the range of 1 to 6. The outcome list for the thrown fair dice consists of numbers 1,2,3,4,5,6. The sample space for the thrown fair dice is $S = \{1,2,3,4,5,6\}$.

**THINK SMART**

A coin has only two faces, that is the head and tail. What is the sample space for one flip of the coin?

**FLASHBACK**

Notation set { }

Set A = {odd number less than 10}

A = {1, 3, 5, 7, 9}

**DO YOU KNOW ?**

1. Experiment is a procedure to observe possible outcome.
2. The outcome is the possible result of an experiment.
3. Sample space is all the possible outcomes of an experiment.





**COGNITIVE STIMULATION** Group

**Aim:** Using a tree diagram to denote outcomes.

**Materials:** Two empty boxes labelled *A* and *B*, 4 pieces of card labelled 2, 3, 5 and 7

**Steps:**

1. Form a group of 5.
2. Place the card labelled 2 into box *A*.
3. Place the card labelled 3, 5, 7 into box *B*.
4. One student takes one card from box *A* and one card from box *B*.
5. Write the pair of numbers in the table below.

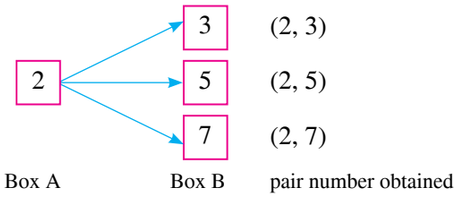
| | Member 1 | Member 2 | Member 3 | Member 4 | Member 5 |
|---|---|---|---|---|---|
| Box A | | | | | |
| Box B | | | | | |

6. Place both the cards into the original boxes.
7. Repeat steps 4 to 6 until all the members have a pair of numbers. Write the results in the table.
8. List the outcomes using in the notation set, { }.

**Discussion:**

The similarities and differences of the paired number that is obtained by each member.

A tree diagram can help you determine probability.




When you pick a card randomly, you might get pairs like the ones shown in the tree diagram above. The possible outcomes of the activity is known as sample space, $S = \{(2,3), (2,5), (2,7)\}$.

**TIPS**

A tree diagram can be used to show the flow of a process. It is used to organise and calculate the probability of an event happening.

**Sample space** is a set of all possible outcomes of an experiment.

## Event of an experiment

**COGNITIVE STIMULATION** Group

**Aim:** Identifying event

**Materials:** Two red balls, two yellow balls and a box

**Steps:**

1. Form groups of 4.
2. Mark every ball with the symbols $M_1$ and $M_2$ for the red balls. $K_1$ and $K_2$ for the yellow balls.
3. Place all the balls into the box.
4. One member takes two balls from the box, one by one.
5. Write the label of the ball that is taken in the table below.
6. Place both the balls back into the box.
7. Repeat steps 4 to 6 for each member. Complete the table below.

| | Member 1 | Member 2 | Member 3 | Member 4 |
|---|---|---|---|---|
| First ball | | | | |
| Second ball | | | | |
| Outcome | | | | |

**Discussion:**

The possible outcomes list that fulfils the following conditions.

(i) Two balls are of the same colour.

(ii) At least one ball is red.

The discussion in the activity above requires you to list the possible outcomes that fulfils two conditions. The first condition is that both the balls are of the same colour. The second condition is at least one of the balls is red. The outcomes list that fulfils the conditions is known as event.

**Event** is a set of possible outcomes that fulfils certain conditions for a sample space and is a subset for the sample space.

**FLASHBACK**

Set $A = \{1, 3, 5, 7, 9\}$

Set $B = \{2, 4, 6, 8\}$

Number of elements:

Set $A$, $n(A) = 5$

Set $B$, $n(B) = 4$

**EXAMPLE 1**

One letter is randomly chosen from the word SEMPURNA. List the possible outcomes and write the sample space for the experiment. State the number of elements in the sample space.

*Solution:*

The word SEMPURNA is made up of eight different letters. The possible outcomes are S, E, M, P, U, R, N, A. Sample space, $S = \{S, E, M, P, U, R, N, A\}$. The number of elements in the sample space, $n(S) = 8$.





**EXAMPLE 2**

A number is chosen randomly from the prime numbers between 20 to 40. List the possible outcomes and write the elements in the sample space for this experiment. State the number of elements in the sample space.

*Solution:*

Prime numbers are between 20 to 40 are 23, 29, 31, 37.

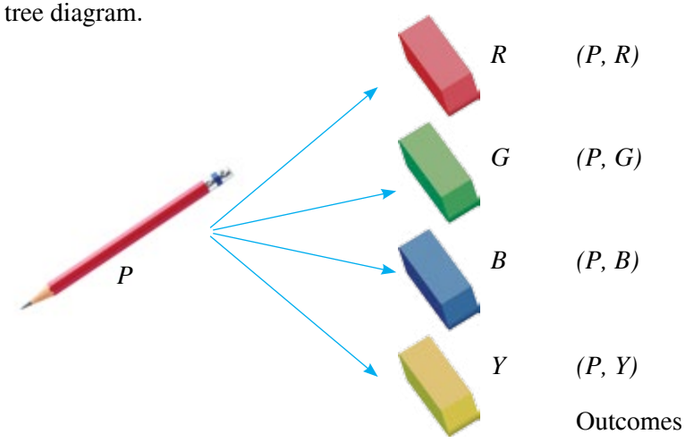
Sample space, $S = \{23, 29, 31, 37\}$. Number of elements in sample space, $n(S) = 4$.

**EXAMPLE 3**

A school cooperative sells brand $P$ pencil. The erasers that are sold are red, green, blue and yellow. Palin wants to buy a pencil and one eraser. With the help of the tree diagram, list the possible outcomes and write the elements in the sample space of the items bought by Palin. State the number of pairs.

*Solution:*

Step 1: Draw the tree diagram.




Step 2: List of elements in the sample space, $S = \{(P,R), (P,G), (P,B), (P,Y)\}$.

Therefore, number of elements in sample space, $n(S) = 4$

**EXAMPLE 4**

One card has been chosen from the box containing cards with numbers 1 to 9. Determine whether the event below may occur.

(i) Number bigger than 5. (ii) Two digits number.

(iii) Factor of 15.

*Solution:*

Sample space , $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(i) May occur (ii) May not occur

(iii) May occur

**EXAMPLE 5**

In a telematch, the contestants must take one card that has the letters $K, A, S, U, T$ from a jar. List the elements in the sample space for the event, if

(a) consonant are chosen (b) vowel are chosen

***Solution:***

Sample space, $S = \{K, A, S, U, T\}$

(a) Consonants = $\{K, S, T\}$ (b) Vowels = $\{A, U\}$

**EXAMPLE 6**

Jar $A$ has a card labelled $I$. Jar $B$ has four cards labelled $I, K, A$ and $N$. One card from Jar $A$ and one card from Jar $B$ are picked.

(a) List the the elements in the sample space

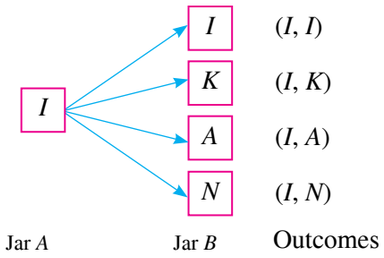
(b) List the elements in the sample space if

(i) the letters are the same.

(ii) at least one consonant is picked.

***Solution:***

Step 1: Draw the tree diagram




Step 2: Write the answers.

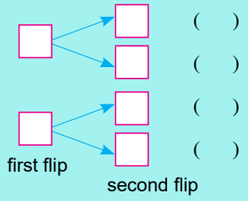
(a) $S = \{(I, I), (I, K), (I, A), (I, N)\}$

(b) (i) Event $X = \{(I, I)\}$ (ii) Event $Y = \{(I, K), (I, N)\}$

**THINK SMART**

A coin is flipped two times consecutively. The tree diagram below shows the possible end result.

1. State the sample space for both flips.
2. What is the probability of obtaining heads in both flips?




## 13.2.2 Probability of an event

**LEARNING STANDARD**

Construct probability models for an event,and hence make connection between theoretical probability and experimental probability.

Throwing a fair dice has six possible outcomes.They are numbers 1, 2, 3, 4, 5 and 6. Assuming that each number has an equal chance of appearing in a throw. What is the

(i) chances of getting number 4.

(ii) chances of getting an odd number.





The possible outcomes from a throw of a fair dice:

(i) The number 4 occurs only once. The probability of getting 4 is $\frac{1}{6}$.

(ii) Odd numbers occur three times, that is 2, 3, and 5. The probability of getting an odd number is $\frac{3}{6} = \frac{1}{2}$.

From the two situations above, the number of possible outcomes from a throw of a fair dice is represented by $n(S)$ and number of an events is represented by $n(A)$. The probability of the event, $A$ is $P(A)$.

Then, Probability of an event $A$ is represented by $P(A) = \frac{n(A)}{n(S)}$

The table on the right shows the total sum when two fair dice are thrown.

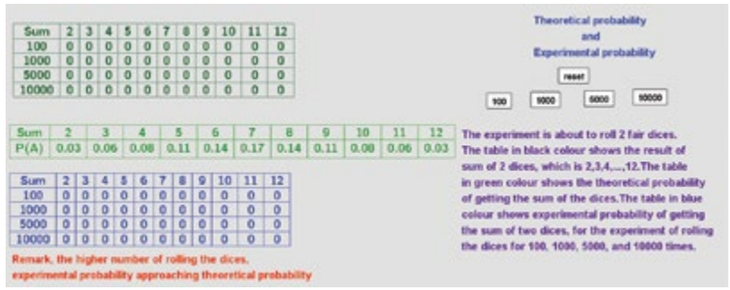
| | + | Dice 1: 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Dice 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

From the table, when two fair dice are thrown, the sum of 5 appears 4 times. The probability of getting a sum of 5 is $\frac{4}{36} = \frac{1}{9}$.

This probability is called **theoretical probability**.

In the experiment of throwing two fair dice thirty-six times, the sum of 2 fair dice with the value 5 appears 12 times. The probability of obtaining sum of 5 is $\frac{12}{36} = \frac{1}{3}$. This probability is called **experimental probability**.

If the number of trials of throwing two fair dice is large enough, the probability of the experimental probability, $\left(\frac{1}{3}\right)$ converges to the theoretical probability, $\left(\frac{1}{9}\right)$ as in the diagram below.




**QR CODE**

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms285 to read about theory vs experiment.

### 13.2.3 Determining probability

LEARNING STANDARD

Determining the probability of an event.

Probability of an event $A$ is determined by using,

$$P(A) = \frac{n(A)}{n(S)}$$

**EXAMPLE 7**

An apple is picked from a box that contains 25 green apples and 35 red apples. Calculate the probability of getting a green apple.

***Solution:***

Number of green apples = 25

Number of red apples = 60

Assume $A$ is an event of getting a green apple.

The probability of getting a green apple.

$$P(\text{green apple}) = \frac{\text{NUMBER OF GREEN APPLES}}{\text{TOTAL NUMBER OF APPLES}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

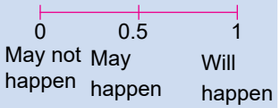
$$= \frac{25}{60}$$

$$= \frac{5}{12}$$

**TIPS**

Probability can be written in the form of fraction or decimal number.

**TIPS**




**EXAMPLE 8**

Pramjit gets RM5 pocket money every Tuesday, Wednesday and Thursday. Calculate the probability of him getting the amount of RM5 in four weeks?

***Solution:***

Assume A is an event of obtaining pocket money.

Total of Tuesday, Wednesday, and Thursday in 4 weeks, $n(A) = 12$

Total days in 4 weeks, $n(S) = 28$ days

Probability of getting RM5 pocket money in 4 weeks, $P(A) = \frac{n(A)}{n(S)}$

$$= \frac{12}{28}$$

$$= \frac{3}{7}$$

**SELF PRACTICE 13.2**

1. A bicycle shop has a stock of 35 bicycles. If the shop sold 15 bicycles in January, calculate the probability of selling a bicycle in that month.





2. The Meteorological Department predicted that rain will fall in the eastern states every three days in the months of November and December. Calculate the probability of rain falling in the months of November and December.

3. A supermarket held a lucky draw for a week in conjunction with its 10th anniversary. The supermarket has set a condition that every purchase of RM50 is eligible to submit one entry. The supermarket recorded the distribution of gift coupons on the average of 30 pieces a day for a week. Danial, a food stall owner, spends RM450 throughout the draw period. Calculate the probability of Danial will win the lucky draw.

## 13.3 Complement of An Event Probability

### 13.3.1 Describing complement of an event

LEARNING STANDARD

Describe the complement of an event in words and by using set notations.

**COGNITIVE STIMULATION** Individual

**Aim:** Identifying complement of an event.

**Materials:** Nine cards numbered in multiples of three, magnetic board, and magnet bar

**Steps**:

1. Arrange the first nine numbers of multiples of three on the magnetic board

| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
|---|---|---|---|---|---|---|---|---|

2. List the element $A$. $A$ is an event when an even number is picked .

$A = \{ \square, \square, \square, \square \}$

3. List the element $A'$. $A'$is an event of when not an even number is picked

$A' = \{ \square, \square, \square, \square, \square \}$

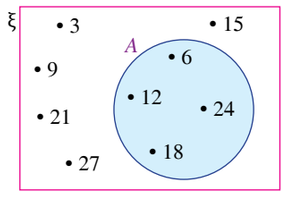
4. (i) Calculate the probability of picking an even number. $P(A)$.
   (ii) Calculate the probability of picking not an even number.$P(A')$.

**Discussion:**

(i) Relate $P(A)$ and $P(A')$.

(ii) Relationship between sample space, $S$ and universal set $\xi$.

From the activity above, the universal set, $\xi$ is the first nine numbers of multiples of three. $A$ is a subset of a universal set. $A'$ is the complement set of $A$. The relationship between set $A$ and the universal set is shown in the Venn diagram on the right. The complement of an event $A$ in a sample space $S$, consists of all outcomes that is not the outcome of $A$.



In the sample space probability, $S$ is a universal set. If set $A$ represents event $A$, then set $A'$ is complement of an event for event $A$.

Probability of picking an even number, $P(A) = \frac{4}{9}$.

Probability of picking not an even number, $P(A') = \frac{5}{9}$.

$$P(A) + P(A') = \frac{4}{9} + \frac{5}{9}$$
$$= \frac{9}{9}$$
$$= 1$$

Then $P(A) + P(A') = 1$.

Thus $P(A') = 1 - P(A)$, where $0 \leqslant P(A) \leqslant 1$.

**TIPS**

If $P(A) = 0$, event $A$ will not take place

If $P(A) = 1$, event $A$ is will to take place

**EXAMPLE 9**

An employee at a florist shop arranges 15 bouquets according to the number of flowers, arranged in odd numbers 1 to 30 in ascending order. $A$ is the event of selling a flower bouquet that has a perfect square number of flowers. Determine the complement of an event, $A'$ in

(i) words

(ii) set notations

***Solution:***

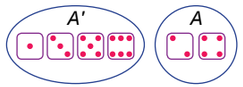
Sample space, $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

Event $A = \{9, 25\}$

(i) $A'$ = event of a number which is not a perfect square.

(ii) $A' = \{1, 3, 5, 7, 11, 13, 15, 17, 19, 21, 23, 27, 29\}$

**DO YOU KNOW?**

1. For the event of getting a head when flipping a coin, the complement event is getting a tail.
2. For the event of selecting a day in a week, if {Monday, Thursday} is selected, the complement is {Sunday, Tuesday, Wednesday, Friday and Saturday}.
3. 

4. Set $A = \{2, 4\}$
   Set $A' = \{1, 3, 5, 6\}$
   $P(A) = \frac{2}{6} = \frac{1}{3}$
   $P(A') = \frac{4}{6} = \frac{2}{3}$

## 13.3.2 Probability of the complement of an event

**LEARNING STANDARD**

Determine the probability of the complement of an event.

**EXAMPLE 10**

One number was chosen randomly from a set of integers from 1 to 20. $A$ is an event of choosing prime numbers. Determine the complement of probability $A$.

***Solution:***

Sample space, $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

Event $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

Probability of an event $A$, $P(A) = \frac{n(A)}{n(S)}$

$$= \frac{8}{20}$$
$$= \frac{2}{5}$$

**ATTENTION**

$P(A) + P(A') = 1$

$P(A) = 1 - P(A')$

$P(A') = 1 - P(A)$





**Method 1:**

Complement probability, $P(A')$

$P(A') = 1 - P(A)$

$= 1 - \frac{8}{20}$

$= \frac{12}{20}$

Then, $P(A') = \frac{12}{20}$

$= \frac{3}{5}$

**Method 2:**

Event $A' = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$

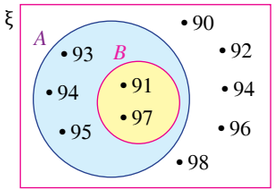
$P(A') = \frac{n(A')}{n(S)}$

$= \frac{12}{20}$

$= \frac{3}{5}$

**EXAMPLE 11**

The Venn diagram on the right shows the elements in a universal set. Calculate the probability $P(A')$.




***Solution:***

Number of elements in the complement set, $n(A') = 5$

Number of elements in the universal set is 10

The probability, $P(A') = \frac{n(A')}{n(S)}$

$= \frac{5}{10}$

$= \frac{1}{2}$

**SELF PRACTICE 13.3**

1. A container has 5 red bean buns, 8 *sambal* buns and 4 chocolate buns. One bun is taken randomly from the container. If $A$ is an event getting a chocolate bun, determine the complement of the event $A$ in

   (a) words (b) set notations

2. A container contains a number of red and blue pens. The probability of choosing one blue pen from the container is $\frac{3}{5}$. Calculate the probability of choosing one red pen from the same container.

3. A souvenir shop sells 25 cups, 30 picture frames and 15 other souvenirs within two weeks. $Z$ is the event of selling a cup. Calculate the probability of selling other souvenirs.

4. Ali has RM73. A shop selling shoes gives Ali a choice by offering three pairs of shoes priced below RM50 a pair, four pairs of shoes priced between RM50 to RM70 a pair and five pairs of shoes priced at RM70 a pair. If $B$ is an event where Ali buys a pair of shoes, express the complement of an event $B'$ in

   (a) words (b) set notations

5. A sum of 10% of oranges from three boxes of oranges is found to be rotten. $C$ is a non-rotten orange event. If an orange box contains 30 oranges, calculate the probability of taking one non-rotten orange at random.

## 13.4 Simple Probability

### 13.4.1 Solving problems

**LEARNING STANDARD**

Solve problems involving the probability of an event.

**EXAMPLE 12**

A shirt manufacturer manages to produce 80 pieces of shirts in one month. He sells 15 pieces of shirts in one week. If the profit from selling 15 pieces of shirts is RM135, calculate:

(a) the probability of the shirts sold in one month

(b) the profit obtained in two months

(c) the probability of the shirts not sold in one month.

*Solution:*

**Understand the problem**

(a) The probability of shirts sold.

(b) Profit earned within two months.

(c) The probability of shirts not sold in one month.

**Planning a strategy**

Sample space, $S$ = Number of shirts produced,

$n(S) = 80$

Event $A$ = The number of shirts in a month

$n(A) = 60$

**Implementing the strategy**

(a) $P(A) = \dfrac{n(A)}{n(S)}$

$= \dfrac{60}{80}$

$= \dfrac{3}{4}$

(b) Number of shirts sold in two months $= \dfrac{3}{4} \times 80 \times 2$

$= 120$ pieces

Total profit $= \dfrac{120}{15} \times \text{RM}135$

$= \text{RM}1\,080$

(c) $P(A') = 1 - P(A)$

$= 1 - \dfrac{3}{4}$

$= \dfrac{1}{4}$

The number of shirts not sold $= \dfrac{1}{4} \times 80$

$= 20$ pieces

**Conclusion**

(a) The probability of shirts sold within a month is $\frac{3}{4}$.

(b) Total profit is RM1 080.

(c) The number of shirts not sold in a month is 20 pieces.

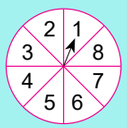
**THINK SMART**

The table below shows the usage of laptops and tablets according to gender at a college.

| Gender | Laptop | Tablet | Total |
|---|---|---|---|
| Male | 19 | 71 | 90 |
| Female | 84 | 4 | 88 |
| Total | 103 | 75 | 178 |

- What is the probability that a student chosen is a laptop user?
- What is the probability that a female student who uses a tablet is chosen?

**THINK SMART**




The picture above shows a number wheel. The needle in the number wheel is spun and stops randomly. Calculate the probability the needle stopping at

(i) an even number

(ii) an odd number

(iii) a prime number





**SELF PRACTICE 13.4**

1. In a crossword competition, a contestant sends 15 entry forms. The probability that the contestant wins is $\frac{3}{25}$. Determine the total number of entry forms submitted in the competition?

2. A set of letters that forms the word MENJUSTIFIKASI is inserted into a box. One letter is taken from the set randomly. Calculate
   (a) the probability of getting a vowel.
   (b) the probability of the complement of choosing a vowel.

3. A container contains 35 red marbles and a few blue marbles. A marble is randomly taken from the container. The probability of taking the red marble is $\frac{7}{15}$. Calculate
   (a) the probability of choosing a blue marble
   (b) the number of blue marbles
   (c) the probability of choosing blue marbles if 8 red marbles are added.

**GENERATING EXCELLENCE**

1. A box contains cards with the letters that form the word PEMBELAJARAN. One card is taken from the box randomly.
   (a) List the sample space for the experiment.
   (b) List all the vowels
   (c) Calculate the probability of taking a consonant.

2. A basket contains 6 mini blue cones, 10 mini yellow cones and a few mini green cones. One cone is taken randomly from the basket. The probability of getting a mini blue cone is $\frac{1}{4}$. Calculate
   (a) the total number of mini cones in the basket.
   (b) the probability of choosing a mini yellow cone.

3. The probability of Aiman shooting an arrow accurately is 85%. In one minute, Aiman is able to take 3 shots. Calculate the number of non-accurate shots that is taken by Aiman in one hour.

4. A box contains 3 balls that are marked with three vowels *a*, *e*, *i*. One ball is taken randomly from the box and the vowel obtained is written down. The ball is placed back into the box and the second ball is taken randomly from the box. With the help of a tree diagram,
   (a) list the sample space for the experiment.
   (b) list all element of the complement of the event of obtaining different vowels.
   (c) calculate the probability of the complement of an event for experiment (b).

5. Box *A* has a piece of card with the first multiple of 2 and box *B* has three pieces of cards with the first three terms of the multiple of 3. A card is drawn randomly from box *A* and box *B*. With the aid of a tree diagram, list the sample space for this experiment and calculate the probability that a person gets
   (a) at least one number of multiple of two.
   (b) at least one number of multiple of three.
   (c) an odd number.

6. Hazrin's hobby is collecting stamps. He has a collection of 75 pieces of stamps from Indonesia, Singapore, Thailand, the Philippines and Malaysia. A stamp is selected at random. The probability of getting a stamp from Thailand and the Philippines is $\frac{3}{5}$. If the number of stamps from Singapore and Indonesia equals the number of stamps from Malaysia, calculate the probability of selecting stamps from Malaysia.

## CHAPTER SUMMARY

### SIMPLE PROBABILITY

**Sample space**
The sample space is the set of all the possible outcomes of an experiment and is represented with the letter *S*.

**Event**
An event is a result of an experiment

$$P(A) = \frac{\text{The number of ways event } A \text{ may occur}}{\text{Total number of possible outcomes}}$$

The probability for the complement of an event, $P(A')$

$$P(A) + P(A') = 1$$

$$P(A') = 1 - P(A)$$

$$0 \leqslant P(A) \leqslant 1$$





## SELF REFLECTION

**At the end of this chapter, I will be able to:**

| | |
|---|---|
| 1. Perform simple probability experiments, and thus determine the ratio $\frac{\text{frequency of an event}}{\text{number of trials}}$ as the experimental probability of an event. | |
| 2. Make conclusions about the experimental probability of an event when the number of trials is large. | |
| 3. Determine the sample space and events of an experiment. | |
| 4. Construct probability model for an event, and thus make a connection between theoretical probability and experimental probability. | |
| 5. Determine the probability of an event. | |
| 6. Describe the complement of an event in words and by using set notations. | |
| 7. Determine the probability of a complement of an event. | |
| 8. Solve problems involving the probability of an event. | |

## MINI PROJECT

Divide your class into five groups. Each group will create five types of games and set up five probability questions for each game.

Group 1 : Model of 2 pieces of fair dice.
Group 2 : Model of 2 pieces of coins of different values.
Group 3 : Rotation Board.
Group 4 : A black box containing numbered cards.
Group 5 : Snake and ladder board game.

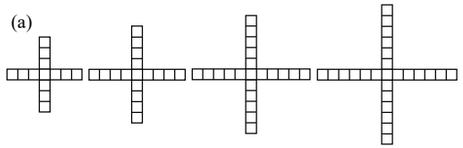
Place the games at the corner of your class.

# Answers

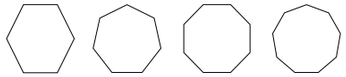
## CHAPTER 1 PATTERN AND SEQUENCES

### SELF PRACTICE 1.1

1. (a)



(b)



2. (a) Pattern: add 7 to the previous number.
(b) Pattern: subtract 3 from the previous number.
(c) Pattern: add 4 to the previous number.
(d) Pattern: divide the previous number by 2.
(e) Pattern: subtract $\frac{1}{4}$ from the previous number.
(f) Pattern: multiply the previous number by –3.

3. (i) 37, 55, 73, 91, 109... Pattern: add 18
+18
(ii) 28, 46, 64, 82, 100... Pattern: add 18
+18

4. 1, [1], 2, [3], [5], [8], [13]

5. [2] [6] [6] [2]

### SELF PRACTICE 1.2

1. (a) Pattern
(b) Pattern
(c) Not a pattern
(d) Not a pattern
(e) Not a pattern
(f) Not a pattern

2. (a) 34, 28, [22], 16, [10], [4], ...
(b) [128], [64], 32, 16, [8], 4, ...
(c) 0.07, [0.28], 1.12, [4.48], 17.92, ...
(d) $1\frac{1}{10}$, 1, [$\frac{9}{10}$], [$\frac{4}{5}$], [$\frac{7}{10}$], ...
(e) 0.2, 2.4, 28.8, [345.6], [4147.2], ...
(f) [−400], −80, −16, [−3.2], [−0.64], ...
(g) [$\frac{3}{4}$], [$\frac{2}{3}$], [$\frac{7}{12}$], [$\frac{1}{2}$], [$\frac{5}{12}$], ...
(h) −8.1, [−6.1], −4.1, −2.1, [−0.1], ...

3. (a) 42, [49], [56], [63], [70], [77], ...
(b) 96, [48], [24], [12], [6], [3], ...

### SELF PRACTICE 1.3

1. (a) Multiply the previous number by 3.
(b) Divide the previous number by 2.

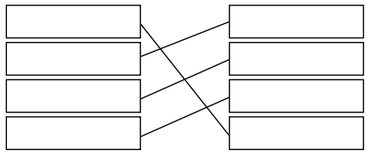
2. (a) $2^n$ $n = 1, 2, 3, ...$
(b) $5 + 3n$ $n = 0, 1, 2, 3, ...$
(c) $3 + 3n$ $n = 0, 1, 2, 3, ...$
(d) $3 - 2n$ $n = 0, 1, 2, 3, ...$

3. (a) $T_7 = 45$, $T_{11} = 77$ (b) $T_7 = 13$, $T_{11} = 19$ (c) $T_7 = -7.3$, $T_{11} = -9.7$

4. (a) 30 minutes (b) 10:00 a.m. (c) 3:00 p.m.

### GENERATING EXCELLENCE

1.



2. (a) add 6
(b) subtract 4
(c) multiply 3
(d) divide 6

3. (a) **Number**
+2

**Words**
Add 2 to the previous number.

**Algebraic Expression**
$2x$ $x = 1, 2, 3, 4, ...$

(b) **Number**
÷2

**Words**
Divide the previous number by 2.

**Algebraic Expression**
$\frac{100}{2^n}$ $n = 0, 1, 2, 3, ...$

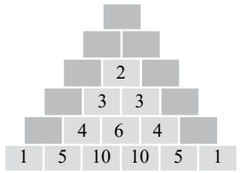
4. (a) 1, 3, 5, [7], 9, [11], ...
(b) [−80], [−40], −20, −10, −5
(c) 268, [235], [202], 169, 136, [103], ...
(d) $\frac{1}{2}$, [$\frac{5}{12}$], $\frac{1}{3}$, [$\frac{1}{4}$], $\frac{1}{6}$



5. (a) $x = 2$
(b) (i) The pattern is −7
(ii) Subtract 7 from the previous number.
(iii) $9 - 7n$ , $n = 0, 1, 2, 3, ...$

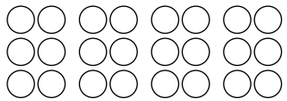
6. 0, 1, 1, [2], [3], [5], ...

7.




8. (a) $x = 3$
(b) −61

9. (a) Add 6
(b) 6, 12, 18, ...
(c)



(d) 36 buttons

10. 841.86 m²

11.

| Ubat | 1 | 2 | 3 |
|---|---|---|---|
| Fever | 8:30 a.m. | 4:30 p.m. | 12:30 a.m. |
| Antibiotics | 8:30 a.m. | 8:30 p.m. | |
| Flu | 8:30 a.m. | | |

## CHAPTER 2 FACTORISATION AND ALGEBRAIC FRACTIONS

### SELF PRACTICE 2.1

1. (a) $(a + 2)(a + 1)$ (b) $(4x - 3)(4x - 3)$

2. (a) $3x + 6$ (b) $32x - 12$
(c) $2a + 10$ (d) $6p^2 - 8p$
(e) $-\frac{rs}{4} + r$ (f) $-2pr + 4pq$
(g) $15bc - 18$ (h) $14ef + 21e$
(i) $16g + 8g^2h$

3. (a) $a^2 + 3a + 2$ (b) $x^2 - x - 20$
(c) $10 + 3m - m^2$ (d) $12p^2 - 11p + 2$
(e) $12r^2 - 11r + 2$ (f) $8r^2 - 3s^2 - 2rs$
(g) $6d^2 - \frac{5}{2}db + \frac{1}{4}b^2$ (h) $r^2 - 6rs + 9s^2$
(i) $16e^2 - 24e + 9$

4. (a) $17b - 4a + 3$ (b) $-4m - 17mn$
(c) $-5h^2 + 4hj + j^2$ (d) $3x^2 - y^2 + 4xy$

5. (a) $4p^2 - 12p + 9$ (b) $\frac{3}{2}y^2 - \frac{5}{2}y + 1$
(c) $10x^2 - 11x - 6$ (d) $5w^2 + w$

6. $p^2 - 2p + 2$

7. $(5x^2 - 17x + 6)$ m²

8. $\sqrt{33y^2 - 34y + 8}$

### SELF PRACTICE 2.2

1. (a) 1, 2, 4, $4y$ (b) 1, $b$
(c) 1, $w$ (d) 1, 5, $m$
(e) 1, $c$ (f) 1, 2, $b$

2. (a) $5(e + 2)$ (b) $2a(b - 4a)$
(c) $3ab(c + 2a)$ (d) $4x(1 - 3x)$
(e) $f(e + f + g)$ (f) $2x(x - 2y + 3w)$

3. (a) $(b - 9)(b + 9)$ (b) $(a + b)(a - b)$
(c) $(x + 1)(x - 1)$ (d) $(4y - 7)(4y + 7)$
(e) $(m + 7)(m - 1)$ (f) $(2x + 1)(2x - 5)$

4. (a) $(x + 2)(x + 7)$ (b) $(x - 2)(x + 9)$
(c) $(x - 8)(x + 3)$ (d) $(m - 2)(m + 13)$
(e) $(y - 5)(y + 3)$ (f) $(k - 4)^2$
(g) $(m - 6)(2m + 1)$ (h) $(3f - 2)^2$
(i) $(2m - 4)(m + 4)$ (j) $(2x - 7)(x + 1)$
(k) $(6y - 5)(2y + 3)$ (l) $(5p - 4)(p + 2)$
(m) $(5m - 4)(-m - 2)$ (n) $(-p + 2)(3p - 2)$
(o) $(-2x + 3)(3x + 5)$

5. (a) $(p - r)(q - w)$ (b) $(x + y)(x + 6)$
(c) $(3a + c)(b - 3d)$ (d) $(h + j)(a - b)$
(e) $(m - n)(j + y)$ (f) $(3x + 4p)(3y - z)$

6. (a) $2y^2 + 3y - 8$ (b) 1 section

### SELF PRACTICE 2.3

1. (a) $(2b - 5)(2b + 1)$ (b) $(m - 1)(m + 7)$
(c) $(p - 12)(p + 2)$ (d) $7x^2 - 7x - 3$
(e) $4c^2 - 2c + 9$

2. (a) $\frac{6y}{5}$ (b) $\frac{2m + 7n}{m - 2n}$ (c) $\frac{r + s}{2r + 3s}$

3. (a) $\frac{5p - 2}{p^2}$ (b) $\frac{2s}{9}$ (c) $\frac{12 - 3z}{4(x + y)}$

4. (a) $\frac{9u + 20v}{12}$ (b) $\frac{5t - 12s}{30st}$ (c) $\frac{6s + 4r - 8}{3rs - 6s}$

5. (a) $\frac{4m + 3n}{36}$ (b) $\frac{6m + n^2}{6m^2n}$ (c) $\frac{20 + 3d}{5d^2g}$

6. (a) $\frac{x - 1}{y}$ (b) $\frac{2a + 5}{4}$ (c) $\frac{1}{m - n}$
(d) $\frac{1}{2k + 1}$ (e) $\frac{c - 3}{2}$

7. (a) $\frac{6}{(a-3)(3+a)}$ (b) $\frac{hy}{(k-2)(h+3)}$

   (c) $\frac{6m^2n}{(m-n)(n-2m)}$ (d) $\frac{2rs-8r}{rs+5s-2r-10}$

8. (a) $\frac{2}{m(x-a)}$ (b) $\frac{5r}{s(1-2r)}$

   (c) $\frac{x+3}{5x}$ (d) $\frac{-2f(e+2f)}{3e(e-3f)}$

9. (a) $\frac{5a(a+b)}{3b(2a+3)}$ (b) $\frac{3}{2a}$

   (c) $\frac{y}{3x^2}$ (d) $\frac{1}{eg}$

10. (a) $\frac{xy(x+1)}{(x+y)^2}$ (b) $\frac{2q(p+1)}{2p-1}$

    (c) $\frac{pr}{(r-1)(q+r)}$ (d) $\frac{t(2t+1)}{(2t-1)(s-u)}$

### GENERATING EXCELLENCE

1. (a) $3a+6b$ (b) $n^2-3n-10$
   (c) $a^2+4ab+4b^2$ (d) $16x^2-8xy+y^2$
   (e) $6v^2+\frac{v}{3w}-\frac{2}{9w^2}$ (f) $13h^2-10hk+k^2$

2. (a) $6m(2-3m)$ (b) $(y+9)(y-9)$
   (c) $4ab(1-2a)$ (d) $(x-4y)(x+4y)$
   (e) $(s-4)(s-2)$ (f) $(x+1)(x+3)$
   (g) $(x-3)(x+5)$ (h) $(x+2)(x+4)$
   (i) $(2c-b)(3d-e)$

3. (a) $\frac{3a-2b+2}{4v}$ (b) $\frac{12ec-25abd}{20abc}$

   (c) $\frac{20-3f}{5f^2g}$ (d) $\frac{pn+2p+nm}{m^2p}$

   (e) $\frac{15x^2+2y^2-2y}{24xyz}$ (f) $\frac{9rsz+4-2r}{36yz}$

4. $(k+4)$ cm

5. $4x^2+4x-6$

6. (i) 25 pieces (ii) 21 units

7. (i) $2x^2+14x-10$
   (ii) RM$(16x^3+112x^2-80x)$

8. (i) $\frac{7+4x}{2}$ (ii) 2 hour

9. $\frac{3x^2+8x+4}{18}$ cm$^2$

10. (i) $xy$ (ii) $(xy-2x)$m (iii) $18xy$

## CHAPTER 3 ALGEBRAIC FORMULAE

### SELF PRACTICE 3.1

1. (a) $m=z+qp$ (b) $u=v-2$
   (c) $x=\frac{7w}{3y}$ (d) $b=\frac{4}{3a}-5$
   (e) $u=\frac{3}{5q+5}$ (f) $v=\frac{5}{2w+4}$
   (g) $b=\frac{(2a-5)^2}{3}$ (h) $w=6t$
   (i) $m=-\frac{\sqrt{4p-8}}{3}$ (j) $r=\frac{4s-7}{3}$

2. $z=29.75x+40.5y$

3. (a) (i) $c=16$ (b) (i) $p=2$
   (ii) $d=\frac{1}{2}$ (ii) $q=2$
   (c) (i) $m=-6$ (d) (i) $n=\sqrt{13}$
   (ii) $n=3$ (ii) $m=\frac{1}{16}$
   (e) (i) $u=6$ (f) (i) $p=0$
   (ii) $r=\frac{3}{2}$ (ii) $q=\frac{9}{2}$
   (iii) $s=4$ (iii) $r=-4$
   (g) (i) $a=\frac{1}{3}$ (h) (i) $s=12$
   (ii) $b=2$ (ii) $t=\sqrt{50}$
   (iii) $c=6$ (iii) $u=\sqrt{\frac{7}{4}}$

4. (a) $z=5.9x+3.6y$ (b) $b=\frac{24p-7}{q}$
   (c) $P=0.85(35m+76n)$ (d) $x=$ RM$0.1st$

### GENERATING EXCELLENCE

1. (a) $A=x^2$ (b) $p=5+3h$
   (c) $a=\frac{v_2-v_1}{t}$

2. (a) $q=\frac{m-p}{-3}$ (b) $w=p-x$
   (c) $g=\frac{2e-3h}{4}$ (d) $q=m-8p$
   (e) $v=\sqrt{\frac{w}{3}}$ (f) $n=\sqrt{\frac{8m}{3}}$
   (g) $v=36w^2-1$ (h) $k=\frac{16}{f^2}+7$





3. (a) (i) $w=-2$ (b) (i) $b=\frac{2}{9}$
   (ii) $x=-\frac{15}{19}$ (ii) $c=8$
   (iii) $y=29$ (iii) $d=3$
   (c) (i) $p=-\frac{1}{6}$ (d) (i) $s=-\frac{1}{5}$
   (ii) $q=-\frac{1}{19}$ (ii) $t=-46$
   (iii) $r=1\frac{1}{6}$ (iii) $u=-\frac{2}{11}$

4. $z=65xy$

5. $t=13\frac{1}{3}s$ minutes

6. $x=4$
   $y=7$

## CHAPTER 4 POLYGONS

### SELF PRACTICE 4.1

1. (a) Irregular polygon (b) Irregular polygon
   (c) Regular polygon (d) Regular polygon
   (e) Irregular polygon (f) Regular polygon
   (g) Regular polygon (h) Regular polygon
   (i) Regular polygon

2. (a) One axis of simmetry (b) 2 axis of simmetry
   (c) No axis of simmetry (d) No axis of simmetry

3.

| Name of polygon | Number of sides | Number of vertices | Number axis of symmetry |
|---|---|---|---|
| Hexagon | 6 | 6 | 6 |
| Heptagon | 7 | 7 | 7 |
| Octagon | 8 | 8 | 8 |
| Nonagon | 9 | 9 | 9 |

4. Students answer

5. Students answer

### SELF PRACTICE 4.2

1.

| Number of triangles in a polygon | Sum of interior angle |
|---|---|
| 3 | 540° |
| 4 | 720° |
| 5 | 900° |
| 6 | 1 080° |
| 7 | 1 260° |

2. (a) Interior angle: $a, g, e, c$
   Exterior angle: $b, d, f, h$
   (b) Interior angle: $a, b, c, d, e$
   Exterior angle: $f, g, h, i, j$

3. (a) $x=150°$ (b) $x=100°$
   (c) $x=22°$ (d) $x=54°$

4. (a) $p=80°$ (b) $p=68°$
   $q=55°$ $q=100°$
   $r=125°$ $r=88°$

5. (a) $a+b+c=300°$ (b) $a+b+c=170°$
   (c) $a+b+c=265°$ (d) $a+b+c=254°$

6. (a) 7 sides (b) 8 sides (c) 9 sides

7. (a) Decagon (b) $y=144°$

8. $x=117°$

### GENERATING EXCELLENCE

1. (a) Students answer (b) Students answer

2. (a) $p=40°$ (b) $p=45°$ (c) $p=75°$
   $q=135°$ $q=95°$ $q=140°$
   $r=95°$ $r=50°$ $r=105°$

3. (a) $x=50°$ (b) $x=42.5°$ (c) $x=80°$

4. (a) $\frac{360°}{45°}=8$ sides (b) $\frac{360°}{36°}=10$ sides

   (c) $\frac{360°}{40°}=9$ sides (d) $\frac{360°}{30°}=12$ sides

5. (a) $x+y=215°$ (b) $x+y=180°$
   (c) $a+b+c+d=425°$

6. Students answer

7. 17 sides

8. $p+q=276°$

9. $\angle CBM=58°$

10. (a) $h=20°$
    (b) Interior angle $=140°$
    Exterior angle $=40°$
    (c) Number of sides, $n=\frac{360°}{40°}=9$, nonagon

11. $x=54°$

12. Cannot, because the sum of interior angles is $(n-2)\times 180°$ when $n=3, 4, \ldots$

13. 12 sides

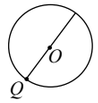
14. $x=72°$

15. $x=12°$

## CHAPTER 5 CIRCLES

**SELF PRACTICE 5.1**

1. (i) centre of circle
   (ii) diameter
   (iii) minor sector
   (iv) radius
   (v) minor arc
   (vi) chord
   (vii) minor segment
2. Student answer
3. (a) 

   (b) 

4. Student answer
5. Student answer

**SELF PRACTICE 5.2**

1. $KL = 24$ cm
2. $KOM = 45$ cm

**SELF PRACTICE 5.3**

1. (a) 44.00 cm (b) 352.00 cm
   (c) 28.91 cm (d) 308.00 mm
2. (a) 7.80 cm (b) 3.90 cm
3. (a) 1 386.00 $m^2$ (b) 9 856.00 $mm^2$
   (c) 154.00 $cm^2$ (d) 6.16 $cm^2$
4. (a) 3.50 cm (b) 22.00 cm
5. 18.87 $cm^2$
6. 102.75 $cm^2$
7. (a) 14.00 cm (b) 7.04 cm
   (c) 14.70 cm (d) 31.51 cm
8. (a) 10.8° (b) 7.2° (c) 25°
9. 70 cm
10. $25\pi - 49$

**GENERATING EXCELLENCE**

1. (a) 3 cm (b) 3 cm (c) 4 cm
2. 51.71 $m^2$
3. 66 $cm^2$
4. 122.9 $cm^2$
5. 550 $cm^2$
6. 2 827.8 $cm^2$

## CHAPTER 6 THREE-DIMENSIONAL GEOMETRY SHAPES

**SELF PRACTICE 6.1**

1. (a) (i) One circular base.
   (ii) One curved surface that meets at the vertex of the cone.
   (b) (i) 6 square surfaces.
   (ii) sides and edges.
   (c) (i) One vertex.
   (ii) sides and edges.
   (d) (i) Two circular bases that are congruent and parallel.
   (ii) A curved side surface that lists two sites.
2. (a) cone (b) pyramid (c) sphere

**SELF PRACTICE 6.2**

1. (a) 
   (b) 
   (c) 
   (d) 
2. (a) Hexagonal prism (b) Pyramid
   (c) Triangular prism (d) Triangular prism

**SELF PRACTICE 6.3**

1. (a) 282.86 $cm^2$ (b) 754.29 $cm^2$
   (c) 84 $cm^2$
2. (a) 1 257.14 $cm^2$ (b) 66 980.57 $mm^2$
   (c) 15 150 $cm^2$
3. (a) 455.71 $m^2$ (b) 361.43 $cm^2$
   (c) 1 428.57 $cm^2$

**SELF PRACTICE 6.4**

1. (a) 576.19 $cm^3$ (b) 618.67 $cm^3$ (c) 142.48 $cm^3$
2. (a) 157.14 $cm^3$ (b) 183.43 $cm^3$ (c) 146.79 $cm^3$
3. 2 192.67 $cm^3$
4. 2 block of pyramid

**GENERATING EXCELLENCE**

1. (a) Cube (b) Triangular prism
   (c) Hexagonal prism
2. 8 cm
3. (a) 60 mm (b) 8.5 cm (c) $h = 9$ cm
4. (a) 4 790.76 $cm^3$ (b) 13 967 marble





5. 30 tube
6. 81 pieces of sweet
7. 770 $cm^2$
8. 45 cm

## CHAPTER 7 COORDINATES

**SELF PRACTICE 7.1**

1. (a) 4 units (b) 5 units
   (c) 8 units (d) 14 units
2. (a) 4.47 cm (b) 1341.64 cm
   (c) 12.37 units (d) 11.4 units
3. (a) 4 units (b) 18 units
   (c) 7 units (d) 1 units
4. (a) $a = 3, b = 3$ (b) $a = 1, b = 4$
   (c) $a = -2, b = 2$ (d) $a = -4, b = 1$
5. (a) 5.66 units (b) 5.83 units
   (c) 7.07 units (d) 12.53 units
6. 5.39 units
7. 7 units
8. 5.83 units
9. $a = 2, b = 7$
10. (a) (4, 1) (b) (2, −1)
    (c) (5, 2) (d) (0, 0)
11. 15.91 units
12. 15.71 units

**SELF PRACTICE 7.2**

1. (a) $B$ (b) $B$
2. (a) (4 , 8) (b) (4 , 2) (c) (2 , 5)
3. (a) (3 , 6) (b) (6 , 4)
   (c) (4 , 0) (d) (−2, 3)
4. (a) (−1, 4) (b) (3, −2)
   (c) (4, 1) (d) (3, 0)
5. $P(-4, 6)$
   $R(-4, -4)$
6. (a) $a = 4$ , $b = 3$ (b) 8 units (c) (3 , 4)
7. (a) $m = 4$ , $n = 6$ (b) (−2 , 6) (c) (2 , −6)
8. $A(3,-1)$
9. $s = 8$ , $u = 3$
10. (a) $a = 1$ (b) $B(7, 1)$

**SELF PRACTICE 7.3**

1. (a) (−2, 1) (b) (0 , 5)
   (c) (1, 3) (d) 4.47 units
2. (a) 14.4 units (b) (−2, −2) (c) (−2, 4)
3. (a) (4, 3) (b) (4 , −3) (c) 6 units
4. (a) $K(-4,1)$ (b) (0, 1)
   $L(4, 1)$
5. (2, 3), 3.6 units

**GENERATING EXCELLENCE**

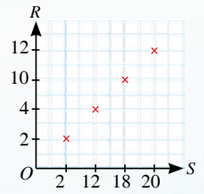
1. (a) $K$ (b) $A$
   (c) $H$ (d) $D$
2. (−4 , 5)
3. $P'(6, 2)$
   $Q'(3, -4)$
   $R'(-3 , 0)$
   Distance $P'Q' = 6.7$ units
   Distance $R'Q' = 7.2$ units
4. 20 $units^2$
5. 10 $units^2$
6. 8 $units^2$

## CHAPTER 8 GRAPHS OF FUNCTIONS

**SELF PRACTICE 8.1**

1. (a) (9, 18) (b) Many to many
2. $b = 7$
3. (a) Function (b) Function
   (c) Not a function
4. (a) Function (b) Not a function
5. (a) {(10,2), (12, 4), (18, 10), (20, 12)}
   (b)

| ***S*** | 10 | 12 | 18 | 20 |
|---|---|---|---|---|
| ***R*** | 2 | 4 | 10 | 12 |

   (c) 

   (d) $R = S - 8$

6. $a = 9$ $b = 15$

7. (a) domain $\{-5, 2, 4\}$, range $\{0, 8, 15, 16\}$ (b) domain $\{-4, 0, 1, 4\}$, range $\{-5, -3, 1, 2, 4\}$

## SELF PRACTICE 8.2

1. (a)

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 2 | 5 | 8 | 11 | 14 |

(b)

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 2 | 8 | 18 | 32 |

(c)

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | −6 | 1 | 2 | 3 | 10 | 29 |

2. (a)

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −8 | −6 | −4 | −2 | 0 | 2 | 4 |

(b)

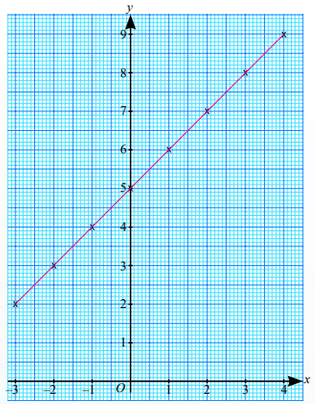
| $x$ | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | −4 | −5 | −2 | 5 | 16 |

(c)

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | −30 | −9 | −6 | −3 | 18 | 75 | 186 |

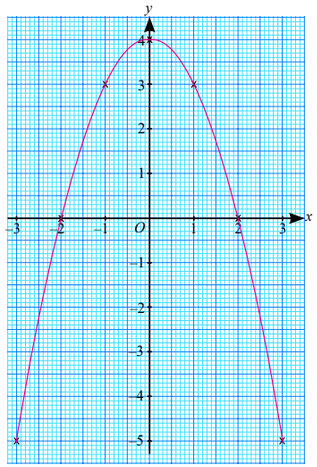
3. (a)

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



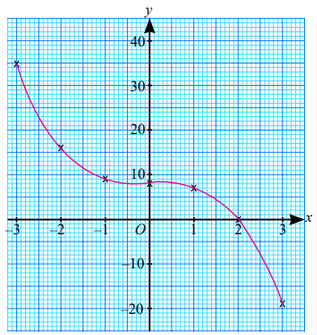
(b)

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | −5 | 0 | 3 | 4 | 3 | 0 | −5 |



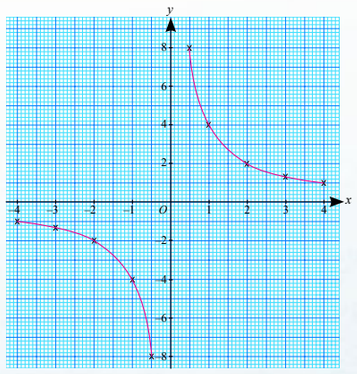
(c)

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 35 | 16 | 9 | 8 | 7 | 0 | −19 |



(d)

| $x$ | −4 | −3 | −2 | −1 | −0.5 | 0.5 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | −1 | −1.33 | −2 | −4 | −8 | 8 | 4 | 2 | 1.33 | 1 |





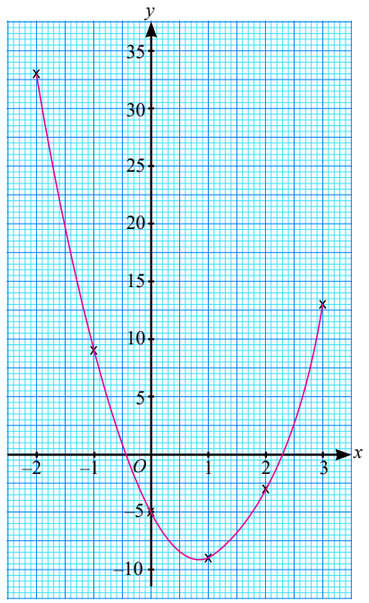


4. (a) (i) 30 km (ii) 42 km
(b) RM82.80

5. (a)

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 33 | 9 | −5 | −9 | −3 | 13 |

(b)(c)



(d) 2.3, −0.4

## GENERATING EXCELLENCE

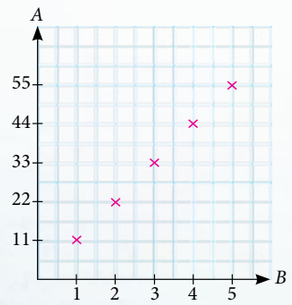
1. (a) Yes (b) No pair

2. (a) Ordered pairs:
$\{(1, 11), (2, 22), (3, 33), (4, 44), (5, 55)\}$

Table:

| $B$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $A$ | 11 | 22 | 33 | 44 | 55 |

Graph:



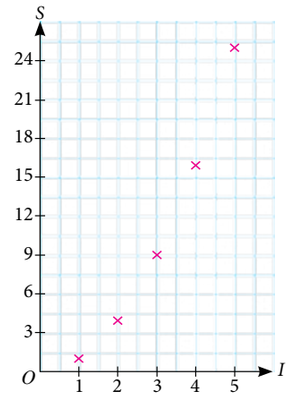
Equation: $f(x) = 11x$

(b) Ordered pairs:
$\{(1, 1), (2, 4), (3, 9), (4, 16), (5, 25)\}$

Table:

| $I$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $S$ | 1 | 4 | 9 | 16 | 25 |

Graph:



Equation: $f(x) = x^2$

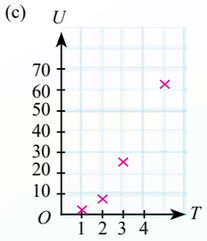
3. (a) (i) $L$ (ii) $r$
(b) $L = 4\pi r^2$

4. (a) $\{(1, 1), (2, 8), (3, 27), (4, 64)\}$

(b)

| $T$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $U$ | 1 | 8 | 27 | 64 |

(c)



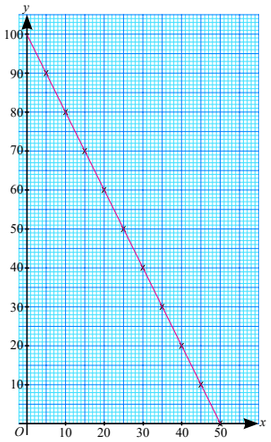
(d) $y = x^3$ or $f(x) = x^3$

5. (a) (i) RM96 (ii) RM90 (iii) RM80

(b)

| $x$ | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 10 | 0 |

(c)

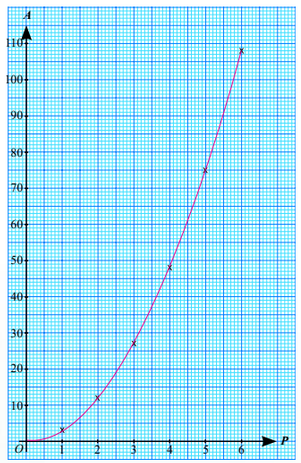


(d) (i) Day to -50 (ii) Day to -28

6. (a)

| $p$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $A$ | 0 | 3 | 12 | 27 | 48 | 75 | 108 |

(b)



(c) (i) 80 m² (ii) 75 m²

7. (a)

| Number of T-shirts | 10 | 30 | 50 | 70 |
|---|---|---|---|---|
| Cost (RM) | 100 | 200 | 300 | 400 |

(b) RM 50
(c) RM 390
(d) 72 pieces

8. (a) 15 m
(b) 1.5 second and 6.5 second
(c) 8 second
(d) 16 m
(e) Ascending and descending

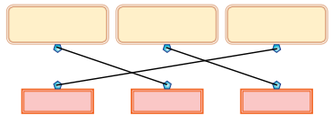
9. (a) RM8
(b) Company A because it only charges RM4 per hour used whereas company B charges RM5 per hour used.
(c) Company B with RM7, Zarul could use the bicycle for 3 hours whereas company A allows use for 2 hours 30 minutes only.
(d) the first 2 hours.
(e) RM10

10. (a) (i) RM 3 (ii) RM2.80
(b) Berjaya Company because it charges RM40 whereas Maju company charges RM5.
(c) Umai has to choose Berjaya Company because he can speed longer than with Maju Company.

## CHAPTER 9 SPEED AND ACCELERATION

### SELF PRACTICE 9.1

1.



2. (a) 120 km (b) 1543.5 km
(c) 16.5 m (d) 666.67 km

3. 8.29 m/s

4. hour 1154

5. (a) 833.33 m/min (b) 2.88 km/h
(c) 1.83 km/minute

6. 114.29 km/h

7. 93.24 km/h

### SELF PRACTICE 9.2

1.

| Situation | Acceleration | True / False |
|---|---|---|
| (a) | | True |
| (b) | | False |
| (c) | | True |
| (d) | | False |

2. (a) 100 km/h (b) –360 km/h

3. 0.5 $ms^{-2}$

4. 8.75 $cms^{-2}$

### GENERATING EXCELLENCE

1. Constant Speed: lift,clock, fan
Irregular Speed: wave, wind, mini bus

2. (a) 180 km/h² (b) –200 km/h²
(c) 120 km/h²

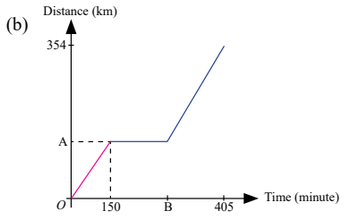
3. (a) 30 minute (b) 7:10 morning

4. (a) $A$ = 185 km, $B$ = 185 min





(b)



(c) 52.44 km/j

5. (a) 0.38 m/min² (b) 5.2 min
(c) 1 m/min (d) 180 second

6. $x$ = 315 km

## CHAPTER 10 GRADIENT OF A STRAIGHT LINE

### SELF PRACTICE 10.1

1. (a) Vertical distance = 4 m
Horizontal distance = 6 m
(b) Vertical distance = 12 m
Horizontal distance = 3 m
(c) Vertical distance = 16 m
Horizontal distance = 2 m

2. (a) $AB$
Vertical = 3 unit
Horizontal = 3 unit
(b) $CD$
Vertical = 8 unit
Horizontal = 1 unit
(c) $PQ$
Vertical = 6 unit
Horizontal = 4 unit

3. Vertical = 36 cm
Horizontal = 36 cm

4. (a) Vertical = 6 units
Horizontal = 5 units
(b) Vertical = 4 units
Horizontal = 5 units
(c) Vertical = 4 units
Horizontal = 2 units
(d) Vertical = 4 units
Horizontal = 4 units
(e) Vertical = 6 units
Horizontal = 1 units
(f) Vertical = 9 units
Horizontal = 3 units

5. (a) $y$ – intercept = 4
$x$ – intercept = 8
(b) $y$ – intercept = −7
$x$ – intercept = 3
(c) $y$ – intercept = 3
$x$ – intercept = −5
(d) $y$ – intercept = –2
$x$ – intercept = −1

6. (a) $AB$ (b) $EF$

7. (a) Negative (b) Positive
(c) Negative (d) Positive

8. (a) 2 (b) 4 (c) $\frac{2}{7}$

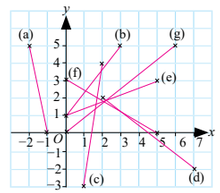
9. (a) 2 (b) $\frac{1}{2}$
(c) $-\frac{5}{6}$ (d) $-\frac{4}{7}$

10. (a) 3 (b) 9
(c) $\frac{5}{3}$ (d) $-\frac{3}{4}$

11. (a) $-\frac{1}{4}$ (b) $-\frac{10}{9}$
(c) $\frac{8}{3}$ (d) −1

### GENERATING EXCELLENCE

1.



(a) Negative (b) Positive
(c) Positive (d) Negative
(e) Positive (f) Negative
(g) Positive

2. $\frac{11}{7}$

3. (c) has the highest gradient

4. (a) −8 (c) −12
(e) 2 (f) −5

5. −3

6. $Q(-3, 4)$

7. $x$ – intercept = 9

8. $\frac{4}{3}$

9. (−6, 0)

10. (a) 0.67 (b) 9.01 m

11. (a) 20 $ms^{-1}$ (b) $v$ = 42 m/s

12. Gradient = $\frac{3}{2}$
Surface area = 14.42 m²

# CHAPTER 11 ISOMETRIC TRANSFORMATIONS

## SELF PRACTICE 11.1

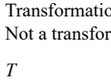
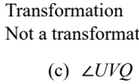
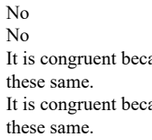
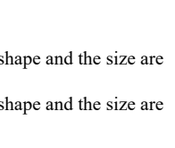
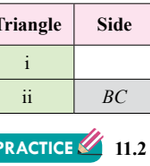
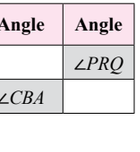
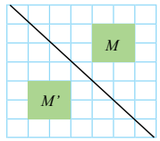
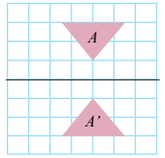
1. (a) Transformation (b) Transformation
   (c) Not a transformation (d) Not a transformation
2. (a) $T$ (b) $QR$ (c) $\angle UVQ$
3. (a) No
   (b) No
   (c) It is congruent because the shape and the size are these same.
   (d) It is congruent because the shape and the size are these same.
4.

| Triangle | Side | Line | Angle | Angle |
|---|---|---|---|---|
| i | | $RQ$ | | $\angle PRQ$ |
| ii | $BC$ | | $\angle CBA$ | |

## SELF PRACTICE 11.2

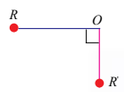
1. (a) Translation (b) No
   (c) Translation (d) No
2. (a) $(7, -1)$ (b) $(9, 3)$
   (c) $(2, -4)$ (d) $(3, -8)$
3. (a) $(-2, -8)$ (b) $(2, -9)$
   (c) $(7, -4)$ (d) $(-8, -6)$
4. (a) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} -6 \\ -8 \end{pmatrix}$
   (c) $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$ (d) $\begin{pmatrix} -3 \\ -7 \end{pmatrix}$
5. (a) $(5, -8)$ (b) $(2, 7)$
   (c) $(3, 3)$ (d) $(-5, -17)$
6. (a) $(1, -7)$ (b) $(7, -8)$

## SELF PRACTICE 11.3

1. (b)  (c) 
2. (a)  (b) 
3. (a)  (b) 
4. $P(-1, 7)$ $Q(-4, -2)$
   $R(5, -3)$ $S(8, 7)$
5. (a)  (b) 
6. (a) Reflection on line $y = 6$
   (b) Reflection on line $x = -2$
   (c) Reflection on line $y = 3$
   (d) Reflection on line $y = 1$
7. (a) Reflection on the $y$ −axis
   (b) Reflection on the $x$ −axis
   (c) Reflection on the $y$ −axis
   (d) Reflection on line $x = 3$
8. (a) $(-3, 7)$ (b) $(7, 4)$

## SELF PRACTICE 11.4

1. (a) 90° clockwise rotation, centre $P$.
   (b) 90° anticlockwise rotation, centre $P$.
   (c) 90° anticlockwise rotation, centre $P$.
   (d) 90° clockwise rotation, centre $P$.
2. (a) 90° clockwise rotation, at point $P$.
   (b) 180° clockwise rotation / anticlockwise rotation at point $P$.
   (c) 180° clockwise rotation / anticlockwise rotation at point $T$.
   (d) 90° rotation, clockwise rotation at the origin.
3. 90° anticlockwise rotation at the origin $O$.




Rotation 180° at centre $O$.


4. Coordinate:
   $P(-4, 0)$ $R(-1, -2)$
   $Q(0, 2)$ $S(-1, 6)$

## SELF PRACTICE 11.5

1. (a) Isometric transformation
   (b) Not an isometric transformation
   (c) Isometric transformation
   (d) Isometric transformation
2. (a) Isometric transformation
   (b) Isometric transformation
   (c) Isometric transformation
3. $A$: Translation
   $B$: Rotation
   $C$: Reflection
4. $K$, $L$ and $M$
5. $x = 100°$





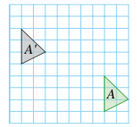
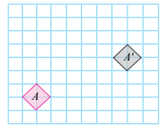
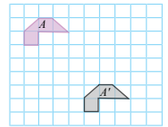
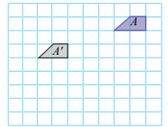
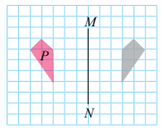
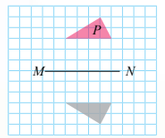
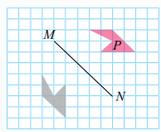
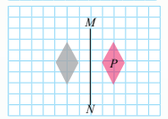
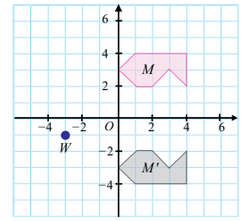
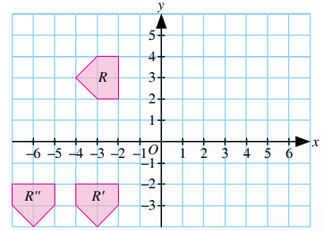
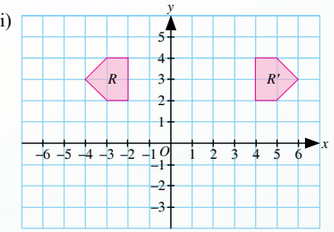
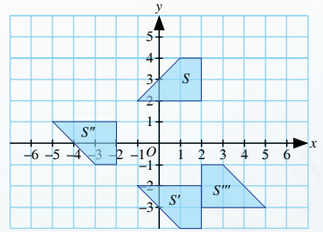
## SELF PRACTICE 11.6

1. (a) ✓ (b) × (c) × (d) ×
2. (a) No order (b) 2nd order
   (c) 4th order (d) 1st order
3.

| Clockwise | Anticlockwise |
|---|---|
| (i) order 2 | order 4 |
| (ii) order 2 | order 4 |
| (iii) order 5 | order 1 |

4. (2,8)

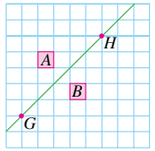
## GENERATING EXCELLENCE

1. (a) $F$ (b) $U$
2. (a) translation because there is a change in position
   (b) not a translation
   (c) translation because there is a change in position
3. (a)  (b) 
   (c)  (d) 
4. (a) $(-2, 0)$ (b) $(1, -3)$
   (c) $(-7, 2)$ (d) $(-5, 2)$
   (e) $(-4, -2)$ (f) $(2, -5)$
5. (a)  (b) 
   (c)  (d) 
6. Coordinate:
   $C'(5, 2)$
   $D'(7, -2)$
   $E(2, -4)$
   $F(3, 1)$
7. Coordinate:
   $K'(3, -1)$
   $L'(1, 3)$
   $M(0, 3)$
   $N(3, -4)$
8. (a) ✓ (b) ✓ (c) × (d) ✓
9. (i) 
   (ii) $W' = (-3, 1)$
10. (a) 90° clockwise rotation, centre $(-1, 1)$.
    (b) 180° anticlockwise rotation, centre $(0, 2)$.
    (c) 180° clockwise rotation, centre $(1, 0)$.
    (d) 90° clockwise rotation, centre $(1, -1)$.
11. (i) $OBC$ (ii) $OCD$ (iii) $ODA$
12. (a) $P \rightarrow Q$ Reflection on the line $x = 4$
    $Q \rightarrow R$ Translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$
    $R \rightarrow S$ 90° clockwise rotation, centre (5, 1)
    (b) (0,4)
13. $A(1, -1)$ → $(1, 1)$ → $(7, -5)$ → $(6, -2)$ → $(8, -4)$ → $(8, 2)$ → $(4, 2)$ → $(6, -2)$
14. (a) (i) 
    (ii) 
    (b) 

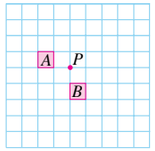
$R$ is a pentagon and $S$ is a square

15. (i) Translation $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

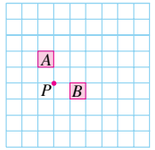
(ii) Reflection on the line GH as shown in the diagram below



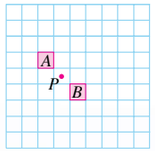

(iii) 90° clockwise rotation, centre *P* as shown in the diagram below



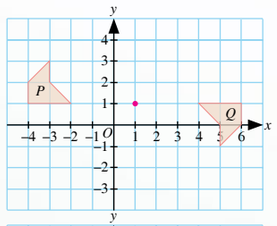

(iv) 90° anticlockwise rotation, centre *P* as shown in the diagram below.



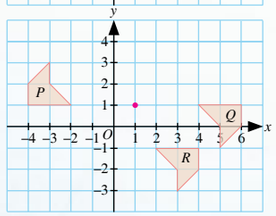

(v) 180° clockwise/ anticlockwise rotation, centre *P* as shown in the diagram below




16. (a)




(b)




(c) 180° rotation, centre origin

17. Zainun will go through a clockwise symmetrical order 3 or a symmetrical order 1 anticlockwise, Fauziah will go through a symmetrical order 1 clockwise or a symmetrical order 3 anticlockwise.

## CHAPTER 12 MEASURES OF CENTRAL TENDENCIES

### SELF PRACTICE 12.1

1. (a) 1 (b) RM7 and RM8
(c) 60
2. L size
3. (a) 0.5 (b) 32 and 37
(c) RM2 (d) Yellow
4. (a) 7 (b) 42.5 (c) 10
5. 30
6. (a) 5 (b) L
7. (a) 7 (b) 3.06
8. (a) 2 (b) 8
9. 1 day
10. (a)

| Age (Year) | Tally | Frequency |
|---|---|---|
| 6 - 10 | / | 1 |
| 11 - 15 | / | 1 |
| 16 - 20 | //// / | 6 |
| 21 - 25 | ~~////~~ | 4 |
| 26 - 30 | //// | 4 |
| 31 - 35 | //// | 4 |

(b)

| Number of ping pong balls | Tally | Frequency |
|---|---|---|
| 10 - 19 | / | 1 |
| 20 - 29 | // | 2 |
| 30 - 39 | ~~////~~ / | 6 |
| 40 - 49 | ~~////~~ /// | 8 |
| 50 - 59 | // | 2 |
| 60 - 69 | / | 1 |

11. (a) mean = 11.3
median = 8.5
mode = 2
(b) median is used because there is an extreme value 40.
12. (a) mean = 7.2
median = 7
mode = 6
(b) mean is used because there are no extreme values.
13. (a) mean is used because there are no extreme values in the data.
(b) median is used because it is numeric data and there are extreme values in the data.
14. (a) Mean or median (b) Mode
(c) Mean





### GENERATING EXCELLENCE

1. 4
2. (a) 70 (b) 12
3. (a) 83 marks (b) 8 packets
4. 20
5. (a) mean = 6.3
median = 6
mode = 6
(b) (i) 8.3, 8, 8 (ii) 12.7, 12, 12
(iii) 4.3, 4, 4 (iv) 3.2, 3, 3
6. 16
7. (a) (i) 284 (ii) 90
(b) 66
8. (a) (i) 32.27 (b) 60%
(ii) 32
(iii) 32
9. (a) (i) 32.17 (b) $\frac{3}{5}$
(ii) 32
(iii) 32
10. (a) $x + 2 + y + 6 + 14 = 40$
$x + y + 22 = 40$
$x + y = 40 - 22$
$x + y = 18$
(b) 6.4
(c) (i) 7 (ii) 10
11. Gold - Yip
Silver - Ravi
Bronze - Malek
12. (a) mean because the mean mark is 85. He has obtained an A in History.
(b) 74 because out of that 5 tests Joshua obtained 74 marks twice.

## CHAPTER 13 SIMPLE PROBABILITY

### SELF PRACTICE 13.1

1. Students answer

### SELF PRACTICE 13.2

1. $\frac{3}{7}$
2. $\frac{1}{3}$
3. $\frac{3}{70}$

### SELF PRACTICE 13.3

1. (a) $A'$ = event of not setting a chocolate cake
(b) $A' = \{ K_1, K_2, K_3, K_4, K_5, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8 \}$
2. $\frac{2}{5}$
3. $\frac{9}{14}$
4. (a) $B'$ = event of Ali not buying a shoe
(b) $B' = \{\ \}$
5. $\frac{9}{10}$

### SELF PRACTICE 13.4

1. 125 forms
2. (a) $\frac{3}{7}$ (b) $\frac{4}{7}$
3. (a) $\frac{8}{15}$ (b) 40 seeds (c) $\frac{40}{83}$

### GENERATING EXCELLENCE

1. (a) $S = \{ P, E, M, B, E, L, A, J, A, R, A, N \}$
(b) $\{ E_1, E_2, A_1, A_2, A_3 \}$
(c) $\frac{7}{12}$
2. (a) 24 cone (b) $\frac{7}{12}$
3. 27 aim
4. (a) $S = \{(a, a), (a, e), (a, i), (e, a), (e, e), (e, i), (i, a), (i, e), (i, i)\}$
(b) $S = \{(a, a), (e, e), (i, i)\}$
(c) $\frac{2}{3}$
5. (a) 1 (b) 1 (c) $\frac{2}{3}$
6. $\frac{1}{5}$

# Glossary

**Acceleration** (*Pecutan*) Increase in speed with time.

**Algebraic expression** (*Sebutan Algebra*) A mathematical expression that consists of variables, numbers and operations.

**Algebraic expression** (*Ungkapan Algebra*) The combination of constant and variables, connected by signs and fundamental operations.

**Algebraic formula** (*Rumus algebra*) A mathematical rule or relationship that uses numbers, letters and algebraic expression in the form of an equation.

**Algebraic fraction** (*Pecahan Algebra*) Fraction with algebraic expression in the numerator and denominator.

**Anticlockwise** (*Lawan arah jam*) In the opposite direction to the movement of the hands of a clock.

**Average speed** (*Laju purata*) Total distance divided by total time taken.

**Axis of symmetry** (*Paksi simetri*) The line that divides an object, shape or diagram into two congruent forms.

**Centre** (*Pusat*) Midpoint in a space.

**Centre of rotation** (*Pusat putaran*) In a rotation, the point that does not move, the rest of the plane rotates around this point.

**Chord** (*Perentas*) A line segment that connects two points on a circumference.

**Circle** (*Bulatan*) A two dimensional shape on a plane that is always the same distance from the centre.

**Circumference** *(Lilitan*) Perimeter of a circle.

**Clockwise** (*Ikut arah jam*) Motion that proceeds in the same direction as a clock's hand.

**Coefficient** (*Pekali*) A multiplicative factor of a term, polynomial or expression.

**Common factor** (*Faktor sepunya*) A factor that divides two or more number exactly.

**Complementary angle** (*Sudut pelengkap*) Either of the two angles whose sum is 90°.

**Congruency** (*Kekongruenan*) The same shape and size.

**Coordinate** (*Koordinat*) A pair of numbers used to indicate position relative to *x*-axis and *y*-axis.

**Cross section** (*Keratan rentas*) Intersection of a solid body in a three-dimensional space with a line or plane.

**Deceleration** (*Nyahpecutan*) Negative acceleration or reduction in speed.

**Denominator** (*Penyebut*) The number that appears on the bottom of a fraction.

**Dependent variable** (*Pemboleh ubah bersandar*) Variable that is the subject in a algebra formula.

**Diameter** (*Diameter*) A straight line going through the centre of circle or sphere that connects two points on the circumference.

**Distance** (*Jarak*) How far apart the length or width of two points.

**Even numbers** (*Nombor genap*) Integer divisible by two.

**Event** (*Peristiwa*) A set of outcomes of an experiment.

**Expansion** (*Kembangan tunggal*) When a linear algebraic expression is multiplied by an algebraic term or a number.

**Expansion of two brackets** (*Kembangan dua kurungan)* When two linear algebraic expression is multiplied.

**Exterior angle** (*Sudut peluaran*) Angle formed by one side of a polygon and a line extended from an adjacent side.

**Extreme value** (*Nilai ekstrem*) Refers to very extreme deviation of values in a sample.

**Factor** (*Faktor*) A number, term or algebraic expression that divides the number, term and algebraic expression given exactly.

**Frequency table** (*Jadual kekerapan*) Tabulated data.

**Function** (*Fungsi*) Relationship between two variables in a equation.



**Geometrical characteristics** (*Sifat geometri*) Relating to geometry or principles of geometry.

**Gradient** (*Kecerunan*) Ratio of the vertical distance to the horizontal distance.

**Graph of function** (*Graf fungsi*) A graph of a certain function.

**Horizontal distance** (*Jarak mengufuk*) The length parallel to the plane.

**Hypotenuse** (*Hipotenus*) The longest side of a right angled triangle, opposite the right angle.

**Image** (*Imej*) Reflection of an object.

**Inclination** (*Kecondongan*) Gradient or slope of a line.

**Independent variable** (*Pemboleh ubah tidak bersandar*) Variable that is not the subject in a algebra formula.

**Intercept** (*Pintasan*) The point where a line or curve crosses the *x*-axis or *y*-axis.

**Interior angle** (*Sudut pedalaman*) The angle inside a shape.

**Isometry** (*Isometri*) Transformation of an object into an image that is congruent.

**Linear equation** (*Persamaan linear*) An algebraic equation in which each term has an exponent of one.

**Linear function** (*Fungsi linear*) A function that has variables with exponents equal to one.

**Mean** (*Min*) In a data set, the sum of all the data points, divided by the number of data points.

**Measure of central tendency** (*Sukatan kecenderungan memusat*) A single value that describes a set of data such as mean, median and the mode.

**Median** (*Median*) Middle number in a set listed from the least to the greatest.

**Midpoint** (*Titik tengah*) A point on a line segment that divides it into two equal parts.

**Mode** (*Mod*) Most frequently occuring number(s) in a set.

**Net** (*Bentangan*) A net is a two-dimensional figure that can be folded into a three-dimensional object.

**Non-linear function** (*Fungsi bukan linear*) A function that has variables with exponents greater than one.

**Non-uniform speed** (*Laju tak seragam*) Different distance covered in equal intervals of time.

**Number pattern** (*Pola nombor*) A list of numbers that follow a certain sequence or pattern.

**Numerator** (*Pengangka*) The number that appears on the top of a fraction.

**Object** (*Objek*) Shape, diagram before transformation.

**Odd numbers** (*Nombor ganjil*) Integers not divisible by two.

**Origin** (*Asalan*) The point where the *x*-axis and *y*-axis intersect (0,0).

**Perfect square** (*Kuasa dua sempurna*) An integer that is the square of an integer. Example $1^2 = 1$, $2^2 = 4$, $3^3 = 9$. Thus, 1, 4 and 9 are perfect squares.

**Plot** (*Plot*) To mark the coordinates, to draw the graph.

**Position** (*Kedudukan*) Spatial location.

**Probability of an event** (*Kebarangkalian suatu peristiwa*) The ratio of the chance that an event will occur to total number of outcomes.

**Radius** (*Jejari*) The line segment from the centre to its perimeter of a circle or sphere.

**Ratio** (*Nisbah*) A comparison of two quantities that can also be in the form of fraction.

**Reflection** (*Pantulan*) Flip over a line.

**Regular polygon** (*Poligon sekata*) Polygon that is equilateral and equiangular.

**Relation** (*Hubungan*) Relationship between two or more variables.

**Sample space** (*Ruang sampel*) The set of all possible outcomes of an experiment.

**Sector** (*Sektor*) A sector is the part of a circle enclosed by two radii of a circle and their intercepted arc.

**Segment** (*Tembereng*) The part of a circle made by a line and a connecting arc.

**Sequence** (*Jujukan*) A list of numbers in a special order.

**Side** (*Sisi lurus*) A line segment to form a polygon.

**Speed** (*Laju*) Rate of change of distance with time.

**Stationary** (*Keadaan pegun*) Not moving.

**Steepness** (*Kecuraman*) Gradient or slope of a line.

**Subject of a formula** (*Perkara rumus*) The single variable to which every variable in the formula is equal.

**Supplementary angle** (*Sudut penggenap*) Two angles that add up to 180°.

**Symmetry** (*Simetri*) The correspondence in size, form and arrangement of parts on the opposite sides of a plane, line or point.

**Three-dimensional shape** (*Bentuk tiga dimensi*) Shape that has three dimensions such as length, width and height.

**Transformation** (*Transformasi*) The process of changing position, orientation or size of an object by translation, reflection, rotation and dilation.

**Translation** (*Translasi*) Every point of the object must be moved in the same direction and same distance.

**Two-dimensional shape** (*Bentuk dua dimensi*) Shape that has two dimensions such as length and width.

**Uniform speed** (*Laju seragam*) Same distance covered in equal intervals of time.

**Value of table** (*Jadual nila*i) A table of dependent and independent variables of a function.

**Variable** (*Pemboleh ubah*) Liable to change or take different values.

**Vector** (*Vektor*) A quantity having direction as well as magnitude.

**Vertical distance** (*Jarak mencancang*) The length of the height.



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