KEMENTERIAN
PENDIDIKAN
MALAYSIA

DUAL LANGUAGE PROGRAMME

MATHEMATICS
FORM 3

KSSM

RUKUN NEGARA

Bahawasanya Negara Kita Malaysia
mendukung cita-cita hendak;

Mencapai perpaduan yang lebih erat dalam kalangan
seluruh masyarakatnya;

Memelihara satu cara hidup demokrasi;

Mencipta satu masyarakat yang adil di mana kemakmuran negara
akan dapat dinikmati bersama secara adil dan saksama;

Menjamin satu cara yang liberal terhadap
tradisi-tradisi kebudayaannya yang kaya dan pelbagai corak;

Membina satu masyarakat progresif yang akan menggunakan
sains dan teknologi moden;

MAKA KAMI, rakyat Malaysia,
berikrar akan menumpukan
seluruh tenaga dan usaha kami untuk mencapai cita-cita tersebut
berdasarkan prinsip-prinsip yang berikut:

KEPERCAYAAN KEPADA TUHAN
KESETIAAN KEPADA RAJA DAN NEGARA
KELUHURAN DAN PERLEMBAGAAN
KEDAULATAN UNDANG-UNDANG
KESOPANAN DAN KESUSILAAN

(Sumber : Jabatan Penerangan, Kementerian Komunikasi dan Multimedia Malaysia)

KURIKULUM STANDARD SEKOLAH MENENGAH

MATHEMATICS
FORM 3

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Contents

Introduction

This Form 3 Mathematics Textbook is prepared based on *Kurikulum Standard Sekolah Menengah (KSSM)*. This book contains 9 chapters arranged systematically based on Form 3 Mathematics *Dokumen Standard Kurikulum dan Pentaksiran (DSKP)*.

At the beginning of each chapter, pupils are introduced to materials related to daily life to stimulate their thinking about the content. The Learning Standard and word lists are included to provide a visual summary of the chapter's content.

Special features of this book are:

	Description
What will you learn?	Contains learning standards that pupils need to achieve in each chapter.
Why do you learn this chapter?	Applications of knowledge in related career fields.
Exploring Era	Historical background or origin of the content.
WORD BANK	Word list contained in each chapter.
Brainstorming — Individual, In pairs, In groups	Helps pupils to understand the basic mathematical concept via individual, pair or group activities.
BULLETIN	Provides additional information about the content learned.
QUIZ	Questions that test pupils' ability to understand basic concepts in each chapter.
REMINDER	Additional facts that pupils need to be reminded of and common mistakes to be avoided.
TIPS	Exposes pupils to additional knowledge that they need to know.
SMART MIND	Challenging tasks for enhancement of critical and creative thinking skills.

	Description
SMART TECHNOLOGY	Exposes pupils to the use of technology in mathematics.
DISCUSSION CORNER	Develops pupils' mathematical communication skills.
FLASHBACK	Helps pupils to recall what they have learnt.
SMART FINGER	Shows how to use scientific calculators.
PROJECT	Enables pupils to carry out and present project work.
MIND TEST	Assesses pupils' understanding on the concepts they have learnt.
	Questions to enhance pupils' higher order thinking skills.
Dynamic Challenge	Provides diversified tasks which incorporate the elements of LOTS, HOTS, TIMSS and PISA.
	Enables pupils to scan a QR Code using a mobile device to access further information.
EXPLORING MATHEMATICS	Covers applicable concepts of digital tool calculators, hands on activities and games that aim to effectively enhance pupils' understanding.
CONCEPT MAP	Overall chapter summary.
SELF-REFLECT	Pupils self-assess their achievement.
Checking Answers	Checking answers using alternative methods.
STEM	Activities with elements of Science, Technology, Engineering and Mathematics.

Symbols and Formulae

SYMBOLS

$\sqrt{\ }$	root
π	pi
$a : b$	ratio of a to b
$A \times 10^n$	standard form where $1 \leqslant A < 10$ and n is an integer
$=$	is equal to
$\approx$	is approximately equal to
$\neq$	is not equal to
$>$	is more than
$\geqslant$	is more than or equal to
$<$	is less than
$\leqslant$	is less than or equal to
Δ	triangle
$\angle$	angle
$^\circ$	degree
$'$	minute
$''$	second

FORMULAE

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$I = Prt$

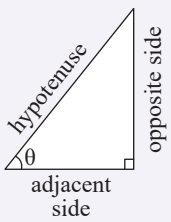
$MV = P(1 + \frac{r}{n})^{nt}$

$A = P + Prt$

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

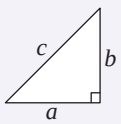
$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$



$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Pythagoras theorem:



$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$a^2 = c^2 - b^2$

Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Gradient, $m = \frac{\text{vertical distance}}{\text{horizontal distance}}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = -\frac{y\text{-intercept}}{x\text{-intercept}}$

http://bukutekskssm.my/Mathematics/F3/Index.html

Download a free QR Code scanner application to your mobile device. Scan QR Code or visit the website http://bukutekskssm.my/Mathematics/F3/Index.html to download files for brainstorming. Then, save the downloaded file for offline use.

Note: Students can download the free GeoGebra and Geometer's Sketchpad (GSP) software to open related files.

CHAPTER 1

Indices

What will you learn?

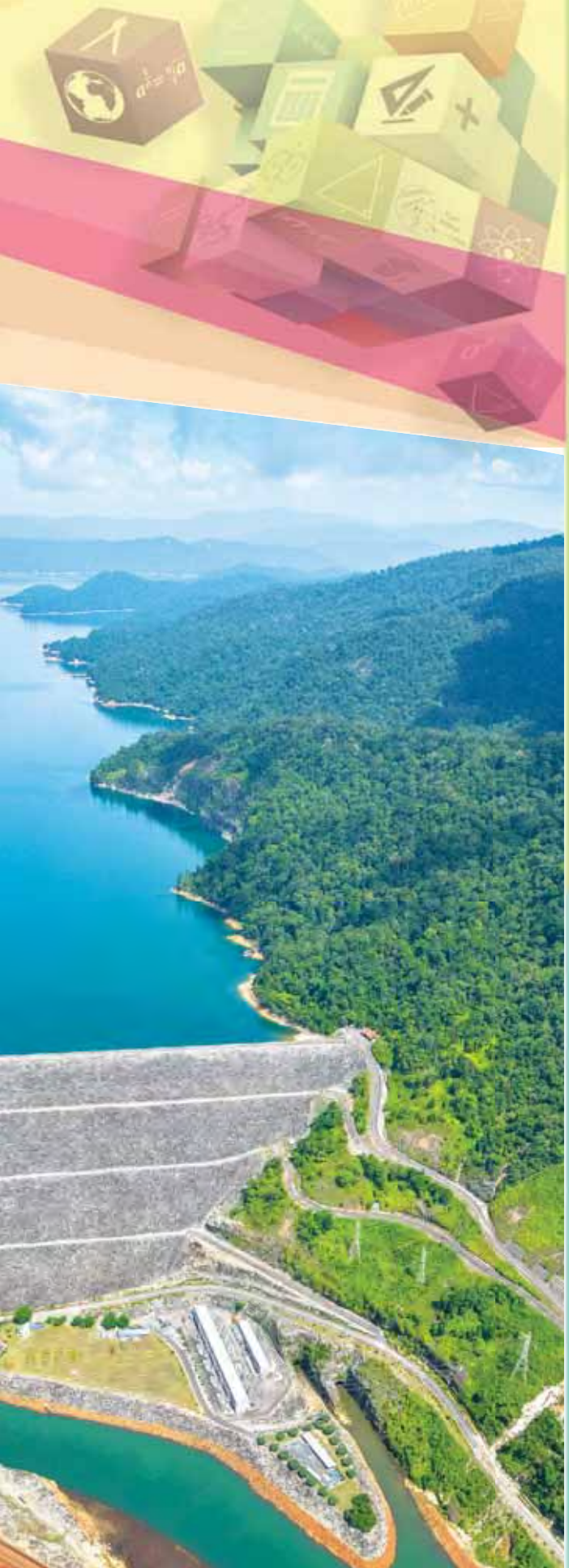
1.1 Index Notation

1.2 Law of Indices

Why do you learn this chapter?

- Writing a number in index notation enables the number stated in a simple and easily understood form. Various operations of mathematics that involve numbers in index notation can be performed by using laws of indices.
- Concept of index is used in the fields of science, engineering, accounting, finance, astronomy, computer and so on.

Kenyir Lake, located in the district of Hulu Terengganu, in Terengganu, is the biggest man-made lake in Southeast Asia. Kenyir Lake is a world famous tourist destination known for its unique natural beauty. Kenyir Lake is an important water catchment area. Kenyir Lake, which was built in the year 1985, supplies water to Sultan Mahmud Power Station. The estimated catchment area at the main dam is 2 600 km^2 with a reservoir volume of 13 600 million cubic metres. During rainy season, the volume of water in the catchment area will increase sharply. What action should be taken to address this situation?



Exploring Era

Index notation is an important element in the development of mathematics and computer programming. The use of positive indices was introduced by Rene Descartes (1637), a well-known French mathematician. Sir Isaac Newton, a well-known British mathematician, developed the field of index notation and introduced negative indices and fractional indices.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter1.pdf

WORD BANK

• base	• *asas*
• factor	• *faktor*
• index	• *indeks*
• fractional index	• *indeks pecahan*
• power	• *kuasa*
• root	• *punca kuasa*
• index notation	• *tatatanda indeks*

1.1 Index Notation

What is repeated multiplication in index form?

LEARNING STANDARD

Represent repeated multiplication in index form and describe its meaning.

The development of technology not only makes most of our daily tasks easier, it also saves expenses in various fields. For instance, the use of memory cards in digital cameras enable users to store photographs in a large number and to delete or edit unsuitable photographs before printing.

DISCUSSION CORNER

Discuss the value of the capacity of a pen drive.

BULLETIN

The nuclear fission of uranium U-320 follows the pattern 3^0, 3^1, 3^2, …

In the early stage, memory cards were made with a capacity of 4MB. The capacity increases over time to meet the demands of users. Do you know that the capacity of memory cards is calculated using a special form that is 2^n?

In Form 1, you have learnt that $4^3 = 4 \times 4 \times 4$. The number 4^3 is written in index notation, 4 is the **base** and 3 is the **index** or **exponent**. The number is read as '4 to the power of 3'.

Hence, a number in index notation or in index form can be written as;

a^n ← Index
Base

You have also learnt that $4^2 = 4 \times 4$ and $4^3 = 4 \times 4 \times 4$. For example;

$\underbrace{4 \times 4}_{\text{Repeated two times}} = 4^{2}$ → The value of index is 2

Repeated two times → The value of index is the same as the number of times 4 is multiplied repeatedly.

$\underbrace{4 \times 4 \times 4}_{\text{Repeated three times}} = 4^{3}$ → The value of index is 3

Repeated three times → The value of index is the same as the number of times 4 is multiplied repeatedly.

Example 1

Write the following repeated multiplications in index form a^n.

(a) $5 \times 5 \times 5 \times 5 \times 5 \times 5$

(b) $0.3 \times 0.3 \times 0.3 \times 0.3$

(c) $(-2) \times (-2) \times (-2)$

(d) $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

(e) $m \times m \times m \times m \times m \times m \times m$

(f) $n \times n \times n \times n \times n \times n \times n \times n$

REMINDER

$2^5 \neq 2 \times 5$ $\quad 4^3 \neq 4 \times 3$

$a^n \neq a \times n$

Solution:

(a) $\underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}_{\text{repeated six times}} = 5^6$

(b) $\underbrace{0.3 \times 0.3 \times 0.3 \times 0.3}_{\text{repeated four times}} = (0.3)^4$

(c) $\underbrace{(-2) \times (-2) \times (-2)}_{\text{repeated three times}} = (-2)^3$

(d) $\underbrace{\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}}_{\text{repeated five times}} = \left(\frac{1}{4}\right)^5$

(e) $\underbrace{m \times m \times m \times m \times m \times m \times m}_{\text{repeated seven times}} = m^7$

(f) $\underbrace{n \times n \times n \times n \times n \times n \times n \times n}_{\text{repeated eight times}} = n^8$

From the solution in Example 1, it is found that the value of index in an index form is the same as the number of times the base is multiplied repeatedly. In general,

$$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ factors}} \; ; \; a \neq 0$$

MIND TEST 1.1a

1. Complete the following table with base or index for the given numbers or algebraic terms.

5^3 $\quad (-4)^7$ $\quad \left(\frac{1}{2}\right)^{10}$ $\quad m^6$ $\quad \left(-\frac{3}{7}\right)^4$ $\quad n^0$ $\quad (0.2)^9$ $\quad x^{20}$ $\quad \left(2\frac{1}{3}\right)^2$ $\quad 8$

Base	Index
5	
	7
$\frac{1}{2}$	
	6
n	
	9
	4
x	
	2
8	

2. State the following repeated multiplications in index form a^n.

(a) $6 \times 6 \times 6 \times 6 \times 6 \times 6$

(b) $0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5$

(c) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

(d) $(-m) \times (-m) \times (-m) \times (-m) \times (-m)$

(e) $1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{3}$

(f) $\left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right)$

3. Convert the numbers or algebraic terms in index form into repeated multiplications.

(a) $(-3)^3$ (b) $(2.5)^4$ (c) $\left(\frac{2}{3}\right)^5$ (d) $\left(-2\frac{1}{4}\right)^3$

(e) k^6 (f) $(-p)^7$ (g) $\left(\frac{1}{m}\right)^8$ (h) $(3n)^5$

How do you convert a number into a number in index form?

LEARNING STANDARD

Rewrite a number in index form and vice versa.

A number can be written in index form if a suitable base is selected. You can use repeated division method or repeated multiplication method to convert a number into a number in index form.

FLASHBACK

$4 \times 4 \times 4 = 4^3$

Example 2

Write 64 in index form using base of 2, base of 4 and base of 8.

Solution:

Repeated Division Method

(a) Base of 2

• 64 is divided repeatedly by 2.

$n = 6$

2)	64
2)	32
2)	16
2)	8
2)	4
2)	2
	1

The division is continued until 1 is obtained.

Hence, $64 = 2^6$

(b) Base of 4

• 64 is divided repeatedly by 4.

$n = 3$

4)	64
4)	16
4)	4
	1

Hence, $64 = 4^3$

(c) Base of 8

• 64 is divided repeatedly by 8.

$n = 2$

8)	64
8)	8
	1

Hence, $64 = 8^2$

Repeated Multiplication Method

(a) Base of 2

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

4

8

16

32

64

Hence, $64 = 2^6$

(b) Base of 4

$4 \times 4 \times 4$

16

64

Hence, $64 = 4^3$

(c) Base of 8

$8 \times 8 = 64$

Hence, $64 = 8^2$

DISCUSSION CORNER

Which method is easier to convert a number into a number in index form? Is it the repeated division or repeated multiplication method? Discuss.

Example 3

Write $\frac{32}{3\,125}$ in index form using base of $\frac{2}{5}$.

Solution:

Repeated Division Method

$n = 5$

2) 32	
2) 16	
2) 8	
2) 4	
2) 2	
1	

$n = 5$

5) 3 125	
5) 625	
5) 125	
5) 25	
5) 5	
1	

Hence, $\frac{32}{3\,125} = \left(\frac{2}{5}\right)^5$

Repeated Multiplication Method

$\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$

$\frac{4}{25}$

$\frac{8}{125}$

$\frac{16}{625}$

$\frac{32}{3\,125}$

Hence, $\frac{32}{3\,125} = \left(\frac{2}{5}\right)^5$

MIND TEST 1.1b

1. Write each of the following numbers in index form using the stated base in brackets.

(a) 81 [base of 3] (b) 15 625 [base of 5] (c) $\frac{64}{125}$ $\left[\text{base of } \frac{4}{5}\right]$

(d) 0.00032 [base of 0.2] (e) – 16 384 [base of (– 4)] (f) $\frac{1}{16}$ $\left[\text{base of } \left(-\frac{1}{4}\right)\right]$

How do you determine the value of the number in index form , a^n?

The value of a^n can be determined by repeated multiplication method or using a scientific calculator.

Example 4

Calculate the values of the given numbers in index form.

(a) 2^5

$2 \times 2 \times 2 \times 2 \times 2$

4×2

8×2

16×2

32

Hence, $2^5 = 32$

(b) $(0.6)^3$

$0.6 \times 0.6 \times 0.6$

0.36×0.6

0.216

$0.6^3 = 0.216$

Hence, $0.6^3 = 0.216$

QUIZ

$(m)^4 = 16$
What are the possible values of m?

Example 5

SMART FINGER

(a) $5^4 = 625$ → 5 ^ 4 =

(b) $(-7)^3 = -343$ → ((–) 7) ^ 3 =

(c) $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$ → (2 ab/c 3) ^ 4 =

(d) $\left(1\frac{3}{5}\right)^2 = \frac{64}{25}$ → (1 ab/c 3 ab/c 5) ^ 2 =

(e) $(-0.5)^6 = 0.015625$ → ((–) 0 . 5) ^ 6 =

REMINDER

Negative or fractional base must be placed within brackets when using a calculator to calculate values of given numbers.

DISCUSSION CORNER

Calculate questions (c), (d) and (e) in Example 5 without using brackets. Are the answers the same? Discuss.

MIND TEST 1.1c

1. Calculate the value of each of the following numbers in index form.

(a) 9^4 (b) $(-4)^5$ (c) $(2.5)^3$ (d) $(-3.2)^3$

(e) $\left(\frac{3}{8}\right)^5$ (f) $\left(-\frac{1}{6}\right)^4$ (g) $\left(1\frac{2}{3}\right)^2$ (h) $\left(-2\frac{1}{3}\right)^3$

1.2 Law of Indices

What is the relationship between multiplication of numbers in index form with the same base and repeated multiplication?

LEARNING STANDARD

Relate the multiplication of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.

Brainstorming 1 In pairs

Aim: To identify the relationship between multiplication of numbers in index form with the same base and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Exhibit three examples in the mathematics corner for other groups to give feedback.

Multiplication of numbers in index form	Repeated multiplication
(a) $2^3 \times 2^4$	3 factors, 4 factors, 7 factors (overall) $(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$ $2^3 \times 2^4 = 2^{\square}$ (7 = 3 + 4) $2^3 \times 2^4 = 2^{3+4}$
(b) $3^2 \times 3^3$	2 factors, 3 factors, 5 factors (overall) $(3 \times 3) \times (3 \times 3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$ $3^2 \times 3^3 = 3^{\square}$ $3^2 \times 3^3 = 3^{\square}$

Multiplication of numbers in index form	Repeated multiplication
(c) $5^4 \times 5^2$	$\overbrace{(5 \times 5 \times 5 \times 5)}^{4 \text{ factors}} \times \overbrace{(5 \times 5)}^{2 \text{ factors}} = \overbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}^{6 \text{ factors (overall)}} = 5^6$ $5^4 \times 5^2 = 5^{\square}$ $5^4 \times 5^2 = 5^{\square\square}$

Discussion:

What is your conclusion regarding the relationship between multiplication of numbers in index form and repeated multiplication?

From Brainstorming 1, it is found that:

$2^3 \times 2^4 = 2^{3+4}$
$3^2 \times 3^3 = 3^{2+3}$
$5^4 \times 5^2 = 5^{4+2}$

In general, $a^m \times a^n = a^{m+n}$

DISCUSSION CORNER

Given,
$a^m \times a^n = b^m \times b^n$.
Is $a = b$? Discuss.

Example 6

Simplify each of the following.

(a) $7^2 \times 7^3$ (b) $(0.2)^2 \times (0.2)^4 \times (0.2)^5$ (c) $2k^2 \times 4k^3$ (d) $3m^4 \times \frac{1}{6}m^5 \times 12m$

Solution:

(a) $7^2 \times 7^3$
$= 7^{2+3}$
$= 7^5$

(b) $(0.2)^2 \times (0.2)^4 \times (0.2)^5$
$= (0.2)^{2+4+5}$
$= (0.2)^{11}$

REMINDER

$a = a^1$

(c) $2k^2 \times 4k^3$
$= (2 \times 4)(k^2 \times k^3)$
Operation of the coefficients
$= 8k^{2+3}$
$= 8k^5$

(d) $3m^4 \times \frac{1}{6}m^5 \times 12m$
$= (3 \times \frac{1}{6} \times 12)(m^4 \times m^5 \times m^1)$
$= 6m^{4+5+1}$
$= 6m^{10}$

SMART MIND

If $m^a \times m^b = m^8$, such that $a > 0$ and $b > 0$, what are the possible values of a and b?

MIND TEST 1.2a

1. Simplify each of the following.

(a) $3^2 \times 3 \times 3^4$ (b) $(-0.4)^4 \times (-0.4)^3 \times (-0.4)$

(c) $\left(\frac{4}{7}\right) \times \left(\frac{4}{7}\right)^3 \times \left(\frac{4}{7}\right)^5$ (d) $\left(-1\frac{2}{5}\right)^2 \times \left(-1\frac{2}{5}\right)^3 \times \left(-1\frac{2}{5}\right)^5$

(e) $4m^2 \times \frac{1}{2}m^3 \times (-3)m^4$ (f) $n^6 \times \frac{4}{25}n^2 \times \frac{5}{4}n^3 \times n$

(g) $-x^4 \times \frac{25}{4}x \times \frac{12}{5}x^2$ (h) $-\frac{1}{2}y^5 \times (-6)y^3 \times \frac{1}{3}y^4$

How do you simplify a number or an algebraic term in index form with different bases?

TIPS

Group the numbers or algebraic terms with the same base first. Then add the indices for the terms with the same base.

Example 7

Simplify each of the following.

(a) $m^3 \times n^2 \times m^4 \times n^5$

(b) $(0.3)^2 \times (0.2)^2 \times 0.3 \times (0.2)^5 \times (0.3)^3$

(c) $p^2 \times m^3 \times p^4 \times n^3 \times m^4 \times n^2$

(d) $-m^4 \times 2n^5 \times 3m \times \frac{1}{4}n^2$

Solution:

(a) $m^3 \times n^2 \times m^4 \times n^5$

$= m^3 \times m^4 \times n^2 \times n^5$ ← Group the terms with the same base.

$= m^{3+4} \times n^{2+5}$

$= m^7 \times n^7$ ← Add the indices for terms with the same base.

$= m^7 n^7$

(b) $(0.3)^2 \times (0.2)^2 \times 0.3 \times (0.2)^5 \times (0.3)^3$

$= (0.3)^2 \times (0.3)^1 \times (0.3)^3 \times (0.2)^2 \times (0.2)^5$

$= (0.3)^{(2+1+3)} \times (0.2)^{(2+5)}$

$= (0.3)^6 \times (0.2)^7$

(c) $p^2 \times m^3 \times p^4 \times n^3 \times m^4 \times n^2$

$= m^3 \times m^4 \times n^3 \times n^2 \times p^2 \times p^4$

$= m^{3+4} \times n^{3+2} \times p^{2+4}$

$= m^7 n^5 p^6$

(d) $-m^4 \times 2n^5 \times 3m \times \frac{1}{4}n^2$

$= (-1 \times 2 \times 3 \times \frac{1}{4})\, m^4 \times m^1 \times n^5 \times n^2$

$= -\frac{3}{2}m^{4+1} n^{5+2}$

$= -\frac{3}{2}m^5 n^7$

REMINDER

$-a^n \neq (-a)^n$

Example:
$-3^2 \neq (-3)^2$
$-9 \neq 9$

MIND TEST 1.2b

1. State in the simplest index form.

(a) $5^4 \times 9^3 \times 5 \times 9^2$

(b) $(0.4)^2 \times (1.2)^3 \times (0.4) \times (1.2)^5 \times (1.2)$

(c) $12x^5 \times y^3 \times \frac{1}{2}x \times \frac{2}{3}y^4$

(d) $-2k^5 \times p^6 \times \frac{1}{4}p^5 \times 3k$

What is the relationship between division of numbers in index form with the same base and repeated multiplication?

LEARNING STANDARD

Relate the division of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.

Brainstorming 2

In pairs

Aim: To identify the relationship between division of numbers in index form with the same base and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Present your findings.

Division of numbers in index form	Repeated multiplication
(a) $4^5 \div 4^2$	$\frac{4^5}{4^2} = \frac{\overbrace{4 \times 4 \times 4 \times 4 \times 4}^{5 \text{ factors}}}{\underbrace{4 \times 4}_{2 \text{ factors}}} = \underbrace{4 \times 4 \times 4}_{3 \text{ factors (Remainder)}} = 4^3$ $4^5 \div 4^2 = 4^{\square}$ (3 = 5 – 2) $4^5 \div 4^2 = 4^{5-2}$
(b) $2^6 \div 2^2$	$\frac{2^6}{2^2} = \frac{\overbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2}^{6 \text{ factors}}}{\underbrace{2 \times 2}_{2 \text{ factors}}} = \underbrace{2 \times 2 \times 2 \times 2}_{4 \text{ factors (Remainder)}} = 2^4$ $2^6 \div 2^2 = 2^{\square}$ $2^6 \div 2^2 = 2^{\square}$
(c) $(-3)^5 \div (-3)^3$	$\frac{(-3)^5}{(-3)^3} = \frac{\overbrace{(-3) \times (-3) \times (-3) \times (-3) \times (-3)}^{5 \text{ factors}}}{\underbrace{(-3) \times (-3) \times (-3)}_{3 \text{ factors}}} = \underbrace{(-3) \times (-3)}_{2 \text{ factors (Remainder)}} = (-3)^2$ $(-3)^5 \div (-3)^3 = (-3)^{\square}$ $(-3)^5 \div (-3)^3 = (-3)^{\square}$

Discussion:

What is the relationship between division of numbers in index form and repeated multiplication?

From Brainstorming 2, it is found that:

$4^5 \div 4^2 = 4^{5-2}$
$2^6 \div 2^2 = 2^{6-2}$
$(-3)^5 \div (-3)^3 = (-3)^{5-3}$

In general, $a^m \div a^n = a^{m-n}$

SMART MIND

Given $m^{a-b} = m^7$ and $0 \leqslant a \leqslant 10$. If $a > b$, state the possible values of a and b.

Example 8

Simplify each of the following.

(a) $5^4 \div 5^2$ (b) $(-3)^4 \div (-3)^2 \div (-3)$ (c) $m^4n^3 \div m^2n$

(d) $25x^2y^3 \div 5xy$ (e) $12m^{10} \div 4m^5 \div m^2$ (f) $-16p^8 \div 2p^5 \div 4p^2$

Solution:

(a) $5^4 \div 5^2$
$= 5^{4-2}$
$= 5^2$

(b) $(-3)^4 \div (-3)^2 \div (-3)$
$= (-3)^4 \div (-3)^2 \div (-3)^1$
$= (-3)^{4-2-1}$
$= (-3)^1$
$= -3$

(c) $m^4n^3 \div m^2n$
$= m^4n^3 \div m^2n^1$
$= m^{4-2}n^{3-1}$
$= m^2n^2$

(d) $25x^2y^3 \div 5xy$

$= 25x^2y^3 \div 5x^1y^1$

$= \frac{25}{5} x^{2-1} y^{3-1}$ ← Operation of the coefficients

$= 5x^1y^2$

$= 5xy^2$

(e) $12m^{10} \div 4m^5 \div m^2$

$= \frac{12}{4} (m^{10} \div m^5 \div m^2)$

$= 3(m^{10-5}) \div m^2$

$= 3m^{5-2}$

$= 3m^3$

(f) $-16p^8 \div 2p^5 \div 4p^2$

$= \frac{-16}{2} (p^8 \div p^5) \div 4p^2$

$= -8p^{8-5} \div 4p^2$

$= -8p^3 \div 4p^2$

$= -\frac{8}{4} (p^3 \div p^2)$

$= -2p^{3-2}$

$= -2p^1$

$= -2p$

MIND TEST 1.2c

1. Simplify each of the following.

(a) $4^5 \div 4^4$ (b) $7^{10} \div 7^6 \div 7^2$ (c) $\frac{m^8n^6}{m^4n}$

(d) $\frac{27x^4y^5}{9x^3y^2}$ (e) $m^7 \div m^2 \div m^4$ (f) $-25h^4 \div 5h^2 \div h$

2. Copy and complete each of the following equations.

(a) $8^{\square} \div 8^4 \div 8^3 = 8$ (b) $m^4n^{\square} \div m^{\square} n^5 = m^2n$

(c) $\frac{m^{10} n^4 \times m^{\square} n^2}{m^7n} = m^5n^{\square}$ (d) $\frac{27x^3y^6 \times xy^{\square}}{\square x^2y^3} = 3x^{\square} y^5$

3. If $\frac{2^x \times 3^y}{2^4 \times 3^2} = 6$, determine the value of $x + y$.

What is the relationship between a number in index form raised to a power and repeated multiplication?

LEARNING STANDARD

Relate the numbers in index form raised to a power, to repeated multiplication, and hence make generalisation.

Brainstorming 3

In pairs

Aim: To identify the relationship between a number in index form raised to a power and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Present your findings.

Index form raised to a power	Repeated multiplication in index form	Conclusion
(a) $(3^2)^4$	$\overbrace{3^2 \times 3^2 \times 3^2 \times 3^2}^{4 \text{ factors}}$ $= 3^{\overbrace{2+2+2+2}^{4 \text{ times}}}$ ← 2 is added 4 times $= 3^{2(4)}$	$(3^2)^4 = 3^{2(4)}$ $= 3^8$

Index form raised to a power	Repeated multiplication in index form	Conclusion
(b) $(5^4)^3$	3 factors $5^4 \times 5^4 \times 5^4$ $= 5^{4+4+4}$ (3 times) $= 5^{4(3)}$ 4 is added 3 times	$(5^4)^3 = 5^{\square}$ $= 5^{\square}$
(c) $(4^3)^6$	6 factors $4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3$ $= 4^{3+3+3+3+3+3}$ (6 times) $= 4^{3(6)}$ 3 is added 6 times	$(4^3)^6 = 4^{\square}$ $= 4^{\square}$

Discussion:

What is your conclusion regarding the index form raised to a power and repeated multiplication in index form?

The conclusion in Brainstorming 3 can be checked using the following method.

Example (a)

$(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$
$= 3^{2+2+2+2}$
$= 3^8$
$3^{2(4)} = 3^{2 \times 4}$
$= 3^8$

Example (b)

$(5^4)^3 = 5^4 \times 5^4 \times 5^4$
$= 5^{4+4+4}$
$= 5^{12}$
$5^{4(3)} = 5^{4 \times 3}$
$= 5^{12}$

Example (c)

$(4^3)^6 = 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3$
$= 4^{3+3+3+3+3+3}$
$= 4^{18}$
$4^{3(6)} = 4^{3 \times 6}$
$= 4^{18}$

From Brainstorming 3, it is found that:

$(3^2)^4 = 3^{2(4)}$
$(5^4)^3 = 5^{4(3)}$
$(4^3)^6 = 4^{3(6)}$

In general, $(a^m)^n = a^{mn}$

SMART MIND

Given,
$m^{rt} = 3^{12}$
What are the possible values of m, r and t if $r > t$?

Example 9

1. Simplify each of the following.

(a) $(3^4)^2$ (b) $(h^3)^{10}$ (c) $((-y)^6)^3$

2. Determine whether the following equations are **true** or **false**.

(a) $(4^2)^3 = (4^3)^2$ (b) $(2^3)^4 = (2^2)^6$ (c) $(3^2)^6 = (27^2)^4$

Solution:

1. (a) $(3^4)^2$
$= 3^{4(2)}$
$= 3^8$

(b) $(h^3)^{10}$
$= h^{3(10)}$
$= h^{30}$

(c) $((-y)^6)^3$
$= (-y)^{6(3)}$
$= (-y)^{18}$

2. (a) $\underbrace{(4^2)^3}_{\text{left}} = \underbrace{(4^3)^2}_{\text{right}}$

Left:
$(4^2)^3 = 4^{2(3)} = 4^6$

Right:
$(4^3)^2 = 4^{3(2)} = 4^6$

Same

Hence, $(4^2)^3 = (4^3)^2$ is **true**.

(b) $\underbrace{(2^3)^4}_{\text{left}} = \underbrace{(2^2)^6}_{\text{right}}$

Left:
$(2^3)^4 = 2^{3(4)} = 2^{12}$

Right:
$(2^2)^6 = 2^{2(6)} = 2^{12}$

Same

Hence, $(2^3)^4 = (2^2)^6$ is **true**.

(c) $\underbrace{(3^2)^6}_{\text{left}} = \underbrace{(27^2)^4}_{\text{right}}$

Left:
$(3^2)^6 = 3^{2(6)} = 3^{12}$

Right:
$(27^2)^4 = (3^{3(2)})^4$
$= 3^{6(4)}$
$= 3^{24}$

Not the same

Hence, $(3^2)^6 = (27^2)^4$ is **false**.

MIND TEST 1.2d

1. Use law of indices to simplify each of the following statements.

(a) $(12^5)^2$ (b) $(3^{10})^2$ (c) $(7^2)^3$ (d) $((-4)^3)^7$

(e) $(k^8)^3$ (f) $(g^2)^{13}$ (g) $((-m)^4)^3$ (h) $((-c)^7)^3$

2. Determine whether the following equations are **true** or **false**.

(a) $(2^4)^5 = (2^2)^{10}$ (b) $(3^3)^7 = (27^2)^4$ (c) $(5^2)^5 = (125^2)^3$ (d) $-(7^2)^4 = (-49^2)^3$

How do you use law of indices to perform operations of multiplication and division?

$(a^m \times b^n)^q$
$= (a^m)^q \times (b^n)^q$
$= a^{mq} \times b^{nq}$

$(a^m b^n)^q = a^{mq}\, b^{nq}$

$(a^m \div b^n)^q$
$= (a^m)^q \div (b^n)^q$
$= a^{mq} \div b^{nq}$

$\left(\frac{a^m}{b^n}\right)^q = \frac{a^{mq}}{b^{nq}}$

Example 10

1. Simplify each of the following.

(a) $(7^3 \times 5^4)^3$ (b) $(2^4 \times 5^3 \times 11^2)^5$ (c) $(p^2q^3r)^4$ (d) $(5m^4n^3)^2$

(e) $\left(\frac{2^5}{3^2}\right)^4$ (f) $\left(\frac{2x^3}{3y^7}\right)^4$ (g) $\frac{(3m^2n^3)^3}{6m^3n}$ (h) $\frac{(2x^3y^4)^4 \times (3xy^2)^3}{36x^{10}y^{12}}$

Solution:

(a) $(7^3 \times 5^4)^3$
$= 7^{3(3)} \times 5^{4(3)}$
$= 7^9 \times 5^{12}$

(b) $(2^4 \times 5^3 \times 11^2)^5$
$= 2^{4(5)} \times 5^{3(5)} \times 11^{2(5)}$
$= 2^{20} \times 5^{15} \times 11^{10}$

(c) $(p^2q^3r)^4$
$= p^{2(4)}\, q^{3(4)} r^{1(4)}$
$= p^8 q^{12} r^4$

(d) $(5m^4n^3)^2$
$= 5^2 m^{4(2)} n^{3(2)}$
$= 25m^8n^6$

(e) $\left(\frac{2^5}{3^2}\right)^4$
$= \frac{2^{5(4)}}{3^{2(4)}}$
$= \frac{2^{20}}{3^8}$

(f) $\left(\frac{2x^3}{3y^7}\right)^4$
$= \frac{2^4 x^{3(4)}}{3^4 y^{7(4)}}$
$= \frac{16x^{12}}{81y^{28}}$

(g) $\frac{(3m^2n^3)^3}{6m^3n}$
$= \frac{3^3 m^{2(3)} n^{3(3)}}{6m^3n^1}$
$= \frac{27m^6n^9}{6m^3n^1}$
$= \frac{9}{2} m^{6-3} n^{9-1}$
$= \frac{9}{2} m^3 n^8$

(h) $\frac{(2x^3y^4)^4 \times (3xy^2)^3}{36x^{10}y^{12}}$
$= \frac{2^4 x^{3(4)} y^{4(4)} \times 3^3 x^{1(3)} y^{2(3)}}{36x^{10}y^{12}}$
$= \frac{16x^{12}y^{16} \times 27x^3y^6}{36x^{10}y^{12}}$
$= \left(\frac{16 \times 27}{36}\right) x^{12+3-10} y^{16+6-12}$
$= 12x^5 y^{10}$

FLASHBACK

$a^m \times a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{mn}$

QUIZ

$m^m = 256$.
What is the value of m?

DISCUSSION CORNER

Why is $1^n = 1$ for all values of n?
Discuss.

MIND TEST 1.2e

1. Simplify each of the following.

(a) $(2 \times 3^4)^2$ (b) $(11^3 \times 9^5)^3$ (c) $(13^3 \div 7^6)^2$ (d) $(5^3 \times 3^4)^5$

(e) $(m^3n^4p^2)^5$ (f) $(2w^2x^3)^4$ (g) $\left(\frac{-3a^5}{b^4}\right)^6$ (h) $\left(\frac{2a^5}{3b^4}\right)^3$

2. Simplify each of the following.

(a) $\left(\frac{11^3 \times 4^2}{11^2}\right)^2$ (b) $\frac{3^3 \times (6^2)^3}{6^4}$ (c) $\left(\frac{4^2}{6^3}\right)^3 \div \frac{4^2}{6^3}$ (d) $\frac{((-4)^6)^2 \times (-5^2)^3}{(-4)^6 \times (-5)^2}$

(e) $\frac{x^2y^6 \times x^3}{xy^2}$ (f) $\frac{(h^3k^2)^4}{(hk)^2}$ (g) $\frac{(m^5\, n^7)^3}{(m^2n^3)^2}$ (h) $\frac{(b^2d^4)^3}{(b^2d^3)^2}$

3. Simplify each of the following.

(a) $\frac{(2m^2n^4)^3 \times (3mn^4)^2}{12m^7n^{12}}$ (b) $\frac{(5xy^4)^2 \times 6x^{10}y}{15x^4y^6}$ (c) $\frac{24d^3e^5 \times (3d^3e^4)^2}{(d^5e^6) \times (6de^2)^3}$

How do you verify $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$; $a \neq 0$?

LEARNING STANDARD

Verify that $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$; $a \neq 0$.

Brainstorming 4

In pairs

Aim: To determine the value of a number or an algebraic term with a zero index.

Steps:

1. Study and complete the following table.
2. What is your conclusion regarding zero index?

Division in index form	Solution		Conclusion from the solution
	Law of indices	Repeated multiplication	
(a) $2^3 \div 2^3$	$2^{3-3} = 2^0$	$\frac{2 \times 2 \times 2}{2 \times 2 \times 2} = 1$	$2^0 = 1$
(d) $m^5 \div m^5$	$m^{5-5} = m^0$	$\frac{m \times m \times m \times m \times m}{m \times m \times m \times m \times m} = 1$	$m^0 = 1$
(c) $5^4 \div 5^4$			
(d) $(-7)^2 \div (-7)^2$			
(e) $n^6 \div n^6$			

Discussion:

1. Are your answers similar to the answers of the other groups?
2. What is your conclusion regarding zero index?

From Brainstorming 4, it is found that:

$2^0 = 1$
$m^0 = 1$

Therefore, a number or an algebraic term with a zero index will give a value of 1.

In general, $a^0 = 1 \; ; a \neq 0$

How do you verify $a^{-n} = \frac{1}{a^n}$?

Brainstorming 5

In groups

Aim: To verify $a^{-n} = \frac{1}{a^n}$.

Steps:

1. Study and complete the following table.

Division in index form	Solution		Conclusion from the solution
	Law of indices	Repeated multiplication	
(a) $2^3 \div 2^5$	$2^{3-5} = 2^{-2}$	$\frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2} = \frac{1}{2^2}$	$2^{-2} = \frac{1}{2^2}$
(b) $m^2 \div m^5$	$m^{2-5} = m^{-3}$	$\frac{m \times m}{m \times m \times m \times m \times m} = \frac{1}{m \times m \times m} = \frac{1}{m^3}$	$m^{-3} = \frac{1}{m^3}$
(c) $3^2 \div 3^6$			
(d) $(-4)^3 \div (-4)^7$			
(e) $p^4 \div p^8$			

Discussion:

1. Are your answers similar to the anwers of the other groups?
2. What is your conclusion?

Scan the QR Code or visit http://bukutekskssm.my/Mathematics/F3/Chapter1AlternativeMethod.mp4 to watch a video that describes alternative method to verify $a^{-1} = \frac{1}{a^n}$.

From Brainstorming 5, it is found that:

$$2^{-2} = \frac{1}{2^2}$$

$$m^{-3} = \frac{1}{m^3}$$

In general, $a^{-n} = \frac{1}{a^n}$; $a \neq 0$

BULLETIN

Negative index is a number or an algebraic term that has an index of a negative value.

TIPS

- $a^{-n} = \frac{1}{a^n}$
- $a^n = \frac{1}{a^{-n}}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

REMINDER

$2a^{-n} \neq \frac{1}{2a^n}$

SMART MIND

$\left(-\frac{4}{9}\right)^{-6} = x^y$

What are the values of x and y?

Example 11

1. State each of the following terms in positive index form.

(a) a^{-2} (b) x^{-4} (c) $\frac{1}{8^{-5}}$

(d) $\frac{1}{y^{-9}}$ (e) $2m^{-3}$ (f) $\frac{3}{5}n^{-8}$

(g) $\left(\frac{2}{3}\right)^{-10}$ (h) $\left(\frac{x}{y}\right)^{-7}$

2. State each of the following in negative index form.

(a) $\frac{1}{3^4}$ (b) $\frac{1}{m^5}$ (c) 7^5

(d) n^{20} (e) $\left(\frac{4}{5}\right)^8$ (f) $\left(\frac{m}{n}\right)^{15}$

3. Simplify each of the following.

(a) $3^2 \times 3^4 \div 3^8$ (b) $\frac{(2^4)^2 \times (3^5)^3}{(2^8 \times 3^6)^2}$ (c) $\frac{(4xy^2)^2 \times x^5y}{(2x^3y)^5}$

Solution:

1. (a) $a^{-2} = \frac{1}{a^2}$ (b) $x^{-4} = \frac{1}{x^4}$ (c) $\frac{1}{8^{-5}} = 8^5$ (d) $\frac{1}{y^{-9}} = y^9$

(e) $2m^{-3} = \frac{2}{m^3}$ (f) $\frac{3}{5}n^{-8} = \frac{3}{5n^8}$ (g) $\left(\frac{2}{3}\right)^{-10} = \left(\frac{3}{2}\right)^{10}$ (h) $\left(\frac{x}{y}\right)^{-7} = \left(\frac{y}{x}\right)^7$

2. (a) $\frac{1}{3^4} = 3^{-4}$ (b) $\frac{1}{m^5} = m^{-5}$ (c) $7^5 = \frac{1}{7^{-5}}$ (d) $n^{20} = \frac{1}{n^{-20}}$

(e) $\left(\frac{4}{5}\right)^8 = \left(\frac{5}{4}\right)^{-8}$ (f) $\left(\frac{m}{n}\right)^{15} = \left(\frac{n}{m}\right)^{-15}$

3. (a) $3^2 \times 3^4 \div 3^8$
$= 3^{2+4-8}$
$= 3^{-2}$
$= \frac{1}{3^2}$

(b) $\frac{(2^4)^2 \times (3^5)^3}{(2^8 \times 3^6)^2}$
$= \frac{2^8 \times 3^{15}}{2^{16} \times 3^{12}}$
$= 2^{8-16} \times 3^{15-12}$
$= 2^{-8} \times 3^3$
$= \frac{3^3}{2^8}$

(c) $\frac{(4xy^2)^2 \times x^5y}{(2x^3y)^5}$
$= \frac{4^2x^2y^4 \times x^5y^1}{2^5x^{15}y^5}$
$= \frac{16}{32}x^{2+5-15}y^{4+1-5}$
$= \frac{1}{2}x^{-8}y^0$
$= \frac{1}{2x^8}$

TIPS

$y^0 = 1$

$y^1 = y$

MIND TEST 1.2f

1. State each of the following terms in positive index form.

(a) 5^{-3} (b) 8^{-4} (c) x^{-8} (d) y^{-16} (e) $\frac{1}{a^{-4}}$

(f) $\frac{1}{20^{-2}}$ (g) $3n^{-4}$ (h) $-5n^{-6}$ (i) $\frac{2}{7}m^{-5}$ (j) $\left(-\frac{3}{8}\right)^{m-4}$

(k) $\left(\frac{2}{5}\right)^{-12}$ (l) $\left(-\frac{3}{7}\right)^{-14}$ (m) $\left(\frac{x}{y}\right)^{-10}$ (n) $\left(\frac{2x}{3y}\right)^{-4}$ (o) $\left(\frac{1}{2x}\right)^{-5}$

2. State each of the following terms in negative index form.

(a) $\frac{1}{5^4}$ (b) $\frac{1}{8^3}$ (c) $\frac{1}{m^7}$ (d) $\frac{1}{n^9}$ (e) 10^2

(f) $(-4)^3$ (g) m^{12} (h) n^{16} (i) $\left(\frac{4}{7}\right)^9$ (j) $\left(\frac{x}{y}\right)^{10}$

3. Simplify each of the following.

(a) $\frac{(4^2)^3 \times 4^5}{(4^6)^2}$ (b) $\frac{(2^3 \times 3^2)^3}{(2 \times 3^4)^5}$ (c) $\frac{(5^2)^5}{(2^3)^{-2} \times (5^4)^2}$

(d) $\frac{3m^2n^4 \times (mn^3)^{-2}}{9m^3n^5}$ (e) $\frac{(2m^2n^2)^{-3} \times (3mn^2)^4}{(9m^3n)^2}$ (f) $\frac{(4m^2n^4)^2}{(2m^{-2}n)^5 \times (3m^4n)^2}$

How do you determine and state the relationship between fractional indices and roots and powers?

LEARNING STANDARD

Determine and state the relationship between fractional indices and roots and powers.

Relationship between $\sqrt[n]{a}$ and $a^{\frac{1}{n}}$

In Form 1, you have learnt about square and square root as well as cube and cube root. Determine the value of x for

(a) $x^2 = 9$ (b) $x^3 = 64$

TIPS

♦ $9 = 3^2$ ♦ $64 = 4^3$

Solution:

(a) $x^2 = 9$

$\sqrt{x^2} = \sqrt{3^2}$ — Square roots are used to eliminate squares.

$x = 3$

(b) $x^3 = 64$

$\sqrt[3]{x^3} = \sqrt[3]{4^3}$ — Cube roots are used to eliminate cubes.

$x = 4$

Did you know that the values of x in examples (a) and (b) above can be determined by raising the index to the power of its reciprocal?

(a) $x^2 = 9$

$x^{2\left(\frac{1}{2}\right)} = 9^{\frac{1}{2}}$ — The reciprocal of 2 is $\frac{1}{2}$.

$x^1 = 3^{2\left(\frac{1}{2}\right)}$

$x = 3$

(b) $x^3 = 64$

$x^{3\left(\frac{1}{3}\right)} = 64^{\left(\frac{1}{3}\right)}$ — The reciprocal of 3 is $\frac{1}{3}$.

$x^1 = 4^{3\left(\frac{1}{3}\right)}$

$x = 4$

BULLETIN

$\frac{1}{a}$ is the reciprocal of a.

From the two methods to determine the values of x in the examples above, it is found that:

$$\sqrt[2]{x} = x^{\frac{1}{2}}$$
$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

In general, $\sqrt[n]{a} = a^{\frac{1}{n}}; a \neq 0$

SMART MIND

What is the solution for $\sqrt{-4}$? Discuss.

Example 12

1. Convert each of the following terms into the form $a^{\frac{1}{n}}$.

(a) $\sqrt[2]{36}$ (b) $\sqrt[3]{-27}$ (c) $\sqrt[5]{m}$ (d) $\sqrt[7]{n}$

2. Convert each of the following terms into the form $\sqrt[n]{a}$.

(a) $125^{\frac{1}{5}}$ (b) $256^{\frac{1}{8}}$ (c) $(-1\,000)^{\frac{1}{3}}$ (d) $n^{\frac{1}{12}}$

3. Calculate the value of each of the following terms.

(a) $\sqrt[5]{-32}$ (b) $\sqrt[6]{729}$ (c) $512^{\frac{1}{3}}$ (d) $(-243)^{\frac{1}{5}}$

Solution:

1. (a) $\sqrt[2]{36} = 36^{\frac{1}{2}}$ (b) $\sqrt[3]{-27} = (-27)^{\frac{1}{3}}$ (c) $\sqrt[5]{m} = m^{\frac{1}{5}}$ (d) $\sqrt[7]{n} = n^{\frac{1}{7}}$

2. (a) $125^{\frac{1}{5}} = \sqrt[5]{125}$ (b) $256^{\frac{1}{8}} = \sqrt[8]{256}$ (c) $(-1\,000)^{\frac{1}{3}} = \sqrt[3]{(-1\,000)}$ (d) $n^{\frac{1}{12}} = \sqrt[12]{n}$

3. (a) $\sqrt[5]{-32} = (-32)^{\frac{1}{5}}$
$= (-2)^{5\left(\frac{1}{5}\right)}$
$= (-2)^1$
$= -2$

(b) $\sqrt[6]{729} = 729^{\frac{1}{6}}$
$= 3^{6\left(\frac{1}{6}\right)}$
$= 3^1$
$= 3$

(c) $512^{\frac{1}{3}} = 8^{3\left(\frac{1}{3}\right)}$
$= 8^1$
$= 8$

(d) $(-243)^{\frac{1}{5}} = (-3)^{5\left(\frac{1}{5}\right)}$
$= (-3)^1$
$= -3$

TIPS

You can use a scientific calculator to check the answers.

MIND TEST 1.2g

1. Convert each of the following terms into the form $a^{\frac{1}{n}}$.

(a) $\sqrt[3]{125}$ (b) $\sqrt[7]{2\,187}$ (c) $\sqrt[5]{-1\,024}$ (d) $\sqrt[10]{n}$

2. Convert each of the following terms into the form $\sqrt[n]{a}$.

(a) $4^{\frac{1}{2}}$ (b) $32^{\frac{1}{5}}$ (c) $(-729)^{\frac{1}{3}}$ (d) $n^{\frac{1}{15}}$

3. Calculate the value of each of the following terms.

(a) $\sqrt[3]{343}$ (b) $\sqrt[5]{-7\,776}$ (c) $262\,144^{\frac{1}{6}}$ (d) $(-32\,768)^{\frac{1}{5}}$

What is the relationship between $a^{\frac{m}{n}}$ and $(a^m)^{\frac{1}{n}}$, $(a^{\frac{1}{n}})^m$, $\sqrt[n]{a^m}$ dan $(\sqrt[n]{a})^m$?

You have learnt that:

$$a^{mn} = (a^m)^n \text{ and } \sqrt[n]{a^1} = a^{\frac{1}{n}}$$

From the two laws of indices above, we can convert $a^{\frac{m}{n}}$ into $(a^m)^{\frac{1}{n}}$, $(a^{\frac{1}{n}})^m$, $\sqrt[n]{a^m}$ and $(\sqrt[n]{a})^m$. Calculate the value of each of the following. Complete the table as shown in example (a).

	$a^{\frac{m}{n}}$	$(a^m)^{\frac{1}{n}}$	$(a^{\frac{1}{n}})^m$	$\sqrt[n]{a^m}$	$(\sqrt[n]{a})^m$
(a)	$64^{\frac{2}{3}}$	$(64^2)^{\frac{1}{3}}$ $= 4\,096^{3\left(\frac{1}{3}\right)}$ $= 16$	$(64^{\frac{1}{3}})^2$ $= 4^{2\left(\frac{1}{2}\right)(2)}$ $= 4^2$ $= 16$	$\sqrt[3]{64^2}$ $= \sqrt[3]{4\,096}$ $= 16$	$(\sqrt[3]{64})^2$ $= 4^2$ $= 16$
(b)	$16^{-\frac{3}{4}}$				
(c)	$243^{-\frac{2}{5}}$				

Are your answers in (b) and (c) the same when you use different index forms? Discuss.

From the activity above, it is found that:

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$$
$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

Example 13

1. Convert each of the following into the form $(a^m)^{\frac{1}{n}}$ and $(a^{\frac{1}{n}})^m$.

(a) $81^{\frac{3}{2}}$ (b) $27^{-\frac{2}{3}}$ (c) $h^{\frac{3}{5}}$

2. Convert each of the following into the form $\sqrt[n]{a^m}$ and $(\sqrt[n]{a})^m$.

(a) $343^{\frac{2}{3}}$ (b) $4\,096^{-\frac{5}{6}}$ (c) $m^{-\frac{2}{5}}$

Solution:

1. (a) $81^{\frac{3}{2}} = (81^3)^{\frac{1}{2}}$ (b) $27^{\frac{2}{3}} = (27^2)^{\frac{1}{3}}$ (c) $h^{\frac{3}{5}} = (h^3)^{\frac{1}{5}}$

$81^{\frac{3}{2}} = (81^{\frac{1}{2}})^3$ $27^{\frac{2}{3}} = (27^{\frac{1}{3}})^2$ $h^{\frac{3}{5}} = (h^{\frac{1}{5}})^3$

2. (a) $343^{\frac{2}{3}} = \sqrt[3]{343^2}$ (b) $4\,096^{-\frac{5}{6}} = \sqrt[6]{4\,096^5}$ (c) $m^{-\frac{2}{5}} = \sqrt[5]{m^2}$

$343^{\frac{2}{3}} = (\sqrt[3]{343})^2$ $4\,096^{-\frac{5}{6}} = (\sqrt[6]{4\,096})^5$ $m^{-\frac{2}{5}} = (\sqrt[5]{m})^2$

MIND TEST 1.2h

1. Complete the following table.

$a^{\frac{m}{n}}$	$729^{\frac{5}{6}}$	$121^{\frac{3}{2}}$	$w^{\frac{3}{7}}$	$x^{\frac{2}{5}}$	$\left(\frac{16}{81}\right)^{\frac{3}{4}}$	$\left(\frac{h}{k}\right)^{\frac{2}{3}}$
$(a^m)^{\frac{1}{n}}$						
$(a^{\frac{1}{n}})^m$						
$\sqrt[n]{a^m}$						
$(\sqrt[n]{a})^m$						

Example 14

1. Calculate the value of each of the following.

(a) $9^{\frac{5}{2}}$ (b) $16^{\frac{5}{4}}$

Solution:

1. (a) $9^{\frac{5}{2}}$ (b) $16^{\frac{5}{4}}$

Method 1 $9^{\frac{5}{2}} = (\sqrt{9})^5 = (3)^5 = 243$ Method 1 $16^{\frac{5}{4}} = (\sqrt[4]{16})^5 = 2^5 = 32$

Method 2 $9^{\frac{5}{2}} = \sqrt{9^5} = \sqrt{59\,049} = 243$ Method 2 $16^{\frac{5}{4}} = \sqrt[4]{16^5} = \sqrt[4]{1\,048\,576} = 32$

MIND TEST 1.2i

1. Calculate the value of each of the following.

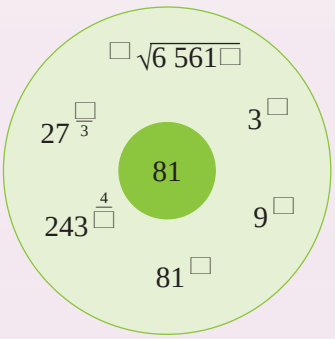
(a) $27^{\frac{2}{3}}$ (b) $32^{\frac{2}{5}}$ (c) $128^{\frac{2}{7}}$ (d) $256^{\frac{3}{8}}$

(e) $64^{\frac{4}{3}}$ (f) $1\,024^{\frac{2}{5}}$ (g) $1\,296^{\frac{3}{4}}$ (h) $49^{\frac{3}{2}}$

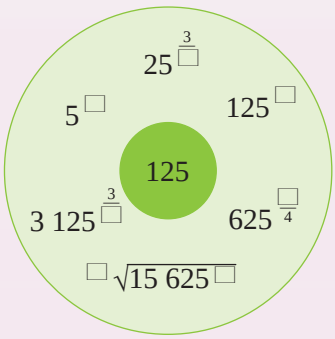
(i) $2\,401^{\frac{1}{4}}$ (j) $121^{\frac{3}{2}}$ (k) $2\,197^{\frac{2}{3}}$ (l) $10\,000^{\frac{3}{4}}$

2. Complete the following diagrams with correct values.

(a)



(b)



How do you perform operations involving laws of indices?

LEARNING STANDARD

Perform operations involving laws of indices.

Law of indices		
$a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{mn}$	$a^0 = 1$ $a^{-n} = \frac{1}{a^n}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$ $a^{\frac{m}{n}} = a^{m(\frac{1}{n})} = (a^{\frac{1}{n}})^m$ $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Example 15

1. Simplify each of the following.

(a) $\dfrac{(-3x)^3 \times (2x^3y^{-4})^2}{108x^4y^3}$ (b) $\dfrac{\sqrt{m}\, n^{\frac{3}{4}} \times (mn^3)^{\frac{1}{3}}}{(m^{-1}\sqrt{n^3})^{\frac{1}{6}}}$ (c) $\dfrac{(2h)^2 \times (16h^8)^{\frac{1}{4}}}{(8^{\frac{1}{3}}h)^{-2}}$

Solution:

(a) $\dfrac{(-3x)^3 \times (2x^3y^{-4})^2}{108x^4y^3}$

$= \dfrac{(-3)^3x^3 \times 2^2x^{3(2)}y^{-4(2)}}{108x^4y^3}$

$= \dfrac{-27x^3 \times 4x^6y^{-8}}{108x^4y^3}$

$= \left(\dfrac{-27 \times 4}{108}\right) x^{3+6-4}\, y^{-8-3}$

$= -1\, x^5\, y^{-11}$

$= -\dfrac{x^5}{y^{11}}$

(b) $\dfrac{\sqrt{m}\, n^{\frac{3}{4}} \times (mn^3)^{\frac{1}{3}}}{(m^{-1}\sqrt{n^3})^{\frac{1}{6}}}$

$= \dfrac{m^{\frac{1}{2}} n^{\frac{3}{4}} \times m^{\frac{1}{3}} n^{3(\frac{1}{3})}}{m^{-1(\frac{1}{6})} n^{\frac{3}{2}(\frac{1}{6})}}$

$= \dfrac{m^{\frac{1}{2}} n^{\frac{3}{4}} \times m^{\frac{1}{3}} n^1}{m^{-\frac{1}{6}} n^{\frac{1}{4}}}$

$= m^{\frac{1}{2}+\frac{1}{3}-(-\frac{1}{6})} n^{\frac{3}{4}+1-\frac{1}{4}}$

$= m^1\, n^{\frac{3}{2}}$

$= mn^{\frac{3}{2}}$

(c) $\dfrac{(2h)^2 \times (16h^8)^{\frac{1}{4}}}{(8^{\frac{1}{3}}h)^{-2}}$

$= \dfrac{2^2h^2 \times 16^{\frac{1}{4}}h^{8(\frac{1}{4})}}{8^{\frac{1}{3}(-2)}h^{(-2)}}$

$= \dfrac{2^2h^2 \times 2^{4(\frac{1}{4})}h^{8(\frac{1}{4})}}{2^{3(\frac{1}{3})(-2)}h^{(-2)}}$

$= \dfrac{2^2h^2 \times 2^1h^2}{2^{-2}h^{-2}}$

$= 2^{2+1-(-2)}\, h^{2+2-(-2)}$

$= 2^5\, h^6$

$= 32\, h^6$

Example 16

1. Calculate the value of each of the following.

(a) $\dfrac{49^{\frac{1}{2}} \times 125^{-\frac{1}{3}}}{\sqrt[4]{2\,401} \times \sqrt[5]{3\,125}}$ (b) $\dfrac{16^{\frac{3}{4}} \times 81^{-\frac{1}{4}}}{(2^6 \times 3^4)^{\frac{1}{2}}}$ (c) $\dfrac{(243^{\frac{4}{5}} \times 5^{\frac{3}{2}})^2}{\sqrt[4]{81} \times \sqrt{25^4}}$

Solution:

(a) $\dfrac{49^{\frac{1}{2}} \times 125^{-\frac{1}{3}}}{\sqrt[4]{2\,401} \times \sqrt[5]{3\,125}}$

$= \dfrac{7^{2(\frac{1}{2})} \times 5^{3(-\frac{1}{3})}}{(7^4)^{\frac{1}{4}} \times (5^5)^{\frac{1}{5}}}$

$= \dfrac{7^1 \times 5^{-1}}{7^1 \times 5^1}$

$= 7^{1-1} \times 5^{-1-1}$

$= 7^0 \times 5^{-2}$

$= 1 \times \dfrac{1}{5^2}$

$= \dfrac{1}{25}$

(b) $\dfrac{16^{\frac{3}{4}} \times 81^{-\frac{1}{4}}}{(2^6 \times 3^4)^{\frac{1}{2}}}$

$= \dfrac{2^{4(\frac{3}{4})} \times 3^{4(-\frac{1}{4})}}{2^{6^3(\frac{1}{2})} \times 3^{4^2(\frac{1}{2})}}$

$= \dfrac{2^3 \times 3^{-1}}{2^3 \times 3^2}$

$= 2^{3-3} \times 3^{-1-2}$

$= 2^0 \times 3^{-3}$

$= 1 \times \dfrac{1}{3^3}$

$= \dfrac{1}{27}$

(c) $\dfrac{(243^{\frac{4}{5}} \times 5^{\frac{3}{2}})^2}{\sqrt[4]{81} \times \sqrt{25^4}}$

$= \dfrac{243^{\frac{4}{5}(2)} \times 5^{\frac{3}{2}(2)}}{81^{\frac{1}{4}} \times 25^{\frac{4}{2}}}$

$= \dfrac{3^{5(\frac{8}{5})} \times 5^3}{3^{4(\frac{1}{4})} \times 5^{2(\frac{4}{2})}}$

$= \dfrac{3^8 \times 5^3}{3^1 \times 5^4}$

$= 3^{8-1} \times 5^{3-4}$

$= 3^7 \times 5^{-1}$

$= \dfrac{3^7}{5}$

$= \dfrac{2\,187}{5}$

$= 437\dfrac{2}{5}$

MIND TEST 1.2j

1. Simplify each of the following.

(a) $\dfrac{\sqrt[3]{c^2d^3e} \times c^{\frac{1}{3}}d^2e^{\frac{2}{3}}}{(c^{-3}d^2e)^2}$ (b) $\dfrac{(mn^2)^3 \times (\sqrt{mn})^4}{(m^6n^3)^{\frac{2}{3}}}$ (c) $\dfrac{\sqrt{25x^3yz^2} \times 4x^2z}{\sqrt{36x^5yz^8}}$

2. Calculate the value of each of the following.

(a) $\dfrac{\sqrt{7^{-4} \times 11^4}}{49 \times 121}$ (b) $\dfrac{(5^{-3} \times 3^6)^{\frac{1}{3}} \times \sqrt[4]{16}}{(125 \times 729 \times 64)^{-\frac{1}{3}}}$ (c) $\dfrac{(2^6 \times 3^4 \times 5^2)^{\frac{3}{2}}}{\sqrt[4]{256} \times \sqrt{729} \times \sqrt[3]{125}}$

(d) $\dfrac{\sqrt[9]{512} \times \sqrt[3]{343} \times \sqrt{121}}{(64)^{\frac{1}{3}} \times (81)^{\frac{3}{4}} \times (14\,641)^{\frac{1}{4}}}$ (e) $\dfrac{(2^4 \times 3^6)^{\frac{1}{2}} \times \sqrt[3]{8} \times \sqrt{81}}{16^{\frac{3}{4}} \times 27^{\frac{1}{3}}}$ (f) $\dfrac{64^{\frac{2}{3}} \times \sqrt[3]{125} \times (2 \times \frac{1}{5})^{-3}}{4^2 \times \sqrt[4]{625}}$

3. Given $m = 2$ and $n = -3$, calculate the value of $64^{\frac{m}{3}} \times 512^{(-\frac{1}{n})} \div 81^{\frac{n}{2m}}$.

4. Given $a = \dfrac{1}{2}$ and $b = \dfrac{2}{3}$, calculate the value of $144^a \div 64^b \times 256^{-\frac{a}{b}}$.

How do you solve problems involving laws of indices?

LEARNING STANDARD

Solve problems involving laws of indices.

FLASHBACK

Common prime factors of 6 and 12 are 2 and 3.

Example 17

Calculate the value of $\sqrt{3} \times 12^{\frac{3}{2}} \div 6$ without using a calculator.

Understanding the problem

Calculate the value of numbers in index form with different bases.

Planning a strategy

Convert each base into prime factors and calculate the value by applying laws of indices.

Implementing the strategy

$$\begin{aligned}
&\sqrt{3} \times 12^{\frac{3}{2}} \div 6 \\
&= 3^{\frac{1}{2}} \times (2 \times 2 \times 3)^{\frac{3}{2}} \div (2 \times 3) \\
&= 3^{\frac{1}{2}} \times 2^{\frac{3}{2}} \times 2^{\frac{3}{2}} \times 3^{\frac{3}{2}} \div (2^1 \times 3^1) \\
&= 3^{\frac{1}{2} + \frac{3}{2} - 1} \times 2^{\frac{3}{2} + \frac{3}{2} - 1} \\
&= 3^1 \times 2^2 \\
&= 12
\end{aligned}$$

Making a conclusion

$\sqrt{3} \times 12^{\frac{3}{2}} \div 6 = 12$

Example 18

Calculate the value of x for the equation $3^x \times 9^{x+5} \div 3^4 = 1$.

Understanding the problem

Calculate the value of variable x which is part of the indices.

Planning a strategy

The question is an equation. Hence, the value on the left side of the equation is the same as the value on the right side of the equation. Convert all the terms into index form with base of 3.

Implementing the strategy

$$\begin{aligned}
3^x \times 9^{x+5} \div 3^4 &= 1 \\
3^x \times 3^{2(x+5)} \div 3^4 &= 3^0 \\
3^{x+2(x+5)-4} &= 3^0 \\
3^{x+2x+10-4} &= 3^0 \\
3^{3x+6} &= 3^0
\end{aligned}$$

$a^m = a^n$, $m = n$

$$\begin{aligned}
3x + 6 &= 0 \\
3x &= -6 \\
x &= \frac{-6}{3} \\
x &= -2
\end{aligned}$$

Making a conclusion

If $3^x \times 9^{x+5} \div 3^4 = 1$, then, $x = -2$

REMINDER

- If $a^m = a^n$ then, $m = n$
- If $a^m = b^m$ then, $a = b$

Checking Answers

You can check the answer by substituting the value of x into the original equation.

$3^x \times 9^{x+5} \div 3^4 = 1$ (Left = Right)

Substitute $x = -2$ into left side of the equation.

$$\begin{aligned}
&3^{-2} \times 9^{-2+5} \div 3^4 \\
&= 3^{-2} \times 9^3 \div 3^4 \\
&= 3^{-2} \times 3^{2(3)} \div 3^4 \\
&= 3^{-2+6-4} \\
&= 3^0 \\
&= 1
\end{aligned}$$

The same value as the value on the right side of the equation.

Example 19

Calculate the possible values of x for the equation $3^{x^2} \times 3^{2x} = 3^{15}$.

Understanding the problem

Calculate the value of x which is part of the indices.

Planning a strategy

All the bases involved in the equation are the same.

Implementing the strategy

$3^{x^2} \times 3^{2x} = 3^{15}$

$3^{x^2 + 2x} = 3^{15}$ — If $a^m = a^n$, then, $m = n$.

$x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$ — Solve the quadratic equation using factorisation method.

$(x - 3)(x + 5) = 0$

$x - 3 = 0$ or $x + 5 = 0$

$x = 0 + 3$ $\quad$ $x = 0 - 5$

$x = 3$ $\quad$ $x = -5$

Making a conclusion

The possible values of x for the equation $3^{x^2} \times 3^{2x} = 3^{15}$ are 3 and –5.

Checking Answers

Substitute the values of x into the original equation.

$3^{x^2} \times 3^{2x} = 3^{15}$

Left Right

Substitute $x = 3$

Left: $3^{(3)^2} \times 3^{2(3)} = 3^9 \times 3^6 = 3^{9+6} = 3^{15}$

Right: 3^{15}

The same value

Substitute $x = -5$

Left: $3^{(-5)^2} \times 3^{2(-5)} = 3^{25} \times 3^{-10} = 3^{25+(-10)} = 3^{15}$

Right: 3^{15}

The same value

Example 20

Solve the following simultaneous equations.

$25^m \times 5^n = 5^8$ and $2^m \times \frac{1}{2^n} = 2$

Solution:

$25^m \times 5^n = 5^8$

$5^{2(m)} \times 5^n = 5^8$

$5^{2m+n} = 5^8$

$2m + n = 8 \rightarrow (1)$

$2^m \times \frac{1}{2^n} = 2$

$2^m \times 2^{-n} = 2^1$

$2^{m+(-n)} = 2^1$

$m - n = 1 \rightarrow (2)$

Equation (1) and (2) can be solved by substitution method.

From (1):

$2m + n = 8$

$n = 8 - 2m \rightarrow (3)$

Substitute (3) into (2)

$m - n = 1$

$m - (8 - 2m) = 1$

$m - 8 + 2m = 1$

$m + 2m = 1 + 8$

$3m = 9$

$m = \frac{9}{3}$

$m = 3$

Substitute $m = 3$ into (1)

$2m + n = 8$

$2(3) + n = 8$

$6 + n = 8$

$n = 8 - 6$

$n = 2$

You can also substitute $m = 3$ into equation (2) or (3).

Hence, $m = 3$ and $n = 2$.

FLASHBACK

Simultaneous linear equations in two variables can be solved using substitution method or elimination method.

Checking Answers

Substitute $m = 3$ and $n = 2$ into original simultaneous equations.

$25^m \times 5^n = 5^8$

Left Right

Left: $25^m \times 5^n = 5^{2(m)} \times 5^n = 5^{2(3)} \times 5^2 = 5^{6+2} = 5^8$

Right: 5^8

The same value

$2^m \times \frac{1}{2^n} = 2$

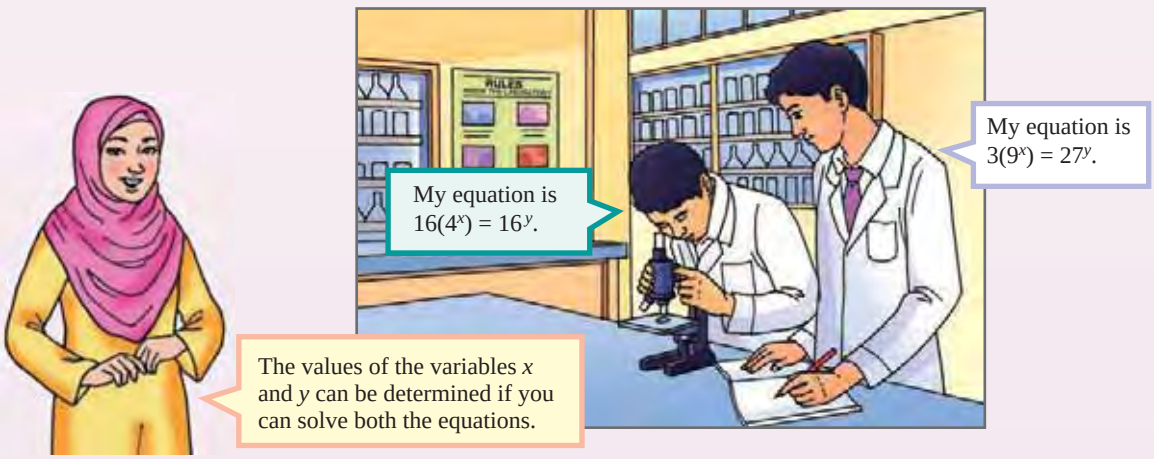
Left Right

Left: $2^m \times \frac{1}{2^n} = 2^3 \times \frac{1}{2^2} = 2^3 \times 2^{-2} = 2^{3+(-2)} = 2^1 = 2$

Right: 2

The same value

Example 21



Chong and Navin performed an experiment to determine the relationship between variable x and variable y. The equation Chong obtained was $16(4^x) = 16^y$, while the equation Navin got was $3(9^x) = 27^y$. Calculate the values of x and y which satisfy the experiment Chong and Navin have performed.

Solution:

$16(4^x) = 16^y$

$4^2(4^x) = 4^{2(y)}$

$4^{2+x} = 4^{2y}$

$2 + x = 2y \rightarrow (1)$

$3(9^x) = 27^y$

$3(3^{2x}) = 3^{3(y)}$

$3^{1+2x} = 3^{3y}$

$1 + 2x = 3y \rightarrow (2)$

Equations (1) and (2) can be solved by elimination method.

Multiply equation (1) by 2 to equate the coefficients of variable x.

$(1) \times 2 : 4 + 2x = 4y \rightarrow (3)$

$(2) \quad : 1 + 2x = 3y$

$(3) - (2) :$

$3 + 0 = y$

$y = 3$

You can also substitute $y = 3$ into equation (2) or (3).

Substitute $y = 3$ into equation (1)

$(1) : 2 + x = 2y$

$2 + x = 2(3)$

$x = 6 - 2$

$x = 4$

Hence, $x = 4$, $y = 3$

Dynamic Challenge

Test Yourself

1. State whether each of the following operations which involves the laws of indices is **true** or **false**. If it is false, state the correct answer.

(a) $a^5 = a \times a \times a \times a \times a$

(b) $5^2 = 10$

(c) $3^0 = 0$

(d) $(2x^3)^5 = 2x^{15}$

(e) $m^0 n^0 = 1$

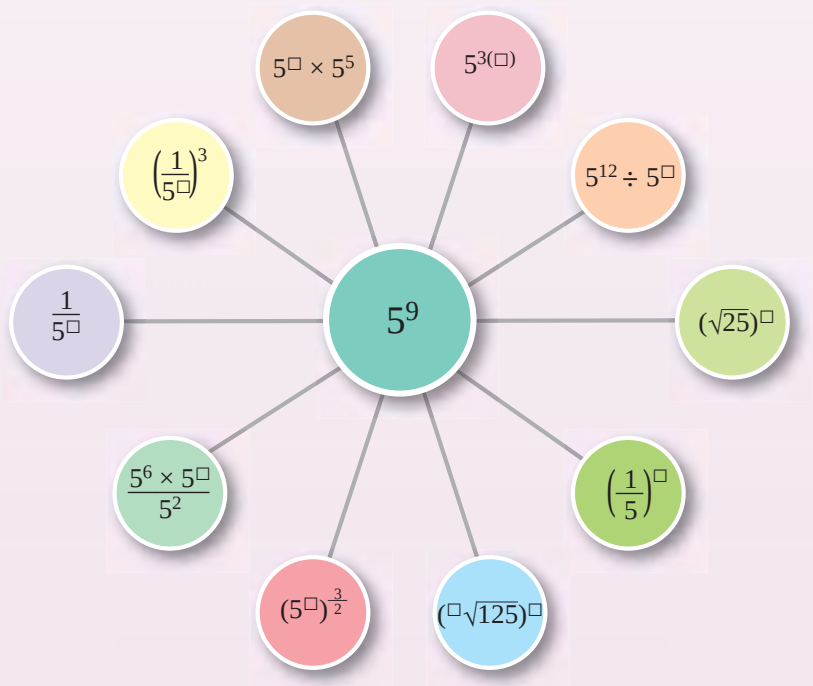
(f) $2a^{-4} = \frac{1}{2a^4}$

(g) $32^{\frac{2}{5}} = (\sqrt[2]{32})^5$

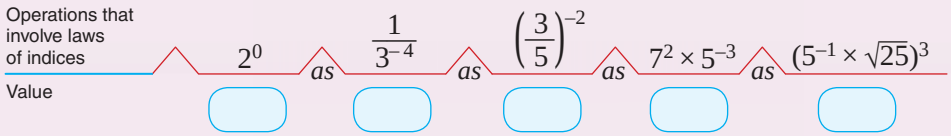
(h) $\left(\frac{m}{n}\right)^{-4} = \left(\frac{n}{m}\right)^4$

(i) $(5m^{\frac{1}{4}})^{-4} = \frac{625}{m}$

2. Copy and complete the following diagram with suitable values.



3. Copy and complete the following diagram.



Skills Enhancement

1. Simplify each of the following.

(a) $(mn^4)^3 \div m^4n^5$

(b) $3x \times \frac{1}{6} y^4 \times (xy)^3$

(c) $\sqrt{xy} \times \sqrt[3]{xy^2} \times \sqrt[6]{xy^5}$

2. Calculate the value of each of the following.

(a) $64^{\frac{1}{3}} \times 5^{-3}$

(b) $7^{-1} \times 125^{\frac{2}{3}}$

(c) $(256)^{\frac{3}{8}} \times 2^{-3}$

(d) $2^4 \times 16^{-\frac{3}{4}}$

(e) $\sqrt{49} \times 3^{-2} \div (\sqrt{81})^{-1}$

(f) $(125)^{\frac{2}{3}} \times (25)^{-\frac{3}{2}} \div (625)^{-\frac{1}{4}}$

3. Calculate the value of x for each of the following equations.

(a) $2^6 \div 2^x = 8$

(b) $3^{-4} \times 81 = 3^x$

(c) $a^x a^8 = 1$

(d) $4 \times 8^{x+1} = 2^{2x}$

(e) $(a^x)^2 \times a^5 = a^{3x}$

(f) $2^x = \frac{2^{10}}{16^x}$

(g) $3^6 \div 3^x = 81^{(x-1)}$

(h) $(m^2)^x \times m^{(x+1)} = m^{-2}$

(i) $25^x \div 125 = \frac{1}{5^x}$

Self Mastery

1. Calculate the value of each of the following without using a calculator.

(a) $4^{\frac{1}{3}} \times 50^{\frac{2}{3}} \times 10^{\frac{5}{3}}$ (b) $5^{\frac{5}{2}} \times 20^{\frac{3}{2}} \div 10^{-2}$ (c) $60^{\frac{1}{2}} \times 125^{\frac{2}{3}} \div \sqrt{15}$

2. Calculate the value of x for each of the following equations.

(a) $64x^{\frac{1}{2}} = 27x^{-\frac{5}{2}}$ (b) $3x^{\frac{2}{3}} = \frac{27}{4}x^{-\frac{4}{3}}$ (c) $25x^{-\frac{2}{3}} - \frac{5}{3}x^{\frac{1}{3}} = 0$

3. Calculate the possible values of x for each of the following equations.

(a) $a^{x^2} \div a^{5x} = a^6$ (b) $2^{x^2} \times 2^{6x} = 2^7$ (c) $5^{x^2} \div 5^{3x} = 625$

4. Solve the following simultaneous equations.

(a) $81^{(x+1)} \times 9^x = 3^5$ and $8^{2x} \times 4(2^{2y}) = 128$ (b) $4(4^x) = 8^{y+2}$ and $9^x \times 27^y = 1$

5. In an experiment performed by Susan, it was found that the temperature of a metal rose from 25°C to T°C according to equation $T = 25(1.2)^m$ when the metal was heated for m seconds. Calculate the difference in temperature between the fifth second and the sixth second, to the nearest degree Celsius.

6. Encik Azmi bought a locally made car for RM55 000. After 6 years, Encik Azmi wishes to sell the car. Based on the explanation from the used car dealers, the price of Encik Azmi's car will be calculated using the formula $\text{RM}55\,000\left(\frac{8}{9}\right)^n$. In this situation, n is the number of years after the car is bought. What is the market value of Encik Azmi's car? State your answer correct to the nearest RM.

7. Mrs Kiran Kaur saved RM50 000 on 1 March 2019 in a local bank with an interest of 3.5% per annum. After t years, Mrs Kiran Kaur's total savings, in RM, is $50\,000\,(1.035)^t$. Calculate her total savings on 1 March 2025, if Mrs Kiran Kaur does not withdraw her savings.

PROJECT

Materials: One sheet of A4 paper, a pair of scissors, a long ruler, a pencil.

Instructions: (a) Carry out the project in small groups.
(b) Cut the A4 paper into the biggest possible square.

Steps:

1. Draw the axes of symmetry (vertical and horizontal only) as shown in Diagram 1.
2. Calculate the number of squares formed. Write your answers in the space provided in Sheet A.
3. Draw the vertical and horizontal axes of symmetry for each square as shown in Diagram 2.
4. Calculate the number of squares formed. Write your answers in the space provided in Sheet A.
5. Repeat step 3 and step 4 as many times as possible.

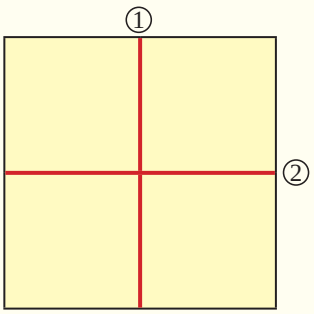


Diagram 1

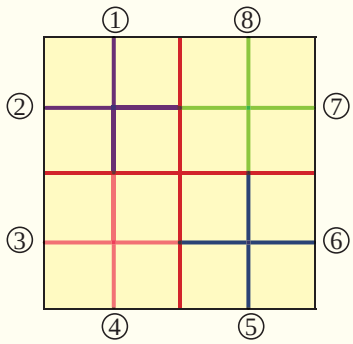


Diagram 2

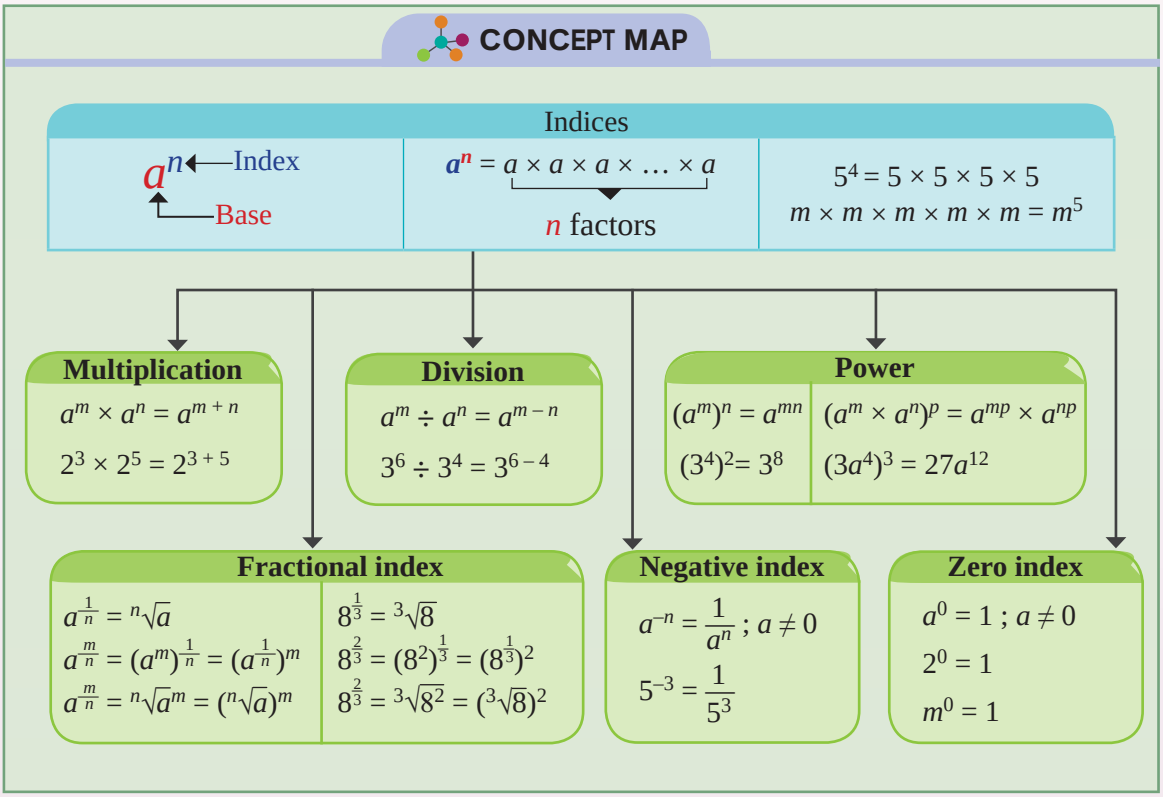
6. Compare your answers with those of other groups.
7. What can you say about the patterns in the column 'Index form' in Sheet A?
8. Discuss the patterns you identify.

Scan the QR Code or visit http://bukutekskssm.my/Mathematics/F3/Chapter1SheetA.pdf to download Sheet A.

Sheet A

Number of axis of symmetry	Index form	Number of square	Index form
0	–	1	2^0
2	2^1	4	2^2
8		16	

CONCEPT MAP



SELF-REFLECT

At the end of this chapter, I can:

1.	Represent repeated multiplication in index form and describe its meaning.		
2.	Rewrite a number in index form and vice versa.		
3.	Relate the multiplication of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.		
4.	Relate the division of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.		
5.	Relate the numbers in index form raised to a power, to repeated multiplication, and hence make generalisation.		
6.	Verify that $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$; $a \neq 0$.		
7.	Determine and state the relationship between fractional indices and roots and powers.		
8.	Perform operations involving laws of indices.		
9.	Solve problems involving laws of indices.		

EXPLORING MATHEMATICS

Do you still remember the Pascal's Triangle that you learnt in the Chapter 1 Patterns and Sequences in Form 2?

The Pascal's Triangle, invented by a French mathematician, Blaise Pascal, has a lot of unique properties. Let us explore two unique properties found in the Pascal's Triangle.

Activity 1

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Sheet 1

Sum	Index form
1	2^0
2	2^1
4	2^2

Sheet 1(a)

Instructions:

1. Carry out the activity in pairs.
2. Construct the Pascal's Triangle as in Sheet 1.
3. Calculate the sum of the numbers in each row. Write the sum in index form with base of 2.
4. Complete Sheet 1(a). Discuss the patterns of answers obtained with your friends.
5. Present your results.

TIPS

$11^5 = 161\,051$

1	5	10	10	5	1
	+1	±1			
1	6	1	0	5	1

Activity 2

11^n	Value
11^0	1
11^1	11
11^2	121
11^3	1 331
11^4	
11^5	
11^6	
11^7	
11^8	
11^9	
11^{10}	

Sheet 2(a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Sheet 2

Instructions:

1. Carry out the activity in small groups.
2. Construct the Pascal's Triangle as in Sheet 2.
3. Take note of the numbers in each row. Each number is the value of index with base of 11.
4. Complete Sheet 2(a) with the value of index with base of 11 without using a calculator.
5. Present your results.
6. Are your answers the same as those of other groups?

CHAPTER 2

Standard Form

What will you learn?

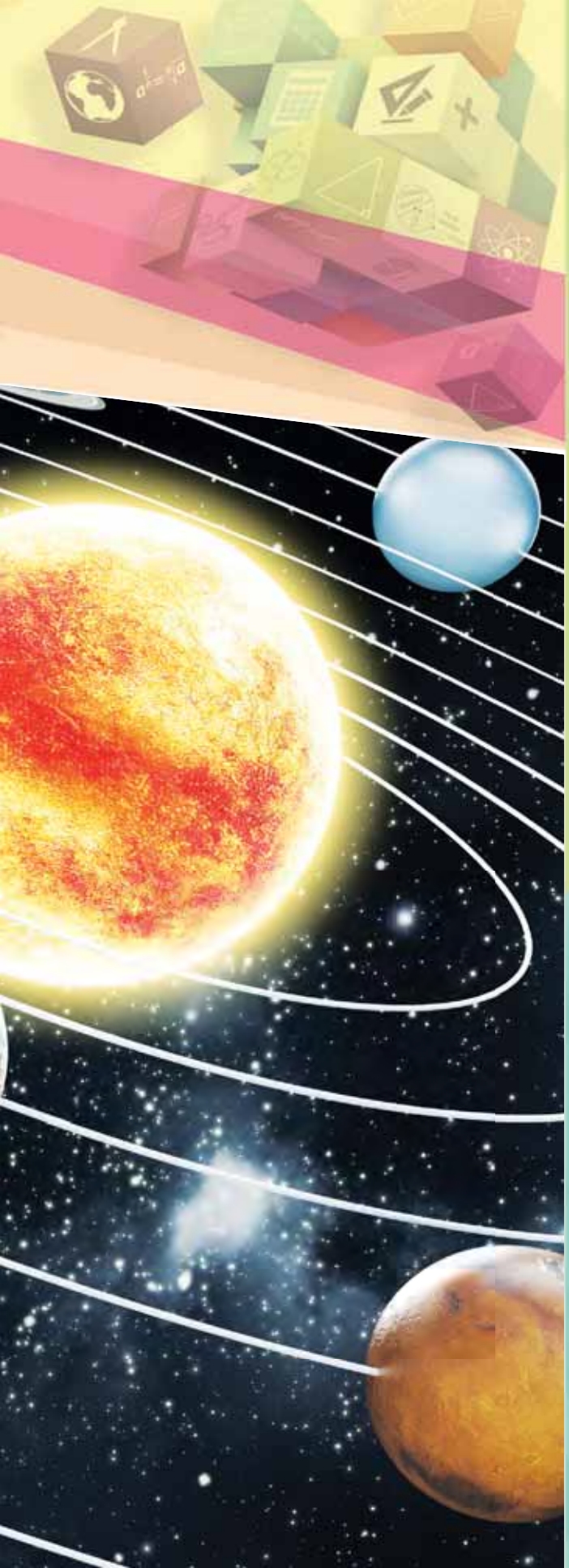
2.1 Significant Figures

2.2 Standard Form

Why do you learn this chapter?

- In scientific field, very big or very small numbers are commonly used. For example in astronomy, the distance between two stars is usually millions of kilometres while in the study of particles, the distance between atoms is extremely small.
- Numbers written in standard form are widely used in the field of science, engineering, astronomy and so on.

Distance in outer space, such as the distance between two stars in the galaxy, is measured in light years. One light year is the distance travelled by light in one year. One light year is equal to 9 500 000 000 000 km, that is 9.5 trillion kilometres. Small units such as nanometre are used for distances closer to zero. Do you know that 1 nanometre is equal to 0.000 000 001 metre?



Exploring Era

The ancient Greeks used a system based on myriad that is ten thousand. Ten myriads is equal to one hundred thousand.

Archimedes (287 BC – 212 BC) created a system of big numbers up to $10^8 \times 10^{16}$.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter2.pdf

WORD BANK

• estimation	• *anggaran*
• significant figure	• *angka bererti*
• standard form	• *bentuk piawai*
• accuracy	• *kejituan*
• single number	• *nombor tunggal*
• round off	• *pembundaran*
• approximation	• *penghampiran*

2.1 Significant Figures

What does significant figure mean and how do you determine the number of significant figures of a number?

LEARNING STANDARD

Explain the meaning of significant figure, and hence determine the number of significant figures of a number.

We use measurement in many situations in our daily life. Examples of frequently used measurements are length, distance, mass, temperature, area and speed.

The **estimation** of a measurement can be done using **approximation**. For example, the distance between the Earth and the Moon is 384 400 km. This value is an estimation calculated using certain methods and stated as an approximation.

The **degree of approximation** of a measurement to the **actual value** shows the level of accuracy of the measurement. The skill in making estimations and approximations can help you in many situations in daily life.

Brainstorming 1

In pairs

Aim: Determine the importance of making estimations and approximations in daily life.

Steps:

1. Read and understand the situations below.

Situation 1

Hashim is interested in a shirt sold in a supermarket with a 50% discount. The original price of the shirt is RM47.90. Hashim estimates the price of the shirt after discount and takes it to the cashier. The cashier informs him that the price of the shirt is RM28.70. Hashim argues that his estimation of the price is not more than RM25. Is Hashim's estimation correct?

Situation 2

Mrs Tan wants to buy 30 metres of cloth costing RM5.85 per metre to make curtains. She makes an estimation of the total price of the cloth and allocates RM180. Is the money allocated by Mrs Tan sufficient?

Discussion:

1. In the two situations above, how did Hashim and Mrs Tan make estimations of the total price?
2. Discuss with your friend the importance of making estimations and approximations.
3. State two other situations that require you to make estimations and approximations.

From Brainstorming 1, it is found that:

Approximating a value to a certain significant figure allows us to make an accurate estimation.

You have understood the importance of making estimation for the purpose of obtaining a value that is near the exact value. Significant figures are used to obtain the approximate value.

The significant figures of an integer or decimal refer to the digits in the number stated accurately to a certain degree of accuracy as required. The number of significant numbers is counted starting from a non-zero digit.

Brainstorming 2

In pairs

Aim: Determine the effect of the position of the zero digit in integers and decimals.

Steps:

1. Study the integer cards below.

3 210	3 201	3 021	0 321
Card 1	**Card 2**	**Card 3**	**Card 4**

Does the position of the zero digit have any effect on the value of digit 3?

2. Study the decimal cards below.

3.210	3.201	3.021	0.321
Card 5	**Card 6**	**Card 7**	**Card 8**

Does the position of the zero digit have any effect on the value of digit 3?

3. Study the decimal cards below.

3.210	3.2100	3.21000	3.210000
Card 9	**Card 10**	**Card 11**	**Card 12**

Does the position of the zero digit have any effect on the value of digit 2?

4. Discuss with your friend the effect of the position of the zero digit on the value of digit 3 in Card 1 to Card 8 and the effect of adding zero digits on the value of digit 2 in Card 9 to Card 12.
5. Present the results of your discussion. Compare your results with other pairs.

Discussion:

What is your conclusion concerning the position of the zero digit in an integer or decimal?

From Brainstorming 2, it is found that:

(a) Card 1, Card 2, Card 3, Card 5, Card 6 and Card 7
- The position of the zero digit between or at the end of the number, maintains the place value of digit 3.

(b) Card 4 and Card 8
- The position of the zero digit as the first digit has changed the place value of digit 3.

(c) Card 9, Card 10, Card 11 and Card 12
- The position of the zero digit at the end of the decimal does not change the place value of digit 2.

FLASHBACK

For digit 9 in the number 5 9 2 7;
- Place value – hundred
- Digit value – 900

In general,

- All non-zero digits are significant figures.
- The digit zero between non-zero digits is a significant figure.
- The digit zero at the end of an integer is a significant figure according to the level of accuracy required.
- The digit zero at the end of a decimal is a significant figure because it determines the level of accuracy of the decimal.
- The digit zero before the first non-zero digit is not a significant figure.

TIPS

♦ Zeros between non-zero digit are significant figures.
For example,
(a) 60 007
(5 significant figures).
(b) 50.0042
(6 significant figures).

♦ For a decimal, all digits before non-zero digit are not significant figures.
For example,
(a) 0.007
(1 significant figure).
(b) 0.005020
(4 significant figures).

♦ For a whole number, zero at the end of the number is not a significant figure unless stated otherwise.
For example,
(a) 8 750 = 8 800
(Rounded off to 2 significant figures).
(b) 8 750 = 9 000
(Rounded off to 1 significant figure).

How do you determine the number of significant figures?

Decimal

Not significant figure: Used only to determine place value of digit 5.

Significant figure: The digit zero between or at end of decimal is a significant figure.

0.00501400

Significant figure: All non-zero digits are significant figures.

Integer

803 000

Significant figure.

Significant figure **depending on** level of accuracy required.

Example 1

Determine the number of significant figures for the numbers below.

(a) 2 763 (b) 5 008 (c) 7 409 (d) 15 000
(e) 0.7803 (f) 0.0809 (g) 12.051 (h) 1.2700

Solution:

(a) 2 763 [4 s.f.]
(b) 5 008 [4 s.f.] → The digit zero between non-zero digit is a significant figure.
(c) 7 409 [4 s.f.] → The digit zero between non-zero digit is a significant figure.
(d) (i) 15 000 [2 s.f.] → If level of accuracy is to the nearest thousand.
(ii) 15 000 [3 s.f.] → If level of accuracy is to the nearest hundred.
(iii) 15 000 [4 s.f.] → If level of accuracy is to the nearest ten.
(iv) 15 000 [5 s.f.] → If level of accuracy is to the nearest one.
(e) 0.7803 [4 s.f.]
(f) 0.0809 [3 s.f.] → The digit zero before first non-zero digit is not significant figure.
(g) 12.051 [5 s.f.]
(h) 1.2700 [5 s.f.] → All zeros after non-zero digit at end of decimal are significant figures.

MIND TEST 2.1a

1. State the number of significant figures for the following numbers.

(a) 2 600 (b) 30 004 (c) 4 000 600 (d) 0.5003
(e) 0.080 (f) 9.0070 (g) 0.002000 (h) 30.0002

TIPS

Significant figure can be written as s.f..

How do you round off a number to certain numbers of significant figures?

LEARNING STANDARD

Round off a number to certain numbers of significant figures.

Do you still remember how to round off a number to a certain place value? The same concept and method are used to round off a number to a certain number of significant figures.

Example 2

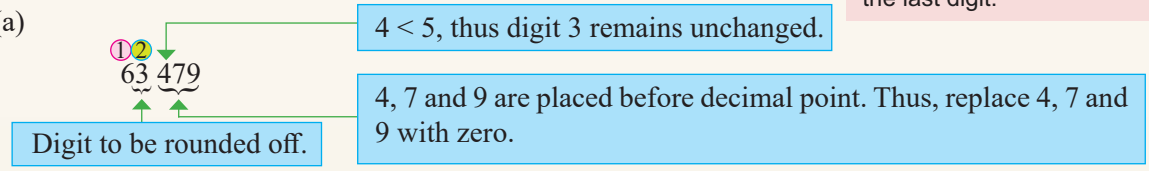
Round off each of the following numbers to 2 significant figures.
(a) 63 479 (b) 2 476 (c) 6 953

TIPS

For integers, the decimal point is placed behind the last digit.

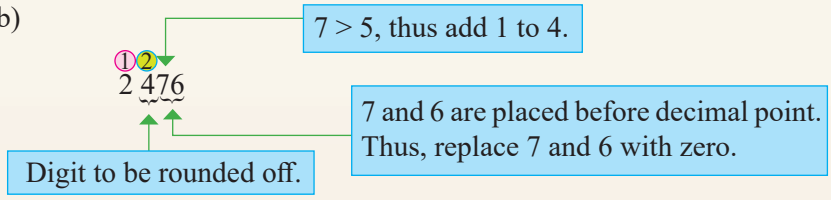
Solution:

(a)



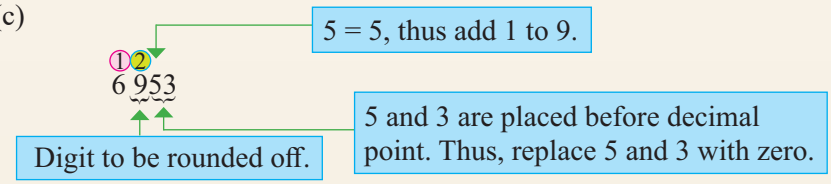
Thus, 63 479 = 63 000 (2 s.f.)

(b)



Thus, 2 476 = 2 500 (2 s.f.)

(c)



Thus, 6 953 = 7 000 (2 s.f.)

FLASHBACK

Round off 38 279 to the
(a) nearest hundred.
(b) nearest thousand.

Solution:

(a) 38 279
+1 (7 > 5)
= 38 300

(b) 38 279
(remain unchanged)
= 38 000

TIPS

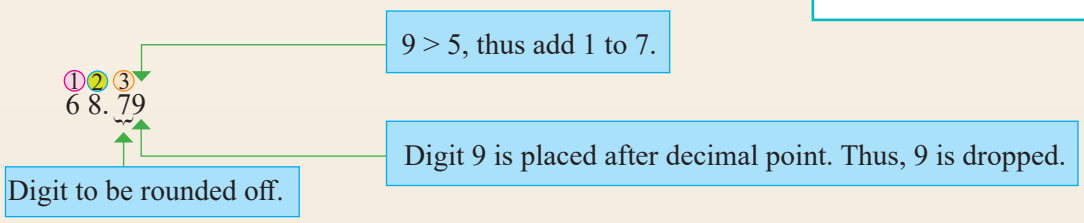
For integers, the first non-zero digit is a significant figure.

Example 3

Round off 68.79 to
(a) 3 significant figures (b) 1 significant figure

Solution:

(a)

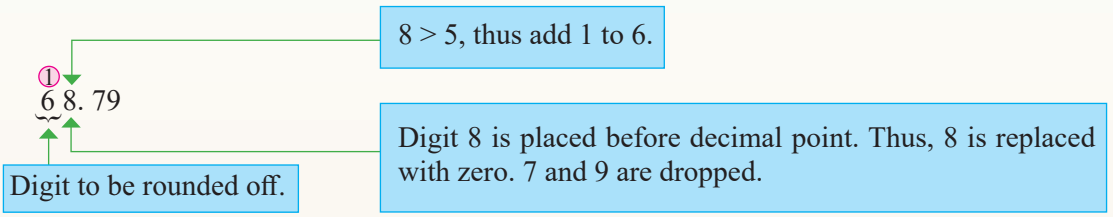


Thus, 68.79 = 68.8 (3 s.f.)

QUIZ

Why should the digits after the digit that is rounded off in a decimal be dropped?

(b)



Thus, 68.79 = 70 (1s.f.)

Example 4

Round off 0.008025 to

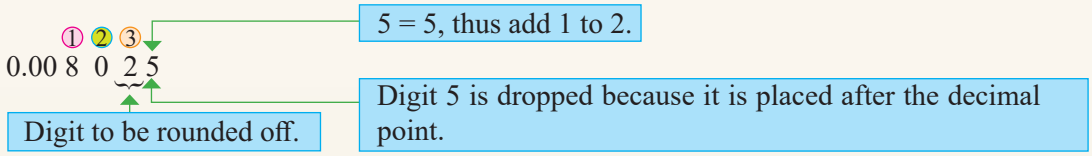
(a) 3 significant figures (b) 2 significant figures

QUIZ

Round off 10.09 to
1 significant figure and
2 significant figures.

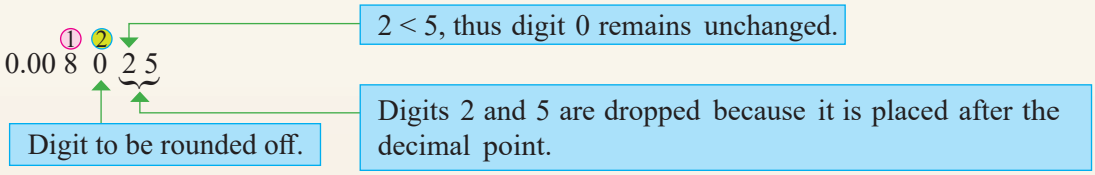
Solution:

(a)



Thus, 0.008025 = 0.00803 (3 s.f.)

(b)



Thus, 0.008025 = 0.0080 (2 s.f.)

MIND TEST 2.1b

1. Complete the table below by rounding off each number below to the given significant figure.

Number	3 significant figures	2 significant figures	1 significant figure
(a) 47 193			
(b) 5 261			
(c) 305.72			
(d) 20.68			
(e) 8.595			
(f) 5.9			
(g) 0.6937			
(h) 0.09184			
(i) 0.005709			

2. Calculate each operation below. State the answer to the significant figures shown in the brackets.

(a) $2.57 \times 4.5 + 0.45$ [4]
(b) $8.59 \div 2.1 - 1.26$ [3]
(c) $14.23 - 2.6 \times 1.2$ [3]
(d) $15.74 + 20.3 \div 2.5$ [2]
(e) $7.63 \times 0.5 \div 4.2 + 5.7$ [3]
(f) $10.25 \div 0.75 - 4.2 \times 0.2$ [2]
(g) $15.62 - 1.72 \times 0.2 + 6.3$ [1]
(h) $4.94 + 5.76 \div 0.26 \times 1.4$ [3]

2.2 Standard Form

How do you recognise and write numbers in standard form?

LEARNING STANDARD

Recognise and write numbers in standard form.

Many scientific fields such as astronomy, biology, physics and engineering frequently use numbers that are too big or too small in their research. These numbers are written in standard form to make writing easier.

Standard form is a way to write a **single number** in the form;

$$A \times 10^n$$

where $1 \leq A < 10$ and n is an integer.

For example, the land area of Malaysia is 330 803 000 000 m^2. This value can be written as 3.308×10^{11} m^2 or 3.30803×10^{11} m^2 or depending on the number of significant figures required.

How do you change a single number to standard form?

When a single number is changed to standard form:

- Numbers with value more than 1 is written as a positive index.
- Numbers with value less than 1 is written as a negative index.

FLASHBACK

- a^n is a positive index.
- a^{-n} is a negative index.

Example 5

Write the following single numbers in standard form.

(a) 28 (b) 280 (c) 2 805.3

Solution:

(a) $28 = 2.8 \times 10$

Place value is tens

Decimal point after first non-zero digit.

(b) $280 = 2.80 \times 100$
$= 2.8 \times 10^2$

Place value is hundreds

(c) $2\,805.3 = 2.8053 \times 1\,000$
$= 2.8053 \times 10^3$

Place value is thousands

Example 6

Write the following decimals in standard form.

(a) 0.325 (b) 0.00325 (c) 0.03025 (d) 0.003005

Solution:

(a) $0.325 = 3.25 \times \frac{1}{10}$
$= 3.25 \times 10^{-1}$

Place value is one tenths

(b) $0.00325 = 3.25 \times \frac{1}{1\,000}$
$= 3.25 \times \frac{1}{10^3}$
$= 3.25 \times 10^{-3}$

Place value is one thousandths

FLASHBACK

$$\frac{1}{a^n} = a^{-n}$$

DISCUSSION CORNER

Is 5.1×10^0 a number in standard form? Discuss.

(c) $0.03025 = 3.025 \times \frac{1}{100}$
$= 3.025 \times \frac{1}{10^2}$
$= 3.025 \times 10^{-2}$

Place value is one hundredths

(d) $0.003005 = 3.005 \times \frac{1}{1\,000}$
$= 3.005 \times \frac{1}{10^3}$
$= 3.005 \times 10^{-3}$

Place value is one thousandths

How do you change a number in standard form to single number?

When a number in standard form is changed to a single number:

- The number will be equal to 10 or more if the index is positive.
- The number will be less than 1 if the index is negative.

Example 7

Write 4.17×10^5 as a single number.

Solution:

$4.17 \times 10^5 = 4.17 \times 100\,000$
$= 417\,000$

FLASHBACK

$10^5 = 10 \times 10 \times 10 \times 10 \times 10$

$10^{-5} = \frac{1}{10^5}$

Example 8

Write 8.063×10^{-5} as a single number.

Solution:

$8.063 \times 10^{-5} = 8.063 \times \frac{1}{100\,000}$
$= 0.00008063$

BULLETIN

1 tera = 1 000 000 000 000

1 nano = 0.000 000 001

Example 9

Determine 3 050 terabytes in bytes. State the answer in standard form.

Solution:

3 050 terabytes $= 3\,050 \times 10^{12}$ bytes
$= (3.05 \times 10^3) \times 10^{12}$ bytes
$= (3.05 \times 10^{3+12})$ bytes ← Use index law $a^m \times a^n = a^{m+n}$
$= 3.05 \times 10^{15}$ bytes

SMART MIND

What is the value of 1 tera in nano?

Example 10

Determine 0.0057 nanometre in metre. State your answer in standard form.

Solution:

0.0057 nanometre $= 0.0057 \times 10^{-9}$ metre
$= (5.7 \times 10^{-3}) \times 10^{-9}$ metre
$= (5.7 \times 10^{-3+(-9)})$ metre ← Use index law $a^m \times a^n = a^{m+n}$
$= (5.7 \times 10^{-3-9})$ metre
$= 5.7 \times 10^{-12}$ metre

Brainstorming 3

In pairs

Aim: Write metric measurements in standard form.

Steps:

1. Complete the table below by writing the single numbers for metric measurements in standard form.

Prefix	Symbol	Value	
		Single number	Standard form
exa	E	1 000 000 000 000 000 000	1×10^{18}
peta	P	1 000 000 000 000 000	
tera	T	1 000 000 000 000	
giga	G	1 000 000 000	
mega	M	1 000 000	
kilo	k	1 000	
hecto	h	100	
deca	da	10	
–	–	1	1×10^{0}
deci	d	0.1	1×10^{-1}
centi	c	0.01	
milli	m	0.001	
micro	μ	0.000 001	
nano	n	0.000 000 001	
pico	p	0.000 000 000 001	
femto	f	0.000 000 000 000 001	
atto	a	0.000 000 000 000 000 001	

Discussion:

A number which is too big or too small in value can be written as a single number or in standard form. Which form will you choose for an arithmetic operation? Give your reasons.

From Brainstorming 3, it is found that:

Standard form makes it easier to write very big and very small numbers in a form that is simple and easy to understand.

MIND TEST 2.2a

TIPS

Use data from Brainstorming 3 to solve question 3.

1. Write the following single numbers in standard form.
   (a) 35 (b) 481 (c) 5 075 (d) 97.25
   (e) 3 124.3 (f) 0.9 (g) 0.23 (h) 0.0375

2. Change the numbers in standard form to single numbers.
   (a) 2.5×10^{0} (b) 3.75×10^{1} (c) 4.23×10^{2}
   (d) 5.07×10^{3} (e) 9.1×10^{4} (f) 6.2×10^{-1}
   (g) 7.29×10^{-2} (h) 1.034×10^{-3} (i) 8.504×10^{-4}

3. Change the following metric measurements to the units given in the brackets. State your answers in standard form.
   (a) 1 050 kilometres [metre] (b) 216 gigabytes [byte]
   (c) 0.75 teralitre [litre] (d) 95 micrometres [metre]
   (e) 123 nanometres [metre] (f) 0.089 femtometre [metre]

How are basic arithmetic operations involving numbers in standard form performed?

LEARNING STANDARD

Perform basic arithmetic operations involving numbers in standard form.

Operations of addition and subtraction

Example 11

Calculate the value of each of the following operations. State your answer in standard form.

(a) $2.73 \times 10^3 + 5.92 \times 10^3$

(b) $4.27 \times 10^5 + 9.35 \times 10^5$

(c) $7.02 \times 10^4 + 2.17 \times 10^5$

(d) $9.45 \times 10^6 - 3.24 \times 10^5$

Solution:

(a) $2.73 \times 10^3 + 5.92 \times 10^3$
$= (2.73 + 5.92) \times 10^3$
$= 8.65 \times 10^3$

Factorise 10^3

(b) $4.27 \times 10^5 + 9.35 \times 10^5$
$= (4.27 + 9.35) \times 10^5$
$= 13.62 \times 10^5$
$= (1.362 \times 10) \times 10^5$
$= 1.362 \times 10^1 \times 10^5$
$= 1.362 \times 10^{1+5}$
$= 1.362 \times 10^6$

FLASHBACK

- $5a^n + 7a^n$
  $= (5 + 7)a^n$
  $= 12a^n$
- $5 \times 10^n + 7 \times 10^n$
  $= (5 + 7)10^n$
  $= 12(10^n)$

(c)

Method 1

$7.02 \times 10^4 + 2.17 \times 10^5$
$= 7.02 \times 10^4 + 2.17 \times 10^1 \times 10^4$
$= 7.02 \times 10^4 + 21.7 \times 10^4$
$= (7.02 + 21.7) \times 10^4$
$= 28.72 \times 10^4$
$= 2.872 \times 10^1 \times 10^4$
$= 2.872 \times 10^{1+4}$
$= 2.872 \times 10^5$

10^5 change to $10^1 \times 10^4$ to simplify calculation.

Method 2

$7.02 \times 10^4 + 2.17 \times 10^5$
$= 7.02 \times 10^{-1} \times 10^5 + 2.17 \times 10^5$
$= 0.702 \times 10^5 + 2.17 \times 10^5$
$= (0.702 + 2.17) \times 10^5$
$= 2.872 \times 10^5$

TIPS

For operations involving addition and subtraction, change index with small value to index with large value as in method 2 of example (c) and example (d).

(d)

Method 1

$9.45 \times 10^6 - 3.24 \times 10^5$
$= 9.45 \times 10^1 \times 10^5 - 3.24 \times 10^5$
$= 94.5 \times 10^5 - 3.24 \times 10^5$
$= (94.5 - 3.24) \times 10^5$
$= 91.26 \times 10^5$
$= 9.126 \times 10^1 \times 10^5$
$= 9.126 \times 10^{1+5}$
$= 9.126 \times 10^6$

Method 2

$9.45 \times 10^6 - 3.24 \times 10^5$
$= 9.45 \times 10^6 - 3.24 \times 10^{-1} \times 10^6$
$= 9.45 \times 10^6 - 0.324 \times 10^6$
$= (9.45 - 0.324) \times 10^6$
$= 9.126 \times 10^6$

SMART MIND

Calculate the following without using a calculator.

- $2.4 \times 10^3 + 1.3 \times 10^5$
- $8.5 \times 10^4 - 1.2 \times 10^2$

Example 12

Calculate the value of each of the following operations. State the answer in standard form.

(a) $3.58 \times 10^{-3} + 9.24 \times 10^{-3}$ (b) $8.21 \times 10^{-4} + 1.49 \times 10^{-5}$

(c) $2.3 \times 10^{-5} - 4.6 \times 10^{-6}$

Solution:

(a) $3.58 \times 10^{-3} + 9.24 \times 10^{-3} = (3.58 + 9.24) \times 10^{-3}$

$= 12.82 \times 10^{-3}$

$= 1.282 \times 10^{1} \times 10^{-3}$

$= 1.282 \times 10^{1 + (-3)}$

$= 1.282 \times 10^{-2}$

(b)

Method 1

$8.21 \times \boxed{10^{-4}} + 1.49 \times 10^{-5}$

$= 8.21 \times \boxed{10^{1} \times 10^{-5}} + 1.49 \times 10^{-5}$

$= 82.1 \times 10^{-5} + 1.49 \times 10^{-5}$

$= (82.1 + 1.49) \times 10^{-5}$

$= 83.59 \times 10^{-5}$

$= \boxed{8.359 \times 10^{1}} \times 10^{-5}$

$= 8.359 \times 10^{1 + (-5)}$

$= 8.359 \times 10^{-4}$

Method 2

$8.21 \times 10^{-4} + 1.49 \times \boxed{10^{-5}}$

$= 8.21 \times 10^{-4} + \boxed{1.49 \times 10^{-1}} \times 10^{-4}$

$= 8.21 \times 10^{-4} + 0.149 \times 10^{-4}$

$= (8.21 + 0.149) \times 10^{-4}$

$= 8.359 \times 10^{-4}$

(c)

Method 1

$2.3 \times \boxed{10^{-5}} - 4.6 \times 10^{-6}$

$= 2.3 \times \boxed{10^{1} \times 10^{-6}} - 4.6 \times 10^{-6}$

$= 23 \times 10^{-6} - 4.6 \times 10^{-6}$

$= (23 - 4.6) \times 10^{-6}$

$= 18.4 \times 10^{-6}$

$= \boxed{1.84 \times 10^{1}} \times 10^{-6}$

$= 1.84 \times 10^{1 + (-6)}$

$= 1.84 \times 10^{-5}$

Method 2

$2.3 \times 10^{-5} - 4.6 \times \boxed{10^{-6}}$

$= 2.3 \times 10^{-5} - \boxed{4.6 \times 10^{-1}} \times 10^{-5}$

$= 2.3 \times 10^{-5} - 0.46 \times 10^{-5}$

$= (2.3 - 0.46) \times 10^{-5}$

$= 1.84 \times 10^{-5}$

SMART TECHNOLOGY

1. Press Mode button a few times until the screen shows:

   Fix Sci Norm
   1 2 3

2. Press 2 to choose Sci, that is, standard form.
3. Enter number of significant figures (s.f.) needed, for example 9.
4. Enter the required operation.

   ♦ $3.2 \times 10^{5} - 4.2 \times 10^{4}$

   Press 3.2 Exp 5 − 4.2 Exp 4.

   Screen display:
   3.2 E5 − 4.2 E4

   Press = 2.78×10^{5}.

   ♦ $4 \times 10^{5} \times 3.7 \times 10^{4}$

   Press 4 Exp 5 × 3.7 Exp 4.

   Screen display:
   4 Exp 5 × 3.7 Exp 4

   Press = 1.48×10^{10}.

5. Extend your exploration to other operations involving other standard forms.
6. Compare the results produced by calculator with answers obtained through manual calculations.

MIND TEST 2.2b

1. Calculate the value of each of the following operations. State your answer in standard form.

(a) $2.4 \times 10^{4} + 3.57 \times 10^{4}$ (b) $8.2 \times 10^{6} - 4.27 \times 10^{6}$

(c) $5.23 \times 10^{7} + 4.98 \times 10^{7}$ (d) $1.2 \times 10^{5} + 3.74 \times 10^{4}$

(e) $5.7 \times 10^{8} - 2.4 \times 10^{7}$ (f) $5.7 \times 10^{3} + 8.02 \times 10^{4}$

(g) $6.5 \times 10^{4} - 7.3 \times 10^{3}$ (h) $5.2 \times 10^{-3} - 4.12 \times 10^{-3}$

(i) $8.74 \times 10^{-5} - 2.65 \times 10^{-5}$ (j) $4.1 \times 10^{-4} + 9.5 \times 10^{-3}$

(k) $8.3 \times 10^{-4} - 6.2 \times 10^{-5}$ (l) $9.42 \times 10^{-6} - 7.35 \times 10^{-7}$

Operations of Multiplication and Division

TIPS

Law of Indices
- Operation of multiplication
  $(A \times 10^m) \times (B \times 10^n) = (A \times B) \times 10^{m+n}$
- Operation of division
  $(A \times 10^m) \div (B \times 10^n) = (A \div B) \times 10^{m-n}$

Example 13

Solve the following operations. State your answers in standard form.

(a) $3 \times 10^5 \times 4.9 \times 10^2$

(b) $7.5 \times 10^{-3} \times 5 \times 10^{-6}$

(c) $\dfrac{5.9 \times 10^5}{2 \times 10^2}$

(d) $\dfrac{6.8 \times 10^{-3}}{4 \times 10^{-6}}$

Solution:

(a) $3 \times 10^5 \times 4.9 \times 10^2$
$= (3 \times 4.9) \times 10^{5+2}$
$= 14.7 \times 10^7$
$= 1.47 \times 10^1 \times 10^7$
$= 1.47 \times 10^{1+7}$
$= 1.47 \times 10^8$

(b) $7.5 \times 10^{-3} \times 5 \times 10^{-6}$
$= (7.5 \times 5) \times 10^{-3+(-6)}$
$= 37.5 \times 10^{-9}$
$= 3.75 \times 10^1 \times 10^{-9}$
$= 3.75 \times 10^{1+(-9)}$
$= 3.75 \times 10^{-8}$

(c) $\dfrac{5.9 \times 10^5}{2 \times 10^2}$
$= \dfrac{5.9}{2} \times 10^{5-2}$
$= 2.95 \times 10^3$

(d) $\dfrac{6.8 \times 10^{-3}}{4 \times 10^{-6}}$
$= \dfrac{6.8}{4} \times 10^{-3-(-6)}$
$= 1.7 \times 10^3$

MIND TEST 2.2c

1. Calculate the value of each of the following operations. State your answer in standard form.

(a) $4 \times 10^5 \times 3.7 \times 10^2$

(b) $7.5 \times 10^{-3} \times 5 \times 10^{-6}$

(c) $6.3 \times 10^5 \times 4.0 \times 10^2$

(d) $5.3 \times 10^{-3} \times 4 \times 10^5$

(e) $(1.08 \times 10^2) \div (2.4 \times 10^4)$

(f) $(9.6 \times 10^{-2}) \div (1.5 \times 10^{-5})$

(g) $(5.9 \times 10^5) \div (2 \times 10^2)$

(h) $(2.58 \times 10^4) \div (0.3 \times 10^{-4})$

2. A mobile swimming pool measures 305 cm × 183 cm × 56 cm. Calculate the maximum volume of water that it can hold in litres. State your answer in standard form and correct to four significant figures.

BULLETIN

1 litre = 1 000 cm^3
1 litre = 0.001 m^3

3. Syazwani wants to transfer 2 terabytes of data to pen drives with a capacity of 32 gigabytes. What is the minimum number of 32-gigabyte pen drives needed?

4. Given 1 millimetre = 10^{-3} metre and 1 micrometre = 10^{-6} metre, state 1 millimetre in micrometre.

DISCUSSION CORNER

Between operation of addition or subtraction and operation of multiplication or division involving standard form, which operation is easier? Why?

How do you solve problems involving numbers in standard form?

LEARNING STANDARD

Solve problems involving numbers in standard form.

Example 14

A ream of paper contains 800 sheets of paper. The thickness of one sheet of paper is 9.4×10^{-3} cm. Given the total thickness of n reams of paper is 225.6 cm, calculate the value of n.

Solution:

Understanding the problem

Number of sheets in 1 ream = 800

Thickness of 1 sheet = 9.4×10^{-3} cm

Thickness of n reams = 225.6 cm

Planning a strategy

- Determine the thickness of 1 ream of paper.
- $n = \dfrac{\text{thickness of } n \text{ reams}}{\text{thickness of 1 ream}}$

Implementing the strategy

Thickness of 1 ream
$= 800 \times 9.4 \times 10^{-3}$ cm
$= 7.52$ cm

Thus,

$$n = \frac{\text{thickness of } n \text{ reams}}{\text{thickness of 1 ream}}$$

$$n = \frac{225.6 \text{ cm}}{7.52 \text{ cm}}$$

$$n = 30$$

Making a conclusion

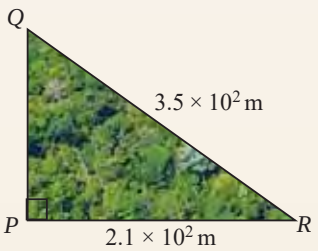
Number of reams is 30.

Example 15

A property firm bought a piece of land in the shape of a right-angled triangle PQR as shown in the diagram.

(a) Calculate the value of PQ, in metres, and state your answer in standard form.

(b) If the cost of one square metre of the land is RM45, calculate the total cost of the land in RM.



Solution:

Understanding the problem

ΔPQR is a right-angled triangle.
QR is the hypotenuse.

Planning a strategy

(a) Calculate PQ using Pythagoras theorem.

(b) Calculate the area of land in the shape of ΔPQR. Multiply total land area by cost of 1 m^2 of land.

Implementing the strategy

(a) $PQ^2 = [(3.5 \times 10^2)^2 - (2.1 \times 10^2)^2]\ m^2$
$= [1.225 \times 10^5 - 4.41 \times 10^4]\ m^2$
$= (7.84 \times 10^4)\ m^2$

$PQ = \sqrt{(7.84 \times 10^4)\ m^2}$
$= 2.8 \times 10^2$ m

(b) Area of $\Delta PQR = \frac{1}{2} \times (2.1 \times 10^2)\text{m} \times (2.8 \times 10^2)\text{m}$
$= 2.94 \times 10^4\ m^2$

Cost of land $= 2.94 \times 10^4 \times$ RM45
$=$ RM1 323 000.00

Making a conclusion

(a) Distance $PQ = 2.8 \times 10^2$ m

(b) Total cost of land = RM1 323 000.00

Example 16

The picture shows the Earth with a diameter of 1.2742×10^4 km. Calculate the surface area of the Earth, in km^2. State the answer in standard form correct to four significant figures. [Surface area of sphere = $4\pi r^2$ and $\pi = 3.142$]

Solution:

Understanding the problem

- Earth is a sphere.
- Diameter of Earth is 1.2742×10^4 km.
- Answer is in standard form correct to four significant figures.

Planning a strategy

- Radius = $\frac{\text{diameter}}{2}$.
- Use formula for surface area of sphere to calculate surface area of Earth.

Implementing the strategy

Radius of Earth $= \left(\frac{1.2742 \times 10^4}{2}\right)$ km
$= 6.371 \times 10^3$ km

Surface area of Earth
$= 4\pi r^2$
$= [4(3.142)(6.371 \times 10^3)^2]$ km^2
$= 510\,130\,608.1$ km^2
$= 5.101 \times 10^8$ km^2 (4 s.f.)

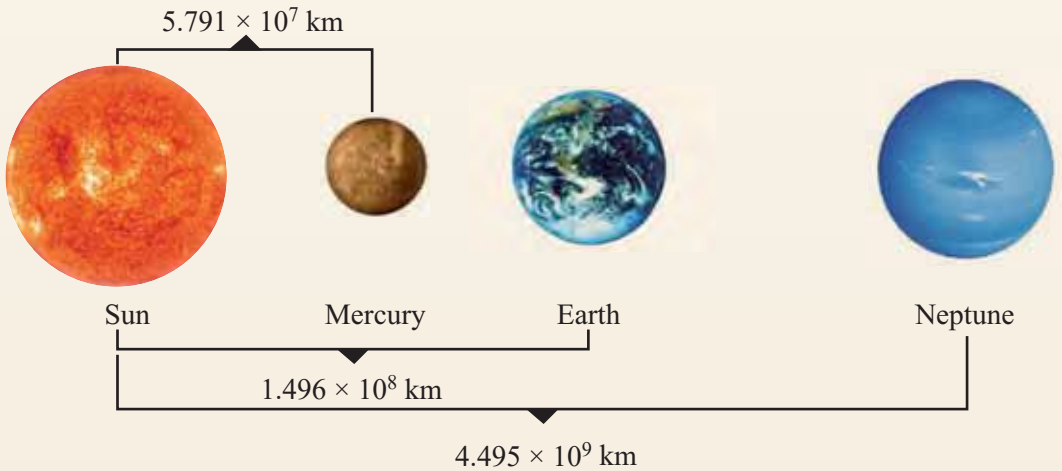
Making a conclusion

Surface area of Earth is 5.101×10^8 km^2

MIND TEST 2.2d

1. The average daily water consumption in a residential area is 6 950 m^3. Calculate the total water consumption, in cubic metres, in the residential area for February 2016. State the answer in standard form correct to three significant figures.

2.



The picture above shows the estimated distance of three planets in the solar system from the Sun on a certain day. Calculate the difference in distance, in km, between

(a) Mercury and Earth (b) Mercury and Neptune (c) Earth and Neptune

State the answers in standard form correct to three significant figures.

Dynamic Challenge

Test Yourself

1. Round off the following numbers and decimals correct to the significant figures stated in the brackets.

(a) 23 725 [2] (b) 54 299 [4] (c) 8 999 [2] (d) 295 197 [2]

(e) 4 854 [1] (f) 5 [3] (g) 0.2763 [2] (h) 35.074 [1]

(i) 423.575 [2] (j) 10.234 [1] (k) 1.0372 [3] (l) 501.724 [3]

2. Given $m = 3.2 \times 10^3$ and $n = 5.43 \times 10^4$, calculate the values of the following operations. State your answers in standard form correct to three significant figures.

(a) $2mn$ (b) $m + n$ (c) $n - m$ (d) $m^2 + n^2$

(e) $\frac{3m}{2n}$ (f) $\frac{m+n}{mn}$ (g) $m^{-2} + n^{-3}$ (h) $n - m^{-3}$

3. Complete the following.

(a) $2.5 \times 10^2 + 1.35 \times 10^4$

$= 2.5 \times 10^{\square} \times 10^4 + 1.35 \times 10^4$

$= \square \times 10^4 + 1.35 \times 10^4$

$= (\square + \square) \times 10^4$

$= \square \times 10^4$

(b) $5.74 \times 10^{-3} + 3.4 \times 10^{-6}$

$= 5.74 \times 10^{-3} + 3.4 \times 10^{\square} \times 10$

$= 5.74 \times 10^{-3} + \square \times 10^{-3}$

$= (\square + \square) \times 10^{-3}$

$= \square \times 10^{-3}$

(c) $1.75 \times 10^2 - 4.2 \times 10^{-1}$

$= 1.75 \times 10^2 - 4.2 \times 10^{\square} \times 10^2$

$= 1.75 \times 10^2 - \square \times 10^2$

$= (\square + \square) \times 10^2$

$= \square \times 10^2$

(d) $3.7 \times 10^{-2} - 4.3 \times 10^{-5}$

$= 3.7 \times 10^{-2} - 4.3 \times 10^{\square} \times 10^{-2}$

$= 3.7 \times 10^{-2} - \square \times 10^{-2}$

$= (\square - \square) \times 10^{-2}$

$= \square \times 10^{-2}$

4. A factory produces 72 thousand packets of chips every week. If the factory operates 6 days a week and 18 hours a day, calculate

(a) the number of packets of chips produced every day. State your answer in standard form.

(b) the average profit per hour if the net profit of one packet of chips is 32 sen. State the answer to the nearest RM.

5. The estimated population of Malaysia for 2018 is 32 million. Given Malaysia's land area is 330 803 km^2, calculate the population density of Malaysia for each square kilometre for 2018.

State your answer correct to the nearest integer.

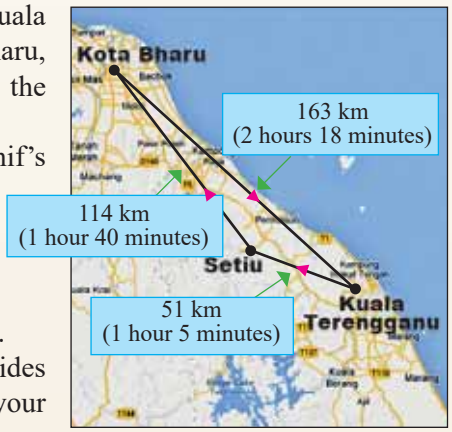
Skills Enhancement

1. A newly built community hall required 6 185 pieces of tiles measuring 30 cm × 30 cm for the floor.

(a) Calculate the floor area of the hall in square metres. State your answer in standard form correct to three significant figures.

(b) Given the cost of one piece of tile is RMl.75, calculate the total cost of the tiles to the nearest RM.

2. Encik Hanif drove his car from Kota Bharu to Kuala Terengganu to visit his son. On the way back to Kota Bharu, Encik Hanif made a stop at Setiu. The map shows the distance and travelling time of Encik Hanif.



(a) Calculate the average speed, in km h^{-1}, of Encik Hanif's car for the journey

(i) from Kota Bharu to Kuala Terengganu

(ii) from Kuala Terengganu to Setiu

(iii) from Setiu to Kota Bharu

State the answers correct to three significant figures.

(b) Encik Hanif is a safety-conscious driver who abides by the speed limit. Is this statement true? State your reasons.

Self Mastery

1. The picture shows three planets in the Solar System.

Mercury
[Diameter = 4 879 km]

Neptune
[Diameter = 49 244 km]

Jupiter
[Diameter = 139 822 km]

(a) Calculate the surface area, in km^2, of all three planets. State the answers in standard form correct to three significant figures.
[Surface area of sphere = $4\pi r^2$ and $\pi = 3.142$]

(b) Based on your answer in (a), calculate the difference in surface area between the largest and smallest planets in the Solar System. State the answer correct to four significant figures.

2.

The diagram above shows two types of A4-sized paper with different masses. GSM means grams per square metre.
Calculate the mass of one piece of A4-sized paper, in grams for

(a) 70 GSM (b) 80 GSM

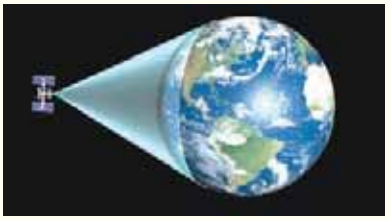
State the answers in standard form correct to three significant figures.

PROJECT

1. Look at the pictures below. Obtain the data relevant to the required measurement. Your answers should be in standard form.

2. You can surf various websites or refer to reference books to obtain interesting data related to the pictures below.

(a) Mass



(b) Population

(c) Distance

(d) Magnitude

3. Obtain other interesting facts that involve calculations in standard form.

4. Present your findings using multimedia applications.

CONCEPT MAP

Standard Form

Significant figure shows the level of **accuracy** of a measurement.

All digits are significant figures **except the zero before the first non-zero digit**.

(a) 0.023 (2 s.f.) (b) 0.102 (3 s.f.)
(c) 1.200 (4 s.f.) (d) 10 518 (5 s.f.)

For **integers**, the value of the significant figure for **zero** as the last digit depends on the **required level of accuracy**.

(a) 93 000 – 5 s.f. (nearest one)
(b) 93 000 – 4 s.f. (nearest ten)
(c) 93 000 – 3 s.f. (nearest hundred)
(d) 93 000 – 2 s.f. (nearest thousand)

Rounding off a number to a certain number of significant figures.

(a) 2 853
3 000 (1 s.f.)
2 900 (2 s.f.)
2 850 (3 s.f.)

(b) 62.54
60 (1 s.f.)
63 (2 s.f.)
62.5 (3 s.f.)

(c) 0.02704
0.03 (1 s.f.)
0.027 (2 s.f.)
0.0270 (3 s.f.)

Standard form is written as $\mathbf{A \times 10^n}$ where $\mathbf{1 \leqslant A < 10}$ and n is an integer.

Changing **single numbers** to **standard form** and vice versa.

(a) $534\,000 = 5.34 \times 10^5$
(b) $0.000\,534 = 5.34 \times 10^{-4}$
(c) $2.763 \times 10^4 = 27\,630$
(d) $2.763 \times 10^{-4} = 0.0\,002\,763$

Basic operations (+, –, ×, ÷) involving numbers in standard form.

(a) $S \times 10^n + T \times 10^n$
$= (S + T) \times 10^n$

(b) $S \times 10^n - T \times 10^n$
$= (S - T) \times 10^n$

(c) $(S \times 10^m) \times (T \times 10^n)$
$= (S \times T) \times 10^{m+n}$

(d) $(S \times 10^m) \div (T \times 10^n)$
$= (S \div T) \times 10^{m-n}$

SELF-REFLECT

At the end of this chapter, I can:

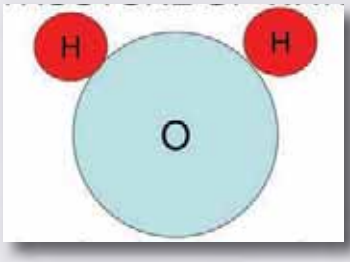
1.	Explain the meaning of significant figure and thus determine the number of significant figures of a number.		
2.	Round off a number to a certain number of significant figures.		
3.	Recognise and write numbers in standard form.		
4.	Perform basic arithmetic operations involving numbers in standard form.		
5.	Solve problems involving numbers in standard form.		

EXPLORING MATHEMATICS

1. Get into groups.

2. By using the various sources available, identify several measurement values in daily life that are very small or very big. For example,

Hard disk
(1 terabyte)



One water molecule
(0.1 nanometer)

One virus
(1 micrometer)

3. Prepare a report on your findings using multimedia applications.

4. Present your report.

5. Obtain additional information from the presentations of the other groups.

6. Discuss the advantages of using standard form in various fields.

CHAPTER 3

Consumer Mathematics: Savings and Investments, Credit and Debt

What will you learn?

3.1 Savings and Investments

3.2 Credit and Debt Management

Why do you learn this chapter?

- Knowledge of savings and investments can help us to manage our finances.
- The concept of savings and investments is used in banking, stocks, real estate, business, finance, accounting and so on.

"Many a little makes a mickle".
The above proverb means it is a good habit to save money for the future. Saving habits that have been practised since childhood can help a person cope with any emergency. Investments made by an individual must be in a timely manner in accordance with the current market.

Exploring Era

The barter system was practised before the use of money in the economy and was the earliest form of business in the world.

History of money development began with the evolution of the human civilisation itself, which was around 2 000 BC.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter3.pdf

WORD BANK

• liquidity	• *kecairan*
• interest	• *faedah*
• debt	• *hutang*
• interest rate	• *kadar faedah*
• credit	• *kredit*
• investment	• *pelaburan*
• loan	• *pinjaman*
• personal loan	• *pinjaman peribadi*
• return	• *pulangan*
• savings	• *simpanan*

3.1 Savings and Investments

What are savings and investments?

LEARNING STANDARD

Recognise various types of savings and investments.

Savings refer to excess money deposited in the safe, money box or drawer. Extra money can also be deposited at a bank that will provide returns based on interest rates and savings periods. There are some common ways of saving in the bank.

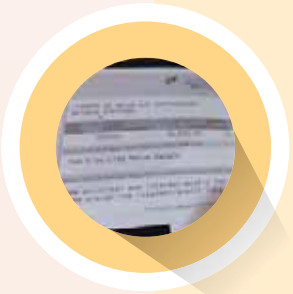
BULLETIN

Credit counselling and credit agencies encourage each individual to save 10% of their monthly income.

Savings Account

- The savings account holder can save any amount according to his ability.
- The account holder receives the interest based on the total amount and duration of savings.
- Interest rates are lower compared to fixed deposit accounts.
- The account holder can withdraw the savings at any time.
- The savings can be withdrawn by using a debit card via an automatic teller machine (ATM).

Fixed Deposit Account



- A sum of money is saved for a certain period of time such as 3 months, 9 months or 1 year tenure.
- Account holders will be offered more competitive interest rates compared to savings accounts.
- Savings cannot be withdrawn before the maturity date.
- If the money is withdrawn before maturity, the actual interest rate that should be received, will be reduced and will be cancelled at a certain time.
- A savings certificate will be issued to the account holder.

Current Account

- Savings in a current account can be used for personal or business purposes.
- The account holder may make payment to another party by cheque.
- Savings in the account will not be paid interest and is subjected to service charges. However, there are certain banks that pay interest to current account holders.
- The current account applicant must submit a referral who is an existing current account holder at the same bank to open the account.
- In addition to cheques, normal withdrawals are usually allowed via debit cards and other channels such as Internet banking, telephone banking and so on.
- The account holder can enjoy an overdraft facility, that is, withdrawing money beyond the balance of the deposit, but with interest charges.

Investment is an alternative step for future returns in the form of current income and capital gains. Types of investments are as follows:

Shares

A company will issue shares for the purpose of raising capital. An individual who purchases shares from a company is the owner of the company under certain conditions. The shareholders will receive returns in the form of dividends and capital gains.

Unit Trust

Trust fund is controlled by a unit trust company that is managed by a qualified professional manager in the field of investment. Those who have no knowledge of the purchase of shares can get help from the unit trust companies to manage their money. Unit trust companies collect money from investors and the money is invested in various potential companies with the aim of providing returns that benefit investors.

Real Estate

Investments on immovable assets such as residential houses, shops, land and others are investments in real estate. Investors should consider various aspects before investing.

Factors to be considered in real estate investment are economic situations, income-generating capabilities that is rent, location and property prospects in the future. Individuals who invest in real estate will receive an investment return in the form of rent and capital gains.

BULLETIN

There are two types of investors, that is, aggressive and moderate investors:

- **Aggressive investors** – invest in stock market.
- **Moderate investors** – buy unit trusts, bonds and equity funds.

TIPS

Luxury Stocks (Blue Chip Stocks) are stocks of large companies with track records of excellent business such as Maybank, TNB and Petronas.

BULLETIN

- Investment returns comprise current income and capital gains.
- Return on current income – rents, dividends, bonus shares.
- Capital gains – additional or increased investment value from its original amount. For example, stock prices rose from RM2.00 to RM2.20, thus the addition of RM0.20 was an increase in investment value.

QUIZ

Is the purchase of life insurance and health insurance considered as an investment or savings?

Brainstorming 1

In groups

Aim: To identify types of savings and investments.

Steps:

1. Get into groups of five or six students. Each group should state the types of savings and investments according to the statements given and explain the characteristics of the savings and investments specified.
2. The information collected should be presented in the form of reports as shown below.

	Type of savings	Type of investment	Description
1. Encik Rizal saves a total of RM300 in the bank.	Savings		Savings account – the amount of money saved is small and can be withdrawn at any time.
2. Cik Zeti is a dealer who keeps a sum of money in the bank with the intention of issuing a cheque to pay the creditor.			
3. Mrs Rani uses the money received from her father to buy a shoplot.			
4. Puan Faridah saved a sum of RM20 000 in the bank to finance the education of her children in the future.			
5. Mr Lee bought 1 000 units of unit trusts.			
6. Ms Sharon bought 4 000 units of Bank Orkid Berhad shares worth RM1.00 per share on the Kuala Lumpur Stock Exchange.			

Discussion:

State the advantages and disadvantages of each type of savings and investments.

From Brainstorming 1, it is found that savings and investments are different.

MIND TEST 3.1a

1. What is the purpose of an individual's savings?
2. Your father has RM5 000 and has not used it for a long time. What is your advice to him? Explain your answer.
3. Besides merchants, why aren't most people interested in opening a current account?

What do you understand about the benefits of savings?

LEARNING STANDARD

Perform calculations involving simple interest and compound interest for savings, and hence explain the impact of changes in period, rate of interest or return and compounding frequency on the future value of savings.

Interest for savings are rewards paid by financial institutions such as banks to depositors. The interests can be divided into two types, namely **simple interest** and **compound interest**.

Simple interest

Simple interest is a reward given to the depositor at a certain rate on the deposit amount (principal) for a certain period of time (in years).

Example 1

Encik Zainal deposited RM4 000 at Bank Bunga Raya with an interest rate of 2% per annum. How much is the interest earned by Encik Zainal after 1 year?

Solution:

The principal deposited by Encik Zainal is RM4 000. So, 2% of RM4 000 is

$$\text{RM4 000} \times \frac{2}{100} = \text{RM80}$$

TIPS

Rate is given in percentage form. Thus, we must divide the rate by a hundred. If the time is given in months, do not forget to change to year by dividing by 12 months.

After 1 year, the interest earned by Encik Zainal is

$$\text{Interest} = \text{RM80} \times 1$$
$$= \text{RM80}$$

The simple interest can be calculated using the following formula:

$$I = Prt$$

I is the interest, P is the principal, r is the rate and t is the time in years.

Example 2

Encik Badrul deposits RM5 000 in a bank with an interest rate of 3% per annum for a period of 2 years. Calculate the total interest that Encik Badrul will receive for the 2-year period.

Solution:

$P = 5\,000 \qquad r = 3\% = \frac{3}{100} = 0.03 \qquad t = 2$

Thus, interest $I = Prt$

$$= \text{RM5 000} \times \frac{3}{100} \times 2$$
$$= \text{RM300}$$

Example 3

Ms Wong deposits RM10 000 in Bank Murni with an interest rate of 4% per annum. Calculate the amount of interest Ms Wong will receive after 6 months.

Solution:

$I = Prt$

$= \text{RM}10\,000 \times \frac{4}{100} \times \frac{6}{12}$

$= \text{RM}200$

What is the impact on the simple interest as a result of changing the savings period?

The changes in savings period in a bank give different returns.

Example 4

Encik Nazrin deposits RM8 000 in Bank Desa with an interest rate of 3% per annum. Calculate the total savings of Encik Nazrin after he has saved for

(a) 2 years (b) 3 years

Solution:

The formula for calculating the interest, $I = Prt$

Principal = RM8 000

Interest rate = 3%

(a) 2 years	(b) 3 years
Interest = $\text{RM}8\,000 \times \frac{3}{100} \times 2 = \text{RM}480$ Total savings at the end of the second year = RM8 000 + RM480 = RM8 480	Interest = $\text{RM}8\,000 \times \frac{3}{100} \times 3 = \text{RM}720$ Total savings at the end of the third year = RM8 000 + RM720 = RM8 720

Based on the example above, it is found that the longer the savings period (at the bank), the higher the amount of interest earned. Therefore, the final amount of savings also increases.

What is the impact if the given interest rates differ for the same principal?

Example 5

Mrs Vanmathy deposits a sum of RM5 000 in a bank. What is the amount of Mrs Vanmathy's savings after 1 year if the interest rate given is
(a) 5% per annum (b) 6% per annum

What is the difference between the amounts of interest earned by Mrs Vanmathy in the above situations?

QUIZ

Why is a fixed deposit account given a higher interest rate than a savings account?

Solution:

Total savings	Interest rate	Savings period (years)	Total interest	Total savings after 1 year
RM5 000	5%	1	RM5 000 $\times \frac{5}{100} \times 1$ = RM250	RM5 000 + RM250 = RM5 250
RM5 000	6%	1	RM5 000 $\times \frac{6}{100} \times 1$ = RM300	RM5 000 + RM300 = RM5 300

The difference between the total interests received is RM300 – RM250 = RM50.

Based on Example 5, for the same principal, when the interest rates increase, the total savings at the end of the year also increases.

TIPS

- The higher the interest rate, the higher the return
- The lower the interest rate, the lower the return

Compound interest

Compound interest is interest that is calculated based on the original principal and also the accumulated interest from the previous period of savings.

Compound interest is different from simple interest in terms of the amount of savings to be used for interest calculation.

For compound interest, the frequency of compounding on the principal can be different. For example, compounded once a year or once every 3 months and so on.

Referring to Example 4(a), if Encik Nazrin is given compound interest which is compounded once a year, what is his savings at the end of the second year?

In the first year, the amount of interest received is

$$\text{RM8 000} \times \frac{3}{100} = \text{RM240}.$$

Thus, the amount of savings at the end of the first year is RM8 240.

For the second year, the amount of savings used for interest calculation is RM8 240 (principal + first year interest).
Thus, interest at the end of the second year is

$$\text{RM8 240} \times \frac{3}{100} = \text{RM247.20}.$$

Therefore, the amount of Encik Nazrin's savings at the end of the second year is

$$\text{RM8 240} + \text{RM247.20} = \text{RM8 487.20}.$$

Scan the QR Code or visit http://bukuteksksm.my/Mathematics/F3/Chapter3Compund.pdf for more information about compound interest.

In general, the formula for calculating compound interest is:

$MV = P\left(1 + \frac{r}{n}\right)^{nt}$

MV = matured value
P = principal
r = yearly interest rate
n = number of periods the interest is compounded per year
t = term in years

Based on Encik Nazrin's example, it was found that:

$$P = 8\,000, \quad r = 0.03, \quad n = 1, \quad t = 2.$$

Thus, the amount of Encik Nazrin's savings at the end of the second year is

$$MV = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= \text{RM}8\,000\left(1 + \frac{0.03}{1}\right)^{(1)(2)}$$
$$= \text{RM}8\,000\,(1.0609)$$
$$= \text{RM}8\,487.20$$

Example 6

At the beginning of a year, Mrs Liew Foong saves RM15 000 in her savings account with a rate of 4% per annum and compounded every 6 months. What is Mrs Liew Foong's total savings at the end of the third year?

Solution:

$P = 15\,000 \quad r = \frac{4}{100} = 0.04 \quad n = 2 \quad t = 3$

$$MV = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 15\,000\left(1 + \frac{0.04}{2}\right)^{(2)(3)}$$
$$= 15\,000\,(1.1262)$$
$$= \text{RM}16\,892.44$$

QUIZ

What is the impact on the total cumulative returns, if the compounding rate increases in a year?

Example 7

A bank offers a 5% interest rate per annum for savings in a fixed deposit account. If Puan Wahidah saves RM10 000 at the beginning of the year, how much money is in her fixed deposit account at the end of the year if the interest is compounded
(a) once every 3 months? (b) once a month?

Solution:

$P = 10\,000 \quad r = \frac{5}{100} = 0.05 \quad t = 1$

$$MV = P\left(1 + \frac{r}{n}\right)^{nt}$$

BULLETIN

Inflation also affects the value of the currency. If the inflation rate increases, the purchasing power of RM1 will reduce.

(a) $n = 4$ Thus, $MV = 10\,000\left(1 + \frac{0.05}{4}\right)^{(4)(1)}$ $= \text{RM}10\,509.45$	(b) $n = 12$ Thus, $MV = 10\,000\left(1 + \frac{0.05}{12}\right)^{(12)(1)}$ $= \text{RM}10\,511.62$

From Example 7, it was found that when the compounding frequency increases, the future value of savings also increases.

Example 8

Mr Charles deposited RM6 000 in a fixed deposit account at Bank Berjaya for 2 years with an interest rate of 6% per annum. What is the difference between the amount of interest Mr Charles earned if he was given compound interest (compounded once every 4 months) compared to simple interest?

Solution:

Simple interest	Compound interest
Interest, $I = Prt$ $= \text{RM}6\,000 \times \frac{6}{100} \times 2$ $= \text{RM}720$	$MV = P\left(1 + \frac{r}{n}\right)^{nt}$ $= 6\,000\left(1 + \frac{0.06}{3}\right)^{(3)(2)}$ $= \text{RM}6\,756.97$ Total accumulated interest RM6 756.97 – RM6 000 = RM756.97

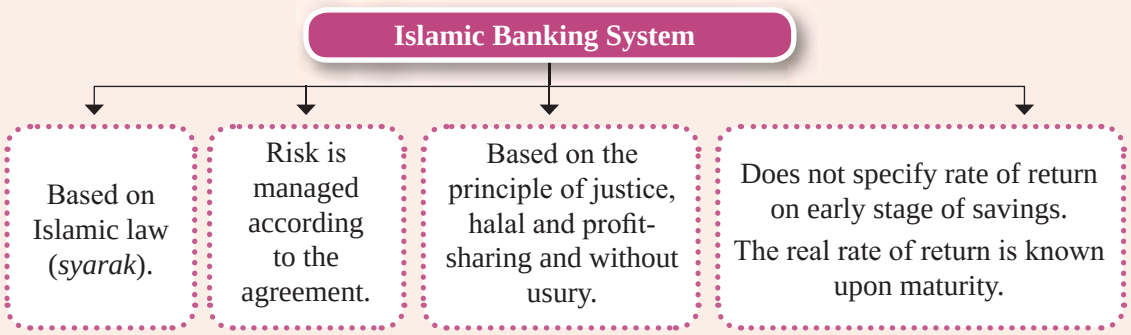
Thus, the difference in the amount between simple interest and compound interest (compounded once every 4 months) is

RM756.97 – RM720 = RM36.97

Based on Example 8, it is clear that savings with compound interest give higher returns than savings with simple interest.

Islamic Banking

Malaysia practises a dual banking system. They are conventional banking system and Islamic banking system.



Example 9

Encik Osman saved RM20 000 in a savings account in an Islamic bank, according to the principle of wadiah for 1 year. By the end of the year, he received a sum of RM20 500 as a return from the savings. An additional RM500 is a hibah (gift) from the bank. Calculate the percentage of hibah obtained by Encik Osman.

BULLETIN

The principle of wadiah
Property or cash received with the agreement of the customer to be deposited in the bank. The bank is responsible for the security of the property or the money.

Solution:

$$\text{Percentage of hibah} = \frac{\text{RM500}}{\text{RM20 000}} \times 100\% = 2.5\%$$

2.5% is only a reference for savings and is not fixed.

MIND TEST 3.1b

1. Puan Nathania deposited RM500 into her savings account that gives an interest rate of 4% per annum and compounded quarterly. How much is Puan Nathania's savings at the end of the fifth year?
2. Mr Chong deposited RM1 000 into his savings account that gives an interest rate of 5% per annum and compounded once every half year. How much is Mr Chong's savings at the end of the third year?
3. Puan Aminah deposited RM100 into her savings account that gives an interest rate of 3% per annum and compounded monthly. How much is Puan Aminah's savings at the end of the second year?

What do you understand by return on investment (ROI)?

LEARNING STANDARD

Perform calculations involving the value of return on investments, and hence explain the factors that affect the return on investments and its impacts.

Return on investment refers to the return value of each ringgit invested by the investor. In other words, return on investment is also a ratio of profit or loss derived from an investment.

In general, investors prefer to assess the return on investment in percentage. Return on investment will reflect the profit or loss achieved by individual investors in investment.

An investment is considered profitable (wise investment) when the present value of the investment and the amount of return received is more than the value of the original investment.

Similarly, when the amount of return and the present value of the return is less than the value of the original investment, then the investment is unprofitable.

BULLETIN

School cooperatives declare dividends at the end of each financial year. The declared dividend determines the return value of the shares purchased by each member of the cooperative.

The formula for calculating return on investment is

$$\text{Return on investment} = \frac{\text{Total return}}{\text{Value of initial investment}} \times 100\%$$

In addition, investors also have an expected rate of return from an investment. For example, an investor expects a rate of return of 10% on their investment. However, the real rate of return to be received may not be as expected.

Investment instruments consist of the unit trust, shares, real estate and so on. Each of these investment instruments will bring returns.

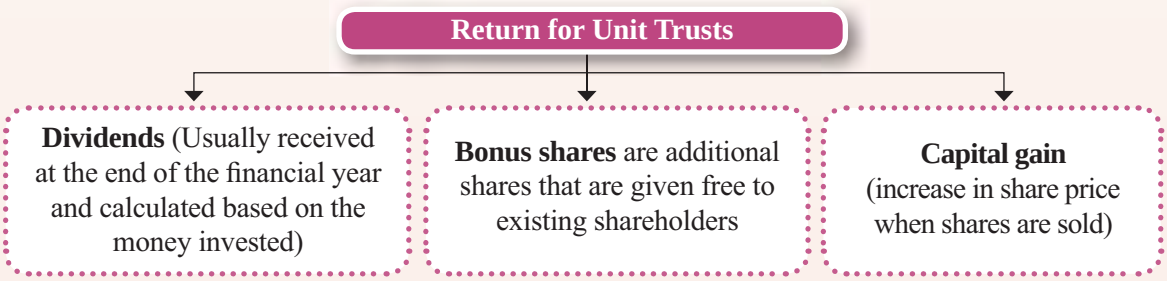
Unit Trust

Unit trust is a good investment alternative for medium term investment (3 to 5 years) and long term (over 5 years).

Investments in unit trusts are low risk as they are managed by professional fund managers regulated by the securities commission and also monitored by Bank Negara Malaysia.

Investments in unit trusts allow investors to diversify their investments with a small capital.

Below is the return for unit trusts.



Example 10

On 1 January 2018, Puan Siti invested 3 000 units valued at RM2.00 per unit in Amanah Saham Bumiputera (ASB). For the financial year ending 31 December 2018, Amanah Saham Bumiputera paid a dividend of 5%. On 1 January 2019, Puan Siti sold all the shares she owned at RM2.20 per unit. What is the return on investment for Puan Siti?

Solution:

Steps for calculating dividend

Initial capital $= 3\,000 \times \text{RM}2.00$
$= \text{RM}6\,000$

Dividend $= \frac{5}{100} \times (3\,000 \text{ units} \times \text{RM}2.00)$
$= \text{RM}300$

Increase in share price $= \text{RM}2.20 - \text{RM}2.00 = \text{RM}0.20$

Capital gain $= \text{RM}0.20 \times 3\,000 \text{ units}$
$= \text{RM}600$

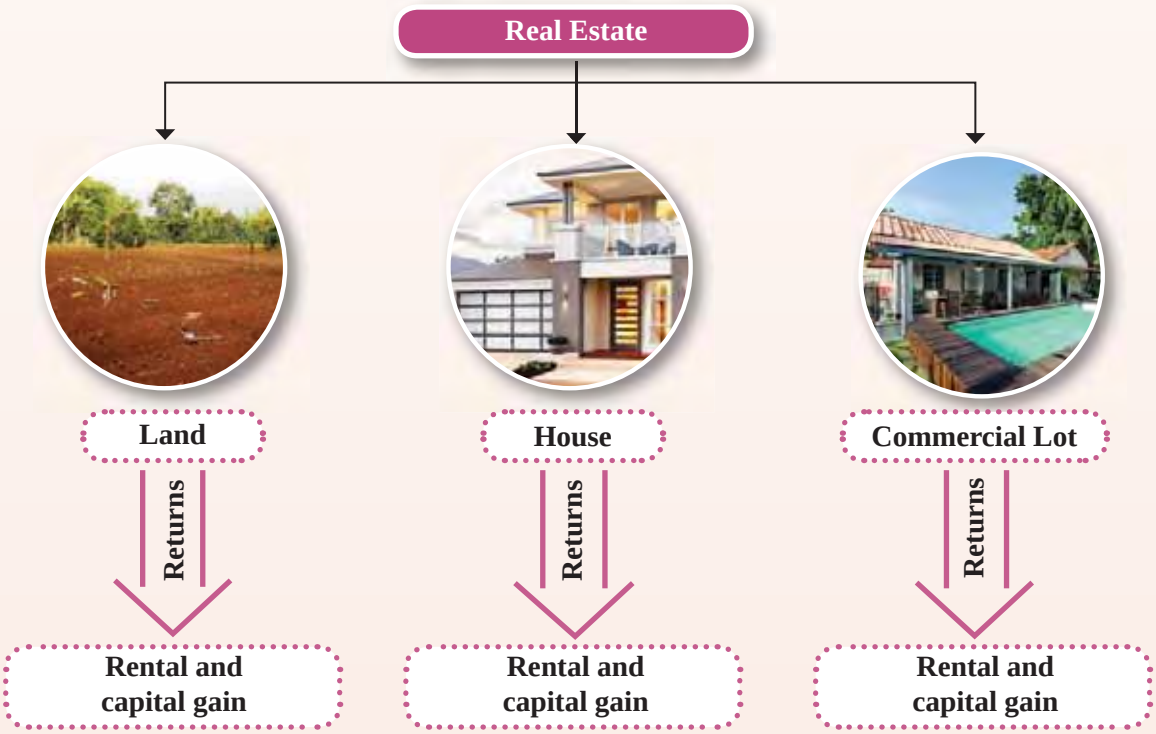
Total return $= \text{RM}300 + \text{RM}600 = \text{RM}900$

Return on investment $= \frac{\text{RM}900}{\text{RM}6\,000} \times 100\% = 15\%$

The return on investment benefits Puan Siti as she receives two types of returns namely dividend and capital gain from an increase in share value from RM2.00 to RM2.20.

What do you understand about return on investment for real estate?

Investment in real estate is one of the investments that bring returns in the form of rent and capital gains. When a property is rented, the owner (investor) of the property will receive return in the form of rent. If the property is sold, the owner (investor) will receive capital gain or capital loss.



Example 11

Encik Yusuf bought a shoplot at a price of RM600 000 on 1 January 2017 in Bangi. He paid 10% of the shoplot's purchase price of RM60 000. The shoplot was rented from 1 January 2017. On 31 December 2026, he sold the shoplot for RM1 300 000. The loan amount still owed to the bank was RM486 000. Meanwhile, the amount that has been amortised to the bank was RM450 000. Other charges involved in the sale and purchase transactions are as follows:

Legal cost	RM15 000
Stamp duty (during sale and purchase)	RM15 000
Agent's commission	RM18 000

BULLETIN

Stamp duty
Tax imposed on documents or letters with legal, commercial or financial implications under the First Schedule, of Stamp Act 1949.

Legal cost
Payment to lawyer to perform the transfer of property for the buyer.

Commission
Fee paid by the property seller to the agent for the sale of real estate.

The total rent collected throughout the possession of the shoplot is RM200 000. Calculate the return on investment obtained by Encik Yusuf.

Solution:

Total rent = RM200 000

Capital gain = RM1 300 000 – RM486 000 – RM60 000 – RM15 000 – RM15 000 – RM18 000 – RM450 000

= RM256 000

Total return = RM200 000 + RM256 000

= RM456 000

$$\text{Return on investment} = \frac{\text{RM456 000}}{\text{RM600 000}} \times 100\%$$

$$= 76\%$$

Example 12

Encik Hussein bought a house on 1 January 2015 in Cheras at RM300 000 and cleared 10% down payment of RM30 000. He expects a return of 30% over 20 years.

Encik Hussein sold the house at a price of RM600 000 after having owned the house for 20 years. The loan amount amortised to the bank was RM475 000. During that period, he managed to earn a rent of RM60 000. Other expenses incurred are as follows:

Stamp duty (during sale and purchase)	RM4 000
Agent's commission	RM2 000
Legal cost during sale and purchase	RM4 000

What is the return on investment for Encik Hussein for 20 years? Did he achieve his expectation to get a return of 30%?

Solution:

Return on investment

= Rent + Capital gain

= RM60 000 + (RM600 000 – RM30 000 – RM475 000 – RM4 000 – RM2 000 – RM4 000)

= RM60 000 + RM85 000

= RM145 000

$$\text{Return on investment} = \frac{\text{RM145 000}}{\text{RM300 000}} \times 100\%$$

$$= 48.33\%$$

Encik Hussein managed to obtain a rate of return of 48.33%. This rate exceeds the expected rate of return of 30%.

In real estate investment there are factors that affect the return on investment. The factors that affect return on investment are as follows:

Factors that affect the return on investment on real estate

The economic situation
- The country's good economic situation will increase real estate prices because the demand for real estate will increase.

Location
- The properties that are strategically located near a vastly developed city centre have higher prices compared to properties in rural areas.

Political situation
- A stable political situation will increase demand for real estate.
- This will indirectly increase real estate prices.
- Political instability will reduce demand for real estate and indirectly cause the fall of real estate prices.

MIND TEST 3.1c

1. What is the meaning of return on investment or ROI?

2.

- On 1 January 2019, Mr Moses bought a homestay at a price of RM250 000.
- The daily rental rate is RM100.
- On average, the homestay will be occupied for 20 days in a month.

(a) Calculate the monthly rentals.

(b) Calculate the return if the homestay is sold for RM480 000 at the end of the year.

3. On 1 January 2018, Rahim invested in 4 000 units of Amanah Saham Bumiputera (ASB) shares valued at RM1 per unit. For the financial year ending 31 December 2018, ASB paid a dividend of 8%.

How much is the dividend received by Rahim for that year?

What factors should be considered before making an investment?

LEARNING STANDARD

Compare and contrast potential risks, return and liquidity of various types of savings and investments.

Three factors that should be taken into account by an investor prior to making an investment are as follows:

Potential investment risk	The uncertainty that losses may be incurred from the investments made.
The level of return	Profit enjoyed by investors from investments.
Liquidity aspect	Relating to how soon the investment or savings could be cashed out.

The table below shows the comparison of various types of savings and investments by individuals in terms of risk levels, return levels and liquidity levels.

Type of investment	Risk level	Return level	Liquidity level
Saving	Risk free	Low	High
Fixed deposits	Risk free	Low	High
Company shares	High	High	Moderate
Real estate	Moderate	High	Low
Unit trust	Low	Moderate	High

TIPS

One way to reduce investment risk is to diversify one's investment portfolio. This helps to offset risks from each investment and thus further reduce the risk in the investment portfolio.

BULLETIN

Portfolio
Various levels of investment type.

MIND TEST 3.1d

1. Explain the relationship between risk and return on investment.
2. The potential risk of saving in a bank is zero as compared to investment. Explain this statement.
3. Why do unit trusts have a high liquidity level?
4. Real estate has a moderate potential risk. Explain.
5. 

Encik Osman sets up a homestay on the lot of land purchased at RM250 000. The overall cost of setting up this homestay is RM500 000.

(a) What type of investment was made by Encik Osman?

(b) State the potential risks, return and liquidity of the investment made by Encik Osman.

(c) In your opinion, was the investment made by Encik Osman a wise move? Justify.

What do you understand about cost averaging strategy?

LEARNING STANDARD

Calculate the average cost per share for the investment of shares using the ringgit cost averaging strategy and explain the benefits of the strategy.

Cost averaging strategy is a technique commonly practised by investors who invest in shares with a fixed amount for a certain period such as monthly, quarterly or yearly regardless of the stock market conditions.

Cost averaging strategies can help investors to buy shares with a lower average cost and the total number of shares owned will be higher within the same investment period, as opposed to buying them in a lump sum or with a single purchase.

For example, Puan Hasniza has RM12 000 and buys shares in a lump sum from Sempurna Company at RM2.00 per share unit. Hence, Puan Hasniza will only have 6 000 units of shares (6 000 units of shares × RM2.00 = RM12 000). However the situation will be different if Puan Hasniza decides to buy shares based on cost averaging strategy.

The table below shows Puan Hasniza's investment chart in a year based on cost averaging strategy for buying shares on a monthly basis.

Month	Investment amount (RM)	Unit price (RM)	Number of unit $= \frac{\text{Investment amount}}{\text{Unit price}}$
January	1 000	2.00	500
February	1 000	1.80	555
March	1 000	1.80	555
April	1 000	1.70	588
May	1 000	1.70	588
June	1 000	1.60	625
July	1 000	1.60	625
August	1 000	1.50	666
September	1 000	1.60	625
October	1 000	2.20	454
November	1 000	2.30	434
December	1 000	1.90	526
	12 000	1.78 (average cost per share unit)	6 741 (number of units owned)

With the cost averaging strategy given above, Puan Hasniza earned 6 741 units by investing RM12 000.

Cost averaging strategy

$$\text{Average Cost} = \frac{\text{Investment amount}}{\text{Number of share units owned}}$$

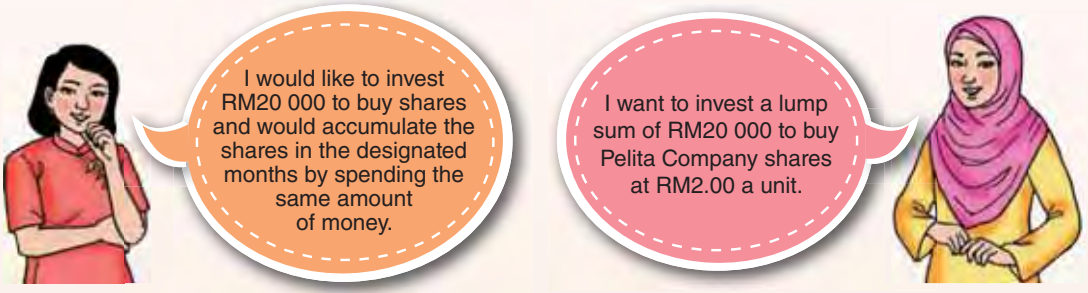
$$\text{Average cost per share unit} = \frac{\text{RM12 000}}{\text{6 741}} = \text{RM1.78}$$

TIPS

When you use cost averaging strategy to invest in shares, the cost of the shares you have bought will not be the highest price nor will it be the lowest price.

Example 13

Below are two investors who plan to invest in shares using different strategies.



Mrs Esther Wong

Puan Linda

Shares acquired by Mrs Esther Wong in the designated months:

Month	January	March	May	August	December
Shares price per unit (RM)	2.00	1.80	1.60	2.10	2.00

(a) Calculate the average cost per unit and the number of shares owned by Puan Linda and Mrs Esther Wong.

(b) Who is a wise investor? Justify your answer.

Solution:

(a) Puan Linda

$$\text{Total shares} = \frac{\text{RM20 000}}{\text{RM2.00}} = 10\,000 \text{ share units}$$

$$\text{Average cost per share} = \frac{\text{RM20 000}}{10\,000 \text{ share units}}$$

$$= \text{RM2.00}$$

Mrs Esther Wong

Month	Total investment	Price per unit (RM)	Number of share unit
January	4 000	2.00	2 000 units
March	4 000	1.80	2 222 units
May	4 000	1.60	2 500 units
August	4 000	2.10	1 904 units
December	4 000	2.00	2 000 units
	20 000		10 626 share units

Total shares = 10 626 share units

$$\text{Average cost per share} = \frac{\text{RM20 000}}{10\,626 \text{ units}}$$

$$= \text{RM1.88}$$

(b) Mrs Esther Wong is a wise investor for practising the cost averaging strategy that helped her to accumulate more shares with the same amount of money.

Advantage of the cost averaging strategy for investors

Able to take advantage of the unit price changes

- When the stock price is low, more units of shares can be purchased.
- Indirectly helps investors to have more shares in the long term.

The average cost of a share unit bought by an investor can be reduced in the long term.

Not influenced by emotions

- Invest consistently on a periodic basis the same amount of money without being influenced by emotions caused by share price fluctuation.

Lower the risk of loss

- Total investment is carried out on a regular and consistent manner, which helps investors to purchase based on current situation and avoid losses associated with lump sum investment.

MIND TEST 3.1e

1. What do you understand about the cost averaging strategy in purchasing shares?

2. Below are two investors who invested using different strategies.

Investor 1 → Mr Derick invested a lump sum of RM24 000 to purchase Wawasan shares at RM2.00 per share unit.

Investor 2 → Encik Sulaiman has RM24 000 and invested consistently on a periodic basis RM2 000 each month to purchase Wawasan shares.

Month	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Share price per unit (RM)	2.00	1.80	1.70	1.60	2.10	1.50	2.20	2.00	2.00	1.60	1.70	1.80

The table above shows the share price bought by Encik Sulaiman on a monthly basis.

(a) Who is a wise investor? Justify your answer.

(b) Calculate the average cost per share unit and the number of shares owned by Encik Sulaiman.

(c) State the advantages of using cost averaging strategy in purchasing shares.

How do you solve problems involving savings and investments?

LEARNING STANDARD

Solve problems involving savings and investments.

Example 14

Encik Zaidi, Mr Leong and Mr Navin had retired from their jobs and each received RM400 000 as gratuity from their company. They use different investment methods to invest.

Who is a wise investor?

Encik Zaidi

Encik Zaidi bought a medium-cost apartment in Kajang worth RM150 000 and receives a rental of RM800 per month. The balance is kept in a fixed deposit account with an interest rate of 4% per annum.

Mr Leong

Mr Leong invested RM400 000 in Manis Company shares at RM2.00 per unit. The company declared a dividend of 8% for that year.

Mr Navin

Mr Navin deposited RM200 000 into a savings account with an interest rate of 1% per annum. The balance RM200 000 is kept in a fixed deposit account at 4% interest rate per annum.

Solution:

Encik Zaidi

Level of risk	The level of risk for property (apartments) and fixed deposits is low.		
Return on investment	Receives a return on investment in the form of rent and interest.		
	Rental	**Savings interest**	**Return on investment (ROI)**
	RM800 × 12 = RM9 600	$\frac{4}{100} \times$ RM250 000 = RM10 000	**ROI** = $\frac{\text{RM19 600}}{\text{RM400 000}} \times 100\%$ = 4.9%
Liquidity	Savings are easily converted to cash while property takes time to sell.		

Mr Leong

Level of risk	The level of risk in buying shares is high.
Return on investment	Receives **dividends** and bonuses depending on the **performance** of the company invested in. If he sells his shares, it is uncertain that he will be able to sell the shares at a higher price compared to the purchase price. This depends on the economic situation and performance of the company stock at that period. Dividend = 400 000 × $\frac{8}{100}$ = RM32 000 **ROI** = $\frac{\text{RM32 000}}{\text{RM400 000}} \times 100\%$ = 8%
Liquidity	Moderate.

Mr Navin

Level of risk	The level of risk for both savings and fixed account are low.		
Return on investment	Receives a return on investment in the form of interest only.		
	Savings interest	**Fixed deposit interest**	**Return on investment (ROI)**
	RM200 000 × $\frac{1}{100}$ = RM2 000	$\frac{4}{100} \times$ RM200 000 = RM8 000	**ROI** = $\frac{\text{RM10 000}}{\text{RM400 000}} \times 100\%$ = 2.5%
Liquidity	Savings are easily turned into cash.		

- Mr Leong is a wise investor because his return on investment value is higher than Encik Zaidi and Mr Navin.
- In this example, Encik Zaidi, Mr Leong and Mr Navin each uses the same amount of capital, which is RM400 000. The effectiveness of their investments can be compared based on their returns for the year respectively.

MIND TEST 3.1f

1. The following are two investors who invested their gratuity.

Mr Rasamanie received RM600 000 as gratuity from his company. He bought a double storey shop in Bangi, Selangor and receives a monthly rental of RM3 500.

Monthly rental
RM3 500

Encik Nik Izwan received RM600 000 as gratuity from his company. He saves RM150 000 in a fixed deposit account at a commercial bank with an interest rate of 4% per annum. He also bought share units worth RM150 000.

At the same time, Encik Nik Izwan bought shares in Cepat Maju Company worth RM100 000. The balance of the gratuity is used to buy a medium-cost apartment in Ampang and he receives a monthly rental of RM1 200.

Monthly rental
RM1 200

(a) Explain the investment risk level of both individuals.

(b) Who is a wise investor? Justify your answer.

(c) What factors need to be considered before investing in real estate?

2. In 2015, Mr Wong bought a house at RM540 000. He paid 10% down payment and the balance was paid through a loan. After 20 years, Mr Wong decided to sell the house at RM900 000. The following are the expenses involved.

Monthly instalment amount paid	RM666 000
Stamp duty	RM15 000
Agent's commission	RM8 000
Other expenses	RM18 000

Calculate the return on investment for Mr Wong.

3.2 Credit and Debt Management

What do you understand about credit and debt?

LEARNING STANDARD

Explain the meaning of credit and debt, and hence describe the wise management of credit and debt.

The word credit has several meanings. In the financial world, **credit** means a contractual agreement between the supplier (for instance a bank or financial institution) and the consumer. The consumer can borrow money from the supplier for any use or purchase and agree to repay within a certain period. In short, credit is a postponement of payment facility provided by the supplier to the consumer.

For example, banks offer credit facilities to customers in the form of credit cards. If the customer or credit card owner uses the credit card in a transaction, the bank will pay the seller first and the customer will pay back the bank within a certain period of time.

Credit can also mean the amount of money that can be borrowed. For example, for credit card, if the credit card limit is RM10 000, then the card owner has the ability to buy goods or make transactions up to RM10 000 with the card.

Debt usually means an amount that has been borrowed but has not been settled. If a transaction is made using a credit card, the credit will be converted into debt.

Personal budget is the estimated income and expenditure of an individual for a given period. The practice of making personal budget is strongly encouraged so that an individual can

(a) plan spending prudently

(b) avoid overspending

(c) save

How do you manage credit and debt wisely?

- Credit card users need to settle the debt payment within the period stipulated by the bank to enjoy interest free period.
- Pay the outstanding balance listed on the credit card statement.
- The minimum amount paid by the credit card holder provides opportunity for the bank to charge interest on the balance and may also incur late payment charges.
- Pay within the cash discount period for payment of debts.

Example 15

Encik Syed bought an air conditioner at RM3 200 on 15 July 2018. He lacked RM1 200 in cash but has a Bank Cemerlang credit card. He was aware that the shortage of cash could be paid at the end of the month when he received his salary.

(a) Which credit facility can be used by Encik Syed to overcome the shortage of money?

(b) State the advantages and disadvantages of the payment method you specified in answer (a).

Solution:

(a) Encik Syed can use the credit card facility.

(b) The use of credit cards is more convenient if Encik Syed repays his credit within the interest-free period to avoid any extra charges.

Brainstorming 2

In groups

Aim: The impact of using credit card in buying goods online.

Steps:

1. Get into groups of four or five.
2. Surf any website related to the topics of discussion for more information.
3. The information collected should be presented using an appropriate thinking map to the class.
4. The best thinking map will be displayed in the mathematics corner.

Discussion:

What is the impact of purchasing goods online?

From Brainstorming 2, it is found that purchasing goods online can cause consumers to spend extravagantly and get into debt. Thus, be prudent when buying goods online.

MIND TEST 3.2a

1. What does a personal loan mean?
2. Many people are bankrupt due to credit cards

   What are the ways to overcome the situation above?
3. Puan Zuraidah wants to buy a refrigerator at Hebat Electrical Shop but she lacks RM2 500 in cash. Hebat Electrical Shop provides instant loans for purchases with an interest rate of 4% per annum. Puan Zuraidah also has a credit card. Which credit facility should be used by Puan Zuraidah and state its advantages?

What do you understand about the advantages and disadvantages of credit cards?

LEARNING STANDARD

Investigate and describe the advantages and disadvantages of credit card and ways to use it wisely.

The use of credit cards is increasingly common today. As a consumer it is important for us to realise and understand the advantages and disadvantages of using credit cards.

Advantages of credit card	Disadvantages of credit card
• Credit cardholders can enjoy a reward system in the form of cash rebate or point redemption. • Does not require us to carry a lot of cash. • Easy and efficient payment method. • Convenience of buying goods and services online.	• Incur charges such as annual fees, finance charges (interest), cash advance interest charges and late payment charges. • Overspending. • Some stores do not accept credit payment.

However, not all individuals are eligible for a credit card. There are several conditions that an applicant must adhere to.

- 21 years old and above.
- Minimum income of RM24 000 per annum and meets other requirements set by the bank.
- Requires salary slip or supporting documents.

Credit card users must comply with the obligations as a credit card user when signing the credit card application form.

- Do not give credit card details to strangers.
- Remember the pin number and do not record the pin number on the back of the card.
- Check the transactions in the credit card statement received at the end of the month.

What do you understand about the impact of minimum payment and late payment on credit card use?

LEARNING STANDARD

Investigate and describe the impact of minimum and late payments for credit card usage.

The credit cardholder will receive a financial statement for the credit card monthly. In the statement, there are details such as credit limit, statement date, latest amount, minimum payment amount, type of charges and so on.

The cardholder should pay the statement balance immediately so that no financial charges are incurred. But banks provide flexibility by allowing users to pay in a given period, known as the interest free period. Usually this period is 20 days from the statement date.

TIPS

The credit cardholder must pay the full statement balance of the credit card to enjoy a 20-day interest free period from the statement date.

However, if you pay only part of it, you will lose the interest-free period.

To enjoy this privilege every month, the cardholder must pay the total balance of credit card statement or make a minimum payment in the interest free period. The minimum payment is usually 5% of the total balance of the credit card statement, or a minimum of RM50.

If there is still a balance of the latest amount upon expiry of the interest free period, a finance charge (or interest) will be imposed on the balance based on daily rate. Most banks charge an annual interest rate of between 15% and 18%.

In addition, if no payment is made within the interest free period, then the minimum late payment charge of RM10 or 1% of total outstanding balance as of the statement date will be charged.

Example 16

Encik Ahmad received his credit card statement for January 2019 from Bank Sentosa. The statement shows that Encik Ahmad has a current amount (outstanding balance) of RM5 200. It is assumed that Encik Ahmad did not use his credit card in February.

(a) What is the minimum payment to be paid?

(b) If he only makes a minimum payment for January and the statement date is 15 days from the expiry date of the interest free period, what is the balance shown in his February statement?

(c) If he missed his payment for January, what is the balance shown in his February statement?

Solution:

TIPS

We should use credit cards wisely.

(a) Current amount = RM5 200

5% of the current amount $= \left(\frac{5}{100}\right) \times$ RM5 200 = RM260

This amount exceeds RM50, thus the minimum payment to be paid by Encik Ahmad is RM260.

(b) Outstanding balance = RM5 200 – RM260 = RM4 940

Period subject to financial charges = 15 days = (15 ÷ 365) year

Interest charged = RM4 940 × [(18 ÷ 100) × (15 ÷ 365)] = RM36.54

Current amount (Outstanding balance) in February = RM4 940 + RM36.54 = RM4 976.54

(c) Outstanding balance = RM5 200
Period subject to financial charges = 15 days = (15 ÷ 365) year
Interest charged = RM5 200 × [(18 ÷ 100) × (15 ÷ 365)] = RM38.47
Late payment charges $= \left(\frac{1}{100}\right) \times$ (RM5 200 + RM38.47) = RM52.38
Current amount in February = RM5 200 + RM38.47 + RM52.38 = RM5 290.85

How do you solve problems involving the use of credit cards?

LEARNING STANDARD
Solve problems involving the use of credit cards.

Credit cardholders should be aware of the advantages and disadvantages of credit cards. Several factors have to be taken into account before using credit cards such as the balance of the existing limit, current cash flow and so on.

Example 17

Ms Chin wants to buy a French made handbag online. She surfs the Internet and finds two interesting promotions:

(a) Company L in Singapore offers promotional price of SGD250. For orders outside Singapore, SGD50 shipping charges apply.

(b) Company V in Malaysia offers promotional price of RM799. Delivery is free for all orders to local addresses.

Ms Chin intends to make payment by credit card and she understands that the bank will charge an additional 1% on each transaction from abroad. Assume the current exchange rate for Malaysian ringgit is

RM1 = SGD0.34

As a wise consumer, which offer should Ms Chin choose? Justify your choice.

Solution:

Ms Chin should compare the actual price to be paid if buying from the two companies.

(a) Company L:
Promotional price = SGD250 × (1 ÷ 0.34) = RM735.29
Postal charges = SGD50 × (1 ÷ 0.34) = RM147.06
Additional charges by bank = RM735.29 $\times \left(\frac{1}{100}\right)$ = RM7.35
Actual price to be paid = RM735.29 + RM147.06 + RM7.35 = RM889.70

(b) Company V:
Promotional price = RM799
Actual price to be paid = RM799

Although the promotional price offered by Company L is cheaper, but the actual price payable is higher due to the additional charges incurred for online purchases from Company L. So, Ms Chin should buy from Company V to save RM90.70.

How do you calculate loan repayment and instalment?

LEARNING STANDARD

Calculate the total amount of loan repayment and instalment, with various interest rates and different loan periods.

Each loan will be charged interest on the loan from the date the loan was released to the borrower. There are two calculation methods for loan interest, namely flat interest rate and interest on balance.

Total loan balance is the amount deducted from the initial down payment plus the amount of interest charged.

The monthly instalment is the total amount paid by the borrower to the bank every month to settle the balance of the loan.

Flat interest

In the flat interest method, the interest rate will be calculated on the original loan amount over the term of the loan. So the amount of interest charged per month is fixed.

Example 18

Mrs Lim bought a car worth RM80 000 on credit. She pays 10% down payment and the balance is payable in instalments over 6 years. The flat interest rate imposed by the bank is 4% per annum. What is the amount of repayment and monthly instalment payable by Mrs Lim?

Solution:

Loan amount = Purchase price – down payment
= RM80 000 – RM8 000 = RM72 000

Interest for 6 years = $\text{RM72 000} \times \frac{4}{100} \times 6 \text{ years} = \text{RM17 280}$

Total repayment = RM72 000 + RM17 280 = RM89 280

Total repayment can be calculated with the formula $A = P + Prt$

Monthly instalment = $\frac{\text{RM89 280}}{72 \text{ months}} = \text{RM1 240 per month}$

Example 19

Encik Azlan obtained a personal loan of RM10 000 from Bank Mulia with an interest rate of 4% per annum. The repayment period is 7 years.

What is the monthly instalment to be paid by Encik Azlan?

Solution:

$A = P + Prt$

Loan, P = RM10 000
r = 4%
t = 7 years

Thus, total repayment

$A = \text{RM10 000} + \left(\text{RM10 000} \times \frac{4}{100} \times 7\right)$
$= \text{RM10 000} + \text{RM2 800}$
$= \text{RM12 800}$

Monthly instalment = $\frac{\text{RM12 800}}{84 \text{ months}} = \text{RM152.38}$

Interest on balance

In addition to the flat interest, banks also offer interest on balance for certain types of loans. In the interest on balance method, the amount of interest charged each month on the loan depends on the amount of the loan balance for that month. Since there is monthly instalment payment, the amount of the loan balance will be reduced, thus the amount of interest for each month will also be reduced.

However, it should be noted that for every instalment paid each month, the priority is given to settle the interest amount in that month, and then the balance is used to settle the outstanding balance of the loan amount.

Example 20

Encik Harith obtained a personal loan of RM10 000 from Bank Mulia with an interest rate of 6% on the balance. The repayment period is 7 years while the monthly instalment is RM150.

Calculate the total amount of interest payable by Encik Harith for the first three months.

Solution:

First month

$$\text{First month interest} = \text{RM10 000} \times \frac{6}{100} \times \frac{1}{12}$$
$$= \text{RM50.00}$$

$$\text{Loan at the end of first month} = \text{RM10 000} + \text{RM50}$$
$$= \text{RM10 050}$$
$$\text{Balance after first instalment} = \text{RM10 050} - \text{RM150}$$
$$= \text{RM9 900}$$

Second month

Balance of the loan at the beginning of second month = RM9 900

$$\text{Second month interest} = \text{RM9 900} \times \frac{6}{100} \times \frac{1}{12}$$
$$= \text{RM49.50}$$

$$\text{Loan at the end of second month} = \text{RM9 900} + \text{RM49.50}$$
$$= \text{RM9 949.50}$$
$$\text{Balance after second instalment} = \text{RM9 949.50} - \text{RM150}$$
$$= \text{RM9 799.50}$$

Third month

Balance of the loan at the beginning of third month = RM9 799.50

$$\text{Third month interest} = \text{RM9 799.50} \times \frac{6}{100} \times \frac{1}{12}$$
$$= \text{RM49.00}$$

$$\text{Loan at the end of third month} = \text{RM9 799.50} + \text{RM49.00}$$
$$= \text{RM9 848.50}$$
$$\text{Balance after third instalment} = \text{RM9 848.50} - \text{RM150}$$
$$= \text{RM9 698.50}$$

Total interest for the first three months is RM50.00 + RM49.50 + RM49.00 = RM148.50

Scan the QR Code or visit http://bukuteksksm.my/Mathematics/F3/Chapter3InterestRateCalculation.pdf for more information about loan with flat interest and interest on debts.

How do you solve problems involving loans?

LEARNING STANDARD

Solve problems involving loans.

Example 21

Ameera wants to buy a car and has paid a deposit of RM4 800. The balance will be settled through a vehicle loan.

State the advantages and disadvantages of the vehicle loan chosen by Ameera.

Solution:

Advantages	Disadvantages
• Repayment of vehicle loan in monthly instalments allows Ameera to own the car. • Does not require a lump sum payment.	• The car will be repossessed if instalments are not made. • The total amount of repayment is high due to interest charged on the loan.

Example 22

Mr Vincent is a teacher with a monthly income of RM2 800. He decides to buy a new car to commute to work. He contacts two banks to get a loan of RM40 000. In addition, he needs RM1 500 to cover other expenses every month.

The following are loan packages offered by two banks to Mr Vincent.

Loan details	Bank A	Bank B
Loan amount	RM40 000	RM40 000
Payment period	9 years	6 years
Interest rate	4.5 %	5 %
Guarantor	Not required	Required

TIPS

Do not borrow money from unlicensed moneylenders as the loan:

- will be made according to their own terms and conditions.
- imposes very high interest charges which is compounded daily.
- exposes you and your family to danger if you make late payment.
- forces you to make an additional loan to repay previous loan.

Suggest to Mr Vincent which bank is better suited for his car loan. State your reasons.

Solution:

Understanding the problem

The amount of monthly instalments payable by Mr Vincent provided that it is not burdensome.

Planning a strategy

- Calculate monthly interest.
- Calculate monthly instalment payable.

Implementing the strategy

The amount of money repaid to Bank A	The amount of money repaid to Bank B
$A = P + Prt$ $A = \text{RM}40\,000 + \text{RM}\,40\,000 \times \frac{4.5}{100} \times 9$ Total money repaid = RM40 000 + RM16 200 = RM 56 200 Monthly instalment $= \frac{\text{RM}56\,200}{108 \text{ months}}$ $= \text{RM}520.37$	$A = P + Prt$ $A = \text{RM}40\,000 + \text{RM}40\,000 \times \frac{5}{100} \times 6$ Total money repaid = RM40 000 + RM12 000 = RM52 000 Monthly instalment $= \frac{\text{RM}52\,000}{72 \text{ months}}$ $= \text{RM}722.22$

Making a conclusion

Mr Vincent should choose Bank A because the monthly instalment for Bank A is lower and less burdensome to him. However, different payment terms result in different amount of interest that will be paid. Therefore, Mr Vincent could also choose Bank B as the total interest paid will be less for Bank B.

Dynamic Challenge

Test Yourself

1. What is savings?
2. Specify features related to Fixed Deposit Account.
3. Encik Lipong deposits a sum of RM8 000 into Bank Pantas with an interest rate of 4% over 2 years. What is the amount of savings at the end of the second year?

Skills Enhancement

1. How can cost averaging strategy help an investor?
2. Explain the meaning of investment in real estate.
3. The following conversation is between Ramesh and Ismail regarding the purchase of shares.

Explain three types of return that will be received by Ismail.

4. The following are two types of investments.

Lee Chong bought 3 000 units of shares of a public limited company.	Mokhtar bought 3 000 000 units of unit trusts.

Explain the two differences between the two types of investments above.

5. Encik Shah wants to deposit RM10 000 into a fixed deposit account for 9 months.
The following are the fixed deposit interest rates for different terms offered by a bank to Encik Shah.

Duration	Annual interest rate
1 month	3.0
3 months	3.5
6 months	3.75
9 months	4.00
12 months	4.25

Calculate the amount of interest that will be received by Encik Shah if he is saving for a 9-month term.

6. In 2018, Encik Zainal holds 6 000 units of shares of Syarikat Vision Sdn. Bhd. which is worth RM1 per share unit. During the year, the company declared a 6% dividend and a bonus issue at 1 new share for 2 share units held. At the end of 2018, the share price rose to RM2.30 per unit. Calculate
   (a) the amount of dividend received by Encik Zainal
   (b) the number of bonus share units to be received by Encik Zainal
   (c) the number of share units held by Encik Zainal after receiving the bonus shares

7. Complete the following table.

Deposit amount (RM)	Flat interest rate	Savings period (years)	Total interest accumulated
10 000	5%	2	
5 000		1	150
4 000	6%		720

8. Mr Kishendran deposits RM5 000 into a fixed deposit account with 4% interest rate compounded every 3 months for a period of 3 years. Calculate the amount of interest accrued after the third year.

Self Mastery

1. Mr Oswald Alphonsus borrowed RM15 000 from Bank Yakin to start a tailoring business in Rawang. The bank charges a 5% flat interest rate for a repayment period of 5 years. How much interest will be paid to the bank by Mr Oswald Alphonsus?

2. Mrs Emily Francis saves RM10 000 in a bank. By the end of the eighth year, the money collected amounts to RM19 992.71. If the bank pays an annual interest of x% for a year and is compounded every 6 months, calculate the value of x.

3. Puan Noraini Mitis deposits a certain amount of money into her savings account which provides an interest rate of 2% per annum and compounded quarterly. What is the initial deposit made by Puan Noraini Mitis if the money collected at the end of the fifth year is RM7 734.26?

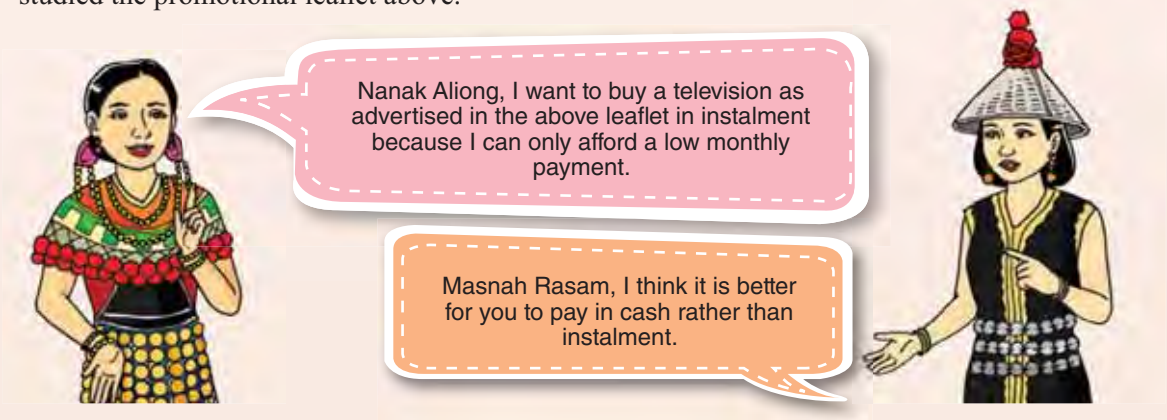
4. Puan Zaiton bought 1 000 share units of Syarikat Pelita Berhad at RM2.00 per unit. At the end of the year, Syarikat Pelita Berhad paid a dividend of 20 sen per unit to all its shareholders. The following year, Puan Zaiton sold all the shares held when the share price rose to RM2.20 per unit. Calculate the total return for Puan Zaiton.

5. Encik Iskandar takes a personal loan of RM20 000 from Bank Cergas with an interest rate of 4% per annum. The repayment term is for 10 years. What is the monthly instalment payable by Encik Iskandar?

6. Puan Balkis takes a personal loan of RM8 000 from Bank Sentosa with interest rate of 4% per annum on the balance. The payback period is 4 years while the monthly instalment is RM110. Calculate the amount of interest payable by Puan Balkis within 2 months.

7. The following is a promotional leaflet offered by Seng Hong Company.

The following conversation took place between Masnah Rasam and Nanak Aliong after they studied the promotional leaflet above.



(a) What is your view on Masnah Rasam's opinion?
(b) Calculate the amount of interest paid and the interest rate on this instalment payments.
(c) If you want to buy a television, how would you purchase it?

8. Ms Kayal borrows from Bank Desa RM*X* with an interest rate of 5% per annum. The payback period is 8 years. If the monthly instalment paid is RM218.75, calculate the amount of money borrowed by Ms Kayal.

9. Mr Murugan has borrowed RM16 000 from Bank Orkid for personal use. He will repay over 5 years with a monthly instalment of RM320. Calculate the yearly interest charged by the bank.

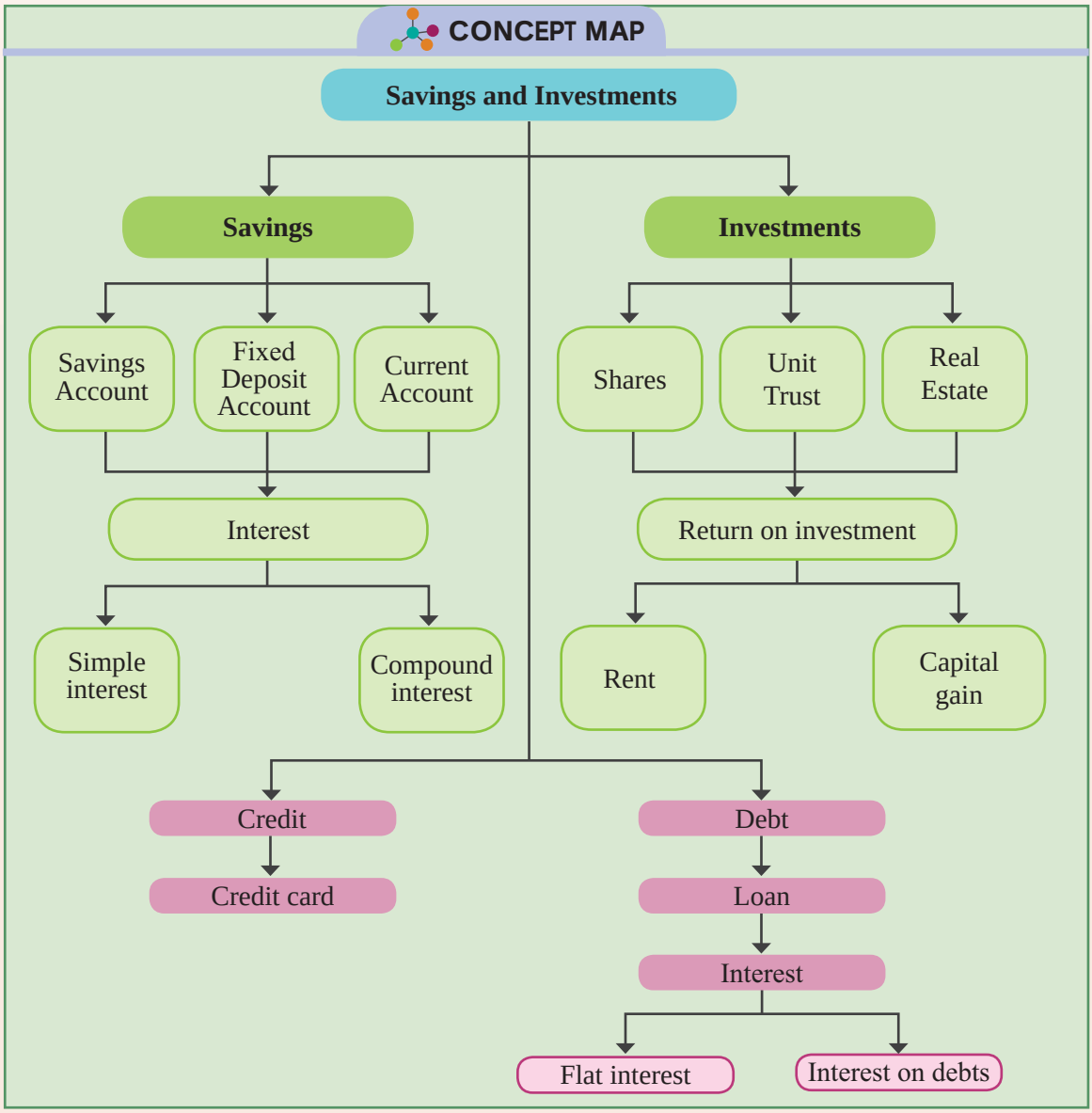
10. Puan Sapiah borrowed RM12 000 from a bank with interest rate of 3% per annum for 5 years. Meanwhile, Puan Shafiqah Ira borrows the same amount of money from another bank with a rate of 4.5% per annum for 5 years. Calculate and state the difference between the total interest paid by Puan Sapiah and Puan Shafiqah Ira.

PROJECT

Assume that you have won RM1 million in a puzzle contest.

1. State the way in which you will invest the money.
2. Explain why you chose this form of investment.

CONCEPT MAP



SELF-REFLECT

At the end of this chapter, I can:

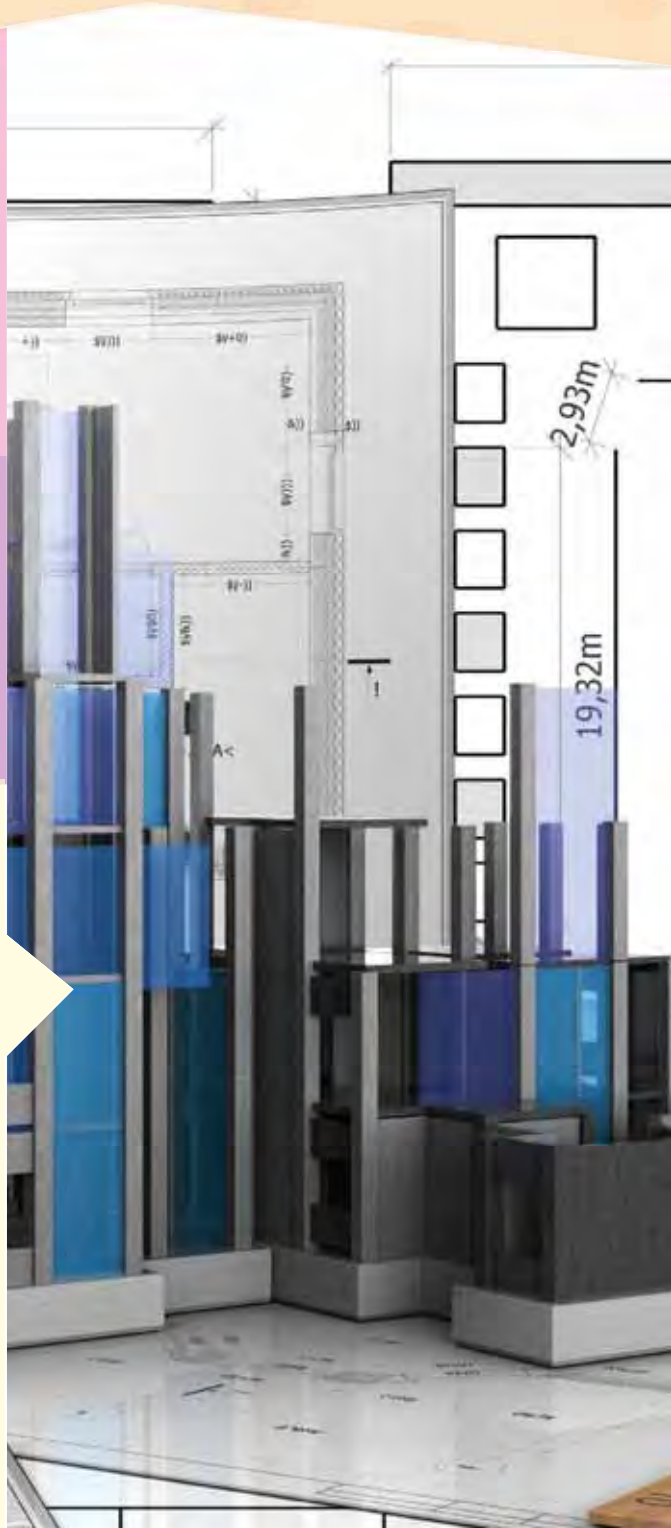
			
1.	Recognise various types of savings and investments.		
2.	Perform calculations involving simple interest and compound interest for savings, and hence explain the impact of changes in period, rate of interest or return and compounding frequency on the future value of savings.		
3.	Perform calculations involving return on investments, and hence explain the factors that affect return on investments and its impacts.		
4.	Compare and contrast potential risks, return and liquidity of various types of savings and investments.		
5.	Calculate the average cost per share for the investment of shares using the ringgit cost averaging strategy and explain the benefits of the strategy.		
6.	Solve problems involving savings and investments.		
7.	Explain the meaning of credit and debt, and hence describe the wise management of credit and debt.		
8.	Investigate and describe the advantages and disadvantages of credit card and ways to use it wisely.		
9.	Investigate and describe the impact of minimum and late payments for credit card usage.		
10.	Solve problems involving the use of credit cards.		
11.	Calculate the total amount of loan repayment and instalment, with various interest rates and different loan periods.		
12.	Solve problems involving loans.		

EXPLORING MATHEMATICS

You can visit the Credit Counselling and Debt Management Agency (AKPK) website to calculate the required period and the amount of interest payable to settle your credit card debt.

CHAPTER 4

Scale Drawings



What will you learn?

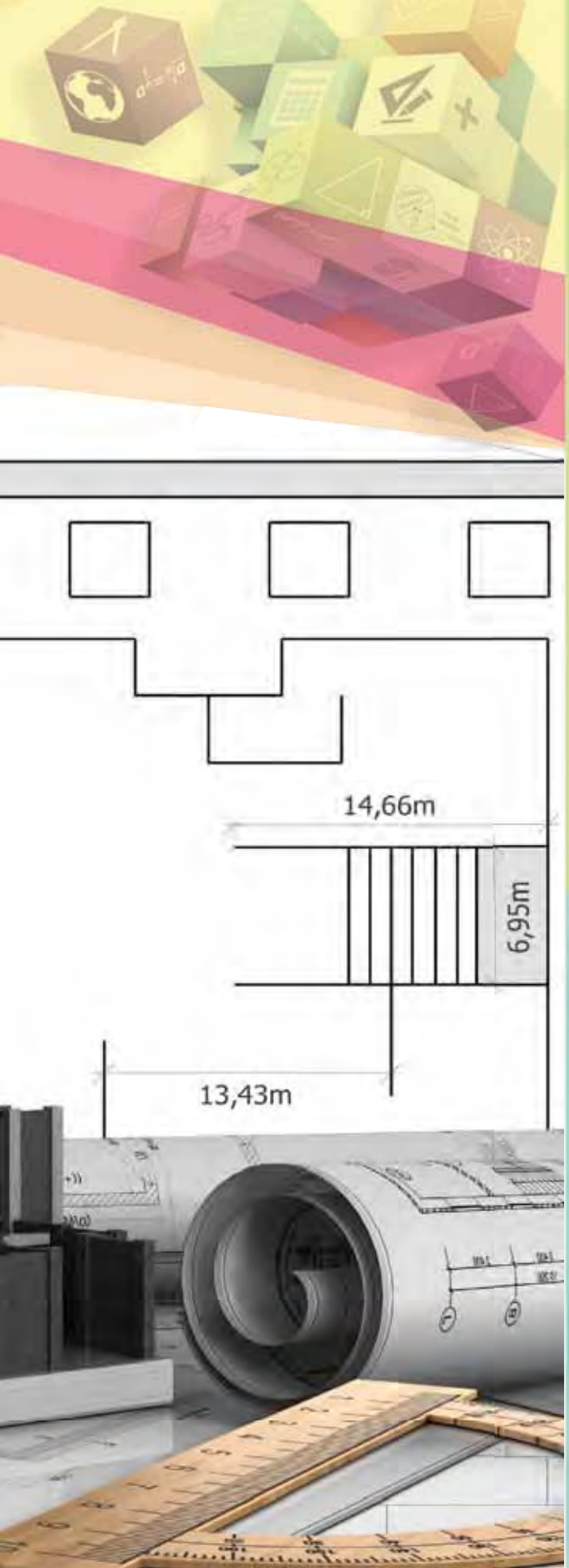
4.1 **Scale Drawings**

Why do you learn this chapter?

- Scale drawings are used to give an idea of object measurements or actual distances.
- Scale drawings are frequently used in architecture, engineering, photography, technological design, and so on.

Housing developers usually prepare models of the housing scheme they are going to build. These models give the buyers an idea of the housing zone and other facilities provided by the developer.

Also, the plans of the houses to be built are drawn using a certain scale with measurements being in proportion to the actual measurements of the houses. These plan drawings allow buyers to choose the type of house to buy based on house size, facilities and also needs and affordability. Have you ever seen your house plan?



Exploring Era

Angkor Wat in Cambodia is one of the famous monuments in Southeast Asia. It was built by Suryavarman II after his victory over Champa and subsequently uniting Kampuchea. This monument symbolises the strength and sovereignty of Suryavarman II's government. Its height is 213 metres and its area is 208 hectares. The altar is at the centre. It has four storeys with every corner having walls measuring 850 metres wide and 1000 metres long built from laterite and sandstones sourced locally. Carvings depicting stories from Ramayana and Mahabharata epics decorate the walls.

http://bukuteksksm.my/Mathematics/F3/ExploringEraChapter4.pdf

WORD BANK

• original	• *asal*
• geometrical shape	• *bentuk geometri*
• degree	• *darjah*
• grid	• *grid*
• sketch	• *lakar*
• object	• *objek*
• size	• *saiz*
• scale	• *skala*

4.1 Scale Drawings

What is the relationship between the actual measurements and the measurements of various sizes of drawings of an object?

LEARNING STANDARD

Investigate and explain the relationship between the actual measurements and the measurements of various sizes of drawings of an object, and hence explain the meaning of scale drawing.

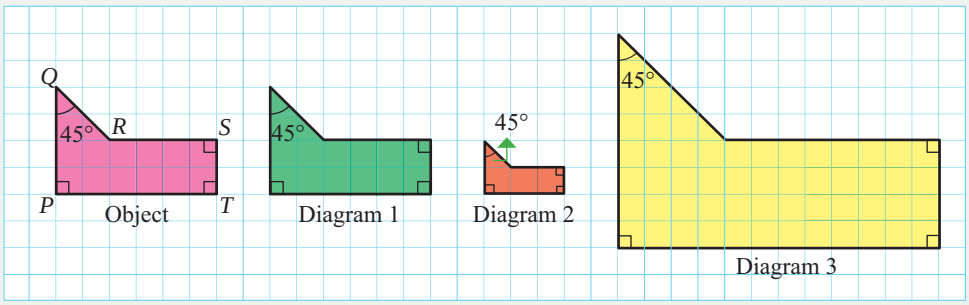
Do you know that the maps found in navigation software are drawn to a certain scale?

The distance between two towns shown in the software is proportional to the actual distance.

For example, in the picture above, the distance between Johor Bahru and Kuala Lumpur is shown using a scale of 1 cm : 50 km.

Example 1

The diagram below shows the drawings representing object *PQRST* drawn to different sizes.



What can you say about the size of Diagram 1, Diagram 2 and Diagram 3 compared to object *PQRST*?

Solution:

Diagram 1: Lengths of all sides and sizes of all angles are the same as object.

Diagram 2: Lengths of all sides reduced by a certain proportion compared to object but sizes of all angles unchanged.

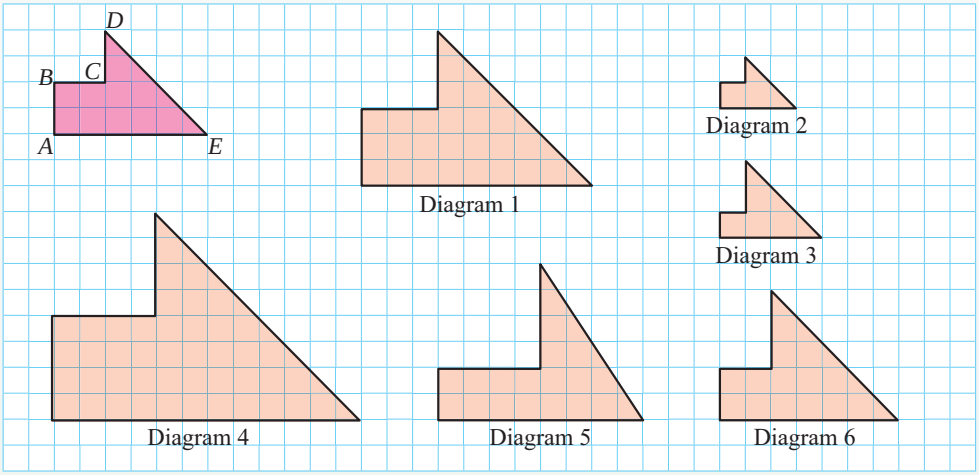
Diagram 3: Lengths of all sides increased by a certain proportion compared to object but sizes of all angles unchanged.

In conclusion, all sides of Diagram 1, Diagram 2 and Diagram 3 follow a certain scale that is proportional to the object whereas angle size remains unchanged. Therefore, Diagram 1, Diagram 2 and Diagram 3 are scale drawings of object *PQRST*.

Scale drawing is the drawing of an object with all measurements in the drawing proportional to the measurements of the object.

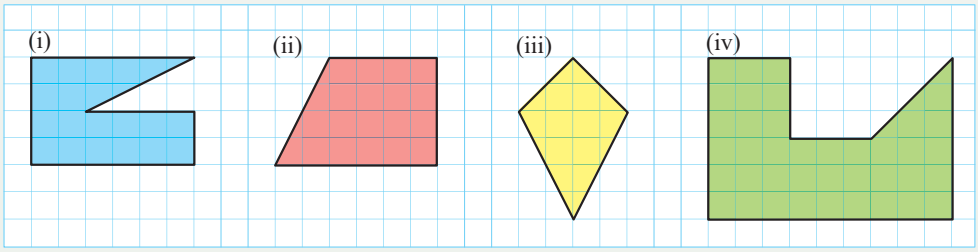
MIND TEST 4.1a

1. The diagram below shows drawings representing object *ABCDE* drawn to different sizes.



State the diagram which is the scale drawing of object *ABCDE*.

2. Using grid paper, draw all the shapes below using
(a) the same size (b) a smaller size (c) a bigger size



How do you interpret the scale of a scale drawing?

LEARNING STANDARD
Interpret the scale of a scale drawing.

The scale used to draw a scale drawing depends on the ratio of measurement of scale drawing to measurement of object, which is

$$\text{Scale} = \frac{\text{Measurement of scale drawing}}{\text{Measurement of object}}$$

This ratio can also be written in the form;

Measurement of scale drawing : Measurement of object

Usually, for scale drawings, we use scale in the form of ratio.

$1 : n$
where n is a positive integer or fraction

$1 : n$ means one unit on the scale drawing will represent n units on the object.

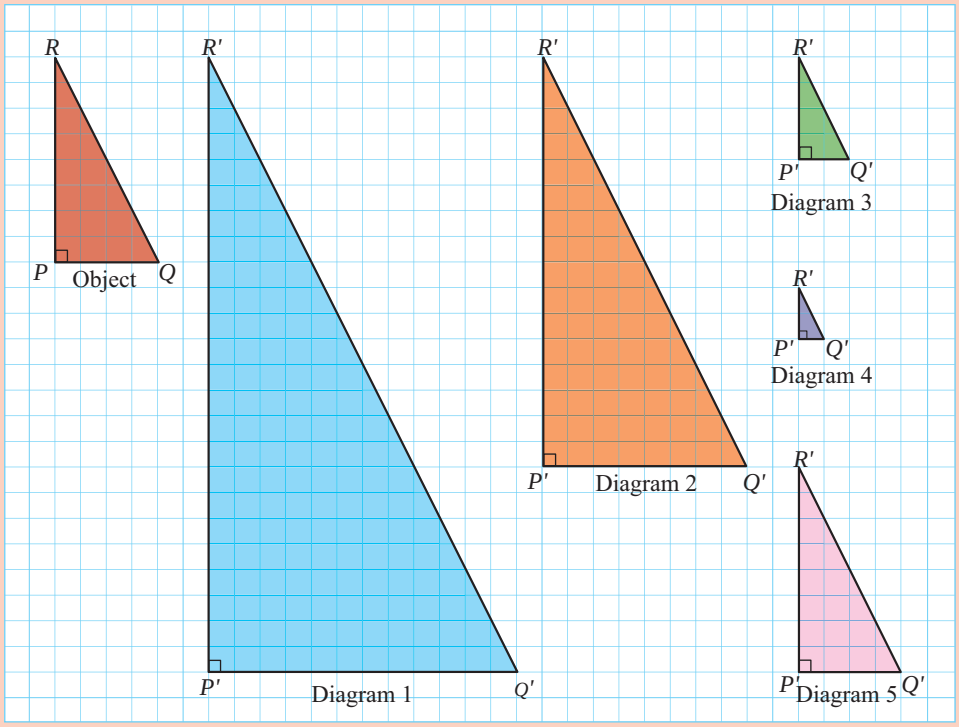
Brainstorming 1

In pairs

Aim: Interpret the scale of a scale drawing.

Steps:

1. Study the diagrams below.



2. Complete the table below based on the diagrams above.

Diagram	Scale drawing		Object		Scale	
	Side	Length (unit)	Side	Length (unit)	Ratio	1 : *n*
Diagram 1	*P'Q'*	12	*PQ*	4	12 : 4	$1 : \frac{1}{3}$
	P'R'	24	*PR*	8	24 : 8	$1 : \frac{1}{3}$
Diagram 2	*P'Q'*		*PQ*			
	P'R'		*PR*			
Diagram 3	*P'Q'*		*PQ*			
	P'R'		*PR*			
Diagram 4	*P'Q'*		*PQ*			
	P'R'		*PR*			
Diagram 5	*P'Q'*		*PQ*			
	P'R'		*PR*			

Discussion:

Discuss the results based on the table above.

From Brainstorming 1, it is found that:

If $n < 1$, then the size of the scale drawing is bigger than the size of the object.
If $n > 1$, then the size of the scale drawing is smaller than the size of the object.
If $n = 1$, then the size of the scale drawing is the same as the size of the object.

How do you determine the scales, measurements of objects or measurements of scale drawings?

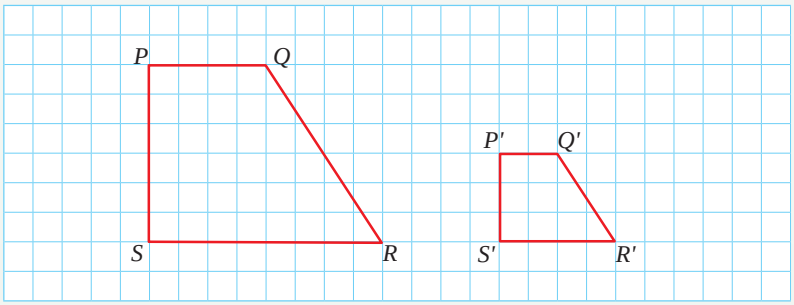
$$\text{Scale} = \frac{\text{Measurement of scale drawing}}{\text{Measurement of object}} = \frac{1}{n}$$

LEARNING STANDARD

Determine the scales, measurements of objects or measurements of scale drawings.

Example 2

The diagram below shows object $PQRS$ and scale drawing $P'Q'R'S'$ drawn on a grid of equal squares. State the scale used in the form $1 : n$.

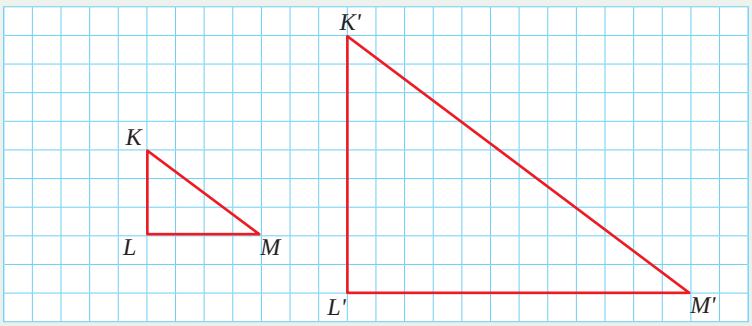


Solution:

$$\text{Scale} = \frac{P'Q'}{PQ} = \frac{2}{4} = \frac{1}{2} \quad \text{or} \quad \text{Scale} = \frac{P'S'}{PS} = \frac{3}{6} = \frac{1}{2} \quad \text{Thus, scale} = 1:2$$

Example 3

The diagram below shows object KLM and scale drawing $K'L'M'$ drawn on a grid of equal squares. State the scale used in the form $1 : n$.



Solution:

$$\text{Scale} = \frac{K'L'}{KL} = \frac{9}{3} = \frac{3}{1} \quad \text{or} \quad \text{Scale} = \frac{L'M'}{LM} = \frac{12}{4} = \frac{3}{1}$$

$$\text{Thus, scale} = 3:1$$

3 divided by 3 will give 1.

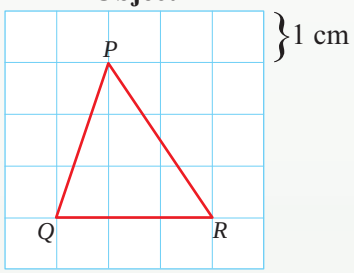
$$= \frac{3}{3} : \frac{1}{3}$$

$$= 1 : \frac{1}{3}$$

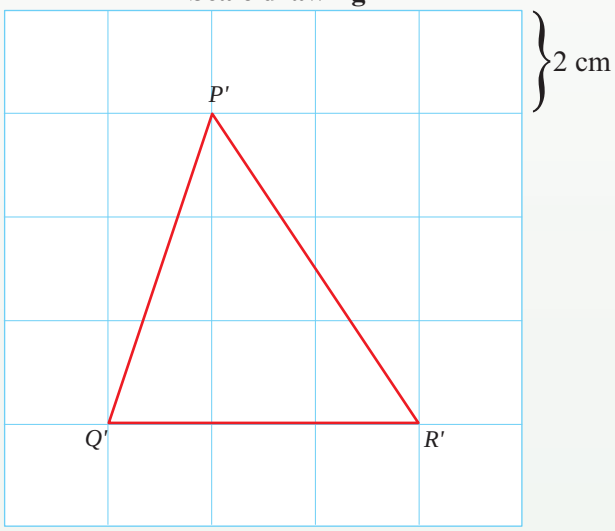
Example 4

The diagram below shows object PQR and scale drawing $P'Q'R'$ drawn on a grid of equal squares of different sizes. Determine the scale used in the form $1 : n$.

Object



Scale drawing



Solution:

Method 1

$$\text{Scale} = \frac{Q'R'}{QR} = \frac{6 \text{ cm}}{3 \text{ cm}} = \frac{2}{1}$$

$$\text{Scale} = 2 : 1$$

$$= 1 : \frac{1}{2}$$

Method 2

Grid size is used because number of units of sides of object and sides of scale drawing is the same.

$$\text{Scale} = \frac{\text{Grid size of scale drawing}}{\text{Grid size of object}} = \frac{2 \text{ cm}}{1 \text{ cm}} = \frac{2}{1}$$

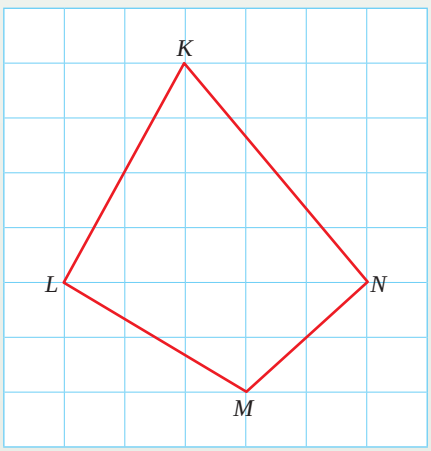
$$\text{Scale} = 2 : 1$$

$$= 1 : \frac{1}{2}$$

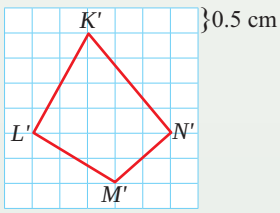
Example 5

The diagram below shows object $KLMN$ and scale drawing $K'L'M'N'$ drawn on a grid of equal squares of different sizes. Determine the scale used.

Object



Scale drawing



FLASHBACK

$K'N' = \sqrt{1.5^2 + 2^2} = 2.5$

$KN = \sqrt{3^2 + 4^2} = 5$

Solution:

Method 1

$$\text{Scale} = \frac{K'N'}{KN} = \frac{2.5 \text{ cm}}{5 \text{ cm}} = \frac{0.5}{1} = \frac{\frac{1}{2}}{1}$$

$$\text{Scale} = \frac{1}{2} : 1$$

$$\frac{1}{2} \times 2 : 1 \times 2$$

$$1 : 2$$

$\frac{1}{2}$ multiplied by 2 to get 1.

Method 2

$$\text{Scale} = \frac{\text{Grid size of scale drawing}}{\text{Grid size of object}} = \frac{0.5 \text{ cm}}{1 \text{ cm}} = \frac{\frac{1}{2}}{1}$$

$$\text{Scale} = \frac{1}{2} : 1$$

$$1 : 2$$

Example 6

A map is drawn to a scale of 1 : 300 000. Calculate the actual length, in km, of a river that is 3 cm long on the map.

Solution:

Method 1

$$\frac{1 \text{ cm}}{300\,000 \text{ cm}} = \frac{3 \text{ cm}}{\text{Actual distance}}$$

$$\text{Actual distance} = \frac{3 \times 300\,000 \text{ cm}}{1 \text{ cm}}$$

$$= 900\,000 \text{ cm}$$

$$= 9 \text{ km}$$

Method 2

Scale drawing : Object

1 : 300 000

1 cm : 300 000 cm

× 3: 1 cm : 3 km (× 3)

3 cm : 9 km

Thus, the actual length of the river is 9 km.

FLASHBACK

1 km = 1 000 m
1 m = 100 cm
1 km = 100 000 cm

Example 7

The map of Johor is drawn to a scale of 1 cm to 10 km. Calculate the actual distance between Kluang and Ayer Hitam if the distance on the map is 2 cm.

Solution:

Method 1

$$\text{Scale} = \frac{\text{Distance on drawing}}{\text{Actual distance}}$$

$$\frac{1 \text{ cm}}{10 \text{ km}} = \frac{2 \text{ cm}}{\text{Actual distance}}$$

$$\text{Actual distance} = \frac{2 \text{ cm } (10 \text{ km})}{1 \text{ cm}}$$

$$\text{Actual distance} = 20 \text{ km}$$

Method 2

Scale drawing : Object

1 cm : 10 km

× 2: 2 cm : 20 km (× 2)

Thus, the actual distance is 20 km.

Example 8

Khairul draws a square to a scale of $1 : \frac{1}{3}$. If the actual length of sides of the square is 6 cm, what is the length of sides, in cm, of the scale drawing?

Solution:

Method 1

$$\text{Scale} = \frac{\text{Side of scale drawing}}{\text{Corresponding side of object}}$$

$$\frac{1}{\frac{1}{3}} = \frac{\text{Side of scale drawing}}{6 \text{ cm}}$$

Side of scale drawing = 3 × 6 cm
Length of side of scale drawing = 18 cm

Method 2

Scale drawing : Object

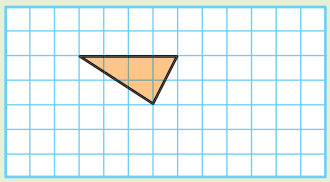
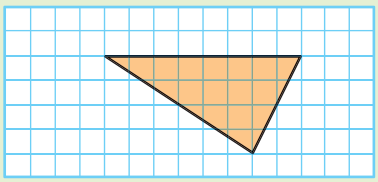
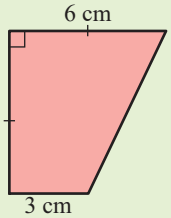
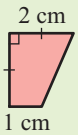
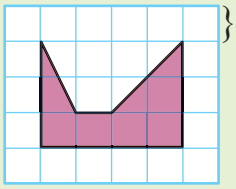
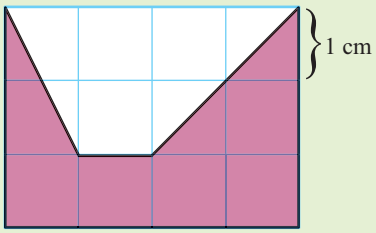
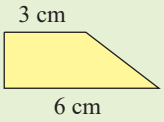
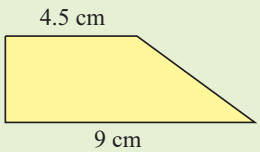
$1 : \frac{1}{3}$ (×18 on the left, ×18 on the right)

18 cm : 6 cm

Thus, the length of side of scale drawing is 18 cm.

MIND TEST 4.1b

1. Determine the scale used for each scale drawing below in the form $1 : n$.

Object	Scale drawing
(a) 	
(b) 	
(c) 	
(d) 	

2. A poster has a length of 24 cm and a width of 8 cm. Calculate the length and width of the scale drawing of the poster, in cm, that is drawn to a scale of 1 : 4.

3. A map is drawn to a scale of 1 : 400 000. What is the actual length, in km, of a river with a length of 2.5 cm on the map?

4. Siew Lin draws a right-angled triangle to a scale of $1 : \frac{1}{3}$. If the hypotenuse of the scale drawing is 18 cm, calculate the length of the hypotenuse of the original triangle.

How do you draw the scale drawings of objects and vice versa?

LEARNING STANDARD

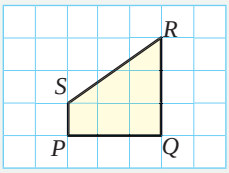
Draw the scale drawings of objects and vice versa.

Drawing the scale drawing of an object

There are three ways to draw the scale drawing of an object.
(a) Use grid paper of the same size for different scales.
(b) Use grid paper of different sizes.
(c) Draw on a blank paper according to the given scale.

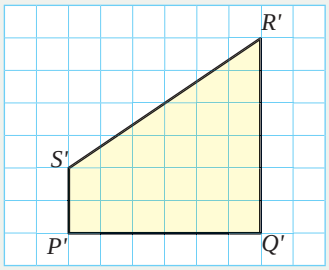
Example 9

Draw the scale drawing of shape *PQRS* on a grid of equal squares using a scale of $1 : \frac{1}{2}$.



Solution:

The scale given is $1 : \frac{1}{2}$. Therefore, every side of the scale drawing is two times longer than the length of sides of object *PQRS*.



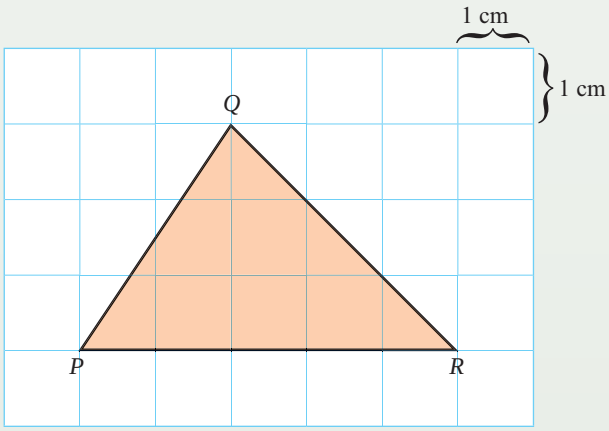
QUIZ

If you have to draw the scale drawing of your school field, what is a suitable scale to be used? Why?

Example 10

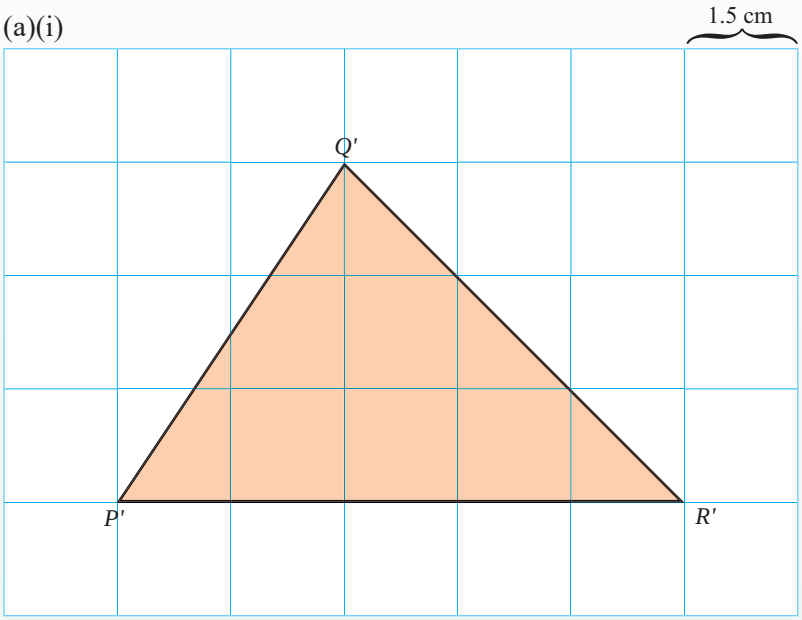
(a) Diagram ΔPQR is drawn on a grid of 1 cm × 1 cm. Redraw ΔPQR on grid paper with dimensions
  (i) 1.5 cm × 1.5 cm
  (ii) 0.5 cm × 0.5 cm

(b) Calculate the scale used in (a)(i) and (a)(ii) in the form 1 : *n*.

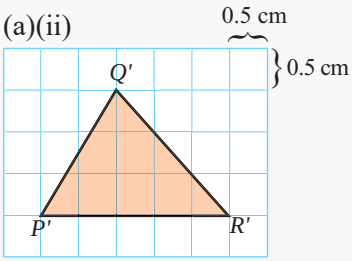


Solution:

(a)(i)



(a)(ii)



(b)(i)

$$\text{Scale} = \frac{\text{Grid size of scale drawing}}{\text{Grid size of object}} = \frac{1.5 \text{ cm}}{1 \text{ cm}}$$

$$\text{Scale} = 1.5 : 1$$
$$= 1 : \frac{2}{3}$$

(b)(ii)

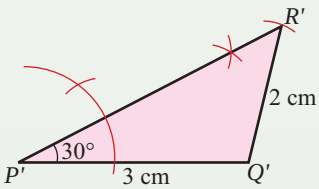
$$\text{Scale} = \frac{\text{Grid size of scale drawing}}{\text{Grid size of object}} = \frac{0.5 \text{ cm}}{1 \text{ cm}}$$

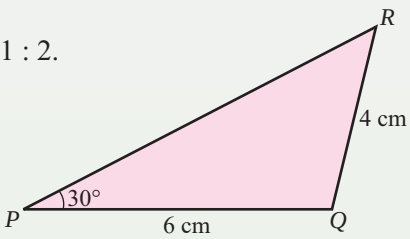
$$\text{Scale} = 0.5 : 1$$
$$= 1 : 2$$

Example 11

Construct the scale drawing of triangle PQR using a scale of 1 : 2.

Solution:





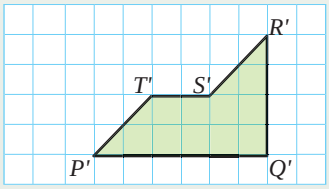
For objects with given angles, the angles of the scale drawing must be accurately drawn and the lengths of sides are drawn to scale.

Scan the QR Code or visit http://bukutekssksm.my/Mathematics/F3/Chapter4Grid.pdf to download grid paper of various sizes.

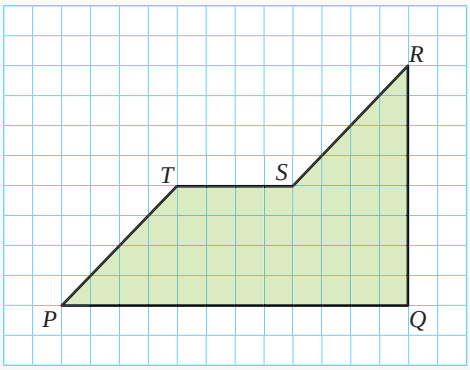
Drawing the objects for a scale drawing

Example 12

The diagram shows a scale drawing drawn on a grid of equal squares to a scale of 1 : 2. Draw the actual object for $P'Q'R'S'T'$.



Solution:



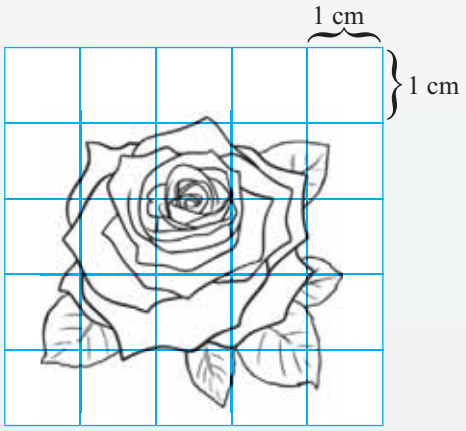
The scale used is 1 : 2, that is the size of scale drawing is two times smaller than the object. Therefore, every side of actual object is two times longer than the sides of the scale drawing.

Example 13

The diagram shows the scale drawing of a flower drawn on 1 cm × 1 cm grids. Draw the actual object on grids of

(a) 0.5 cm × 0.5 cm

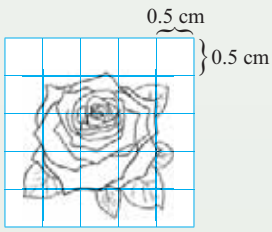
(b) 1.5 cm × 1.5 cm

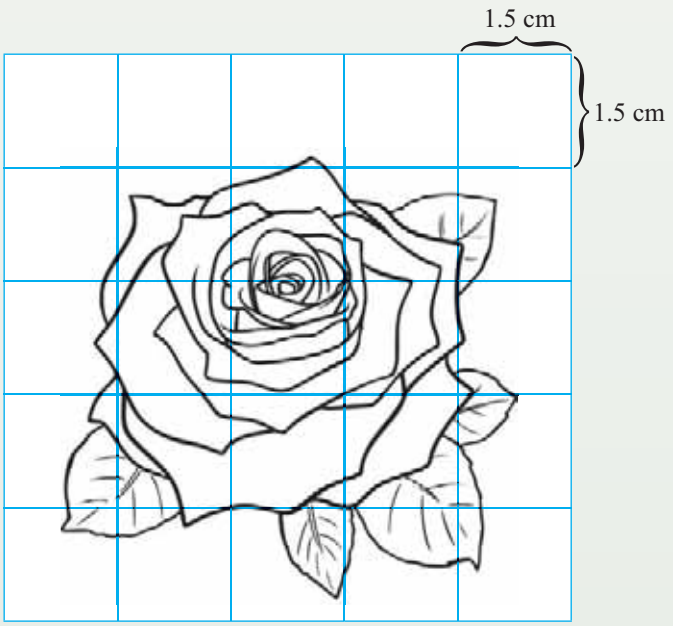


Solution:

Object must be drawn on grids of different sizes. Thus, the number of units of sides of object is the same as the number of units of sides of scale drawing.

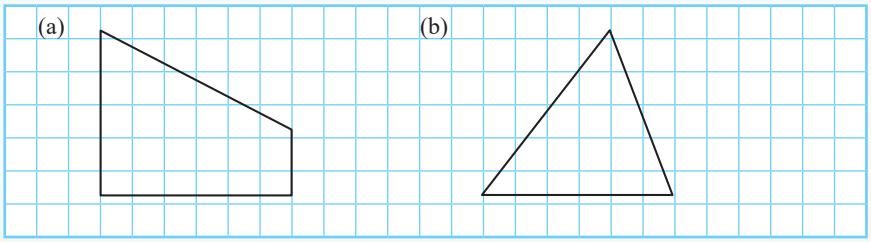
(a)





MIND TEST 4.1c

1. Draw the scale drawing of each object below to a scale of $1 : \frac{1}{2}$ and $1 : 3$.

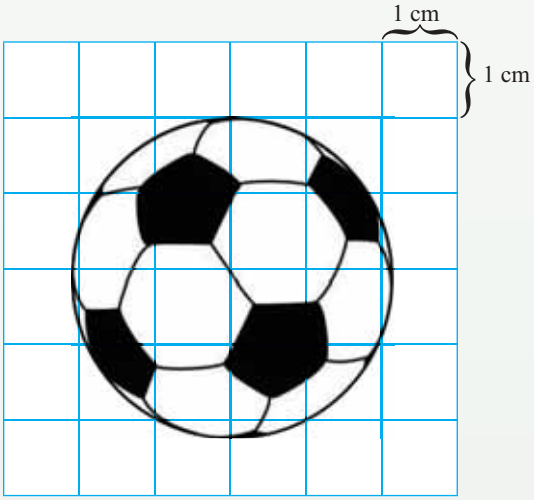


2. (a) The object in the diagram is drawn on 1 cm × 1 cm grid paper. Redraw the shape of the object on a grid paper of

(i) 2 cm × 2 cm

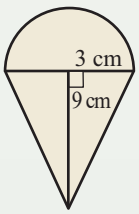
(ii) 0.5 cm × 0.5 cm

(b) Calculate the scale used in (a)(i) and (a)(ii).

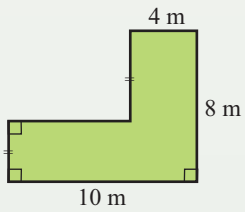


3. Draw the scale drawing of the following shapes to the given scale.

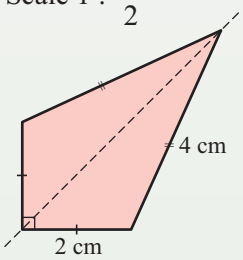
(a) Scale 1 : 3



(b) Scale 1 : 200

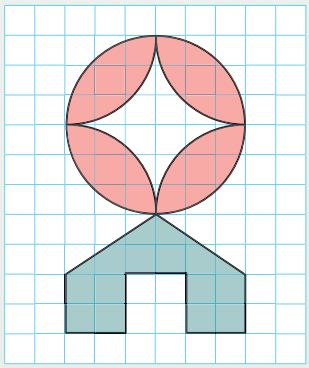


(c) Scale $1 : \frac{1}{2}$



4. The diagram shows the scale drawing of a composite shape that is drawn on a grid of equal squares to a scale of $1 : \frac{1}{2}$.

Draw the actual object for the shape.



How do you solve problems involving scale drawings?

LEARNING STANDARD

Solve problems involving scale drawings.

Example 14

The distance on a map between Bintulu and Miri is 4 cm.

(a) If the scale used to draw the map is 1 cm : 50 km, calculate the actual distance, in km, between Bintulu and Miri.

(b) If the map is redrawn to a scale of 1 : 2 000 000, calculate the distance between Bintulu and Miri on the new map.

(c) Mr Dominic Lajawa and his family wants to visit Miri. If he plans to drive to Miri at a speed of 80 km h^{-1}, calculate the time taken to drive from Bintulu to Miri in hours and minutes.

TIP

If the scale of scale drawing and the requirement of the question are in the same unit, the scale need not be changed to cm.

Solution:

Understanding the problem

- Actual distance for 4 cm drawn to scale of 1 cm : 50 km.
- Distance on scale drawing drawn to scale of 1 : 2 000 000.
- Time in hours and minutes for journey from Bintulu to Miri at speed of 80 km h^{-1}.

Planning a strategy

$$\text{Scale} = \frac{\text{Distance on drawing}}{\text{Actual distance}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Implementing the strategy

(a) $\text{Scale} = \dfrac{\text{Distance on drawing}}{\text{Actual distance}}$

$$\frac{1}{50\text{ km}} = \frac{4\text{ cm}}{\text{Actual distance}}$$

$$\text{Actual distance} = \frac{4\text{ cm}\ (50\text{ km})}{1\text{ cm}}$$

Actual distance = 200 km

(b) $\text{Scale} = \dfrac{\text{Distance on drawing}}{\text{Actual distance}}$

$$\frac{1}{2\,000\,000} = \frac{\text{Distance on drawing}}{200\text{ km}}$$

$$\text{Distance on drawing} = \frac{(200 \times 100\,000)\text{ cm}}{(2\,000\,000)\text{ cm}}$$

Distance on scale drawing = 10 cm

(c) $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

$$= \frac{200\text{ km}}{80\text{ km h}^{-1}}$$

= 2.5 hours

Time taken
= 2 hours 30 minutes

Making a conclusion

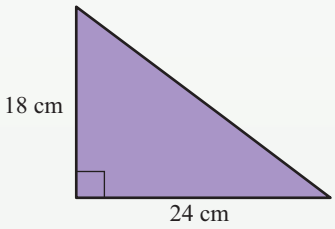
- Actual distance between Bintulu and Miri is 200 km.
- Distance between Bintulu and Miri on the map of scale of 1 : 2 000 000.
- Time taken for Mr Dominic Lajawa to drive from Bintulu to Miri at a speed of 80 km h^{-1} is 2 hours 30 minutes.

FLASHBACK

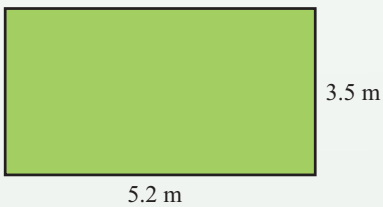
$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

MIND TEST 4.1d

1. The diagram shows a right-angled triangle. A scale drawing of the triangle is drawn to a scale of $1 : \frac{1}{3}$. Calculate the area, in cm^2, of the scale drawing.



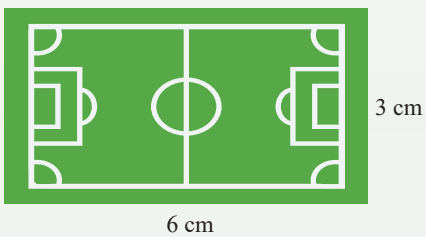
2. The diagram shows a room in the shape of a rectangle. Calculate the perimeter, in cm, of the scale drawing of the room which is drawn to a scale of 1 : 50.



3. The measurements of a rectangular room on a scale drawing are 7 cm × 5 cm. If the scale used is 1 : 400, calculate the actual area of the room in m^2.

4. A regular polygon with an exterior angle of 36° is redrawn using a scale of 1 : 5. If the actual length of sides of the regular polygon is 10 cm, calculate the perimeter of the scale drawing of the regular polygon.

5.



The diagram above shows a scale drawing of a rectangular field.

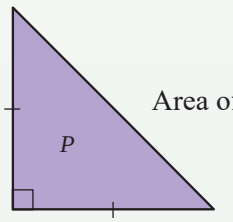
(a) If the scale used is 1 : 2 000, calculate the actual area of the field in square metres.

(b) Mr Dany cuts the grass on the field at a rate of 400 square metres in 8 minutes. Calculate the time, in hours and minutes, that Mr Dany takes to cut all the grass on the field.

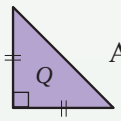
Dynamic Challenge

Test Yourself

1. The diagram below shows triangle P which is the scale drawing of triangle Q with a scale of $1 : n$. Calculate the value of n.

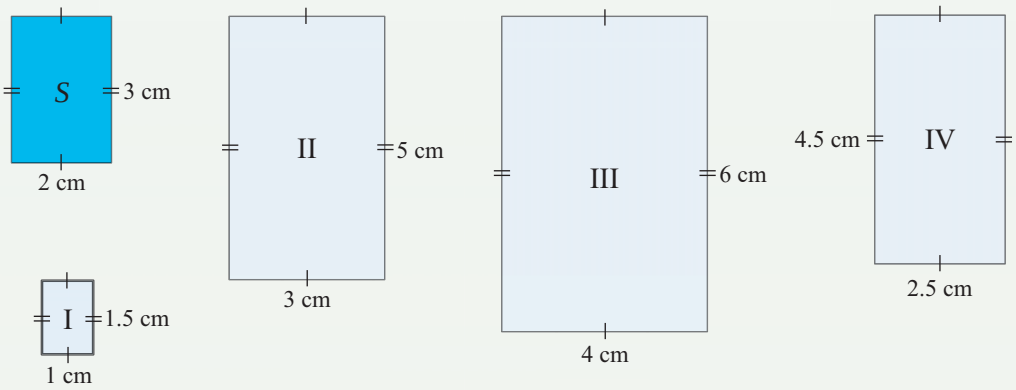


Area of $P = 112.5$ cm^2



Area of $Q = 4.5$ cm^2

2. The diagram below shows five rectangles.



(a) Among rectangles I, II, III and IV, which are the scale drawings of rectangle S drawn to a certain scale?

(b) For each answer in (a), determine the scale used.

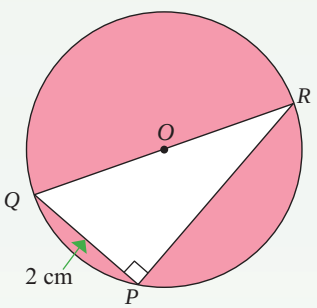
(c) (i) Calculate the area of each rectangle, in cm^2, for your answer in (a).

(ii) Determine the ratio of area of S to area of each answer in (c)(i).
What are your conclusions about the ratios obtained?

3. The diagram shows a scale drawing of a circle with centre O and triangle PQR. It is given that the diameter of the circle is 6 cm and the scale of the drawing is $1 : 3$.

(a) Calculate the actual length of PR in cm. State your answer correct to three significant figures.

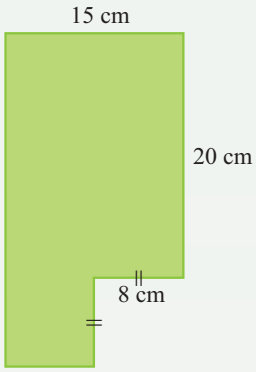
(b) Using your answer in (a), calculate the actual area of the shaded region in cm^2. State the answer correct to four significant figures.



1.

The distance by air from Kuching to Kota Kinabalu on a map is 5.4 cm. It is given that the scale of the map is 1 cm : 150 km. If an aeroplane takes off from Kuching International Airport at 1240 hours and lands at Kota Kinabalu International Airport at 1410 hours, calculate the average speed of the aeroplane in km h^{-1}.

2. The diagram shows the scale drawing of Puan Farah's living room. The scale of the drawing is 1 : 50. Puan Farah wants to lay tiles throughout the entire living room. She intends to use tiles measuring 30 cm × 30 cm which cost RM2.80 a piece. Puan Farah's husband suggests to use tiles of 50 cm × 50 cm at RM6 a piece. Which tile should Puan Farah choose if she wants to save money? State the reasons for your answer.



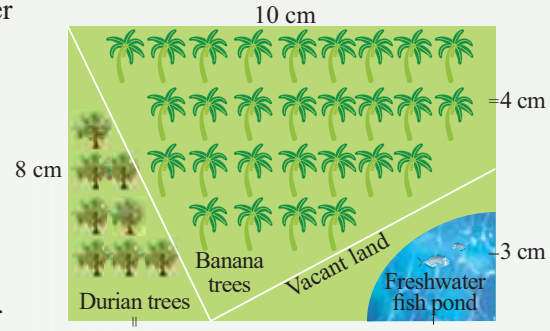
3. The diagram shows the scale drawing of a rectangular farm owned by Pak Hassan. It is given that the scale of the drawing is 1 : 2 000.

(a) Calculate the actual area of the freshwater fish pond to the nearest square metre.
$\left[\pi = \frac{22}{7}\right]$.

(b) Calculate the ratio of the area planted with durian trees to the area planted with banana trees.

(c) Calculate the area, in m^2, of the vacant land.

(d) Pak Hassan wants to fence up his farm. If the cost of one metre of fencing is RM5.50, calculate the total cost of fencing, in RM.



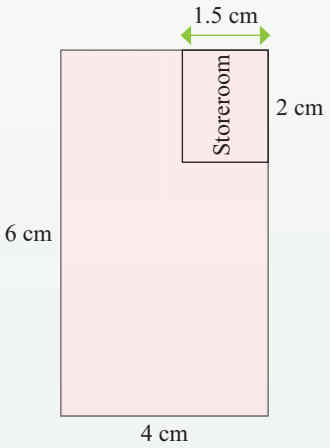
Self Mastery

1. The diagram shows the scale drawing of the floor plan of a shophouse that is drawn to a scale of 1 : 400.

(a) Calculate the actual area of the storeroom, in m^2.

(b) State the ratio of the area of the shophouse to the area of the storeroom.

(c) If the actual height of the shophouse is 3.75 m, calculate the volume, in m^3, of the three-dimensional shophouse.



2.

Diagram 1

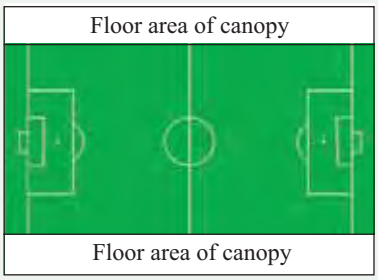


Diagram 2

Diagram 1 shows the scale drawing of a rectangular football field.

(a) If this scale drawing is drawn to a scale of 1 : 1 000, calculate the actual area, in m^2, of the football field.

(b) Sharon wants to redraw the scale drawing in Diagram 1 on a piece of A4-sized paper. What is the maximum scale that Sharon can choose? State the reasons for your answer.

(c) Several canopies will be set up on the football field as in Diagram 2 for a carnival.

(i) If the dimensions of the base of a tent are 5 m × 4 m, calculate the maximum number of tents that can be erected.

(ii) The rent for a tent is RM100 a day. A 25% discount will be given if the tent is rented for five days or more. Calculate the total rent, in RM, if the carnival lasts for one week.

PROJECT

Draw the map of the district where you live using a suitable scale. You can mark the location of your house, school and interesting places in your district using symbols or suitable illustrations. Exhibit your project in the classroom.

CONCEPT MAP

Scale Drawings

Scale drawing is drawing that shows the original object based on a certain **scale**.

$$\text{Scale} = \frac{\text{Measurement of scale drawing}}{\text{Measurement of object}}$$

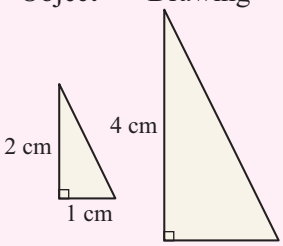
Scale $1 : n$, or $1 : \frac{1}{n}$ where $n = 1, 2, 3, \ldots$

$n < 1$

Scale drawing bigger than object

$1 : \frac{1}{2}$

Object Drawing

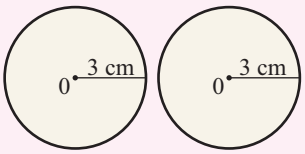


$n = 1$

Scale drawing same size as object

$1 : 1$

Object Drawing

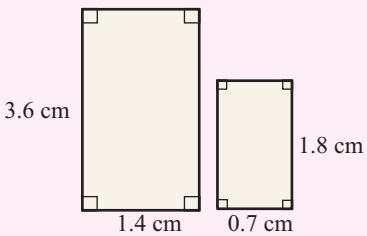


$n > 1$

Scale drawing smaller than object

$1 : 2$

Object Drawing



SELF-REFLECT

At the end of this chapter, I can:

1.	Investigate and explain the relationship between the actual measurements and the measurements of various sizes of drawings of an object, and hence explain the meaning of scale drawing.		
2.	Interpret the scale of a scale drawing.		
3.	Determine the scales, measurements of objects or measurements of scale drawings.		
4.	Draw the scale drawings of objects and vice versa.		
5.	Solve problems involving scale drawings.		

EXPLORING MATHEMATICS

1. Download grid paper of various sizes.

2. Draw your favourite object as shown in Diagram 1 or Diagram 2 on one of the grid papers chosen.

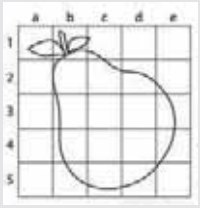


Diagram 1

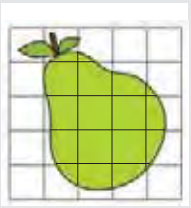


Diagram 2

Scan the QR Code or visit http://bukutekssm.my/Mathematics/F3/Chapter4Grid.pdf to download grid paper of various sizes.

3. Redraw the drawing on all the grid papers of different sizes.

4. Can you easily draw your favourite object on grids of different sizes?

5. Exhibit your work at the mathematics corner of your classroom.

CHAPTER 5

Trigonometric Ratios

What will you learn?

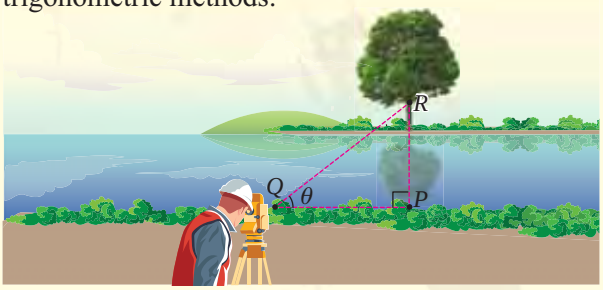
5.1 Sine, Cosine and Tangent of Acute Angles in Right-angled Triangles

Why do you learn this chapter?

- Trigonometric ratios allow problems related to length, height and angle to be solved by using a right-angled triangle.
- Trigonometric concepts are used in the fields of navigation, aviation, engineering, astronomy, construction and so forth.

The river is the main source of water for humans for domestic use.

The width of a river can be calculated by using the trigonometric concepts. The angle from the surveyor's position to the tree with R as the reference point as shown in the diagram below is determined by using a theodolite, an equipment used to measure angles from a long distance. If the length of PQ and the angle PQR is known, thus the width of the river, PR can be calculated easily using trigonometric methods.



Exploring Era

Al-Battani or Muhammad Ibn Jabir Ibn Sinan Abu Abdullah is the father of trigonometry. He was born in Battan, Damascus. He was an Arab prince and the ruler of Syria. Al-Battani was recognised as a well-known astronomer and Islamic mathematician. Al-Battani received early education from his father Jabir Ibn San'an who was also a famous scientist in his time. He successfully advanced trigonometry to a higher level and was the first to compile the table of cotangents.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter5.pdf

WORD BANK

• degree	• *darjah*
• hypotenuse	• *hipotenus*
• cosine	• *kosinus*
• sine	• *sinus*
• tangent	• *tangen*
• Pythagoras theorem	• *teorem Pythagoras*

5.1 Sine, Cosine and Tangent of Acute Angles in Right-angled Triangles

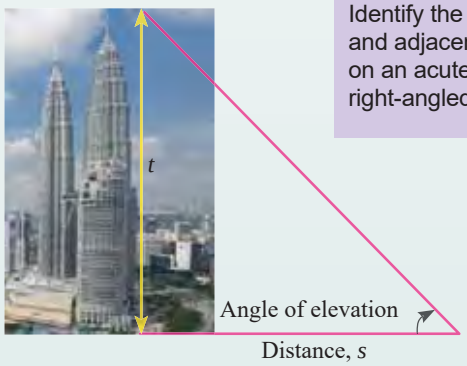
How do you identify the opposite side, adjacent side and hypotenuse?

LEARNING STANDARD

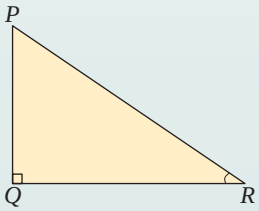
Identify the opposite side and adjacent side based on an acute angle in a right-angled triangle.

Do you know how the height of an object which is difficult to be measured such as buildings and mountains are determined?

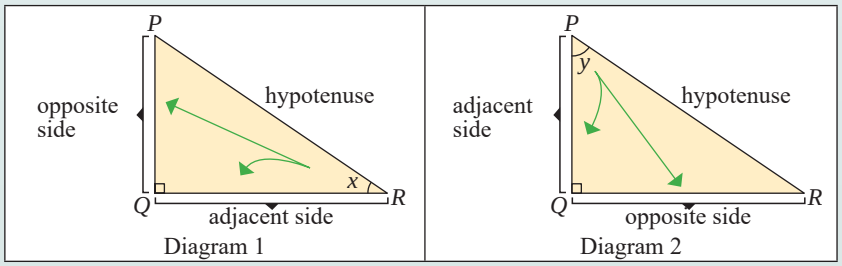
For example, in the diagram on the right, if the distance, s and the angle of elevation is known, then, the height, t of the building can be calculated by using the trigonometric concepts.



The diagram on the right shows a right-angled triangle PQR. As you have learnt in the chapter Pythagoras theorem in Form 1, the side PR is known as the **hypotenuse**, which is the longest side in the right-angled triangle PQR. Do the other two sides PQ and QR have special names like the longest side PR has?



Examine Diagram 1 and Diagram 2 below.



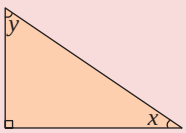
TIPS

Acute angle
$0° < \theta < 90°$

Based on $\angle PRQ$ in Diagram 1, QR is known as the **adjacent side** while PQ is known as **opposite side**.

Based on $\angle QPR$ in Diagram 2, PQ is the adjacent side while QR is the opposite side.

TIPS



$x + y = 90°$
$\angle x$ and $\angle y$ are acute angles.

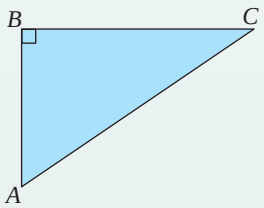
Take note that in both Diagram 1 and Diagram 2, the position of the hypotenuse PR is fixed, which is opposite the 90° angle.

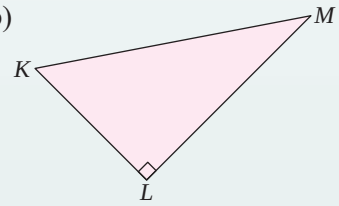
For a right-angled triangle:

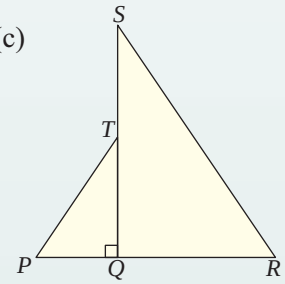
(a) The hypotenuse is the longest side which is opposite the 90° angle.
(b) The adjacent side and the opposite side change based on the position of the referred acute angle.

Example 1

Identify the opposite side, adjacent side and hypotenuse based on the given angle in the table below for all the following right-angled triangles.

(a) 

(b) 

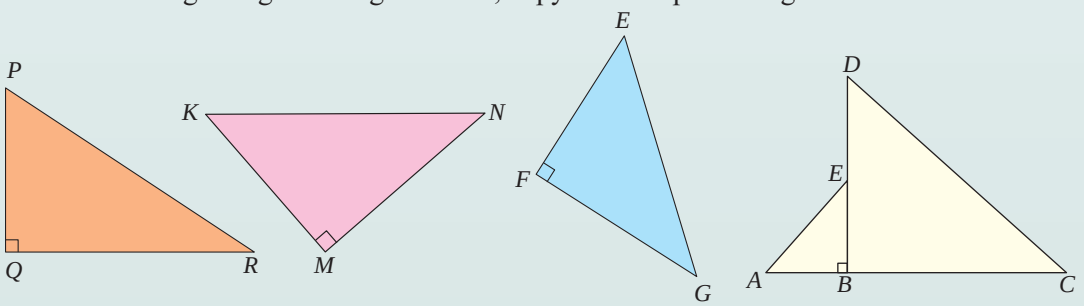
(c) 

Solution:

Triangle	Angle	Hypotenuse	Opposite side	Adjacent side
ΔABC	$\angle BAC$	AC	BC	AB
	$\angle BCA$	AC	AB	BC
ΔKLM	$\angle LKM$	KM	LM	KL
	$\angle LMK$	KM	KL	LM
ΔPQT	$\angle TPQ$	PT	QT	PQ
ΔRQS	$\angle QRS$	RS	QS	QR

MIND TEST 5.1a

1. Based on the right-angled triangles below, copy and complete the given table.



Triangle	Angle	Hypotenuse	Opposite side	Adjacent side
ΔPQR	$\angle QPR$			
	$\angle PRQ$			
ΔKMN	$\angle MNK$			
	$\angle MKN$			
ΔEFG	$\angle FEG$			
	$\angle EGF$			
ΔABE	$\angle BAE$			
	$\angle AEB$			
ΔCBD	$\angle BCD$			
	$\angle BDC$			

What is the relationship between acute angles and the ratios of the sides of right-angled triangles?

LEARNING STANDARD

Make and verify the conjecture about the relationship between acute angles and the ratios of the sides of right-angled triangles, and hence define sine, cosine and tangent.

Brainstorming 1

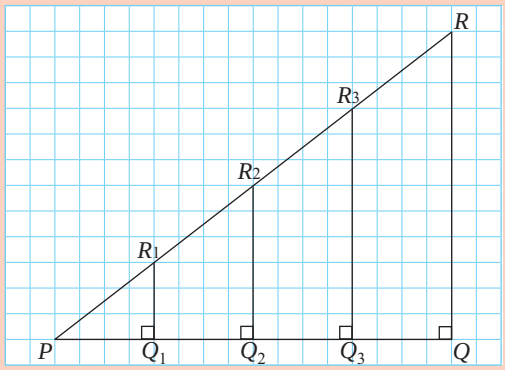
In groups

Aim: To identify the relationship between acute angles and the ratios of the sides of right-angled triangles.

Materials: Square grid paper, ruler and pencil.

Steps:

1. Draw a right-angled triangle PQR, where the length PQ is 16 units and the length QR is 12 units.
2. Draw a few straight lines parallel to RQ. Label them as R_1Q_1, R_2Q_2 and R_3Q_3 as shown in the diagram below.



TIPS

Use the Pythagoras theorem to determine the length of PR_1, PR_2, PR_3 and PR.

3. Complete the table below with the required measurements.

Acute angle	$\frac{\text{Opposite side}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\frac{\text{Opposite side}}{\text{Adjacent side}}$
$\angle QPR$	$\frac{R_1Q_1}{PR_1} = \frac{3}{5}$	$\frac{PQ_1}{PR_1} = \frac{4}{5}$	$\frac{R_1Q_1}{PQ_1} = \frac{3}{4}$
	$\frac{R_2Q_2}{PR_2} =$	$\frac{PQ_2}{PR_2} =$	$\frac{R_2Q_2}{PQ_2} =$
	$\frac{R_3Q_3}{PR_3} =$	$\frac{PQ_3}{PR_3} =$	$\frac{R_3Q_3}{PQ_3} =$
	$\frac{RQ}{PR} =$	$\frac{PQ}{PR} =$	$\frac{RQ}{PQ} =$

Discussion:

1. What is the pattern of your answer to the ratio of the length of the opposite side to the hypotenuse, the ratio of the length of the adjacent side to the hypotenuse and the ratio of the length of the opposite side to the length of the adjacent side?
2. What happens if the size of the angle is changed? Justify your answer.

From Brainstorming 1, it is found that:

Given a fixed acute angle in right-angled triangles of different sizes:
(a) The ratio of the length of the opposite side to the hypotenuse is a constant.
(b) The ratio of the length of the adjacent side to the hypotenuse is a constant.
(c) The ratio of the length of the opposite side to the length of the adjacent side is a constant.

The relationships of the ratios obtained from Brainstorming 1 are trigonometric ratios known as **sine, cosine and tangent**, that is:

$$\text{sine} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{cosine} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\text{tangent} = \frac{\text{opposite side}}{\text{adjacent side}}$$

REMINDER

- sin = sine
- cos = cosine
- tan = tangent

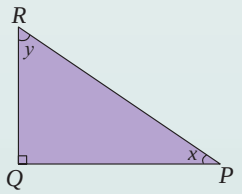
BULLETIN

The word **Trigonometry** originates from Greek words, that is,
Trigonon = triangle
Metron = to measure

Example 2

Complete the following table based on the diagram on the right.

sin x	cos x	tan x	sin y	cos y	tan y

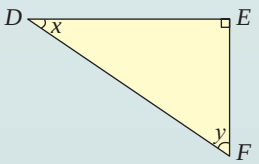


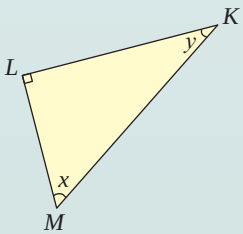
Solution:

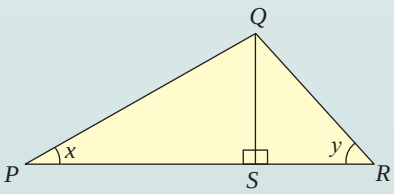
sin x	cos x	tan x	sin y	cos y	tan y
$\frac{QR}{PR}$	$\frac{PQ}{PR}$	$\frac{QR}{PQ}$	$\frac{PQ}{PR}$	$\frac{QR}{PR}$	$\frac{PQ}{QR}$

MIND TEST 5.1b

1. Complete the table based on the right-angled triangles below.







Triangles	sin x	cos x	tan x	sin y	cos y	tan y
ΔDEF						
ΔKLM						
ΔPQR						

What is the impact of changing the size of the angles on the values of sine, cosine and tangent?

LEARNING STANDARD

Make and verify the conjecture about the impact of changing the size of the angles on the values of sine, cosine and tangent.

Brainstorming 2

In pairs

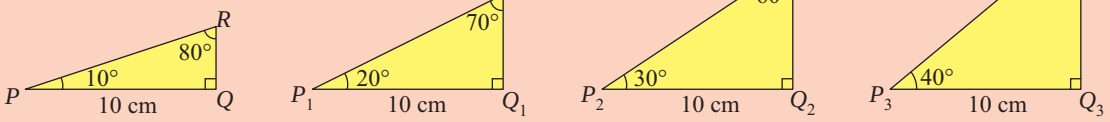
Aim: To identify the impact of changing the size of the angles on the values of sine, cosine and tangent.

Materials: Square grid paper, ruler, protractor and pencil.

Steps:

1. Draw four right-angled triangles as shown below with the base length of 10 cm.

2. Make sure that the angles and lengths of all right-angled triangles are exactly as given.



3. Complete the table below.

sin 10°	sin 20°	sin 30°	sin 40°	sin 50°	sin 60°	sin 70°	sin 80°
$\frac{RQ}{PR} = \frac{1.8}{10.2} = 0.1765$							$\frac{PQ}{PR} = \frac{10}{10.2} = 0.9804$

cos 10°	cos 20°	cos 30°	cos 40°	cos 50°	cos 60°	cos 70°	cos 80°
$\frac{PQ}{PR} = \frac{10}{10.2} = 0.9804$							$\frac{RQ}{PR} = \frac{1.8}{10.2} = 0.1765$

tan 10°	tan 20°	tan 30°	tan 40°	tan 50°	tan 60°	tan 70°	tan 80°
$\frac{RQ}{PQ} = \frac{1.8}{10} = 0.1800$							$\frac{PQ}{RQ} = \frac{10}{1.8} = 5.5556$

Discussion:

1. Based on the values in the table for the trigonometric ratios you have completed, what conclusion can you make?

2. What is your conjecture on
   (a) the value of the sine ratio when the angle approaches 0° and 90°?
   (b) the value of the cosine ratio when the angle approaches 0° and 90°?
   (c) the value of the tangent ratio when the angle approaches 0° and 90°?

From Brainstorming 2, it is found that:

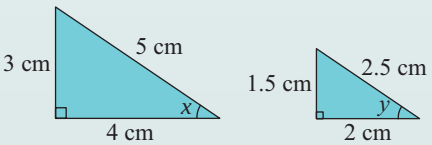
The larger the size of the acute angle
(a) the **larger the value of sine** and its value **approaches 1**.
(b) the **smaller the value of cosine** and its value **approaches zero**.
(c) the **larger the value of tangent**.

TIPS

sin 0° = 0	sin 90° = 1
cos 0° = 1	cos 90° = 0
tan 0° = 0	tan 90° = ∞

Example 3

The diagram on the right shows two right-angled triangles. Determine whether all trigonometric ratios of angle x and angle y are equal. State the reason for your answer.



Solution:

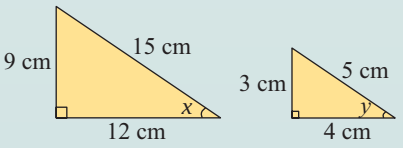
$\sin x = \frac{3}{5}$ $\quad \cos x = \frac{4}{5}$ $\quad \tan x = \frac{3}{4}$

$\sin y = \frac{1.5}{2.5} = \frac{3}{5}$ $\quad \cos y = \frac{2}{2.5} = \frac{4}{5}$ $\quad \tan y = \frac{1.5}{2} = \frac{3}{4}$

The trigonometric ratios of angle x and angle y are equal because the length of corresponding sides of the two triangles are proportional.

MIND TEST 5.1c

1. The diagram on the right shows two right-angled triangles. Determine whether all trigonometric ratios of angle x and angle y are equal. State the reason for your answer.

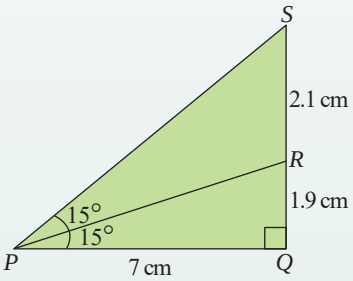


2. The diagram on the right shows a right-angled triangle.

(a) Determine the trigonometric ratio for

i. sin 15° ii. cos 15° iii. tan 15°
iv. sin 30° v. cos 30° vi. tan 30°

(b) Is the increase in the value of the trigonometric ratio for angle 15° and angle 30° proportional to the increase in the angle?



How do you determine the values of sine, cosine and tangent of acute angles?

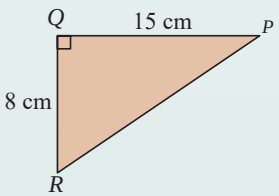
LEARNING STANDARD

Determine the values of sine, cosine and tangent of acute angles.

Example 4

The diagram on the right shows a right-angled triangle PQR. Calculate the value of

(a) length of PR (b) $\sin \angle PRQ$ (c) $\cos \angle PRQ$ (d) $\tan \angle QPR$



Solution:

(a) length of PR (b) $\sin \angle PRQ$ (c) $\cos \angle PRQ$ (d) $\tan \angle QPR$

$PR = \sqrt{15^2 + 8^2}$ $= \frac{15}{17}$ $= \frac{8}{17}$ $= \frac{8}{15}$

$= \sqrt{289}$

$= 17$ cm

FLASHBACK

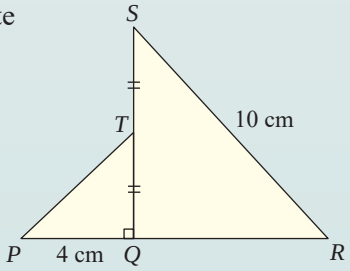
Pythagoras theorem

$c^2 = a^2 + b^2$
$a^2 = c^2 - b^2$
$b^2 = c^2 - a^2$

Example 5

The diagram on the right shows right-angled triangles PQT and RQS. PQR is a straight line. Given that the length of SQ is 6 cm, calculate the value of

(a) length of QR (b) length of PT (c) $\sin \angle QRS$
(d) $\cos \angle TPQ$ (e) $\tan \angle PTQ$ (f) $\tan \angle QSR$



Solution:

(a) length of QR

$QR = \sqrt{10^2 - 6^2}$
$= \sqrt{64}$
$= 8$ cm

(b) length of PT

$PT = \sqrt{4^2 + 3^2}$
$= \sqrt{25}$
$= 5$ cm

(c) $\sin \angle QRS$

$= \frac{6}{10}$
$= \frac{3}{5}$

(d) $\cos \angle TPQ$

$= \frac{4}{5}$

(e) $\tan \angle PTQ$

$= \frac{4}{3}$

(f) $\tan \angle QSR$

$= \frac{8}{6}$
$= \frac{4}{3}$

REMINDER

Ratio value should be given in the simplest term.

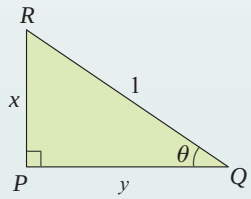
What is the relationship between sine, cosine and tangent?

For right-angled triangles, you have learnt that:

$$\text{sine} = \frac{\text{opposite side}}{\text{hypotenuse}}, \text{cosine} = \frac{\text{adjacent side}}{\text{hypotenuse}} \text{ and tangent} = \frac{\text{opposite side}}{\text{adjacent side}}$$

Do you know that the three trigonometric ratios above are related to one another? Tangent is the ratio of sine to cosine.

Study the diagram below.



It is known that,

(a) $\sin\theta = \frac{x}{1}$, $x = \sin\theta$

(b) $\cos\theta = \frac{y}{1}$, $y = \cos\theta$

(c) $\tan\theta = \frac{x}{y}$

Thus,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

DISCUSSION CORNER

If θ is $\angle QRP$, is the ratio of $\tan\theta$ still $\frac{\sin\theta}{\cos\theta}$? Discuss.

Example 6

If $\sin\theta = 0.6$ and $\cos\theta = 0.8$, calculate the value of $\tan\theta$.

Solution:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$= \frac{0.6}{0.8}$$
$$= \frac{3}{4}$$
$$= 0.75$$

QUIZ

If $\tan\theta = \frac{1}{2}$, state the possible values of $\sin\theta$ and $\cos\theta$.

Example 7

If $\sin\theta = \frac{3}{8}$ and $\tan\theta = \frac{3}{\sqrt{55}}$, calculate the value of $\cos\theta$.

Solution:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\frac{3}{\sqrt{55}} = \frac{\frac{3}{8}}{\cos\theta}$$
$$\cos\theta = \frac{\frac{3}{8}}{\frac{3}{\sqrt{55}}}$$
$$\cos\theta = \frac{\sqrt{55}}{8}$$

SMART MIND

Given that $\sin\theta = x$, determine the possible values of $\cos\theta$ and $\tan\theta$.

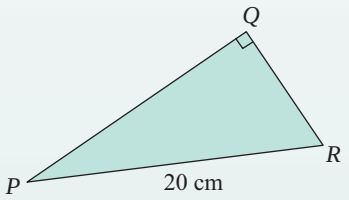
DISCUSSION CORNER

If $\tan\theta = 1$, what type of triangle is being represented?

Example 8

The diagram on the right shows a right-angled triangle PQR.
Given that $PR = 20$ cm and $\sin \angle QPR = \frac{3}{5}$, calculate

(a) the length of QR
(b) $\cos \angle QPR$



Solution:

(a) $\sin \angle QPR = \frac{3}{5}$

$\frac{QR}{PR} = \frac{3}{5}$

$\frac{QR}{20} = \frac{3}{5}$

$QR = \frac{3(20)}{5}$

$QR = 12$ cm

(b) $PQ = \sqrt{20^2 - 12^2}$

$= \sqrt{256}$

$PQ = 16$ cm

Thus, $\cos \angle QPR = \frac{PQ}{PR}$

$= \frac{16}{20}$

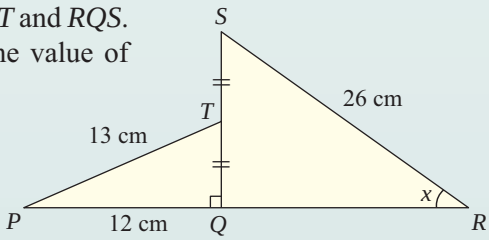
$= \frac{4}{5}$

SMART MIND

Given $\sin \theta = \frac{3}{5}$ and the length of hypotenuse is 20 cm, determine $\cos \theta$ and $\tan \theta$.

Example 9

The diagram on the right shows right-angled triangles PQT and RQS. Given that PQR and STQ are straight lines, calculate the value of $\cos x$.



Solution:

$\cos x = \frac{RQ}{SR}$ → Determine the value of RQ first

$TQ = \sqrt{13^2 - 12^2}$; $SQ = 2TQ$; $RQ = \sqrt{26^2 - 10^2}$

$= \sqrt{25}$ $= 2(5)$ $= \sqrt{576}$

$TQ = 5$ cm $SQ = 10$ cm $= 24$ cm

Thus, $\cos x = \frac{RQ}{SR}$

$= \frac{24}{26}$

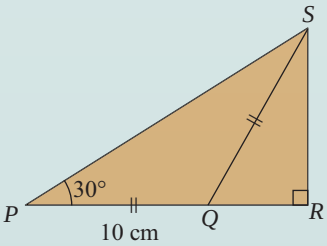
$= \frac{12}{13}$

DISCUSSION CORNER

Given that the hypotenuse of a right-angled triangle is $\sqrt{8}$ cm, determine $\tan \theta$ if $\cos \theta = \frac{1}{\sqrt{2}}$.

Example 10

The diagram on the right shows a right-angled triangle PRS. Given that PQR is a straight line and $\cos 60° = 0.5$, calculate the length of PS. State the answer correct to two decimal places.



Understanding the problem

Calculate the length of PS which is the hypotenuse of ΔPRS.

Planning a strategy

- $PS = \sqrt{PR^2 + SR^2}$
- SR and QR can be calculated if $\angle SQR$ or $\angle QSR$ is known.
- Identify the position of $\cos 60°$.

Making a conclusion

$PS = 17.32$ cm (2 d.p.)

Implementing the strategy

- $\angle QSP = \angle QPS = 30°$

  thus, $\angle PQS = 180 - 30° - 30° = 120°$
- $\angle SQR = 180° - \angle PQS$

  $\angle SQR = 180° - 120° = 60°$
- Given that $\cos 60° = 0.5$

$$\cos 60° = \frac{1}{2}$$

$$\frac{QR}{10} = \frac{1}{2}$$

$$QR = \frac{10\,(1)}{2}$$

$$= 5 \text{ cm}$$

$$SR = \sqrt{10^2 - 5^2} = \sqrt{75} \text{ cm}$$

Thus,

$$PS = \sqrt{SR^2 + PR^2}$$

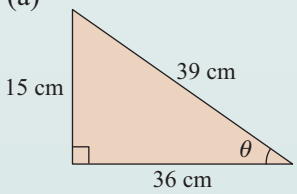
$$PS = \sqrt{(\sqrt{75})^2 + 15^2}$$

$$PS = 17.32 \text{ cm}$$

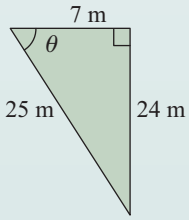
MIND TEST 5.1d

1. Calculate the values of $\sin\theta$, $\cos\theta$ and θ for each of the following right-angled triangles.

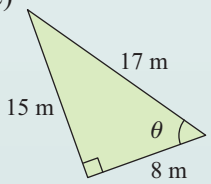
(a)



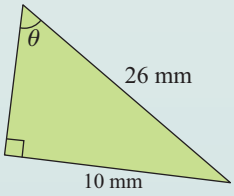
(b)



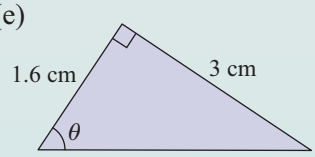
(c)



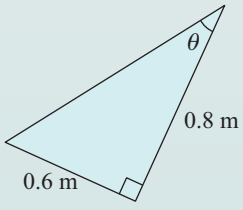
(d)



(e)



(f)



2. Calculate the value of x without drawing any right-angled triangles or using Pythagoras theorem or a calculator.

(a) $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = x$

(b) $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta = x$, $\tan\theta = 1$

(c) $\sin\theta = x$, $\cos\theta = \frac{5}{8}$, $\tan\theta = \frac{\sqrt{39}}{5}$

(d) $\sin\theta = \frac{7}{9}$, $\cos\theta = x$, $\tan\theta = \frac{7}{4\sqrt{2}}$

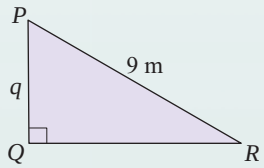
FLASHBACK

Pythagoras triples

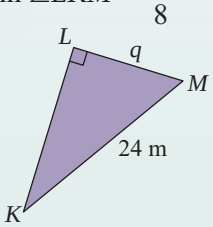
A	B	C
3	4	5
5	12	13
6	8	10
8	15	17
7	24	25
9	40	41

3. Determine the length of side q for each of the right-angled triangles below.

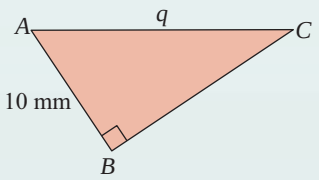
(a) $\sin \angle QRP = \frac{1}{3}$



(b) $\sin \angle LKM = \frac{7}{8}$

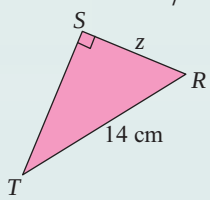


(c) $\sin \angle ACB = \frac{2}{5}$

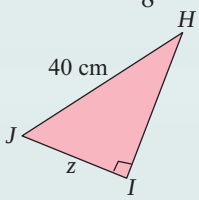


4. Determine the length of side z for each of the right-angled triangles below.

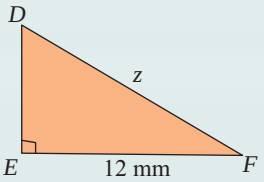
(a) $\cos \angle SRT = \frac{5}{7}$



(b) $\cos \angle HJI = \frac{3}{8}$

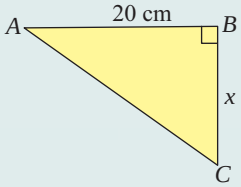


(c) $\cos \angle DFE = 0.4$

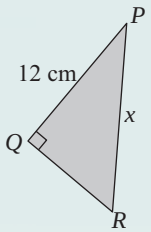


5. Calculate the value of x for each of the right-angled triangles below.

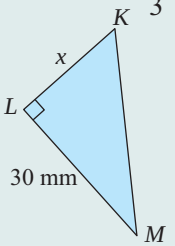
(a) $\tan \angle BAC = 0.9$



(b) $\tan \angle PRQ = \frac{3}{4}$



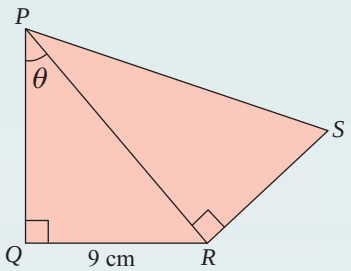
(c) $\tan \angle LKM = \frac{10}{3}$



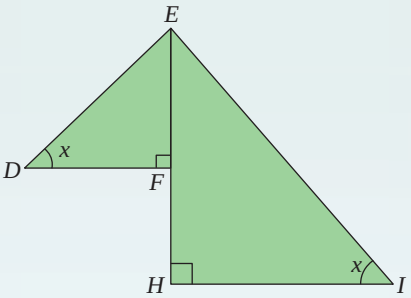
6. The diagram on the right shows right-angled triangles PQR and PRS. Given that $\tan \theta = \frac{3}{4}$ and $PS = \frac{5}{3} PR$, calculate, in cm, the length of

(a) PR

(b) RS



7. The diagram on the right shows right-angled triangles DFE and EHI. If $\tan x = \frac{5}{7}$, $DF = 21$ cm and $EF : EH = 1 : 2$, determine the length of EI in cm.



How do you determine the values of sine, cosine and tangent of 30°, 45° and 60° angles without using a calculator?

LEARNING STANDARD

Determine the values of sine, cosine and tangent of 30°, 45° and 60° angles without using a calculator.

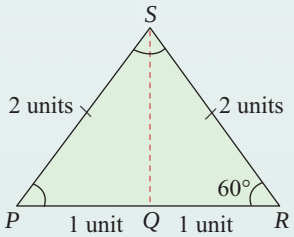


Diagram 1(a)

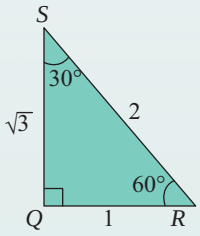


Diagram 1(b)

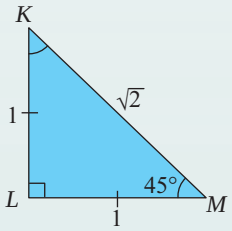


Diagram 2

TIPS

$QS = \sqrt{2^2 - 1^2}$
$QS = \sqrt{3}$
$KM = \sqrt{1^2 + 1^2}$
$KM = \sqrt{2}$

Diagram 1(b) above is half of the equilateral triangle *PRS* where the length of *PQR* is 2 units. Diagram 2 shows an isosceles triangle *KLM*.

The table below shows the values of the trigonometric ratios of 30°, 45° and 60° angles that can be calculated without using a calculator, based on Diagram 1(b) and Diagram 2.

Ratio \ Angle	30°	60°	45°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

BULLETIN

Surd is an irrational number in the root form such as $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{17}$. $\sqrt{3}$ is read as surd three.

Example 11

Calculate the following values without using a calculator.

(a) $\sin 45° + \cos 45°$ (b) $3\cos 30° - 2\sin 60°$ (c) $2\tan 45° - 2\cos 60°$

(d) $(2\sin 60°)(4\cos 30°) - 4\tan 60°$ (e) $(3\tan 30°)(4\sin 60°) + 4\sin 45°$

Solution:

(a) $\sin 45° + \cos 45°$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

$\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{2\sqrt{2}}{2}$
$= \sqrt{2}$

(b) $3\cos 30° - 2\sin 60°$

$= 3\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3\sqrt{3}}{2} - \frac{2\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

(c) $2\tan 45° - 2\cos 60°$

$= 2(1) - 2\left(\frac{1}{2}\right)$

$= 2 - 1$

$= 1$

(d) $(2\sin 60°)(4\cos 30°) - 4\tan 60°$

$= 2\left(\frac{\sqrt{3}}{2}\right)\left(4\right)\left(\frac{\sqrt{3}}{2}\right) - 4\sqrt{3}$

$= (\sqrt{3})(2)(\sqrt{3}) - 4\sqrt{3}$

$= 2(3) - 4\sqrt{3}$

$= 6 - 4\sqrt{3}$

(e) $(3\tan 30°)(4\sin 60°) + 4\sin 45°$

$= 3\left(\frac{1}{\sqrt{3}}\right)\left(4\right)\left(\frac{\sqrt{3}}{2}\right) + 4\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{6}{1} + \frac{4}{\sqrt{2}}$

$= 6 + 2\sqrt{2}$

$= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{4\sqrt{2}}{2}$
$= 2\sqrt{2}$

TIPS

$\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2}$
$= \sqrt{4}$
$= 2$

MIND TEST 5.1e

1. Determine the following values without using a calculator.

(a) $2 \cos 60° + \tan 45°$ (b) $3 \cos 60° + 2 \tan 45°$ (c) $2 \tan 45° + \cos 60°$

(d) $3 \sin 30° - 2 \cos 60°$ (e) $2 \sin 30° - 3 \cos 60°$ (f) $4 \tan 45° - 2 \cos 60°$

(g) $(2 \sin 60°)(3 \cos 60°) + 3 \tan 30°$ (h) $(3 \tan 45°)(4 \sin 60°) - (2 \cos 30°)(3 \sin 30°)$

(i) $4 \tan 45° + (2 \sin 45°)(6 \cos 45°)$ (j) $(5 \tan 60°)(2 \sin 60°) - (3 \sin 45°)(4 \cos 45°)$

What is the unit of measure for angles?

Angles are measured in the unit of degrees (°). Angles can also be expressed in units of degrees (°), minutes (′) and seconds (″), that is,

$$1° = 60'$$
$$1' = 60''$$

Example 12

(a) Convert 30.2° to degrees and minutes.

(b) Convert the angle 43° 30′ to degrees.

Solution:

(a) $30.2° = 30° + 0.2°$
$= 30° + (0.2 \times 60)'$
$= 30° + 12'$
$= 30°\ 12'$

(b) $43°\ 30' = 43° + 30'$
$= 43° + \left(\frac{30}{60}\right)°$
$= 43° + 0.5°$
$= 43.5°$

MIND TEST 5.1f

1. Convert each of the following angles to degrees and minutes.

(a) 37.80° (b) 74.6° (c) 58.1° (d) 60.2°

(e) 41.5° (f) 16.9° (g) 5.4° (h) 72.3°

2. State each of the following angle in degrees.

(a) 65° 54′ (b) 47° 42′ (c) 18° 12′ (d) 69° 24′

(e) 70° 6′ (f) 36° 36′ (g) 35° 30′ (h) 20° 18′

How do you determine the values of sine, cosine and tangent?

LEARNING STANDARD

Perform calculations involving sine, cosine and tangent.

Do you know that a scientific calculator can be used to determine the trigonometric ratio of an angle?

Example 13

Use a scientific calculator to determine the following values correct to four decimal places.

(a) sin 45° 6′ (b) cos 20.7° (c) tan 64° 12′

Solution:

(a) $\sin 45° 6' = 0.7083$

SMART FINGER: sin 4 5 °′ ″ 6 °′ ″ = 0.7083398377

(b) $\cos 20.7° = 0.9354$

SMART FINGER: cos 2 0 . 7 = 0.9354440308

(c) $\tan 64° 12' = 2.0686$

SMART FINGER: tan 6 4 °′ ″ 12 °′ ″ = 2.068599355

BULLETIN

The button °′ ″ should be pressed only when the question is given in degrees and minutes.

MIND TEST 5.1g

1. Use a scientific calculator to determine the following values correct to four decimal places.

(a) $\sin 44°$ (b) $\cos 73.5°$ (c) $\tan 69.5°$ (d) $\sin 51° 24'$ (e) $\cos 30° 21'$ (f) $\tan 56° 24'$

How do you calculate the size of an angle by using trigonometric ratios sine, cosine and tangent?

If the value of the trigonometric ratio is given, you can use a scientific calculator to determine the size of the related angle.

Example 14

Use a scientific calculator to calculate the following x values.

(a) $\sin x = 0.8377$ (b) $\cos x = 0.7021$ (c) $\tan x = 2.4876$

Solution:

(a) $\sin x = 0.8377$

$x = \sin^{-1} 0.8377$

$x = 56.9°$ ← Answer in degrees.

$x = 56° 54'$ ← Answer in degrees and minutes.

REMINDER

If the unit of second is 30″ or more, the minute unit will be added by 1′.

SMART FINGER: shift sin 0 . 8 3 7 7 = 56.89803635 °′ ″ 56° 53′ 52.93″

52.93″ shows the value in seconds. Follow the steps below to round off the answer to the nearest minute.

$56° 53' \underbrace{52.93''}_{+1 \text{ if } > 30}$

$= 56° 54'$

(b) $\cos x = 0.7021$

$x = \cos^{-1} 0.7021$

$x = 45.4°$

$x = 45° 24'$

SMART FINGER shift cos 0 . 7 0 2 1 = 45.40426895 °′ ″ 45° 24′ 15.37″

(c) $\tan x = 2.4876$

$x = \tan^{-1} 2.4876$

$x = 68.1°$

$x = 68° 6'$

SMART FINGER shift tan 2 . 4 8 7 6 = 68.10017426 °′ ″ 68° 6′ 0.63″

MIND TEST 5.1h

1. Using a scientific calculator, calculate the following x values.

(a) $\tan x = 0.2162$ (b) $\cos x = 0.5878$ (c) $\sin x = 0.4062$ (d) $\sin x = 0.9121$

(e) $\cos x = 0.9686$ (f) $\tan x = 3.8027$ (g) $\cos x = 0.5604$ (h) $\sin x = 0.1521$

(i) $\tan x = 0.7199$ (j) $\sin x = 0.9792$ (k) $\tan x = 1.0088$ (l) $\cos x = 0.099$

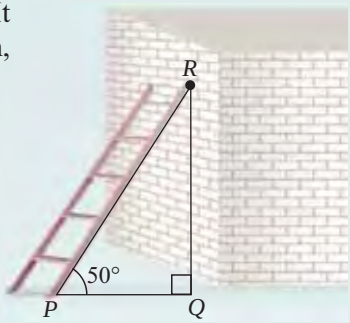
How do you solve problems involving sine, cosine and tangent?

LEARNING STANDARD

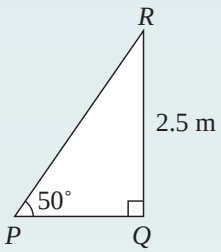
Solve problems involving sine, cosine and tangent.

Example 15

The diagram on the right shows a ladder leaning against a wall. It forms a right-angled triangle PQR. If the height of QR is 2.5 m, calculate the length of the ladder, PR in metres.
(State the answer correct to two decimal places).



Solution:



$$\sin 50° = \frac{QR}{PR}$$

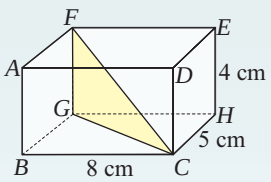
$$\sin 50° = \frac{2.5}{PR}$$

$$PR = \frac{2.5}{\sin 50°}$$

$$PR = 3.26 \text{ m (2 d.p.)}$$

Example 15

The diagram on the right shows a cuboid $ABCDEFGH$. It is given that $BC = 8$ cm, $CH = 5$ cm and the height of $HE = 4$ cm. If right-angled triangle FGC is formed in this cuboid, calculate the value of $\angle FCG$.



Understanding the problem

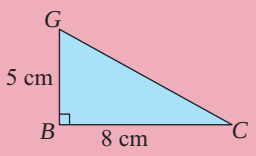
$\angle FCG$ can be calculated if any two sides CG, CF, FG are known.

Planning a strategy

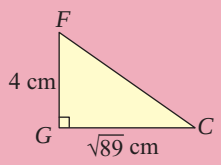
- $FG = EH$
- The length of CG is easier to calculate than the length of CF.
- $\tan \angle FCG = \dfrac{FG}{CG}$

Implementing the strategy

- $FG = EH$
  Thus,
  $FG = 4$ cm



$CG = \sqrt{BC^2 + BG^2}$
$= \sqrt{8^2 + 5^2}$
$CG = \sqrt{89}$



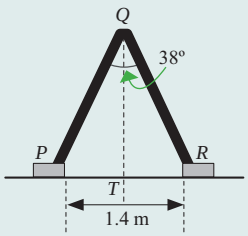
$\tan \angle FCG = \dfrac{4}{\sqrt{89}}$
$\angle FCG = \tan^{-1} \dfrac{4}{\sqrt{89}}$
$\angle FCG = 22.98°$

Making a conclusion

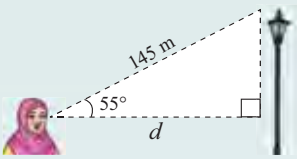
$\angle FCG = 22.98°$
or
$\angle FCG = 22° 59'$

MIND TEST 5.1i

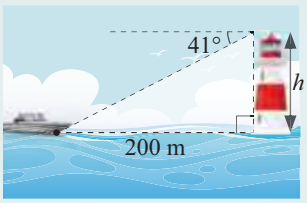
1. A foldable ladder which is placed on the floor forms an isosceles triangle PQR as shown in the diagram on the right. Given that T is the midpoint of PR, $\angle PQR = 38°$ and $PR = 1.4$ m, calculate the length of PQ, correct to two decimal places.



2. The diagram on the right shows Aisyah who is looking at a lamp post. Given that the angle of elevation at the tip of the lamp post from Aisyah's eyes is 55° and the distance between Aisyah's eyes and the tip of the lamp post is 145 metres, calculate the horizontal distance d in metres. State the answer correct to three significant figures.



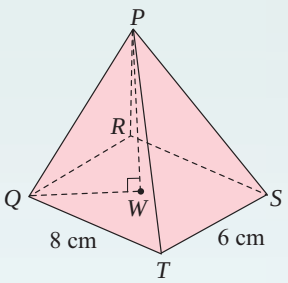
3. The diagram on the right shows the position of a ship and a lighthouse. Given that the angle of depression of the ship from the lighthouse is 41° and the horizontal distance between the lighthouse and the ship is 200 m, calculate the height of the lighthouse, h in metres. State the answer correct to four significant figures.



4. A right pyramid $PQRST$ has a rectangular base $QRST$. Given that W is the midpoint of QS and RT, the lengths of $QT = 8$ cm, $TS = 6$ cm and point P is vertically above point W, calculate

(a) PT, in cm, if $PW = 12$ cm

(b) the value of $\angle PTR$

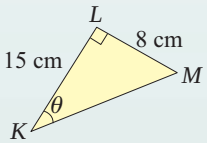


Dynamic Challenge

Test Yourself

1. The diagram on the right shows a right-angled triangle KLM. Calculate

(a) θ in degrees and minutes (b) $\sin(90° - \theta)$ (c) $\cos(90° - \theta)$

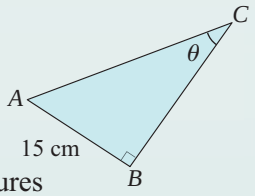


2. The diagram on the right shows a right-angled triangle ABC. Given that $\tan\theta = \frac{5}{12}$, calculate

(a) the length of AC in cm

(b) the value of $(90° - \theta)$

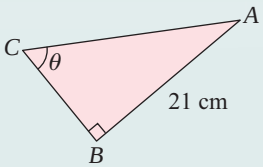
(c) the value of θ in degrees and minutes correct to three significant figures



3. The diagram on the right shows a right-angled triangle ABC. Given that $AB = 21$ cm and $\sin\theta = \frac{7}{9}$, calculate

(a) the length of AC in cm

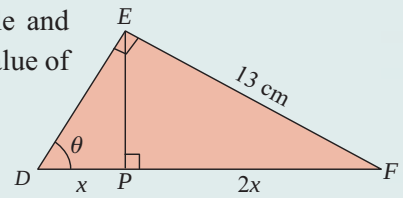
(b) the value of $\angle BAC$. State your answer to the nearest degree.



4. In the diagram on the right, DEF is a right-angled triangle and DPF is a straight line. Given that $PE = 5$ cm, calculate the value of

(a) x in cm

(b) θ in degrees and minutes



Skills Enhancement

1. Calculate the values of the following without using a calculator.

(a) $8\sin 60° - 3\tan 60°$ (b) $(\tan 30°)(2\cos 30°) + 6\sin 30°$

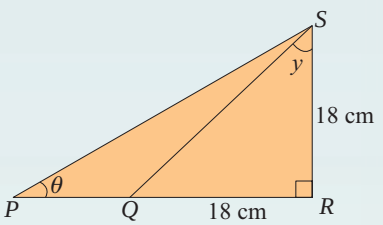
(c) $(8\cos 45°)(\sin 60°) + (8\sin 45°)(\cos 30°)$

2. The diagram on the right shows a right-angled triangle PRS. PQR is a straight line. Given that $QR = RS = 18$ cm and $\tan\theta = \frac{3}{5}$, calculate

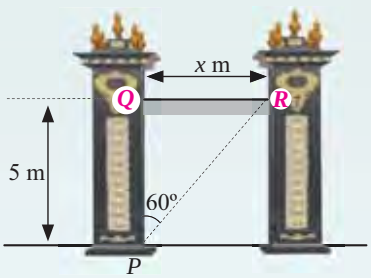
(a) the length of PQ, in cm

(b) the length of PS, in cm, correct to the nearest integer

(c) the value of y

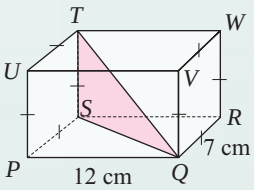


3. A gate has two vertical poles that are connected to a horizontal bridge with a distance of x metres. If the vertical height of the bridge from the ground surface is 5 m and the angle between the pole PQ and the inclined line PR is 60°, determine the value of x, in metres.



Self Mastery

1. The diagram on the right shows a cuboid *PQRSTUVW*. *QRWV* and *PSTU* are squares. Given that *PQ* = 12 cm and *QR* = 7 cm, calculate

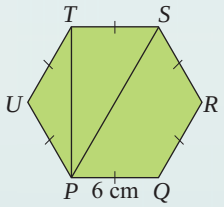


(a) tan ∠*PQS*

(b) the length of *TQ*, in cm, correct to four significant figures

(c) the value of ∠*SQT*, in degrees and minutes

2. The diagram on the right shows a regular hexagon *PQRSTU* with sides 6 cm. Calculate



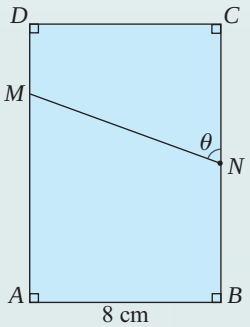
(a) ∠*PTS*

(b) ∠*TPS*

(c) the length of *TP*, in cm, correct to three significant figures

(d) the ratio of area of Δ*PTU* to area of Δ*PTS*

3. The diagram on the right shows a rectangle *ABCD*. It is given that *AB* = 8 cm, *BC* = 2*AB* and *N* is the midpoint of *BC*.



(a) If $MD = \frac{1}{4}AD$, calculate the length of *MN*, in cm. State your answer in surd form.

(b) Calculate the value of θ, in degrees and minutes.

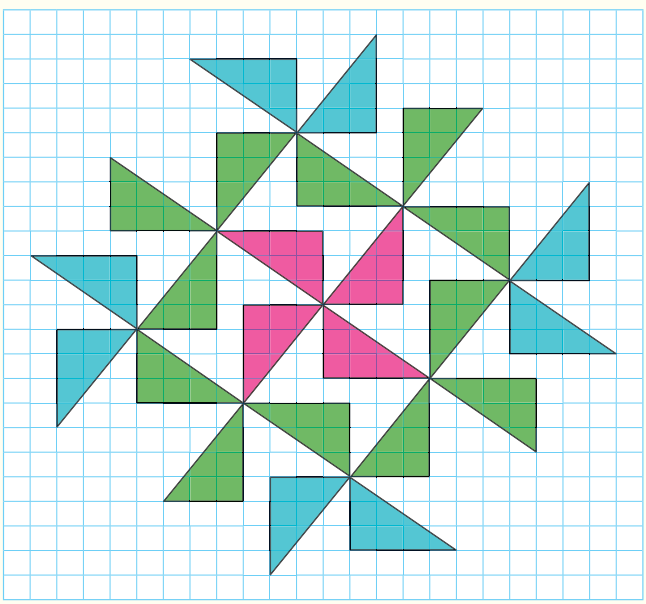
(c) Shahril stated that the ratio of the area of trapezium *CDMN* to the area of trapezium *ABNM* is 1: 2. Is Shahril's statement true? State the reasons for your answer.

PROJECT

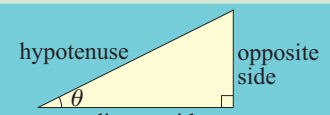
Materials: Grid paper 0.5 cm × 0.5 cm, pencil, ruler and colour pencil.

Steps:

1. Start by drawing a combination of right-angled triangles (pink).
2. Connect each vertex of the original combination with right-angled triangle (green).
3. Continue the pattern obtained in step 2 as many times as possible.
4. Colour and present your work in class.
5. Other groups are encouraged to use right-angled triangles of different sizes as the beginning pattern.



CONCEPT MAP



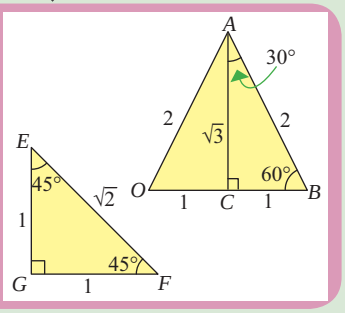
Trigonometric ratio

Ratio of sine	Ratio of cosine	Ratio of tangent
$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$	$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

When the **size** of an **acute angle** θ **increases,**
- the values of **tan** θ **and sin** θ **increase**
- the value of **cos** θ **decreases**

Determine the angles from the values of sine, cosine and tangent

θ	Special angles		
	30°	45°	60°
tan θ	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
sin θ	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$



θ°	0°	90°
sin θ	0	1
cos θ	1	0
tan θ	0	∞

SELF-REFLECT

At the end of this chapter, I can:

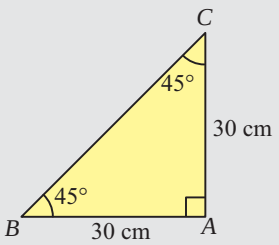
1.	Identify the opposite side and adjacent side based on an acute angle in a right-angled triangle.		
2.	Make and verify the conjecture about the relationship between acute angles and the ratios of the sides of right-angled triangles, and hence define sine, cosine and tangent.		
3.	Make and verify the conjecture about the impact of changing the size of the angles on the values of sine, cosine and tangent.		
4.	Determine the values of sine, cosine and tangent of acute angles.		
5.	Determine the values of sine, cosine and tangent of 30°, 45° and 60° angles without using a calculator.		
6.	Perform calculations involving sine, cosine and tangent.		
7.	Solve problems involving sine, cosine and tangent.		

EXPLORING MATHEMATICS

To measure the height of a pole, the following method can be used.

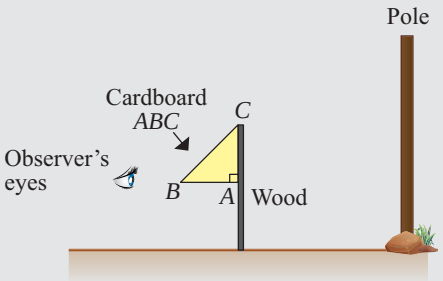
Step 1

Draw a right-angled triangle ABC on a cardboard as shown on the right with $AB = AC = 30$ cm.



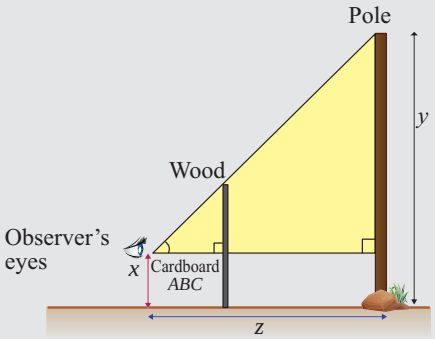
Step 2

$\angle ABC = \angle BCA = 45°$. Cut out the triangle and fix it to a wooden rod. Place the rod parallel to the pole to be measured.



Step 3

Align your view so that BC and the top of the pole are in a straight line. Use the wooden rod to make sure the position of triangle ABC is upright.



Step 4

If x represents the height of the eye from ground level, y represents the height of the pole and z represents the distance between the observer and the pole, then,

$$\tan 45° = \frac{y - x}{z}$$

$$y = z \tan 45° + x$$

The height of the pole can be easily determined without the need to measure the pole itself.

CHAPTER 6

Angles and Tangents of Circles

What will you learn?

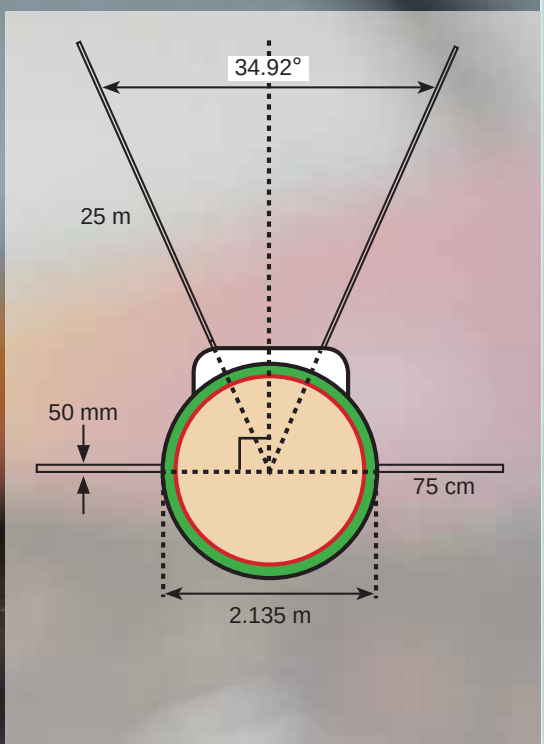
Why do you learn this chapter?

- The circle is a unique shape and it has special properties. Its uniqueness allows circles to be used in various fields.
- The concept of angles and tangents of circles are used in industry, road construction, painting, astronomy, sports and so on.

Shot-put is an athletics event. The shot-put area is circular with a diameter of 2.135 m. The circle is divided into two parts or two semicircles with a white line of 50 mm thickness. Two straight lines are drawn from the centre of the circle at an angle of 34.92° between each other to determine the shot-put area.

Muhammad Ziyad Zolkefli is a national paralympic athlete. He won the gold medal in the T20 shot-put event at the 10th Fazza International Athletics Championship, Grand Prix (GP) World Athletics in Dubai, United Arab Emirates.

Have you ever participated in a shot-put event?



Exploring Era

Thales and Pythagoras are famous Greek mathematicians. Thales' theorem states that when the three vertices of a triangle touch the circumference of the circle and one of the sides of the triangle is the diameter, then the angle subtended by the diameter is 90°. This theory was based on the influence of Ancient Egypt, India and Mesopotamia. Ancient mathematicians studied the circle as it was considered a perfect shape.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter6.pdf

WORD BANK

• diameter	• *diameter*
• arc	• *lengkok*
• circumference	• *lilitan*
• axis of symmetry	• *paksi simetri*
• chord	• *perentas*
• semicircle	• *semi bulatan*
• symmetry	• *simetri*
• tangent	• *tangen*
• alternate segments	• *tembereng selang-seli*
• point of tangency	• *titik ketangenan*

6.1 Angle at the Circumference and Central Angle Subtended by an Arc

What are the angles at the circumference of a circle?

A circle is a unique two-dimensional shape. This is because the number of sides of the circle is infinite. The uniqueness of its shape allow round-shaped objects such as wheels to move easily. Have you ever seen a vehicle wheel in other shapes?

Angles formed in circles also have their own properties.

LEARNING STANDARD

Make and verify conjectures about the relationships between angles at the circumference and central angle subtended by particular arcs, and hence use the relationships to determine the values of angles in circles.

DISCUSSION CORNER

Compare the bicycle wheels below, which wheel allows you to reach your destination faster?

CHAPTER 6

Diagram 1 shows two chords, PQ and QR which meet at point Q at the circumference of the circle.

$\angle PQR$ is the **angle at the circumference** of the circle subtended by the arc PR.

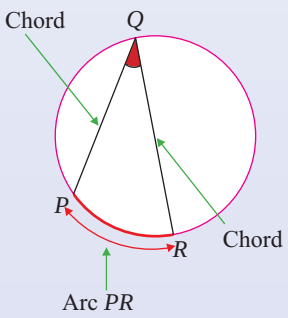


Diagram 1

In Diagram 2,

(a) $\angle PQS$ and $\angle PRS$ are angles at the circumference of the circle subtended by **major arc** PS.

(b) $\angle QPR$ and $\angle QSR$ are angles at the circumference of the circle subtended by **minor arc** QR.

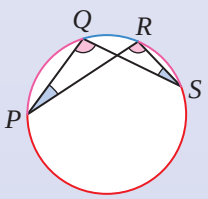
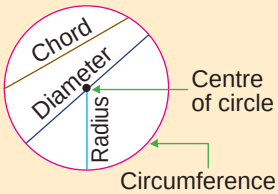
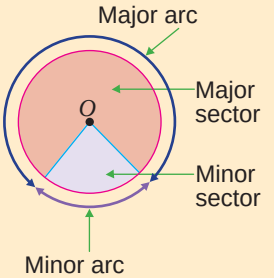


Diagram 2

FLASHBACK





Are angles at the circumference of a circle subtended by the same arc equal?

Brainstorming 1

In groups

Aim: To verify that angles at the circumference subtended by the same arc are equal.

Materials: A4 paper, compasses, protractor, ruler and pencil.

Steps:

1. Draw a circle of radius 5 cm. Draw a chord PQ (Diagram 1).
2. Draw a chord QR that forms 30° at point Q (Diagram 2). Other groups are encouraged to form acute angles between 20° and 40°.
3. Mark the point S on the circumference and draw chords PS and RS (Diagram 3).
4. Measure $\angle PSR$ and record it in the table below.
5. Repeat step 3 with point T and chords PT and RT (Diagram 4).
6. Measure $\angle PTR$ and record it in the table.

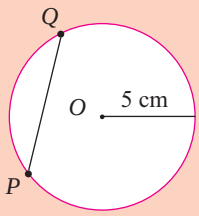


Diagram 1

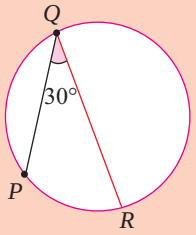


Diagram 2

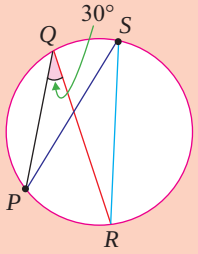


Diagram 3

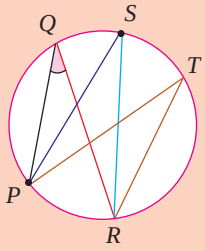


Diagram 4

$\angle PQR$	$\angle PSR$	$\angle PTR$		
30°				

7. You may repeat step 3 with other points on major arc PR. Measure the angle formed and record in the table.
8. Display your group's findings in the Mathematics corner. Give feedback on the findings of other groups.

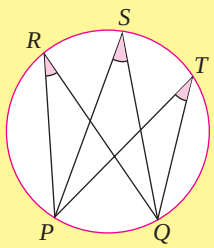
Discussion:

What can you say about the angles at the circumference of the circle subtended by arc PR?

From Brainstorming 1, it is found that:

The angles subtended by arc PR, $\angle PQR$, $\angle PSR$ and $\angle PTR$, are equal.

In general,



Angles at the **circumference subtended** by the **same arc** are **equal**.
$\angle PRQ = \angle PSQ = \angle PTQ$

You can also use dynamic software to verify the properties of angles at the circumference subtended by the same arc.

Brainstorming 2

In pairs

Aim: To verify that angles at the circumference subtended by the same arc are equal.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click on the *Compass Tool* to draw a circle (Diagram 1).
2. Click on *Point Tool* and mark three points (Diagram 2).
3. Click on *Text Tool* and label the three points marked in step 2 (Diagram 3).

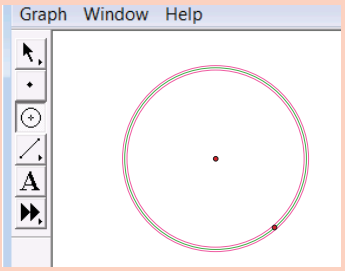


Diagram 1

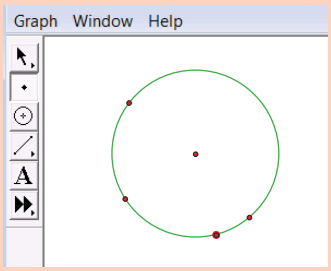


Diagram 2

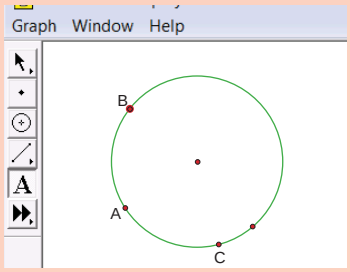


Diagram 3

4. Click on *Straightedge Tool* and draw two straight lines connecting point *A* and point *B* as well as point *B* and point *C* (Diagram 4).
5. Click on *Selection Arrow Tool* and click on points *A*, *B* and *C* (Diagram 5).
6. Click *Measure* and select *Angle*. The value of $\angle ABC$ will be displayed (Diagram 6).

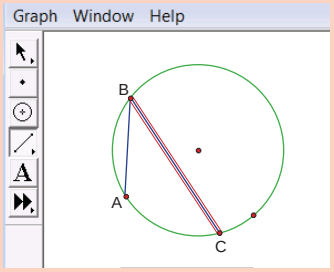


Diagram 4

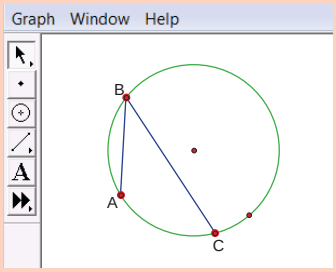


Diagram 5

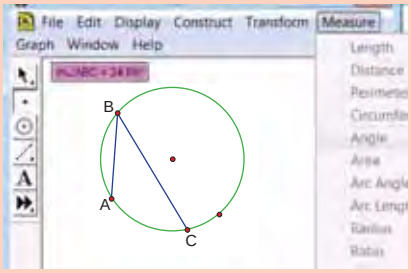


Diagram 6

7. Repeat steps 2 to 4 for point *D* and step 5 to select points *A*, *D* and *C* (Diagram 7).

8. Repeat step 6. The value of $\angle ADC$ will be displayed (Diagram 8). Notice that the values of $\angle ABC$ and $\angle ADC$ are the same.

9. You can try this with another point on the major arc *AC* to determine the value of the angle at the circumference.

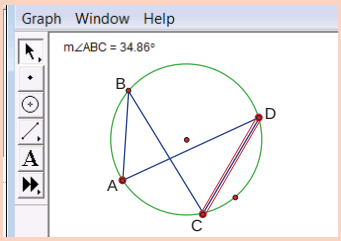


Diagram 7

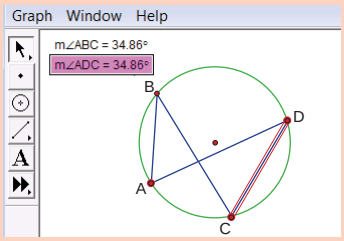


Diagram 8

Discussion:

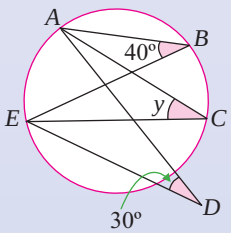
What can be concluded from your observations in the above activities?

From Brainstorming 2, it is found that:

The angles at the circumference subtended by the same arc are equal.

Example 1

Based on the diagram on the right, calculate the value of *y*.



Solution:

$y = \angle ABE = 40°$

BULLETIN

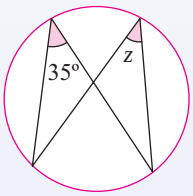
$y = \angle ABE = 40°$.

$\angle ADE \neq 40°$ because $\angle ADE$ is not an angle at the circumference of the circle subtended by arc *AE*.

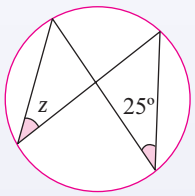
MIND TEST 6.1a

1. Calculate the value of z.

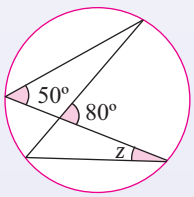
(a)



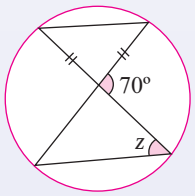
(b)



(c)

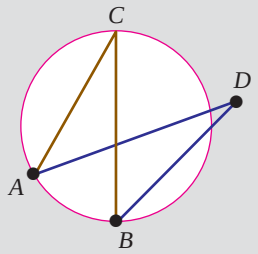


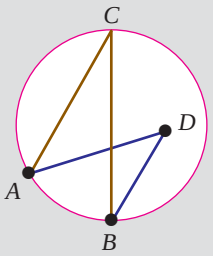
(d)



DISCUSSION CORNER

Is $\angle ADB = \angle ACB$? Discuss.

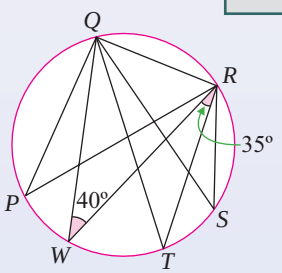




2. In the diagram on the right, chords $QW = RW$. Given that $\angle QWR = 40°$ and $\angle WRT = 35°$, determine the value of

(a) $\angle QSR$ (b) $\angle WQT$

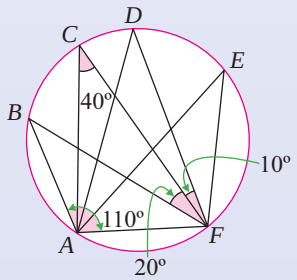
(c) $\angle WRQ$ (d) $\angle QRT$



3. In the diagram on the right, $\angle BAF = 110°$, $\angle ACF = 40°$, $\angle CFD = 10°$ and $\angle BFC = 20°$. Determine the value of

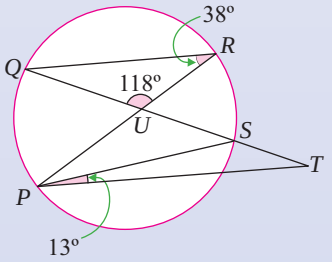
(a) $\angle ABF$ (b) $\angle BFA$

(c) $\angle CAD$ (d) $\angle DAF$



4. In the diagram on the right, $\angle QRP = 38°$, $\angle QUR = 118°$ and $\angle SPT = 13°$. Determine the value of

(a) $\angle RPS$ (b) $\angle PTQ$



CHAPTER 6

Are angles at the circumference of a circle subtended by arcs of the same length equal and are angles at the circumference proportional to the arc length?

Brainstorming 3

In groups

Aim: 1. To verify that angles at the circumference subtended by arcs of the same length are equal.

2. To verify that angles at the circumference is proportional to the length of the arc.

Materials: Compasses, protractor, pencil, ruler and A4 paper.

Steps:

1. Draw a circle of radius 5 cm. Without adjusting the gap of the compasses, divide the circumference of circle into six parts (Diagram 1 - Diagram 3).
2. Draw two angles at the circumference that are subtended by two different parts of the same length and label them (Diagram 4).

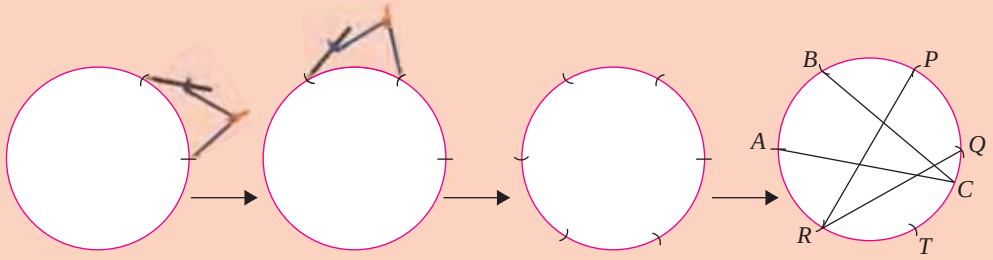


Diagram 1 Diagram 2 Diagram 3 Diagram 4

3. Measure $\angle BCA$ and $\angle PRQ$. Record them in Table 1.
4. Repeat step 1. Draw chords with different arc lengths (Diagram 5). Measure $\angle RPT$ and $\angle BQR$. Record them in Table 2.

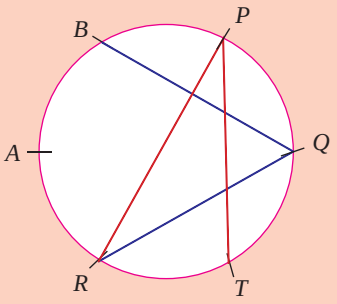


Diagram 5

Arcs	
BA	PQ
$\angle BCA$	$\angle PRQ$

Table 1

Arcs	
RT	$BR = 2RT$
$\angle RPT$	$\angle BQR$

Table 2

Discussion:

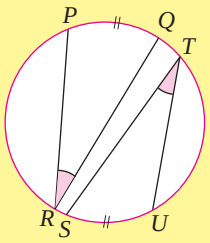
1. What can you conclude about angles at the circumference subtended by arcs of the same length?
2. What is your conclusion on the effects of changing the arc length to the angles subtended at the circumference?

From Brainstorming 3, it is found that:

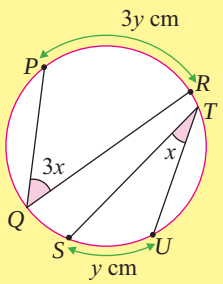
(a) $\angle BCA = \angle PRQ$ [Arc length AB = Arc length PQ].

(b) $\angle BQR = 2 \times \angle RPT$ [Arc length $BR = 2 \times$ Arc length RT].

In general,



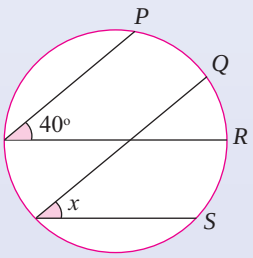
Angles at the **circumference** subtended by **arcs of the same length** are equal. If arc length PQ = arc length SU then $\angle PRQ = \angle STU$.



The size of an angle at the **circumference** subtended by an arc is **proportional** to **the arc length**.

Example 2

The diagram on the right shows a circle with length of arcs $PR = QS$. Determine the value of x. Give reasons for your answer.

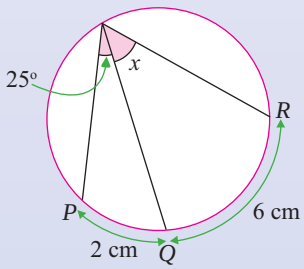


Solution:

$x = 40°$ because $\angle x$ and $\angle 40°$ are at the circumference and length of arcs $PR = QS$.

Example 3

Based on the diagram on the right, determine the value of x.



Solution:

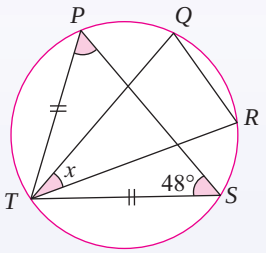
$$\frac{x}{25°} = \frac{6 \text{ cm}}{2 \text{ cm}}$$

$$x = 3\,(25°)$$

$$x = 75°$$

Example 4

Given the length of minor arc PS is two times the length of arc QR, determine the value of x.



Solution:

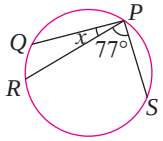
$\angle PTS = 180° - 2(48°)$ — Sum of angles in a triangle is 180°.

$\angle PTS = 84°$

Thus, $x = \frac{84°}{2}$ — $QR = \frac{PS}{2}$

$x = 42°$

QUIZ

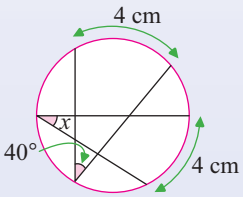


If arc length of $RS = \frac{7}{2}QR$, determine the value of x.

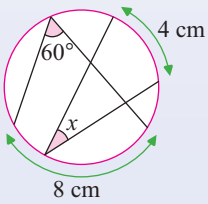
MIND TEST 6.1b

1. Based on the diagrams below, calculate the value of x.

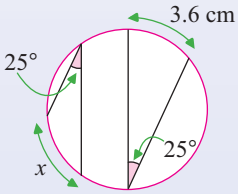
(a)



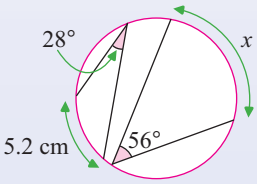
(b)



(c)



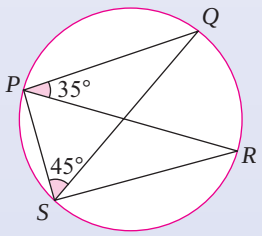
(d)



2. The diagram on the right shows a circle. Given that the length of arcs $RS = 2QR$, $\angle QPR = 35°$ and $\angle PSQ = 45°$, determine the value of

(a) $\angle SPR$

(b) $\angle SRP$

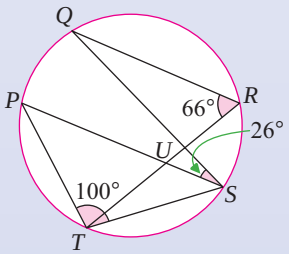


3. In the diagram on the right, the length of arcs $QPT = 3RS$. Given that $\angle QRT = 66°$, $\angle QST = 26°$ and $\angle PTS = 100°$, determine the value of

(a) $\angle RQS$

(b) $\angle TUS$

(c) $\angle TPS$



What is the relationship between angles at the centre of a circle and angles at the circumference that are subtended by the same arc?

Brainstorming 4

In pairs

Aim: To verify the relationship between angles at the centre of a circle and angles at the circumference subtended by the same arc.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click on *Compass Tool* to draw a circle.
2. Use *Point Tool* to place three points around its circumference (Diagram 1).
3. Use *Text Tool* to label all points at the circle with *A*, *B*, *C* and centre as *D* (Diagram 2).
4. Use *Straightedge Tool* to construct lines from one point to another (Diagram 3).

Diagram 1

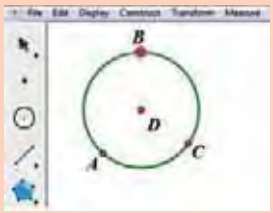


Diagram 2

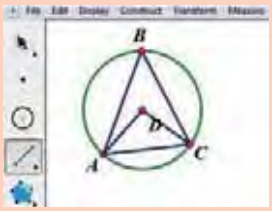


Diagram 3

5. Use *Selection Arrow Tool* to select points *A*, *B* and *C*.
6. Click on the menu *Measure* and select *Angle*. The value of $\angle ABC$ will be displayed.
7. Repeat steps 5 and 6 to get $\angle ADC$. The value of $\angle ADC$ will be displayed (Diagram 4).

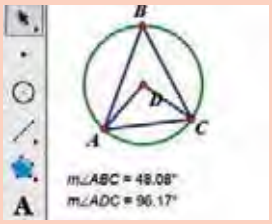


Diagram 4

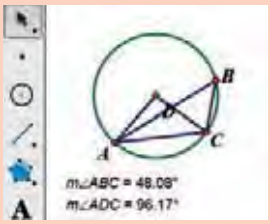


Diagram 5

8. What is the relationship between $\angle ABC$ and $\angle ADC$?
9. Click on point *B* and move it along the circumference of the circle as shown in Diagram 5. Is the value of $\angle ABC$ still the same as the value obtained in step 6?

Discussion:

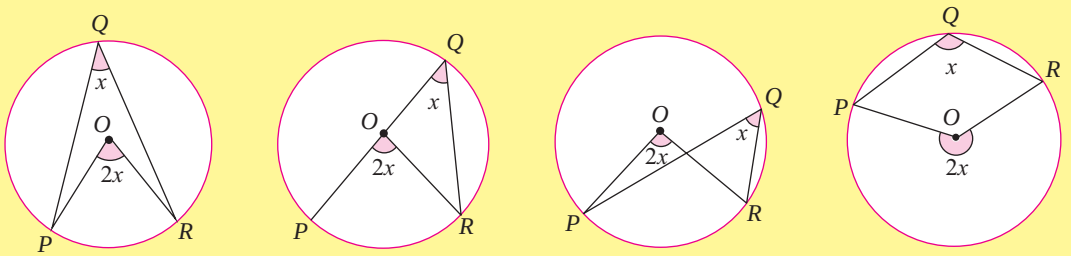
What can you conclude about the relationship between angles at the centre of a circle and angles at the circumference of a circle subtended by the same arc?

From Brainstorming 4, it is found that:

(a) $\angle ADC = 2 \times \angle ABC$

(b) The value of $\angle ABC$ is constant even though point B is moved along the circumference of the circle.

In general,

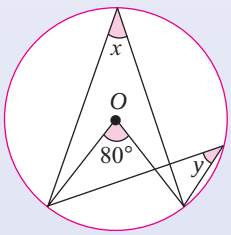


The size of the **angle** at the **centre of a circle** (central angle) subtended by the same arc is **twice** the size of angle at the **circumference**.

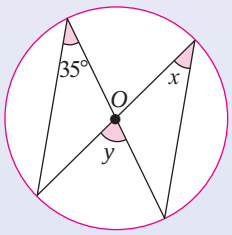
Example 5

Determine the value of x and y for each of the following.

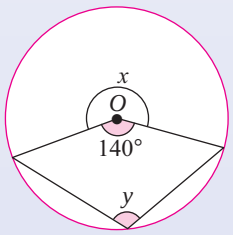
(a)



(b)



(c)



Solution:

(a) $x = \frac{1}{2}(80°)$

$x = 40°$

$y = 40°$

(b) $x = 35°$

$y = 2(35°)$

$y = 70°$

(c) $x = 360° - 140°$

$x = 220°$

$y = \frac{1}{2}(220°)$

$y = 110°$

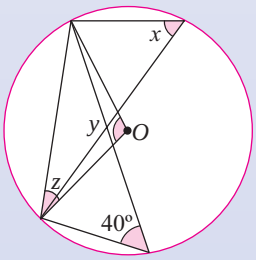
MIND TEST 6.1c

1. The diagram on the right shows a circle with centre O. Determine the value of

(a) x

(b) y

(c) z

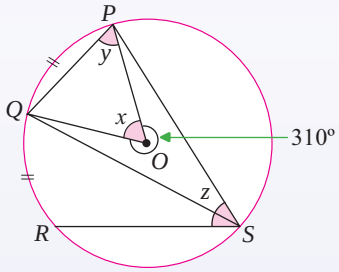


2. The diagram on the right shows a circle with centre O. Given that the length of arcs $PQ = QR$ and major angle $POQ = 310°$, calculate the value of

(a) x

(b) y

(c) z

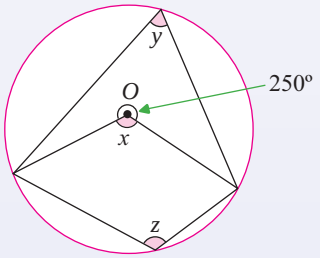


3. The diagram on the right shows a circle with centre O. Calculate the value of

(a) x

(b) y

(c) z

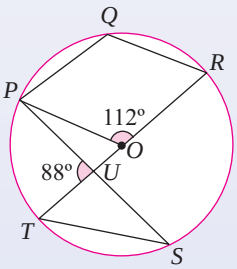


4. The diagram on the right shows a circle with centre O. Given that $\angle POR = 112°$ and $\angle PUT = 88°$, determine the value of

(a) $\angle PQR$

(b) $\angle UST$

(c) $\angle RTS$



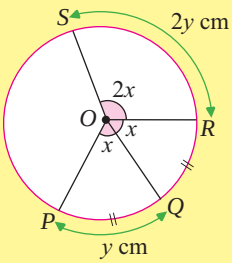
Are the central angles of a circle proportional to the arc length?

You have learned that:

1. Angles at the circumference of a circle subtended by the same arc are equal.

2. Angles at the circumference of a circle subtended by an arc are proportional to its arc length.

Both of the concepts above can also be applied to the central angle. In general,



For central angles of a circle subtended by an arc:

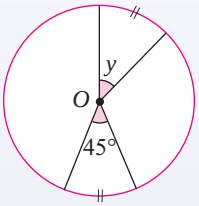
(a) the sizes of the angles are equal if their arc lengths are equal.

(b) the change in size of an angle is proportional to the change in the arc length.

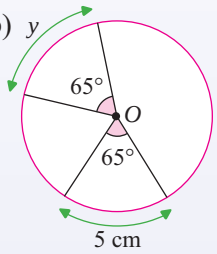
MIND TEST 6.1d

1. Based on the diagrams below, calculate the value of y.

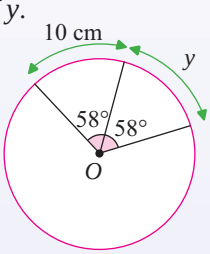
(a)



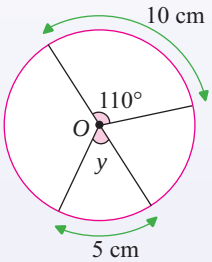
(b)



(c)



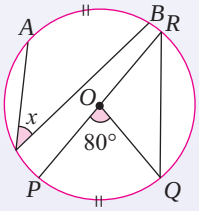
(d)



2. The diagram on the right shows a circle with centre O where the length of arcs $AB = PQ$. Determine

(a) the value of x

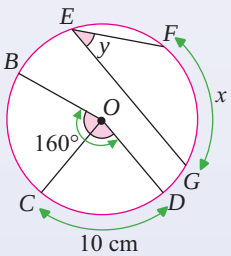
(b) the angle that has the same value as x



3. The diagram on the right shows a circle with centre O. It is given that length of arc $CD = 10$ cm and $\angle BOD = 160°$. If the length of arcs $BCD = 2CD$ and $\angle FEG = \frac{1}{4}\angle BOD$, determine

(a) the value of y, in cm

(b) the length of x, in cm



What is the value of angles at the circumference subtended by the diameter?

Brainstorming 5

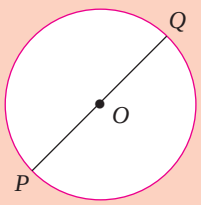
In pairs

Aim: To determine the angles subtended by the diameter.

Materials: Compasses, protractor, pencil, ruler and drawing paper.

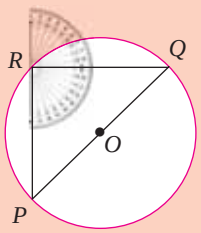
Steps:

1. Draw a circle with centre O and diameter PQ as in the diagram.



2. Draw two chords, PR and QR as in the diagram. Measure the value of $\angle PRQ$.

3. Change the position of point R at the circumference of the circle. Measure the new value of $\angle PRQ$.



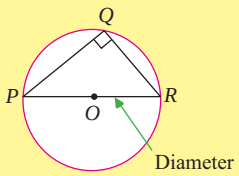
Discussion:

1. What can you conclude about the value of $\angle PRQ$ when the position of point R is changed at the circumference?
2. What is the value of the angle at the circumference of a circle subtended by the diameter?

From Brainstorming 5, it is found that:

For all positions of point R at the circumference of the circle subtended by diameter PQ, the value of $\angle PRQ$ is 90°.

In general,



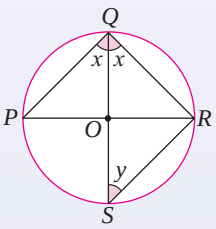
The angle at the circumference of circle subtended by the diameter is 90°. If PQR is a semicircle, then $\angle PQR = 90°$.

DISCUSSION CORNER

Is a diameter a chord? Discuss.

Example 6

The diagram on the right shows a circle with centre O where points P, Q, R and S lie on the circumference of the circle. Given that PR and QS are diameters, calculate the value of y.



Solution:

$PR = QS$

Thus, $2x = 90°$

$x = 45°$

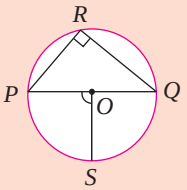
$y + x + \angle QRS = 180°$

$y + 45° + 90° = 180°$

$y = 180° - 45° - 90°$

$y = 45°$

BULLETIN

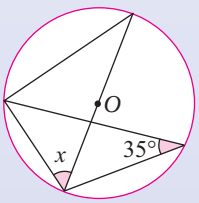


If arc lengths $PRQ = 2PS$ then, $\angle PRQ = \angle POS$

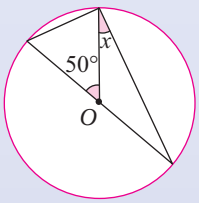
MIND TEST 6.1e

1. The diagrams below show circles with centre O. Calculate the value of x.

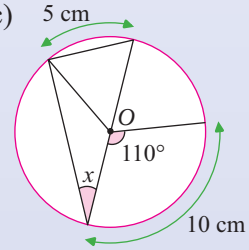
(a)



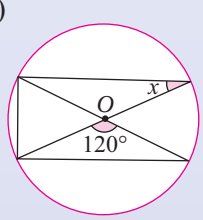
(b)



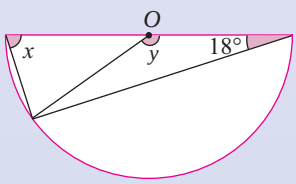
(c)



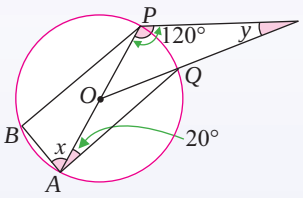
(d)



2. The diagram on the right shows a semicircle with centre O. Determine the value of $x + y$.



3. The diagram on the right shows a circle with centre O. If length of arcs AB = PQ, calculate the value of $x + y$.



How do you solve problems involving angles in circles?

LEARNING STANDARD

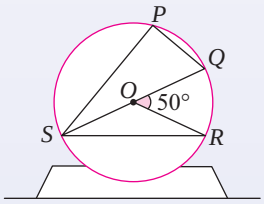
Solve problems involving angles in circles.

Example 7

A sculpture is constructed in the shape of a circle with centre at O as in the diagram. The points on the circumference form arc PQ which is of the same length as arc QR. Line SQ passes through O. Determine the value of

(a) $\angle QSR$

(b) $\angle PQS$



Solution:

(a) $\angle QSR = \frac{1}{2}\angle QOR$

$= \frac{1}{2}(50°)$

$= 25°$

(b) $\angle PSQ = \angle QSR = 25°$

$\angle PQS + 90° + 25° = 180°$

$\angle PQS = 180° - 90° - 25°$

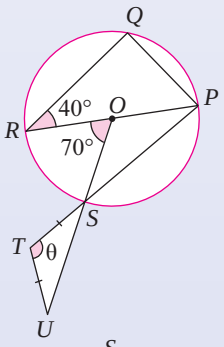
$= 65°$

MIND TEST 6.1f

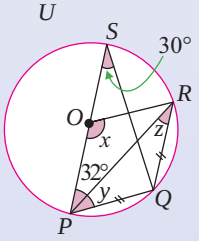
1. The diagram on the right shows a circle with centre O. OSU and PST are straight lines. Given that the diameter of the circle is 16 cm, $\angle ROS = 70°$, $\angle QRP = 40°$ and $ST = TU$,

(a) calculate the value of θ

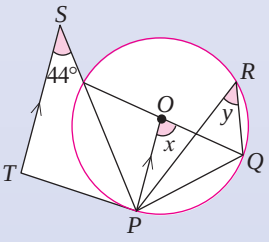
(b) calculate the length of PQ, in cm, correct to three significant figures



2. The diagram on the right shows a circle with centre O. Given that $PQ = QR$, $\angle PSQ = 30°$ and $\angle SPR = 32°$, calculate the value of $x + y + z$.



3. The diagram on the right shows a circle with centre O. Given that TS is parallel to PO and $\angle TSP = 44°$, calculate the value of $x + y$.



6.2 Cyclic Quadrilaterals

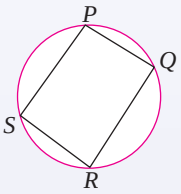
What do you know about cyclic quadrilaterals?

LEARNING STANDARD

Recognise and describe cyclic quadrilaterals.

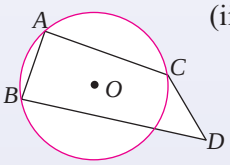
A **cyclic quadrilateral** is a quadrilateral in a circle where **all four vertices** of the quadrilateral lie on the **circumference of the circle**.

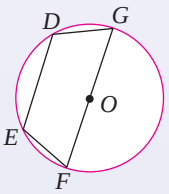
PQRS in the diagram on the right is a cyclic quadrilateral. $\angle P$ and $\angle R$ as well as $\angle S$ and $\angle Q$ are known as **opposite angles** in the cyclic quadrilateral.

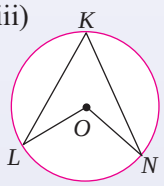


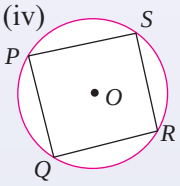
Example 8

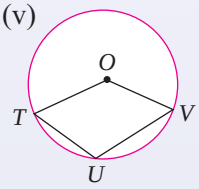
For each of the following circles, *O* is centre of the circle.

(i) 

(ii) 

(iii) 

(iv) 

(v) 

(a) Identify the cyclic quadrilateral found in each of the above circles and explain your answer.

(b) State the opposite angles in each cyclic quadrilateral that you have identified.

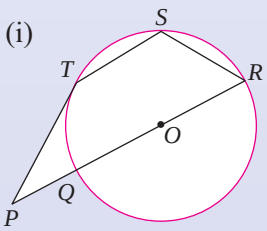
Solution:

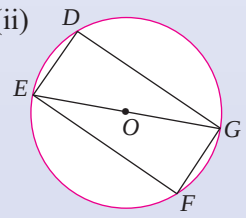
(a) (i) Vertex *D* does not lie on the circumference, hence *ABCD* is not a cyclic quadrilateral.
(ii) All vertices are on the circumference, hence *DEFG* is a cyclic quadrilateral.
(iii) Vertex *O* does not lie on the circumference, hence *KLON* is not a cyclic quadrilateral.
(iv) All vertices are on the circumference, hence *PQRS* is a cyclic quadrilateral.
(v) Vertex *O* does not lie on the circumference, hence *OTUV* is not a cyclic quadrilateral.

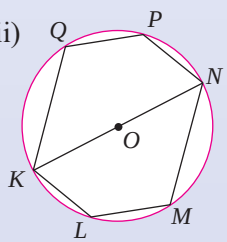
(b) (i) None (ii) $\angle D$ and $\angle F$, $\angle E$ and $\angle G$ (iii) None
(iv) $\angle P$ and $\angle R$, $\angle Q$ and $\angle S$ (v) None

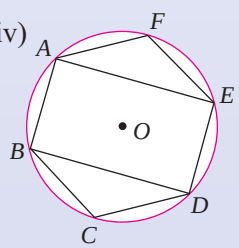
MIND TEST 6.2a

1. For each of the following circles, *O* is the centre of the circle.

(i) 

(ii) 

(iii) 

(iv) 

(a) Identify the cyclic quadrilateral found in each circle above and explain your answer.

(b) State the opposite angles in each cyclic quadrilateral that you have identified.

What are the relationships between angles of a cyclic quadrilateral?

Brainstorming 6

In pairs

Aim: To determine the relationship between opposite interior angles of a cyclic quadrilateral.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click on *Compass Tool* to draw a circle.
2. Click on *Straightedge Tool* to construct four lines from one point to another point on its circumference (Diagram 1).
3. Use *Text Tool* to label all points connecting the line with *A*, *B*, *C* and *D*.
4. Use *Selection Arrow Tool* to select *D*, *A*, and *B*.
5. Click on the menu *Measure* and select *Angle*. The value of $\angle DAB$ will be displayed.
6. Repeat steps 4 and 5 to get $\angle ABC$, $\angle BCD$ and $\angle CDA$ (Diagram 2).

Discussion:

1. What are the relationships between $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle ADC$?
2. What can you conclude about the relationships between the angles of a cyclic quadrilateral?

LEARNING STANDARD

Make and verify conjectures about the relationships between angles of cyclic quadrilaterals, and hence use the relationships to determine the values of angles of cyclic quadrilaterals.

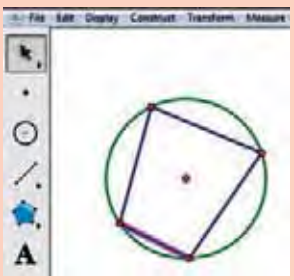


Diagram 1

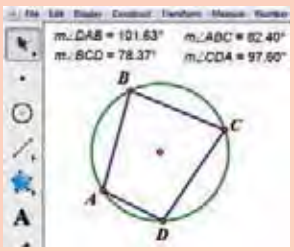


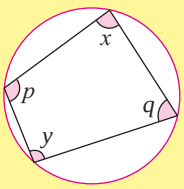
Diagram 2

From Brainstorming 6, it is found that:

(a) $\angle DAB + \angle BCD = 180°$ and $\angle ABC + \angle ADC = 180°$

(b) **The sum of the opposite** interior angles in a cyclic quadrilateral is 180°.

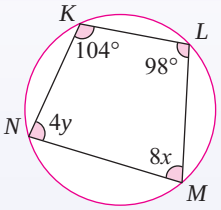
In general,



The sum of opposite interior angles in a cyclic quadrilateral is 180°.
$\angle x + \angle y = 180°$ and $\angle p + \angle q = 180°$

Example 9

The diagram on the right shows a cyclic quadrilateral $KLMN$. Calculate the value of
(a) x (b) y



Solution:

(a) The interior angles $\angle LKN$ and $\angle LMN$ are opposite in the cyclic quadrilateral.

Thus, $\angle LKN + \angle LMN = 180°$

$104° + 8x = 180°$

$8x = 180° - 104°$

$8x = 76°$

$x = \frac{76°}{8}$

$x = 9.5°$

(b) The interior angles $\angle KNM$ and $\angle KLM$ are opposite in the cyclic quadrilateral.

Thus, $\angle KNM + \angle KLM = 180°$

$4y + 98° = 180°$

$4y = 180° - 98°$

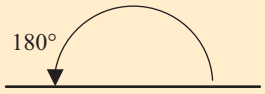
$4y = 82°$

$y = \frac{82°}{4}$

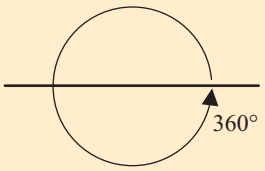
$y = 20.5°$

FLASHBACK

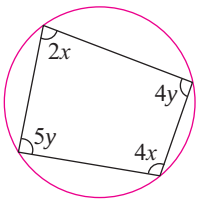
Angle on a straight line is 180°.



Angle of a full rotation is 360°.



QUIZ

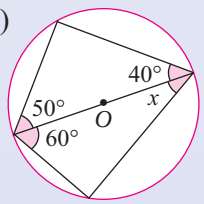


Calculate the value of $x + y$.

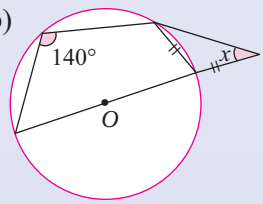
MIND TEST 6.2b

1. The diagrams below show circles with centre O. Calculate the value of x.

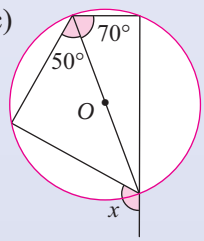
(a)



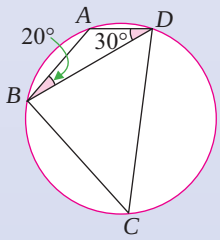
(b)



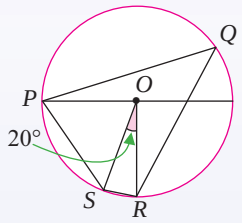
(c)



2. The diagram on the right shows a cyclic quadrilateral $ABCD$. Given that $\angle ADB = 30°$ and $\angle ABD = 20°$, calculate the value of $\angle BCD$.



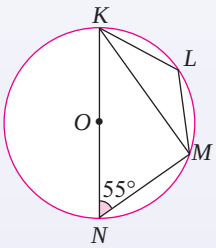
3. The diagram on the right shows a circle with centre O. If ΔPOS is an equilateral triangle and $\angle SOR = 20°$, calculate the value of $\angle PQR$.



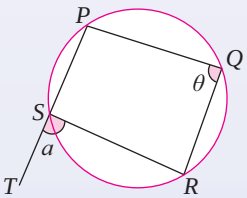
4. The diagram on the right shows a circle with centre O. Given that $\angle KNM = 55°$ and $KL = LM$, determine the value of

(a) $\angle KLM$

(b) $\angle LMN$

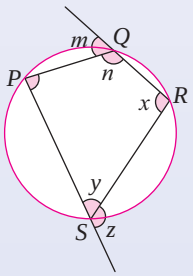


What is the relationship between the exterior angle with the corresponding opposite interior angle?



The diagram shows a cyclic quadrilateral $PQRS$. The chord PS is extended to T. $\angle TSR$, a, is the **exterior angle** of the cyclic quadrilateral $PSRQ$. $\angle PQR$, θ, is known as the **opposite interior angle** corresponding to a.

Example 10



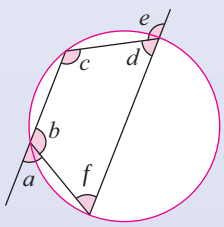
In the diagram on the right, $PQRS$ is a cyclic quadrilateral. Given that m and z are exterior angles, state the opposite interior angles corresponding to m and z.

Solution:

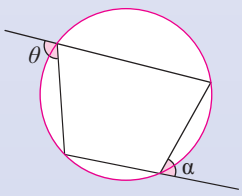
y is the opposite interior angle corresponding to m.
n is the opposite interior angle corresponding to z.

MIND TEST 6.2c

1. Copy and complete the table below based on the diagram on the right.



Exterior angle	Corresponding opposite interior angle



2. Draw a circle as shown in the diagram. Label the corresponding opposite interior angles for the exterior angle θ and α with symbols p and q respectively.

How do you solve problems involving cyclic quadrilaterals?

LEARNING STANDARD

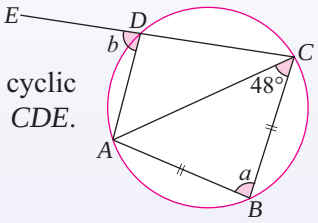
Solve problems involving cyclic quadrilaterals.

Example 11

The diagram on the right shows a cyclic quadrilateral $ABCD$ and a straight line CDE. Calculate the value of

(a) a

(b) b



Solution:

(a) $\angle ACB = \angle CAB = 48°$

$\angle ACB + \angle CAB + a = 180°$

$48° + 48° + a = 180°$

$a = 180° - 48° - 48°$

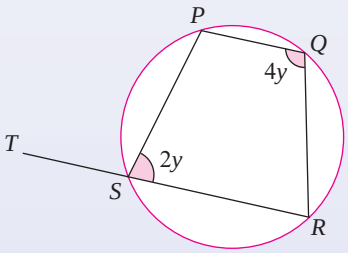
$a = 84°$

(b) $b = a$

Thus, $b = 84°$

Example 12

The diagram on the right shows a cyclic quadrilateral $PQRS$ and a straight line RST. Calculate the value of $\angle PST$.



Solution:

$\angle PQR + \angle PSR = 180°$

$4y + 2y = 180°$

$6y = 180°$

$y = 30°$

$\angle PST = \angle PQR$

$= 4y$

$= 4(30°)$

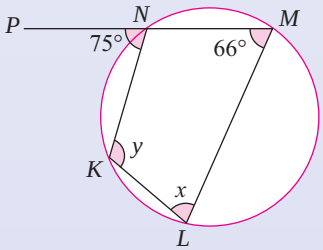
$\angle PST = 120°$

Example 13

The diagram on the right shows a cyclic quadrilateral $KLMN$ and a straight line MNP. Calculate the value of

(a) x

(b) y



Solution:

(a) $\angle PNK$ is an exterior angle. The opposite interior angle corresponding to it is angle x.

Thus,

$x = 75°$

(b) y and $\angle NML$ are opposite interior angles of cyclic quadrilateral $KLMN$.

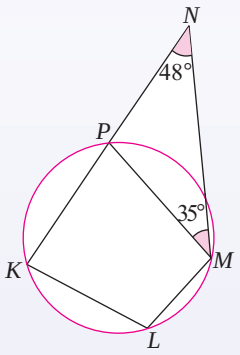
Thus, $y = 180° - \angle NML$

$y = 180° - 66°$

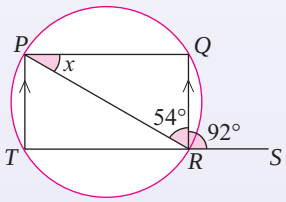
$y = 114°$

MIND TEST 6.2d

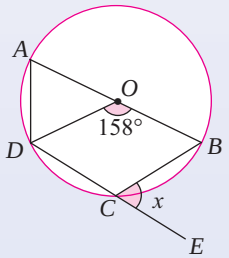
1. The diagram on the right shows a cyclic quadrilateral $KLMP$ and a straight line KPN. Given that $\angle KNM = 48°$ and $\angle NMP = 35°$, calculate the value of $\angle MLK$.



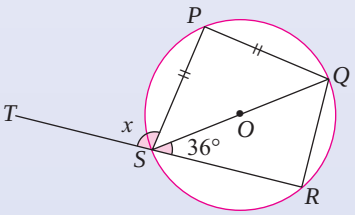
2. The diagram on the right shows a cyclic quadrilateral $PQRT$ and a straight line TRS. The sides PT and QR are parallel. Given that $\angle PRQ = 54°$ and $\angle QRS = 92°$, calculate the value of x.



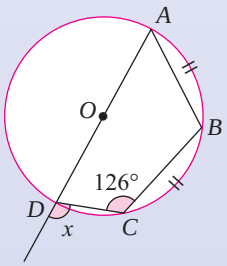
3. In the diagram on the right, the cyclic quadrilateral $ABCD$ lies in a circle with centre O. Calculate the value of x if DCE is a straight line and $\angle DOB = 158°$.



4. The diagram on the right shows a circle with centre O. $PQRS$ is a cyclic quadrilateral. It is given that $\angle QSR = 36°$. If the length of $PS = PQ$ and RST is a straight line, calculate the value of x.



5. The diagram on the right shows a circle with centre O. Given that $\angle BCD = 126°$, length of arcs $AB = BC$ and AOD is a straight line, calculate the value of x.



6.3 Tangents to Circles

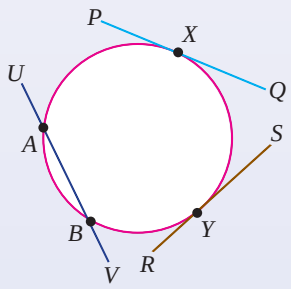
What do you understand about the tangents to circles?

LEARNING STANDARD

Recognise and describe the tangents to circles.

You have learnt that the circle is a unique shape and has many special properties.

In the diagram on the left, point T on the wheel will only touch the road once, when it makes a complete circle. The road serves as a tangent to the wheel which is round and the point T is the point of tangency when it touches the road.



In the diagram on the left, straight lines PQ and RS each touches the circle at point X and point Y while straight line UV passes through point A and point B on the circle. Thus,

(a) PQ and RS – Tangents to the circle.

(b) X and Y – Points of tangency of PQ and RS, respectively.

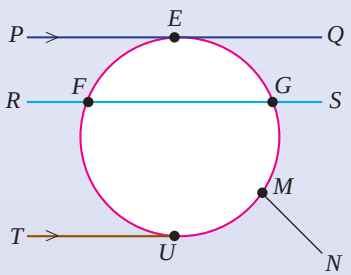
(c) UV – Not a tangent.

(d) A and B – Not points of tangency of UV.

Tangent to a circle is a straight line that touches the circle at only one point. The point of contact between tangent and the circle is the **point of tangency**.

Example 14

Are all straight lines and points shown in the diagram on the right tangents to the circle and points of tangency? State the reasons for your answer.



Solution:

PQ and TU are tangents to the circle because they touch the circle at only one point. Point E and point U are points of tangency of PQ and TU respectively.

RS is not a tangent to the circle because it passes through two points on the circle. Hence, point F and point G are not points of tangency of RS. MN is not a tangent to the circle because it will touch two points on the circle if extended. Thus, point M is not a point of tangency.

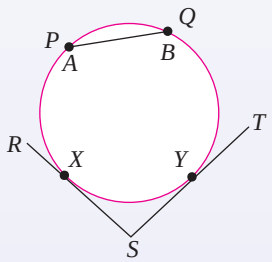
MIND TEST 6.3a

1. In the diagrams below, identify points and lines which are

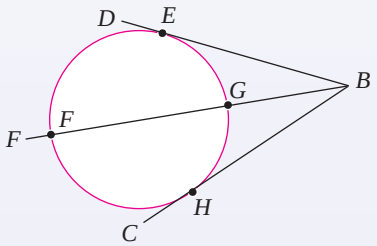
(i) tangents (ii) points of tangency (iii) not a tangent (iv) not a point of tangency

State the reasons for your answer.

(a)



(b)



What do you know about the value of the angle between tangent and radius at the point of tangency?

Brainstorming 7

In pairs

Aim: To measure the angle between tangent and radius of a circle at the point of tangency.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click on the *Compass Tool* to draw a circle (Diagram 1).

2. Click on *Straightedge Tool* to draw a straight line from the centre of the circle to a point on the circumference (Diagram 2).

3. Click on *Arrow Tool* to select point on the circumference and straight line.

4. Click *Construct* and select *Perpendicular Line* (Diagram 3).

5. Use *Point Tool* to mark the points and label them with the *Text tool* as *A*, *B* and *C* (Diagram 4).

6. Use *Selection Arrow Tool* to select *A*, *B* and *C*.

Diagram 1

Diagram 2

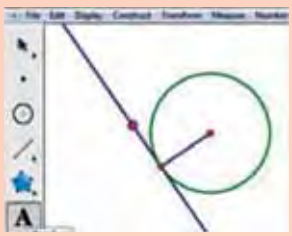


Diagram 3

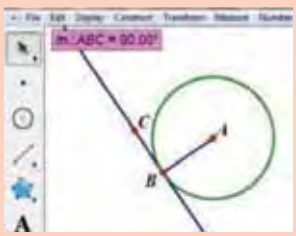


Diagram 4

LEARNING STANDARD

Make and verify conjectures about the angle between tangent and radius of a circle at the point of tangency.

CHAPTER 6

7. Click on the menu *Measure* and select *Angle*. The value of *ABC* will be displayed.

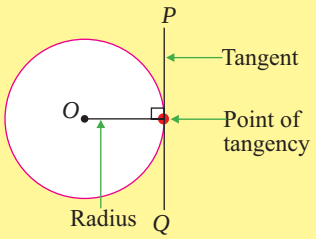
8. Repeat step 2 to step 7 to draw tangent lines on the other side of the circle and determine the angle between tangent and radius at the point of tangency.

Discussion:
What conclusions can you draw about the value of the angle between tangent and radius at the point of tangency?

From Brainstorming 7, it is found that:

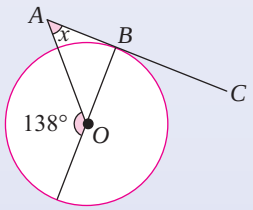
When tangent and radius intersect at the point of tangency, a right angle is formed. Thus $\angle ABC = 90°$.

In general,



The radius of a circle that intersects with tangent to the circle at the point of tangency will form a 90° angle with the tangent.

Example 15



The diagram on the right shows a circle with centre *O* which meets the straight line *ABC* at point *B* only. Calculate the value of *x*.

Solution:

Line *ABC* is a tangent to the circle and it touches the circle at point *B*. Thus, the angle $\angle OBA = 90°$.

$\angle AOB + 138° = 180°$

$\angle AOB = 180° - 138°$

$= 42°$

$x + \angle AOB = 90°$

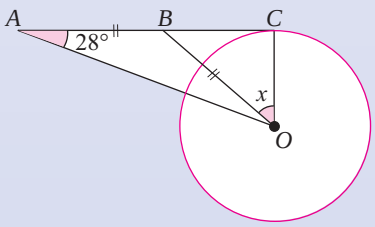
$x = 90° - \angle AOB$

$x = 90° - 42°$

$x = 48°$

MIND TEST 6.3b

1. In the diagram on the right, *ABC* is a straight line and *O* is the centre of the circle. Given that *AB* = *OB* and $\angle BAO = 28°$, calculate the value of *x*.

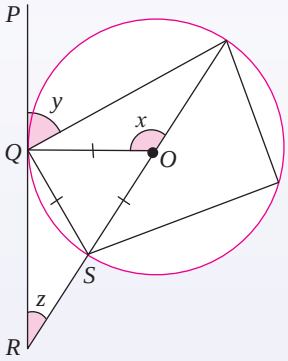


2. The diagram on the right shows a circle with centre O. Given that OQS is an equilateral triangle and PQR is a tangent to the circle, calculate the value of

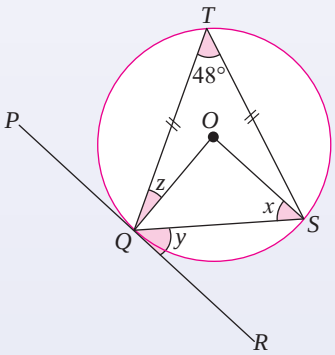
(a) x

(b) y

(c) z



3. In the diagram on the right, O is the centre of the circle and PQR is a tangent to the circle. Given that $QT = ST$ and $\angle QTS = 48°$, calculate the value of $x + y + z$.



What are the properties related to two tangents to a circle?

LEARNING STANDARD

Make and verify conjectures about the properties related to two tangents to a circle.

Brainstorming 8

In pairs

Aim: To determine the properties related to two tangents to a circle.

Materials: Drawing paper, compasses, protractor, ruler and pencil.

Steps:

1. Draw a circle of radius 3 cm with centre O. Draw a straight line 8 cm from the centre O and label as OA (Diagram 1).

2. Draw another circle of radius 7 cm with point A as centre of the circle. Mark the intersection points of both circles as B and C (Diagram 2).

3. Draw straight lines OB, OC, AB and AC (Diagram 3).

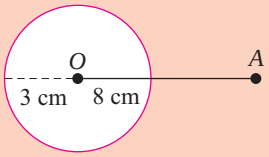


Diagram 1

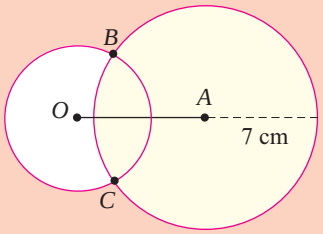


Diagram 2

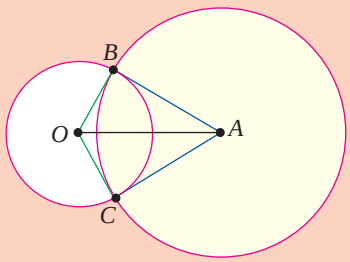


Diagram 3

4. Measure the following and complete the table below.

$\angle AOB$	$\angle AOC$	$\angle OBA$	$\angle OCA$	$\angle OAB$	$\angle OAC$	Length			
						OB	*OC*	*AB*	*AC*

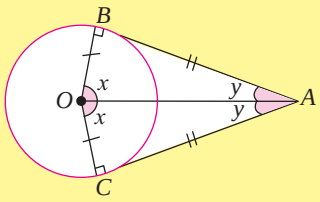
5. Display your group's findings in the Mathematics corner. Compare your group's answers with other groups.

Discussion:
What are your conclusions regarding the pairs of $\angle AOB$ and $\angle AOC$, $\angle OBA$ and $\angle OCA$, $\angle OAB$ and $\angle OAC$ and also the length of lines *OB*, *OC*, *AB* and *AC*?

From Brainstorming 8, it is found that:

(a) $\angle AOB = \angle AOC$, $\angle OBA = \angle OCA$ and $\angle OAB = \angle OAC$
(b) Length of *OB* = length of *OC* and length of *AB* = length of *AC*

In general,



If two tangents to a circle with centre *O* and points of tangency *B* and *C* meet at point *A*, then

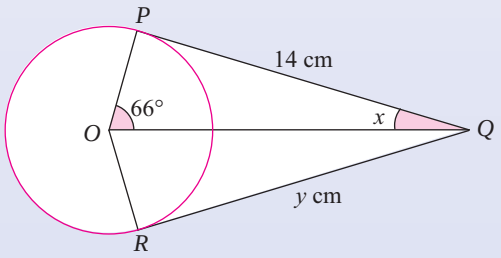
- $BA = CA$
- $\angle BOA = \angle COA$
- $\angle OAB = \angle OAC$

CHAPTER 6

Example 16

The diagram on the right shows a circle centred at *O*. Tangents *PQ* and *RQ* meet at point *Q*. Calculate

(a) the value of *x*
(b) the value of *y*
(c) the radius of the circle



Solution:

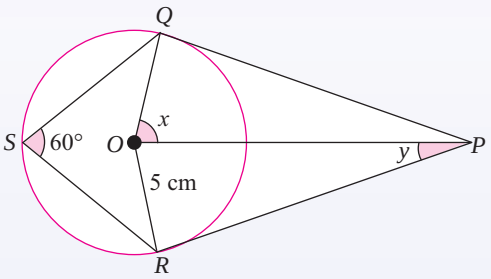
(a) Right-angled triangle ΔOPQ and
$\angle OPQ = 90°$.
Thus, $x + 66° = 90°$
$x = 90° - 66°$
$x = 24°$

(b) Length of $PQ = QR = y$
Thus, $y = 14$ cm

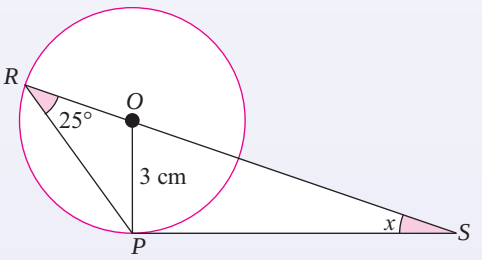
(c) $\tan 24° = \dfrac{OP}{14}$
$OP = 14 \times \tan 24°$
Radius, $OP = 6.233$ cm

MIND TEST 6.3c

1. The diagram on the right shows a circle of radius 5 cm centred at O. Given that PQ and PR are tangents to the circle and $\angle QSR = 60°$, calculate
(a) the value of x (b) the value of y
(c) the length of PQ (d) the length of OP



2. In the diagram on the right, O is the centre of circle with radius 3 cm and ROS is a straight line. Given that $\angle ORP = 25°$ and PS is a tangent to the circle, calculate
(a) the value of x (b) the length of PS
(c) the length of RS



What is the relationship of the angle between tangent and chord with the angle in the alternate segment which is subtended by the chord?

LEARNING STANDARD

Make and verify conjectures about the relationship of angle between tangent and chord with the angle in the alternate segment which is subtended by the chord.

In Diagram 1 (a), PQR is a tangent to the circle.
$\angle x$ is the angle between the chord QS and tangent PQR on a minor segment.

$\angle y$ is the angle of the major segment or alternate segment which is subtended by the chord QS.

In Diagram 1 (b), O is the centre of the circle. OQ and OS are radii of the circle and PQR is a tangent to the circle. Thus,

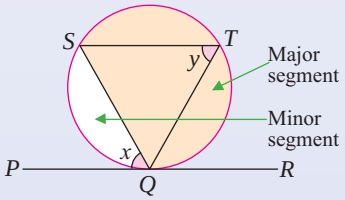


Diagram 1(a)

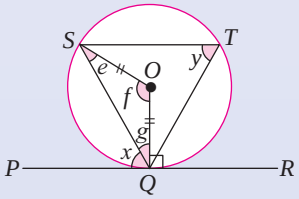


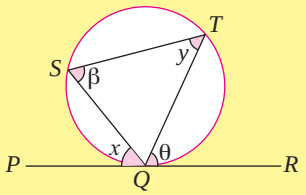
Diagram 1(b)

(a) $x + g = 90°$
$g = 90° - x$
$e = g$
Substitute

(b) $f = 180° - 2g$
$f = 180° - 2(90° - x)$
$f = 180° - 180° + 2x$
$f = 2x$ ← (1)

(c) $y = \frac{f}{2}$ ← (2)
Substitute (1) in (2)
$y = \frac{2x}{2}$
$y = x$

Based on the statements of Diagram 1 (a) and Diagram 1 (b), we can conclude that:

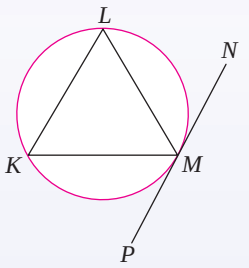


$\angle x = \angle y$ and $\angle \theta = \angle \beta$ because the angles between the chords and the tangents are equal to the angles at the alternate segments subtended by the chords.

Example 17

The diagram on the right shows triangle KLM and PMN is a tangent to the circle. Determine the angles in the alternate segment for

(a) $\angle PMK$ (b) $\angle NML$



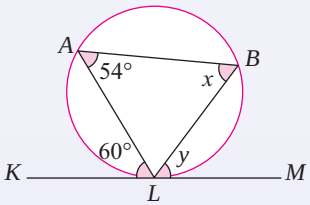
Solution:

(a) $\angle KLM$ (b) $\angle LKM$

Example 18

The diagram on the right shows the triangle ABL inside a circle. Given that KLM is a tangent to the circle, determine the value of

(a) x (b) y



Solution:

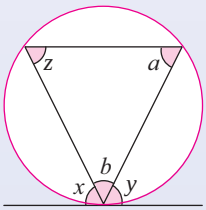
(a) $x = 60°$ because x is an angle in the alternate segment of $\angle KLA$ which is subtended by chord AL.

(b) $y = 54°$ because $\angle LAB$ is an angle in the alternate segment of y which is subtended by chord BL.

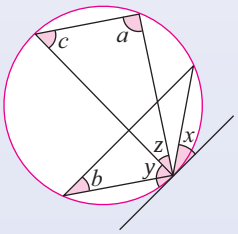
MIND TEST 6.3d

1. State the pair of angles with the same value in the following circles.

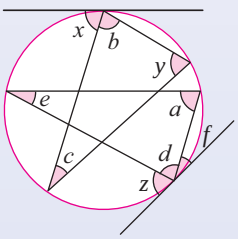
(a)



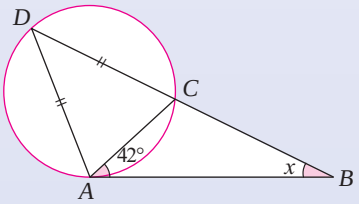
(b)



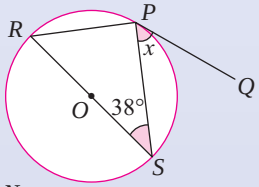
(c)



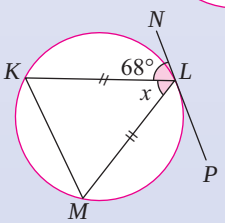
2. The diagram on the right shows a circle where AB is a tangent to the circle. Given $\angle BAC = 42°$, calculate the value of x.



3. The diagram on the right shows a circle with centre O. PQ is a tangent to the circle. Given $\angle PSR = 38°$, calculate the value of x.



4. The diagram on the right shows a circle where PLN is a tangent to the circle. ΔKLM is an isosceles triangle. Given $\angle KLN = 68°$, calculate the value of x.



How do you solve problems involving tangents to circles?

What do you know about common tangents?

LEARNING STANDARD

Solve problems involving tangents to circles.

A common tangent to two circles is a straight line that is a tangent to both the circles.

Notice the following pairs of circles and their common tangents.

1.

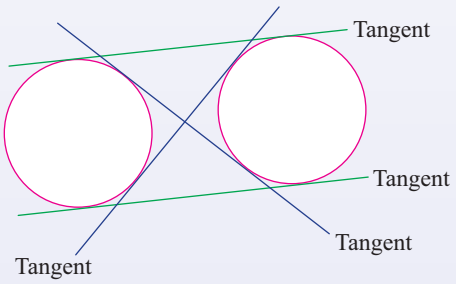


Diagram 1(a)

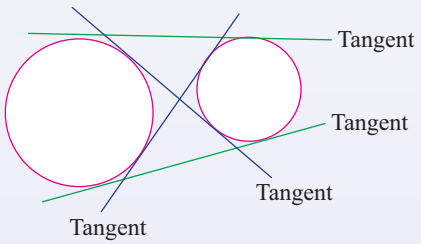


Diagram 1(b)

2.

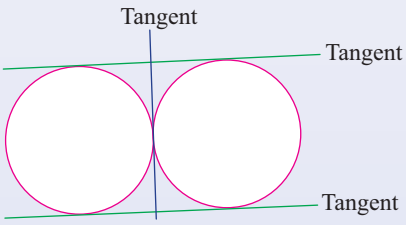


Diagram 2(a)

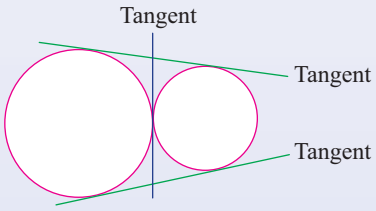


Diagram 2(b)

3.

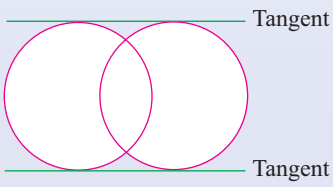


Diagram 3(a)

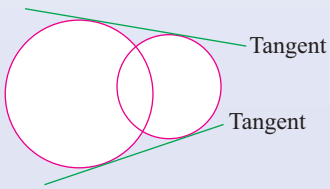


Diagram 3(b)

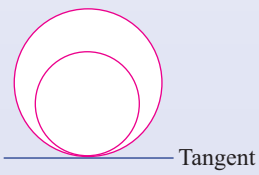


Diagram 3(c)

From the above diagrams it is found that if two circles of equal sizes or different sizes that are

(a) not touching each other, as shown in Diagram l(a) and Diagram l(b), will produce four common tangents

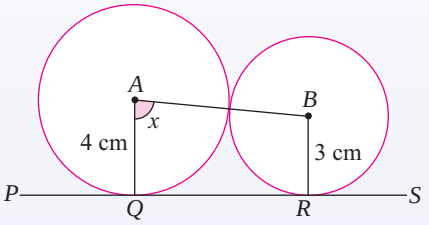
(b) touching at one point, as shown in Diagram 2(a) and Diagram 2(b), will produce three common tangents

(c) intersecting, as shown in Diagram 3(a) and Diagram 3(b), will produce two common tangents

(d) overlapping, as shown in Diagram 3(c), will produce only one common tangent

Example 19

The diagram on the right shows two circles centred at A and B with radius 4 cm and 3 cm respectively. Given that $PQRS$ is a common tangent to both circles, calculate the value of x.

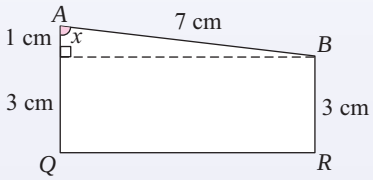


Solution:

$\cos x = \frac{1}{7}$

$x = \cos^{-1}\left(\frac{1}{7}\right)$

$x = 81.79°$

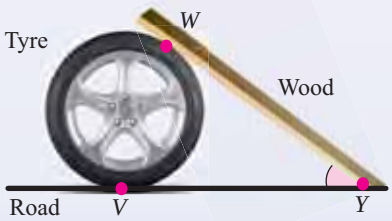


Example 20

A piece of wood is mounted on a tyre as shown in the diagram. It is given that V is the point of contact between the tyre and the road, W is the point of contact between the wood and the tyre while Y is the point of contact between the wood and the road. The diameter of the tyre is 50 cm and the distance of WY is 1.2 metres. Assuming that the road is a straight line, calculate

(a) the distance of VY

(b) the distance between the centre of the tyre and the point Y in metres. State your answer correct to two decimal places



Understanding the problem

(a) VY and WY are tangents to the circle. The diameter of the tyre is 50 cm and the distance WY is 1.2 metres.

(b) The distance between the centre of the tyre to point Y.

Planning a strategy

Draw a diagram and label it with the given values.

Diameter = 50 cm = 0.5 metre
Radius = 25 cm = 0.25 metre
WY = 1.2 metres

Making a conclusion

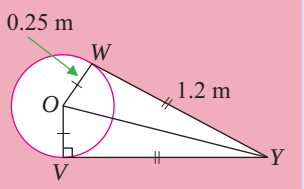
(a) ΔOWY and ΔOVY are congruent.
Thus, $VY = WY = 1.2$ metre.

(b) The distance between the centre of the tyre and point Y, $OY = 1.23$ m.

Implementing the strategy

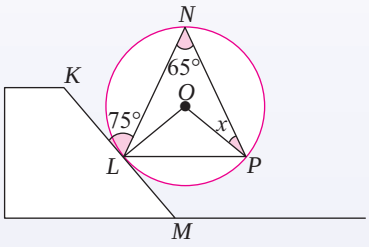
(a) $VY = WY = 1.2$ m.

(b) $OY = \sqrt{1.2^2 + 0.25^2}$
$OY = \sqrt{1.5025}$
$OY = 1.23$ m (2 d.p.)

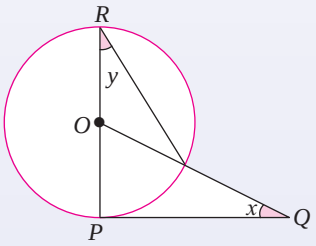


MIND TEST 6.3e

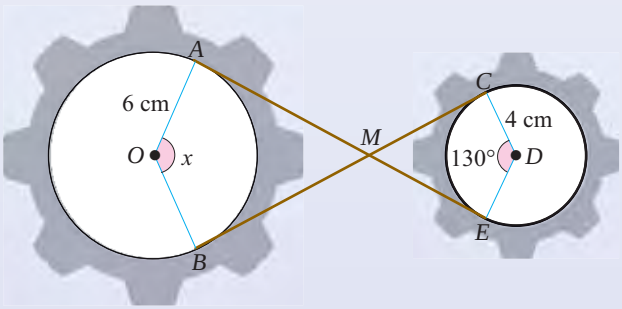
1. The diagram on the right shows the cross section of a barrel and a wall viewed from the top. The barrel is centred at O. The wall KLM touches the barrel at point L. Given that $\angle KLN = 75°$ and $\angle LNP = 65°$, calculate the value of x.



2. The diagram on the right shows a circle with centre O. PQ is a tangent to the circle. Given that $PQ = 2OP$, determine the value of $\angle x$ and $\angle y$. Give your answer in minutes and degrees.



3. The diagram below shows part of the gear system on a machine. Straight chains AE and BC meet both gears at points A, B, C, and E. The gears are circular with centres O and D respectively. Given that $OA = 6$ cm, $DC = 4$ cm and $\angle CDE = 130°$, calculate



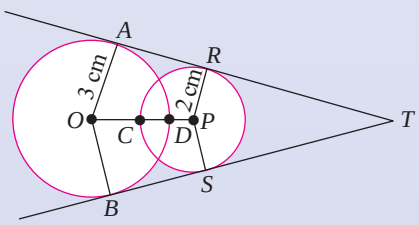
(a) the value of x

(b) the length in cm, correct to four significant figures, of

(i) AM (ii) CM (iii) OD

4. The diagram on the right shows two circles with radius 3 cm and 2 cm centred at O and P respectively. Given the length of $CD = DP$, calculate the length, in cm, correct to two decimal places, of

(a) OP (b) BS (c) BST



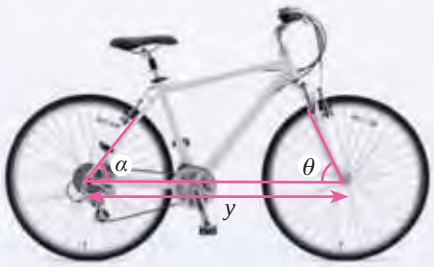
6.4 Angles and Tangents of Circles

How do you solve problems involving angles and tangents to the circle?

LEARNING STANDARD

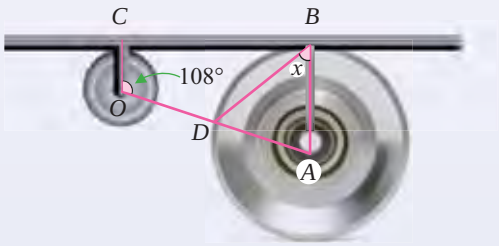
Solve problems involving angles and tangents of circles.

A circle is a familiar shape that we come across in our daily routine. One example is the wheel of the bicycle. Can you calculate the length of y, $\angle\alpha$ and $\angle\theta$?



Example 21

The diagram on the right shows two pulleys centred at O and A respectively, which are suspended from the ceiling BC. The rope ADO connects both pulleys. Calculate the value of x.



Solution:

Understanding the problem

BC is a tangent to the circles at points C and B.

$\angle OCB = \angle ABC = 90°$

$\angle AOC = 108°$

Identify $\angle ABD$, x

Planning a strategy

$\angle OCB + \angle ABC + \angle AOC + \angle OAB = 360°$

$\angle ABD = \angle ADB = x$

Making a conclusion

The value of $x = 54°$

Implementing the strategy

$\angle OAB + 90° + 90° + 108° = 360°$

$\angle OAB = 360° - 90° - 90° - 108°$

$= 72°$

AB and AD are radii. Thus,

$\angle ABD = \angle ADB = x$

$$x = \frac{180° - 72°}{2}$$

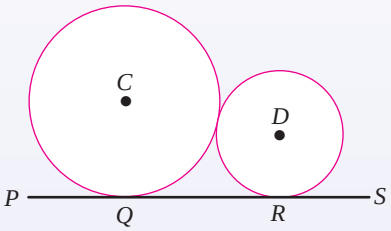
$x = 54°$

MIND TEST 6.4a

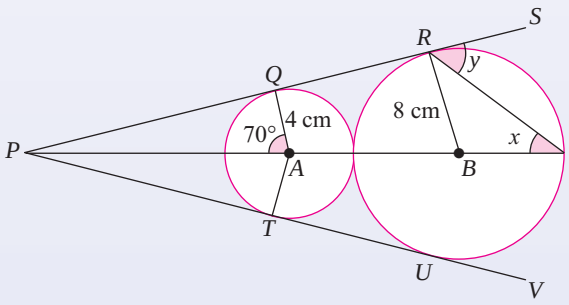
1. The diagram on the right shows two circles with centres C and D. Given radii of the two circles are 6 cm and 3 cm respectively, and $PQRS$ is a common tangent to both circles, calculate

(a) the length of QR, in cm. State the answer correct to three significant figures.

(b) the area of quadrilateral $CDRQ$, in cm^2. State the answer correct to four significant figures.



2. The diagram below shows two circles centred at A and B with radius 4 cm and 8 cm respectively. $PQRS$ and TUV are common tangents to both circles and $\angle PAQ = 70°$.



Calculate

(a) the value of x

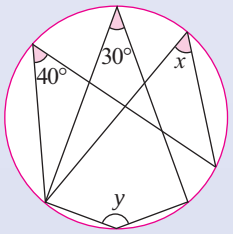
(b) the value of y

(c) the length of QR, in cm, correct to four significant figures

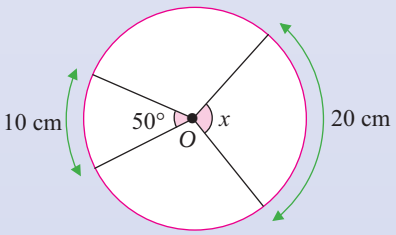
Dynamic Challenge

Test Yourself

1. The diagram on the right shows a circle. Calculate the value of x and y.



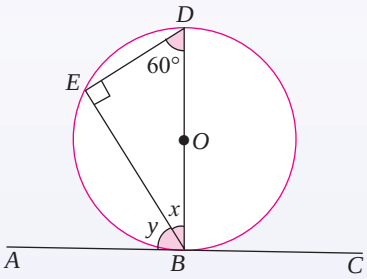
2. The diagram on the right shows a circle with centre O. Calculate the value of x.



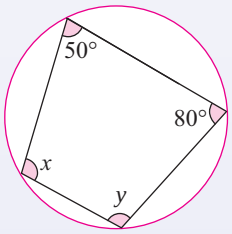
3. The diagram on the right shows a circle with centre O. ABC is a tangent to the circle. Given that the $\angle BDE = 60°$, calculate the value of

(a) x

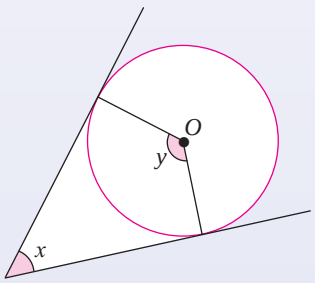
(b) y



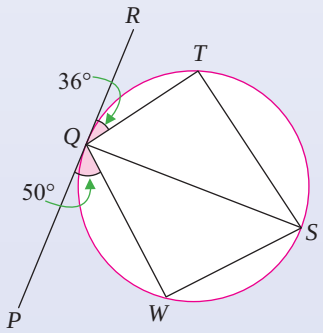
4. The diagram on the right shows a cyclic quadrilateral. Calculate the value of $x + y$.



5. A circle with centre O has two tangents to the circle as shown in the diagram on the right. What is the relationship between angle x and angle y?

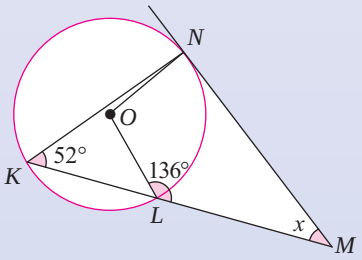


6. The diagram on the right shows a circle. Given that PQR is a tangent to the circle, $\angle RQT = 36°$ and $\angle PQW = 50°$, calculate the value of angle TSW.

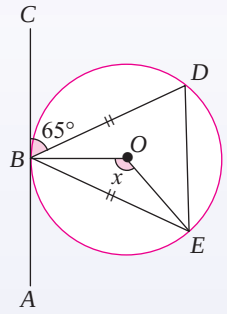


Skills Enhancement

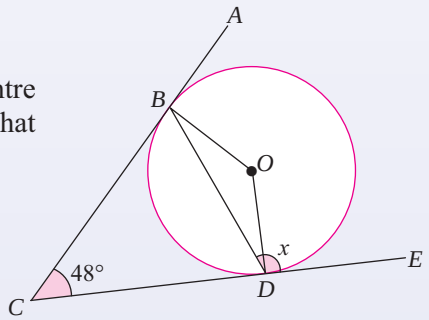
1. In the diagram on the right, O is centre of the circle and MN is a tangent to the circle. Given that $\angle LKN = 52°$ and $\angle MLO = 136°$, calculate the value of x.



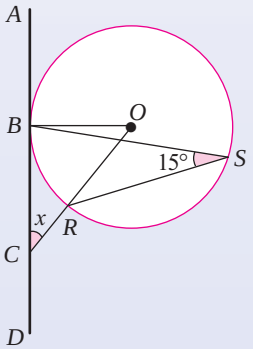
2. The diagram on the right shows a circle with centre O. ABC is a tangent to the circle. Given that $BD = BE$ and $\angle CBD = 65°$, calculate the value of x.



3. The diagram on the right shows a circle with centre O. ABC and CDE are tangents to the circle. Given that $\angle BCD = 48°$, calculate the value of x.



4. The diagram on the right shows a circle with centre O. AD is a tangent to the circle. Given that $\angle BSR = 15°$, calculate the value of x.

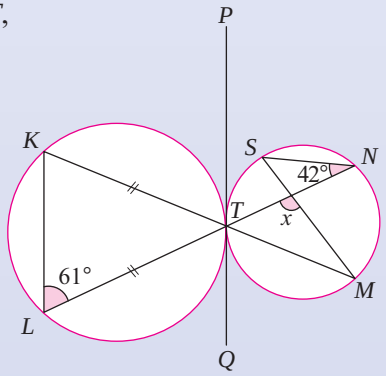


Self Mastery

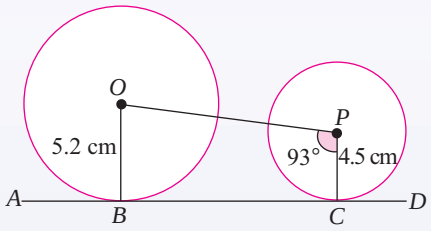
1. The diagram on the right shows two circles. PTQ is a common tangent to both circles. Given the length of $KT = LT$, $\angle KLT = 61°$ and $\angle SNT = 42°$, calculate

(a) $\angle LTQ$

(b) the value of x



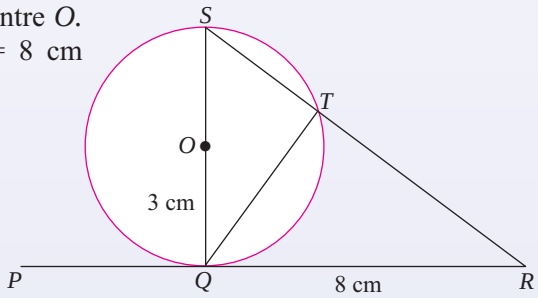
2. The diagram on the right shows two circles centred at O and P respectively. $ABCD$ is a common tangent to both circles. Calculate the area of trapezium $OBCP$, in cm^2, correct to three significant figures.



3. The diagram on the right shows a circle with centre O. Given that radius of the circle is 3 cm, QR = 8 cm and PQR is a tangent to the circle, determine

(a) $\angle TRQ$

(b) the length ST, in cm

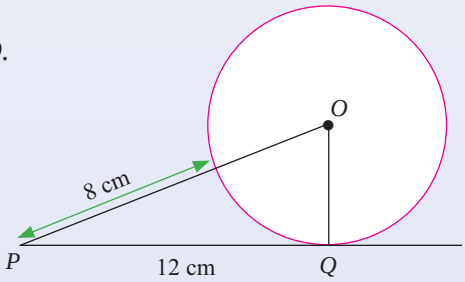


4. The diagram on the right shows a circle with centre O. PQ is a tangent to the circle. Calculate the

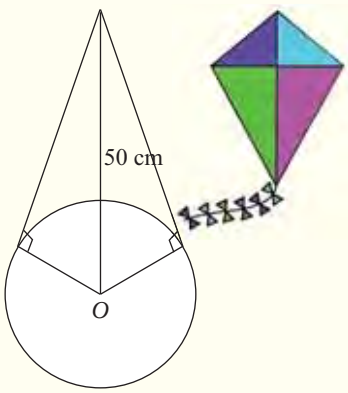
(a) radius of the circle, in cm

(b) length of OP, in cm

(c) area of ΔOPQ, in cm^2



PROJECT

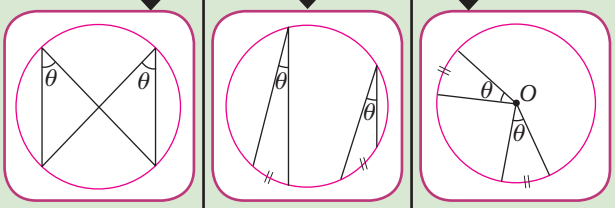


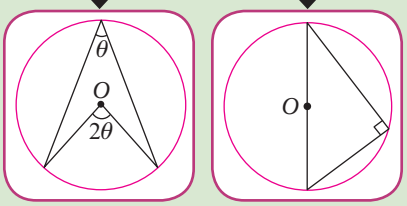
Kite-flying is a traditional game in our country. Kites can be constructed by using the concept of tangents to circles. With the knowledge of congruence and tangency that you have learnt, make a kite which has a length of 50 cm. Look for guidance from the diagram provided on the left.

CONCEPT MAP

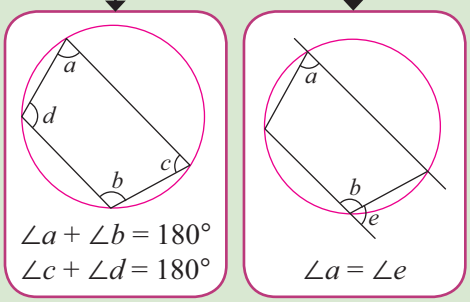
Circles

Properties of angles in a circle

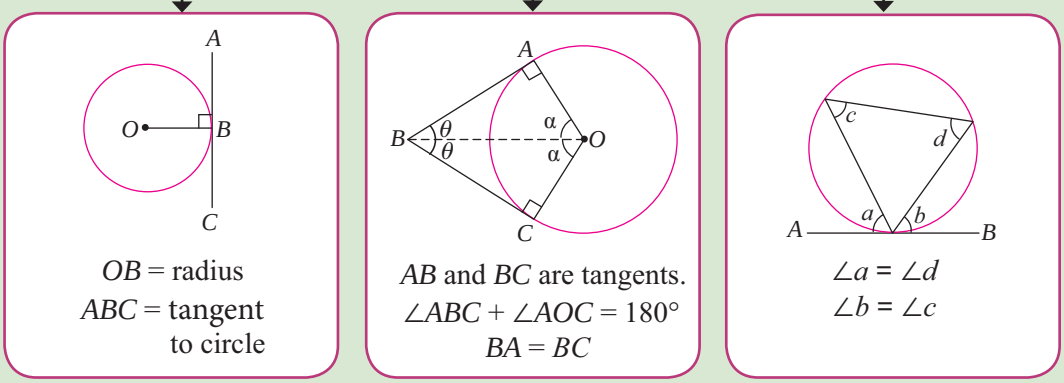




Cyclic quadrilaterals



Tangents to circles



SELF-REFLECT

At the end of this chapter, I can:

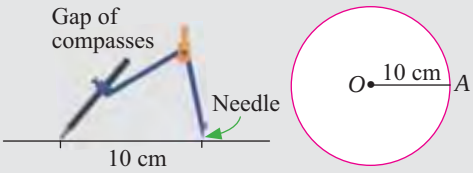
			
1.	Make and verify conjectures about the relationships between angles at the circumference, angle at the circumference and central angle subtended by particular arcs, and hence use the relationships to determine the values of angles in circle.		
2.	Solve problems involving angles in circles.		
3.	Recognise and describe cyclic quadrilaterals.		
4.	Make and verify conjectures about the relationships between angles of cyclic quadrilateral, and hence use the relationships to determine the values of angles of cyclic quadrilateral.		
5.	Solve problems involving cyclic quadrilaterals.		
6.	Recognise and describe the tangents to circles.		
7.	Make and verify conjectures about the angle between a tangent and radius of a circle at the point of tangency.		
8.	Make and verify conjectures about the properties related to two tangents to a circle.		
9.	Make and verify conjectures about the relationship of angles between tangents and chords with the angle in the alternate segment which is subtended by the chord.		
10.	Solve problems involving tangents to circles.		
11.	Solve problems involving angles and tangents of circles.		

EXPLORING MATHEMATICS

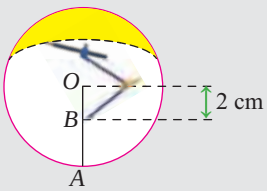
The appearance of the moon changes according to phases. Students can draw shapes of the moon at different phases to be used as decoration.

Materials: Drawing paper, compasses, pencil, ruler and scissors.

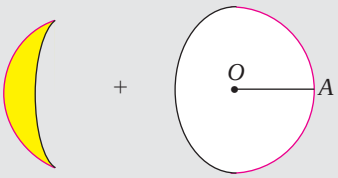
1. Draw a circle with radius 10 cm.



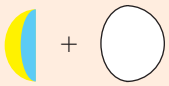
2. Then set the gap of compasses at 10 cm and place its needle at point *B*, which is 2 cm from point *O*. Draw a circle as in the diagram on the right.



3. The shaded part can be cut out and used as a crescent. The unshaded part can be used as a gibbous.



4. Step 1 and 2 can be repeated by changing the distance of *OB*. The distance of *OB* can be extended to 2 cm to obtain two different shapes of moon. The distance can be further extended to from different shapes. The examples are as follows:

Distance of *OB*	Resulting shape
1 cm	
2 cm	

These shapes can be used as a tool to teach the changing phases of the moon or as decoration.

Day	4	8	12	15	19	23	27
Phase of the moon	 Waxing Crescent	 First Quarter	 Waxing Gibbous	 Full Moon	 Waning Gibbous	 Last Quarter	 Waning Crescent

CHAPTER 7

Plans and Elevations

What will you learn?

7.1 Orthogonal Projections

7.2 Plans and Elevations

Why do you learn this chapter?

- The drawing of a plan and the elevations of an object allows the actual shape of the object to be seen in a two-dimensional form from various viewing directions.
- Plans and elevations are used in engineering, industrial construction, graphic design, architecture, computations and so on.

Each building in Putrajaya has its own uniqueness. The Malaysia Energy Commission Headquarters in Putrajaya which is known as the Diamond Building is a very beautiful building with a unique design. The Diamond Building has received the ASEAN Energy Award for its structure and design that maximises the use of sunlight. The Malaysia Green Building Index and Singapore Green Mark Scheme also awarded platinum ratings to recognise the building's design that enable sustainable recycling of rainwater. The uniqueness and creativity of the Diamond Building architecture is distinctive when viewed from various directions. Have you ever visited the Diamond Building?

Exploring Era

Mimar Sinan is one of the greatest and most influential architects. His real name is Sinar bin Abdulmennan bin Dogan Yusuf (1498-1588). He was born into a Christian Turkish family, from the Anatolian region of Agirnas in Kayseri. In 1539, Sinan was awarded the rank of Chief Architects of the Ottoman. Since then, he has been called Mimar Sinan, meaning Sinan the Architect. After Hagia Sophia (Ayasofya in Turkish) was converted into a mosque, Ottoman architects often used the mosque as a benchmark when designing other mosques. This is why most mosques in Turkey are similar in design.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter7.pdf

WORD BANK

• origin	• *asalan*
• geometrical shape	• *bentuk geometri*
• elevation	• *dongakan*
• solid line	• *garis padu*
• dashed line	• *garis sempang*
• orthogon	• *ortogon*
• plan	• *pelan*
• scale	• *skala*
• quadrant	• *sukuan*
• projection	• *unjuran*

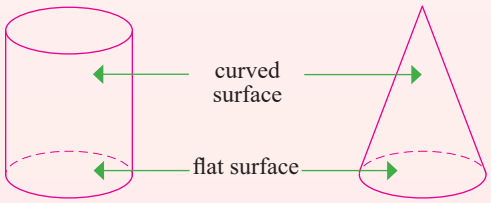
7.1 Orthogonal Projections

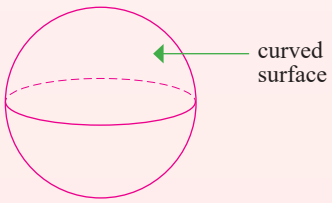
What is a plane and a normal to a plane?

LEARNING STANDARD

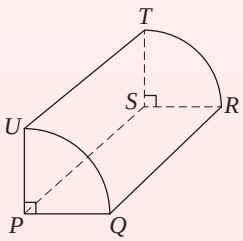
Draw orthogonal projections.

You have studied objects in two and three dimensions. Each of these objects consists of flat surfaces or curved surfaces or both.



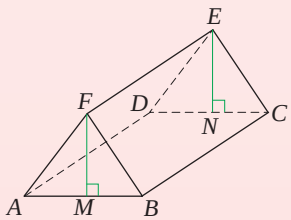


The diagram on the right shows a quarter of a right cylinder with a horizontal base *PQRS*. Both *PSTU* and *PQRS* are planes and *QRTU* is a curved surface.



A **plane** is the **flat surface** of an object. There are three types of planes, namely **horizontal plane**, **vertical plane** and **inclined plane.**

The diagram on the right shows a right prism with a horizontal plane *ABCD*. *ABF* and *CDE* are vertical planes. *BCEF* and *ADEF* are inclined planes. The lines *FM* and *EN* are perpendicular to the lines *AB* and *CD* respectively. The lines *FM* and *EN* are also known as the normal to the plane *ABCD*.



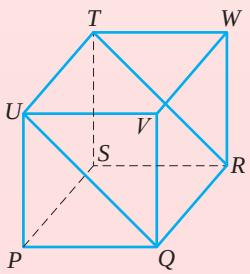
A **normal to a plane** is a straight line that is **perpendicular** or that forms a right angle to any **line on the plane**.

CHAPTER 7

Example 1

The diagram on the right shows a cube. State the normal to the following planes.

(a) *PQRS* (b) *PSTU* (c) *RSTW* (d) *QRTU*



Solution:

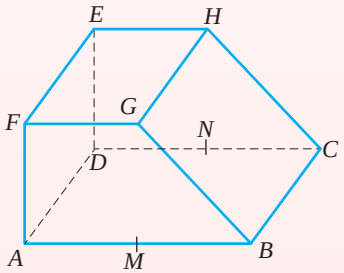
(a) *UP, VQ, WR, TS*
(b) *QP, RS, WT, VU*
(c) *QR, PS, UT, VW*
(d) *PV, SW*

The order of letters to specify a normal is important. *TS* means the line *TS* is perpendicular to the plane *PQRS* at point *S*.

Example 2

The diagram on the right shows a right prism with a rectangular base *ABCD*. *M* and *N* are the midpoints of *AB* and *CD* respectively. Given *FG* = *EH* = *DN* = *NC* = *AM* = *MB*, state the normal to the following planes:

(a) *ABCD* (b) *ADEF*



Solution:

(a) *FA, GM, HN, ED* (b) *BA, CD, GF, HE*

What do you understand about orthogonal projections?

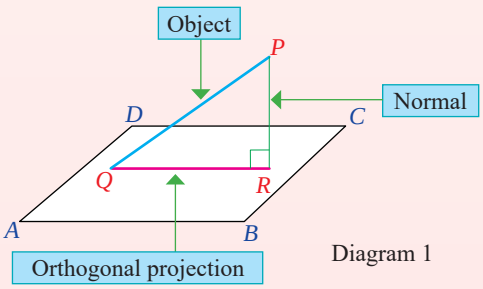


Diagram 1

In Diagram 1, *PQ* is a straight line where point *Q* lies on the horizontal plane *ABCD*. *PR* is a normal line to the plane *ABCD*. The straight line *RQ* which lies on the plane *ABCD* is an orthogonal projection of the straight line *PQ* on the plane *ABCD*.

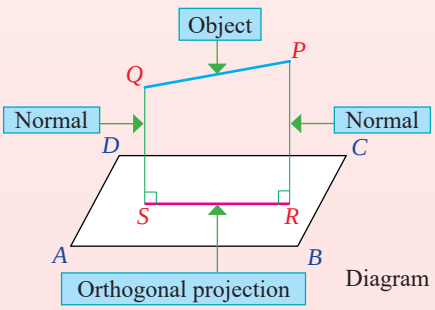


Diagram 2

In Diagram 2, the lines *PR* and *QS* are the normal to the plane *ABCD*. *RS* is an orthogonal projection of the straight line *PQ* on the plane *ABCD*.

Orthogonal projections are **images** formed on a plane when the **projected line** from an object is **perpendicular** to the plane.

In Diagram 1 and Diagram 2, we have identified the orthogonal projection for a line. Diagram 3 and Diagram 4 shows the orthogonal projections of a two-dimensional plane and a three-dimensional object.

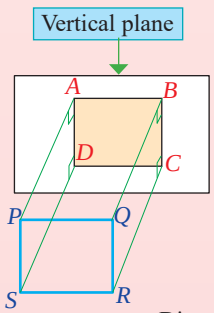


Diagram 3

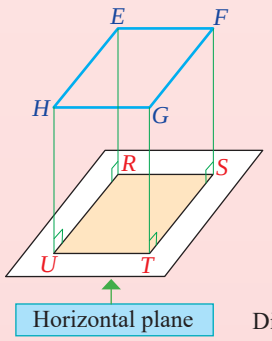


Diagram 4

In Diagram 3, *PQRS* is projected on a vertical plane and in Diagram 4 *EFGH* is projected on a horizontal plane.

Diagram	Object	Normal to the plane	Orthogonal projection on the plane
Diagram 3	*PQRS*	*PA, QB, RC, SD*	*ABCD*
Diagram 4	*EFGH*	*ER, FS, GT, HU*	*RSTU*

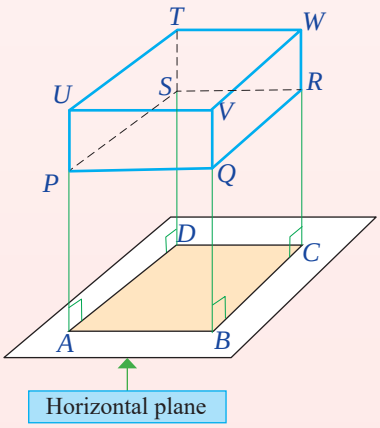


Diagram 5

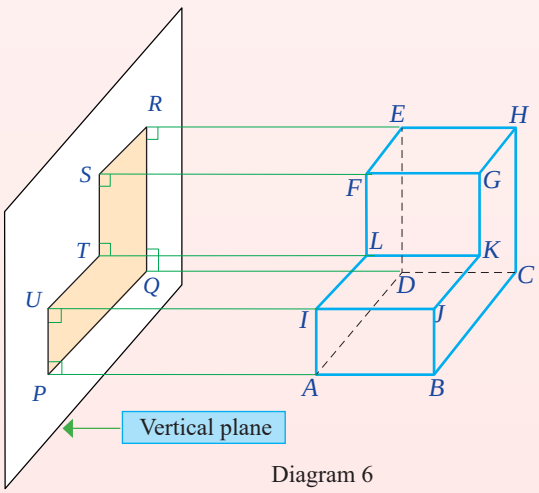


Diagram 6

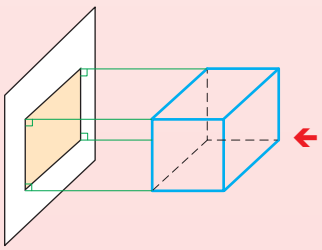
In Diagram 5, a cuboid is projected on a horizontal plane and in Diagram 6 a right prism with the surface *BCHGKJ* as a uniform cross section is projected on a vertical plane.

Diagram	Object	Normal to the plane	Orthogonal projection on the plane
Diagram 5	Cuboid	*PA, QB, RC, SD*	*ABCD*
Diagram 6	Right prism	*AP, IU, LT, DQ, FS, ER*	*PQRSTU*

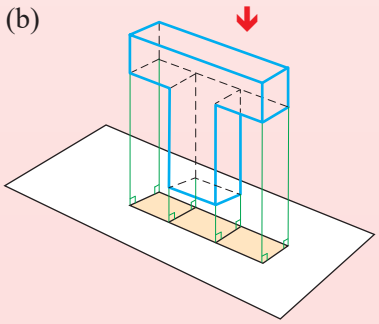
Example 3

Each of the following diagrams shows the projection of an object on a vertical plane or a horizontal plane. Determine whether the resulting projection is an orthogonal projection.

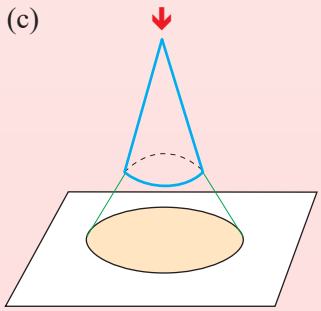
(a)



(b)



(c)



Solution:

(a) Yes

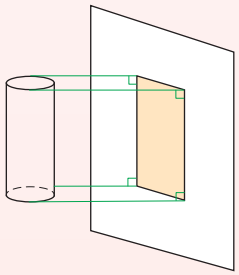
(b) Yes

(c) No, because the lines projected from the object to the plane is not a normal.

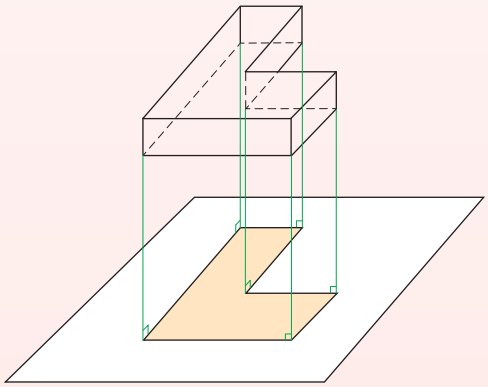
MIND TEST 7.1a

1. Each diagram below shows the object and its projection on a plane. Determine whether the projection is an orthogonal projection.

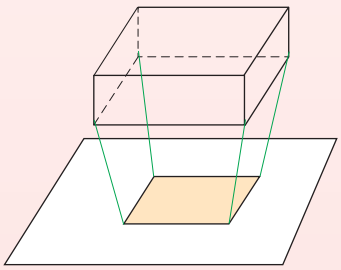
(a)



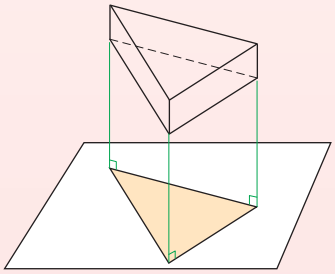
(b)



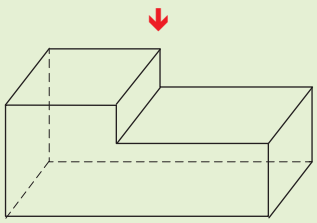
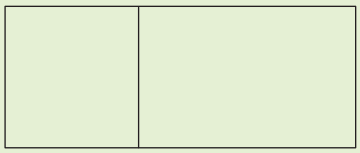
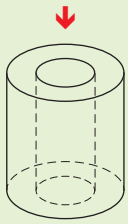
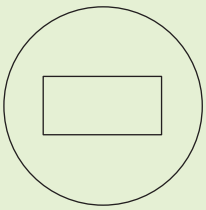
(c)



(d)



2. A student looks at the following object from a given viewing direction. Which of the following combinations shows the correct orthogonal projection?

Object	Orthogonal projection
(a) 	
(b) 	

How do you draw an orthogonal projection?

You can draw an orthogonal projection of an object on a horizontal plane or a vertical plane using the following steps.

1. Identify the type of plane and the direction in which the object that should be projected.
2. Draw normal lines from all vertices of the object to the plane. Make sure all the normal lines are straight and upright so that the length of projected sides and the length of sides of object are the same.
3. Connect the points of intersection of the normal to the plane to draw the shape of the orthogonal projection.
4. Redraw the orthogonal projection with actual measurements. Label all vertices and side lengths.

Example 4

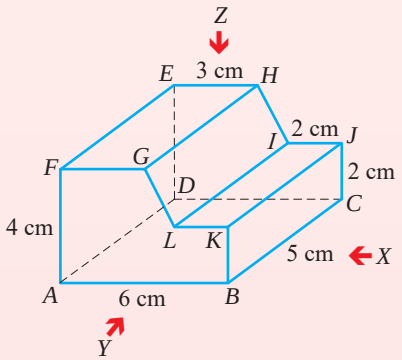
The diagram on the right shows a right prism with rectangular base *ABCD* on a horizontal plane. *ABKLGF* is a uniform cross section of the prism. The sides *AF* and *BK* are vertical.

Draw the orthogonal projection of the object on a

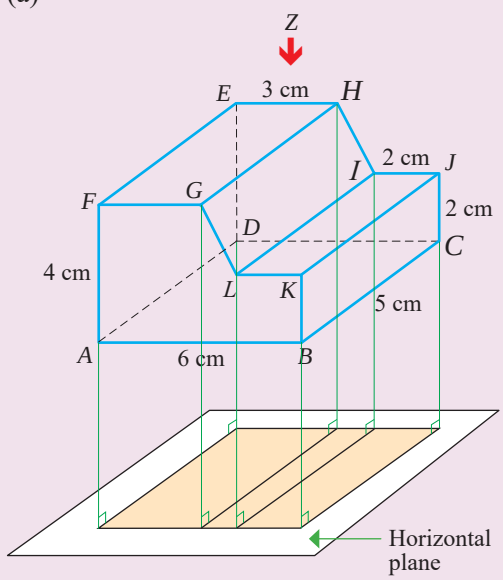
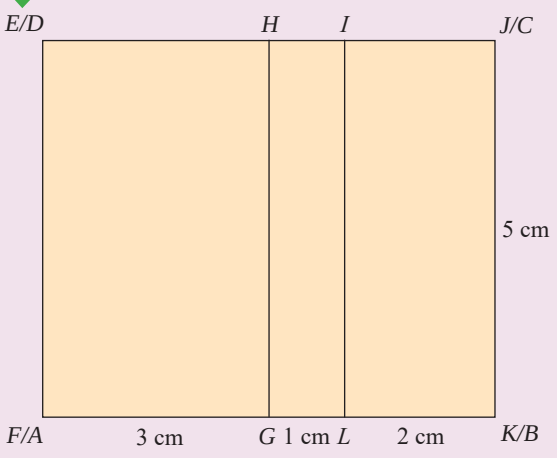
(a) horizontal plane as viewed from *Z*

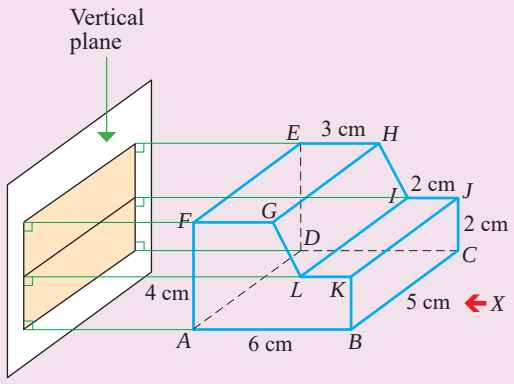
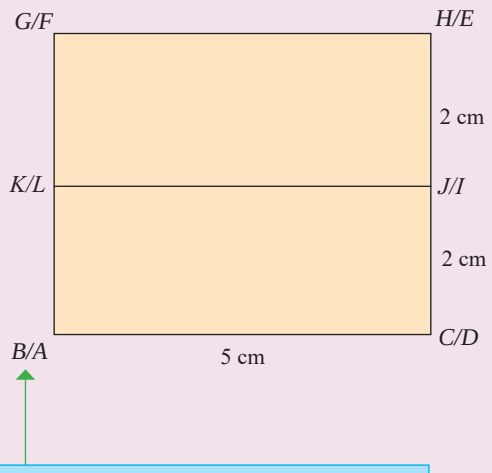
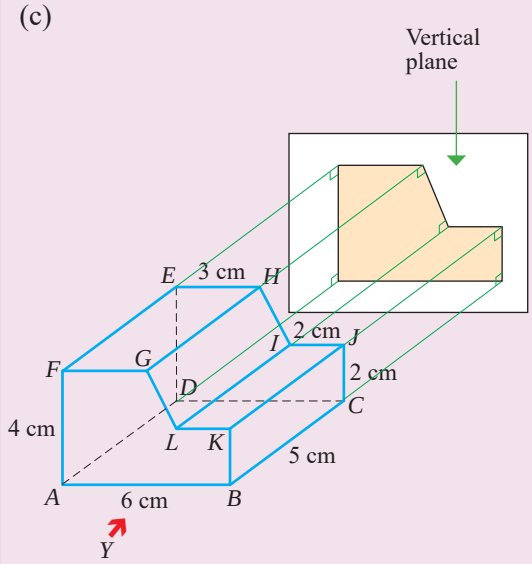
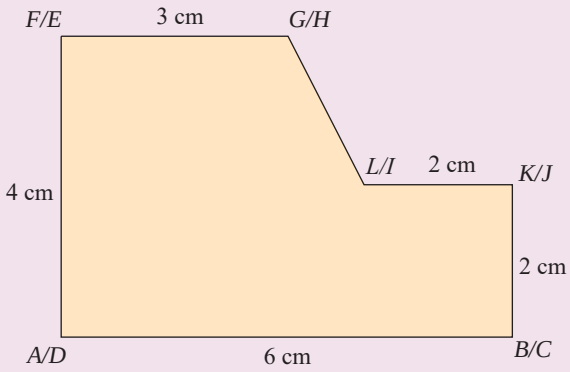
(b) vertical plane as viewed from *X*

(c) vertical plane as viewed from *Y*



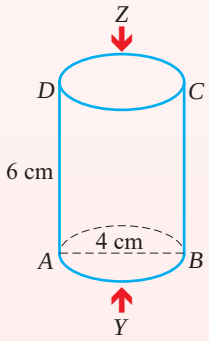
Solution:

Viewing direction	Orthogonal projection
(a) 	The order of letters is following the viewing direction. Point *D* is below point *E* as viewed from *Z*. 

Viewing direction	Orthogonal projection
(b)  	 Point A is behind point B as viewed from X.
(c)  	 Point D is behind point A as viewed from Y.

Example 5

The diagram on the right shows a cylindrical object on a horizontal plane. It is given that the diameter of the cylinder is 4 cm and its height is 6 cm.

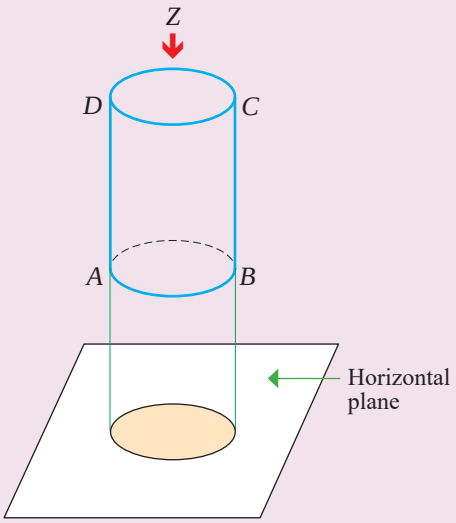
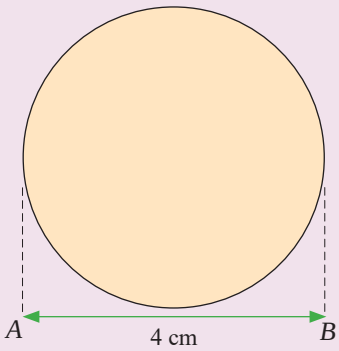
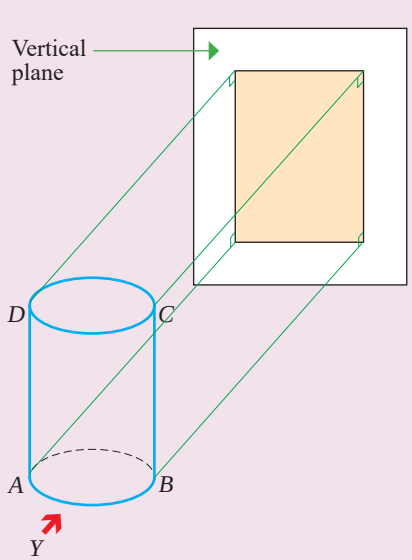
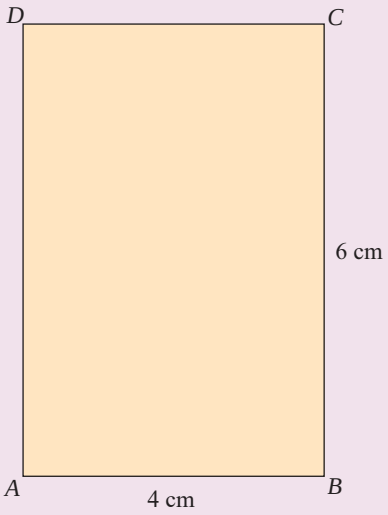


Draw the orthogonal projection of the cylindrical object on a

(a) horizontal plane as viewed from Z

(b) vertical plane as viewed from Y

Solution:

Viewing direction	Orthogonal projection
(a) 	
(b) 	

Brainstorming 1

In groups

Aim: To determine the orthogonal projections of an object.

Materials: Dynamic software, drawing paper.

Steps:

1. Open *View* and select 3D *graphics.*
2. Select the shape of pyramid .
3. Basic display is formed (Diagram 1).
4. Drag the cursor to display and select the four points:
   (a) Point (–2, 0) on the red line.
   (b) Point (–2, 0) on the green line.
   (c) Point (2, 0) on the red line.
   (d) Point (2, 0) on the green line and connect it to the starting point (–2, 0) at the red line (Diagram 2).
5. The display will show a brownish shape (Diagram 3).
6. Drag the cursor up to the blue line (0, 4) (Diagram 4).
7. Select the *3D rotate icon*, select *view in front of* .
8. Place the arrow at the top end of the blue line to see the orthogonal projection on the horizontal plane (Diagram 5).

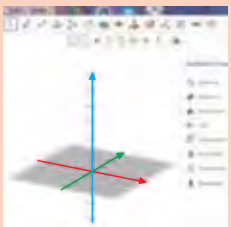


Diagram 1

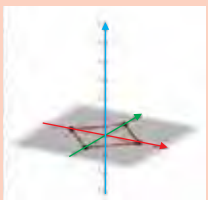


Diagram 2

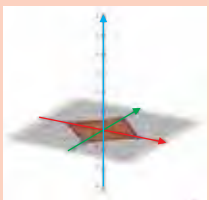


Diagram 3

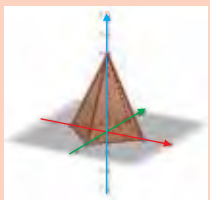


Diagram 4

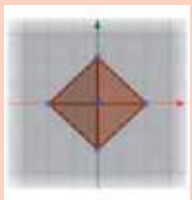


Diagram 5

9. Repeat step 8 on the red line and the green line to see various orthogonal projections on vertical planes.
10. Draw the resulting orthogonal projections as in steps 8 and 9 in the given table.
11. Select a new file. Build other 3D shapes and draw orthogonal projections from different perspectives.

Results of findings:

Pyramid 	Orthogonal projection
The view on the horizontal plane as seen from the blue line	
The view on the vertical plane as seen from the red line	
The view on the vertical plane as seen from the green line	

Discussion:

Discdiscuss the resulting shape of the orthogonal projection as compared to the actual shape of the object.

From Brainstorming 1, it is found that:

Pyramid 	Orthogonal projection
The view on the horizontal plane as seen from the blue line	
The view on the vertical plane as seen from the red line	
The view on the vertical plane as seen from the green line	

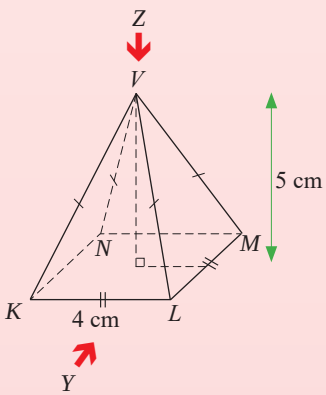
MIND TEST 7.1b

1. Each object below lies on a horizontal plane. Draw orthogonal projections of each object on a

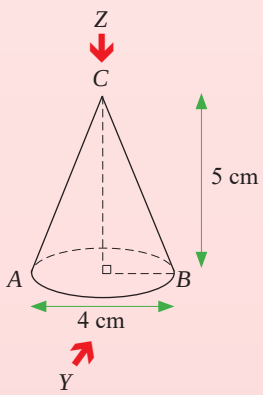
(a) horizontal plane as viewed from Z

(b) vertical plane as viewed from Y

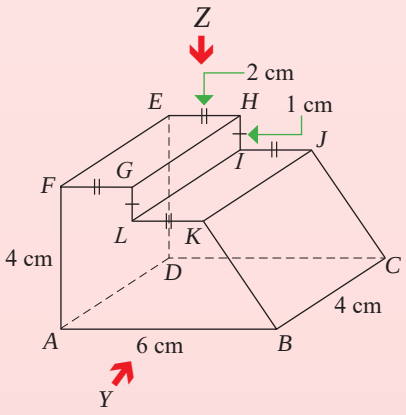
(i)



(ii)



(iii)



How do you compare and contrast objects with their projections?

LEARNING STANDARD

Compare and contrast between objects and the corresponding orthogonal projections.

Brainstorming 2

In groups

Aim: Compare and contrast an object with an orthogonal projection in terms of length of side and size of angle.

Materials: Cardboard, a pencil, a pair of scissors, adhesive tape and drawing paper.

Steps:

1. Draw the following shape according to the size given on a cardboard (Diagram 1).

2. Cut out the shape in Diagram 1 and use adhesive tape to build the shape in Diagram 2.

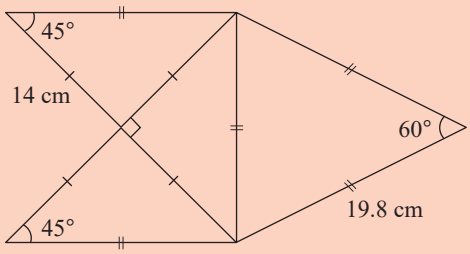


Diagram 1

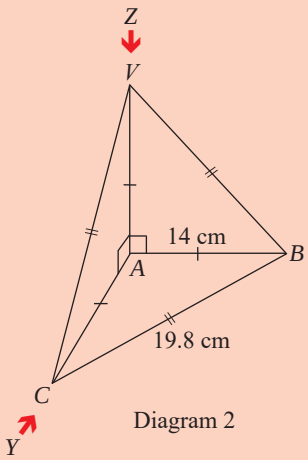
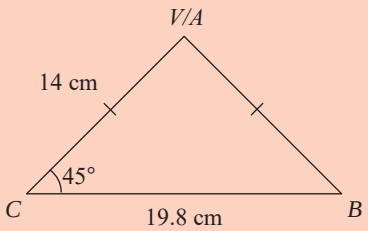


Diagram 2

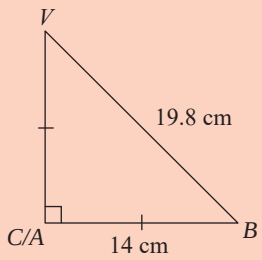
3. Draw an orthogonal projection for the shape that you built on a horizontal plane as viewed from Z and on a vertical plane as viewed from Y.

4. Produce the orthogonal projections on the horizontal plane and the vertical plane as follows:

Projection from direction Z (horizontal plane)



Projection from direction Y (vertical plane)



5. Measure each of the length of sides and angles of the two orthogonal projections you drawn. Complete the table below.

Side	Object	Projection from direction *Z*	Angle	Object	Projection from direction *Z*
AC	14 cm	14 cm	$\angle VCB$	60°	45°
AB			$\angle VBC$		
BC	19.8 cm	19.8 cm	$\angle BAC$	90°	90°
VC	19.8 cm	14 cm	$\angle CAB$		
VB					

Side	Object	Projection from direction *Y*	Angle	Object	Projection from direction *Y*
AV	14 cm	14 cm	$\angle VCB$	60°	90°
AB			$\angle VBC$	60°	45°
BC	19.8 cm	14 cm	$\angle CVB$		
VC			$\angle AVB$	45°	45°
VB	19.8 cm	19.8 cm			

Discussion:

Are all sides and angles of the orthogonal projection of the same size as those of the object? Discuss.

From Brainstorming 2, it is found that:

(a) For orthogonal projections on a horizontal plane from direction *Z*, the lengths of *AC, AB* and *BC*, and the size of $\angle BAC$, $\angle ACB$ and $\angle ABC$ remain unchanged.

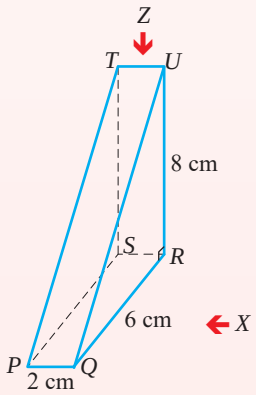
(b) For orthogonal projections on a vertical plane from direction *Y*, the lengths of *AV, AB* and *VB*, and the size of $\angle AVB$ and $\angle ABV$ remain unchanged.

In general,

The **length of sides** and **size of angles** of the **orthogonal projections** of an object can remain unchanged or vary according to the **viewing direction**.

Example 6

The diagram on the right shows a right prism with a rectangular base $PQRS$ which lies on a horizontal plane. The plane URQ is a uniform cross section of the object.

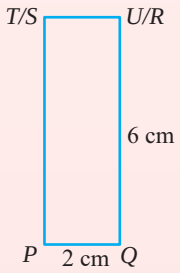


(a) Draw to full scale the orthogonal projection of the prism on
  (i) a horizontal plane as viewed from Z
  (ii) a vertical plane as viewed from X

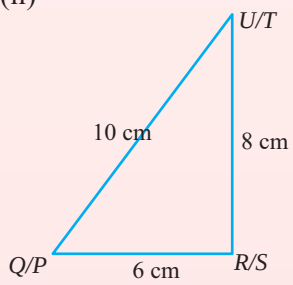
(b) State your conclusion about the length of sides and the size of angles of the object and its orthogonal projections. Explain your conclusions.

Solution:

(a) (i)



(ii)



(b) (i) The length of sides of TU, SR, PQ, PS and QR and the right angle remain unchanged on orthogonal projections as viewed from Z. The length of sides TP and UQ are changed.

(ii) The length of sides of TP, UQ, PS, QR, TS and UR as well as the size of all angles remain unchanged on the orthogonal projection as viewed from X.

MIND TEST 7.1c

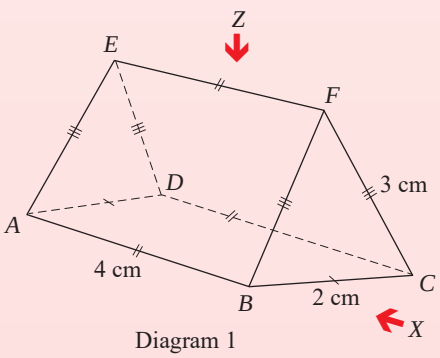


Diagram 1

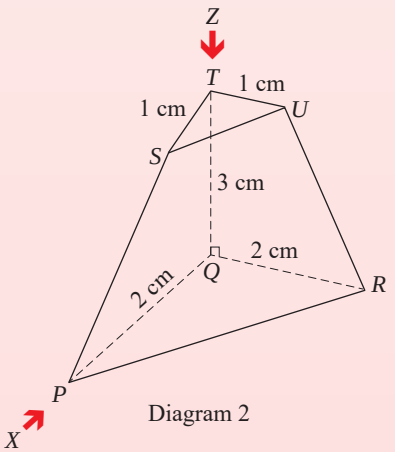


Diagram 2

1. (a) Diagram 1 and Diagram 2 above show two objects placed on a horizontal plane. Draw a full scale orthogonal projection of both objects on a
  (i) horizontal plane as viewed from Z
  (ii) vertical plane as viewed from X

(b) State your conclusion about the length of sides and the size of angles of the objects and their orthogonal projections for Diagram 1 and Diagram 2. Explain your conclusion.

7.2 Plans and Elevations

What are plans and elevations?

LEARNING STANDARD

Draw the plan and elevations of an object to scale.

You have learnt that the orthogonal projection of an object or a solid can be drawn on a horizontal plane and a vertical plane.

The **orthogonal projection** on a **horizontal plane**, which is seen from the top view, is known as a **plan**. The **orthogonal projection** on a **vertical plane**, which is seen from either the side view or the front view, is known as **elevations**. Orthogonal projection drawings give accurate information on the design as well as the size of an object.

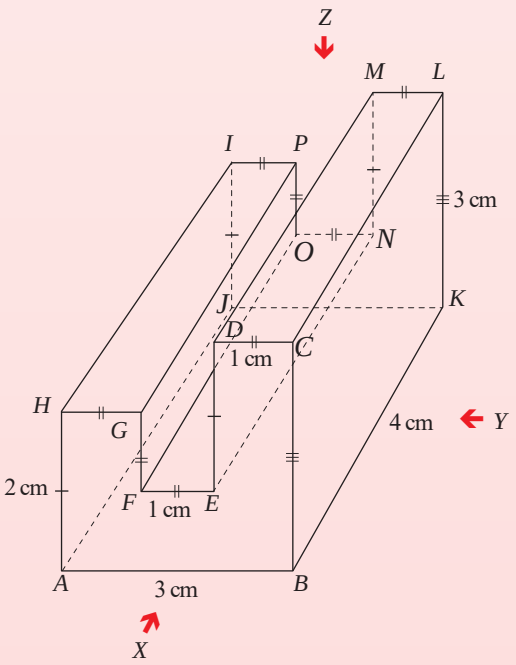
TIPS

Full scale means the actual size.

How do you draw a plan and elevations of an object to scale?

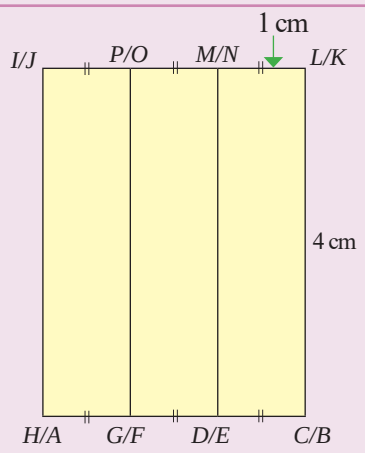
The diagram below shows a right prism with a rectangular base *ABKJ* which lies on a horizontal plane. *ABCDEFGH* is a uniform cross section of the prism. The sides *AH*, *FG*, *ED* and *BC* are vertical. The plan of the right prism can be drawn as viewed from *Z* and the elevations of the object can be drawn as viewed from *X* and *Y*. Plan and elevations should be drawn to full scale.

Right prism (object)



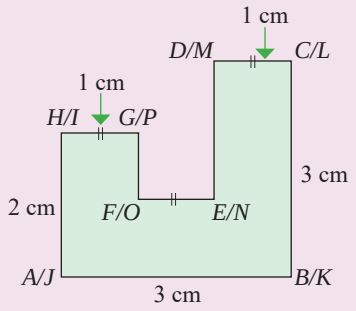
Plan

As viewed from *Z*, which is the view from the top.



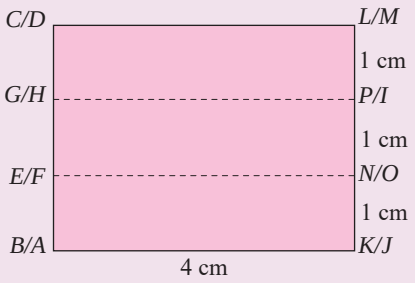
Note:
All sides are drawn with **solid lines** because they are visible from the top.

Front elevation
As viewed from *X*.



Note:
All sides are drawn with **solid lines** because they can be seen when viewed from *X*.

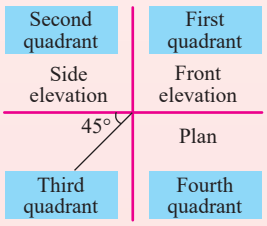
Side elevation
As viewed from *Y*.



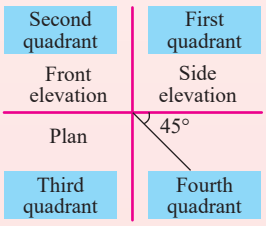
Note:
Lines *GP*, *HI*, *EN* and *FO* are drawn with **dashed lines** because the sides are hidden when viewed from *Y*.

The drawings of a plan, a front elevation and a side elevation of an object can also be combined on a piece of paper which is divided into four quadrants. Here are two commonly used methods.

Method 1



Method 2



The position of the front elevation is at the top of the plan. The side elevation is drawn on the left side or the right side of the front elevation, depending on the viewing direction.

In method 1, the side view is from right to left as in Example 7. Thus, the position of this elevation is on the left side of the front elevation as method 1. In method 2, a side view is from left to right as in example 8. Thus, the position of this elevation is on the right side of the front elevation as method 2.

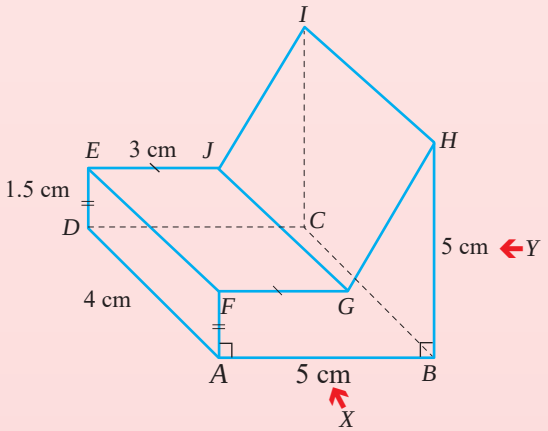
Example 7

The diagram on the right shows a right prism with rectangle *ABCD* that lies on a horizontal plane. *ABHGF* is a uniform cross section of the prism. The sides of *AF* and *BH* are vertical. Draw to full scale,

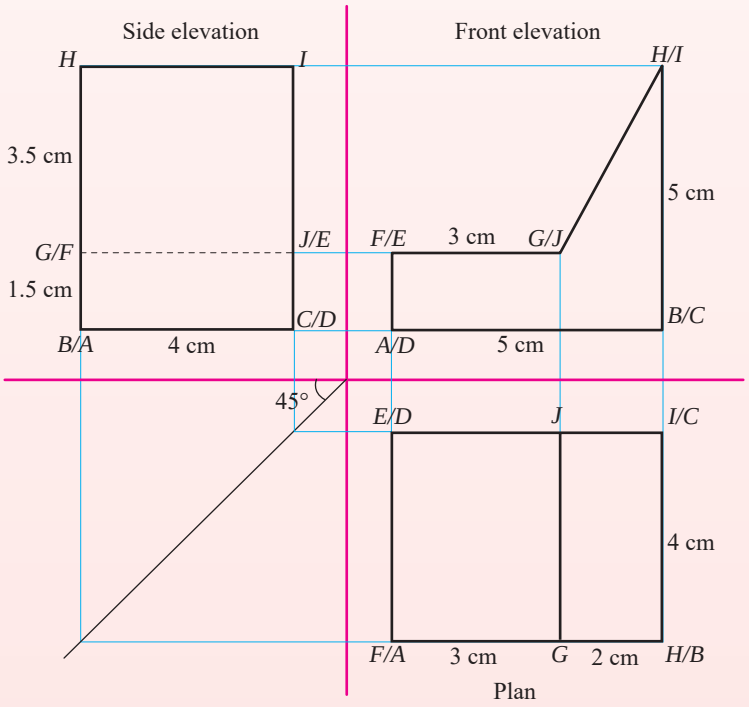
(a) the plan of the prism

(b) the elevation of the prism as viewed from *X*

(c) the elevation of the prism as viewed from *Y*



Solution:



Steps:

1. The direction of the side elevation (direction *Y*) is from right to left, thus the position of the side elevation is in the second quadrant.
2. Draw the plan to full scale in the fourth quadrant.
3. Project sides of the plan with thin solid lines to the first quadrant as a guide to draw the front elevation (direction *X*).
4. Project the sides of the plan and the front elevation to the second quadrant to draw the side elevation.

TIPS

Guide for drawing plan and elevation.

- **Thick** solid lines for visible sides.
- **Dashed** lines for hidden sides.
- **Thin** solid lines for construction lines.

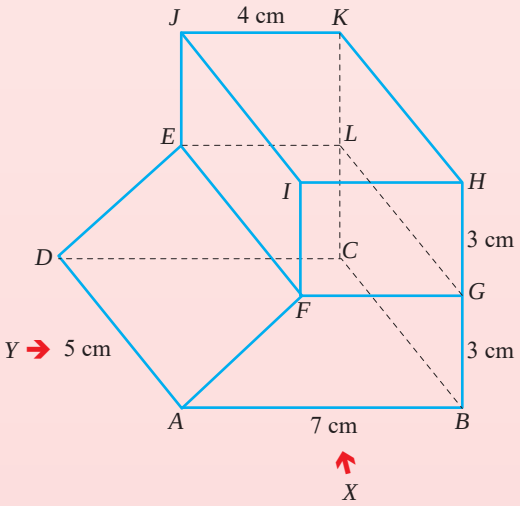
Example 8

The diagram on the right shows a combination of a cuboid and a right prism with rectangle *ABCD* on a horizontal plane. *ABGHIF* is a uniform cross section of the object. *BH* and *FI* are vertical. Draw to full scale,

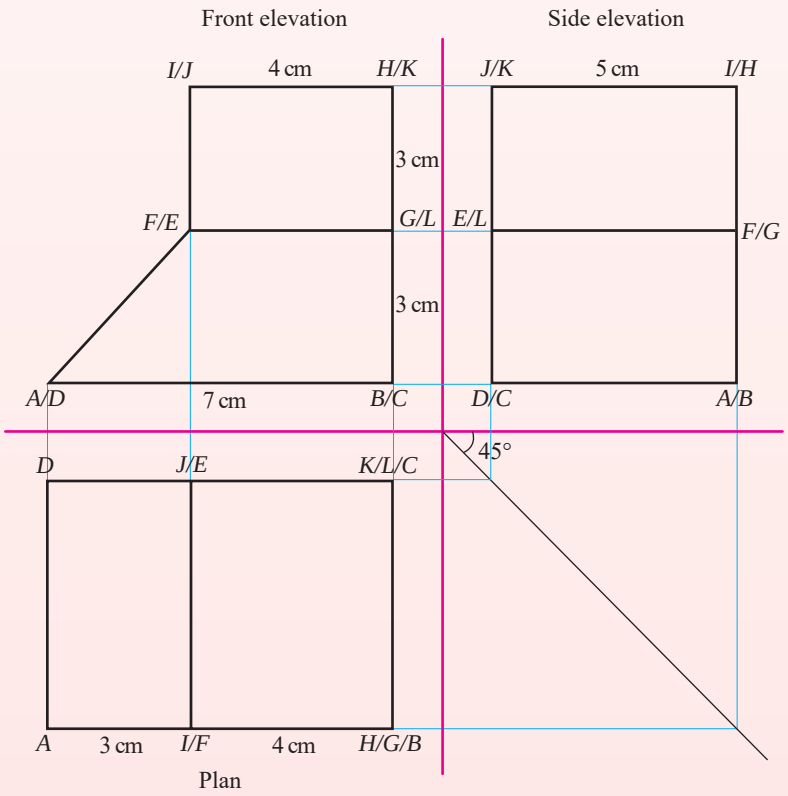
(a) the plan of the object

(b) the elevation of the object as viewed from *X*

(c) the elevation of the object as viewed from *Y*



Solution:



TIPS

The direction of the side elevation (direction *Y*) is from left to right, thus the position of the side elevation is on the first quadrant.

DISCUSSION CORNER

In the subject *Reka Bentuk dan Teknologi (RBT)*, the plan and elevations of an object are drawn with orthographic projection method. Is this method the same as the method you use in this chapter? Discuss.

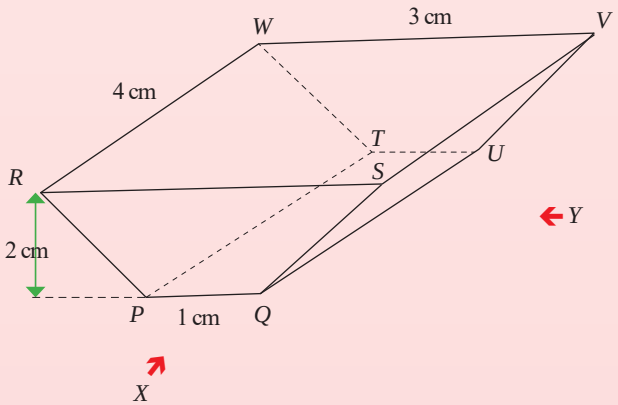
MIND TEST 7.2a

1. The diagram below shows a prism with rectangle *PQUT* on a horizontal plane. *PQSR* is a uniform cross section of the prism. Draw to full scale,

(a) the plan of the prism

(b) the elevation of the prism as viewed from *X*

(c) the elevation of the prism as viewed from *Y*

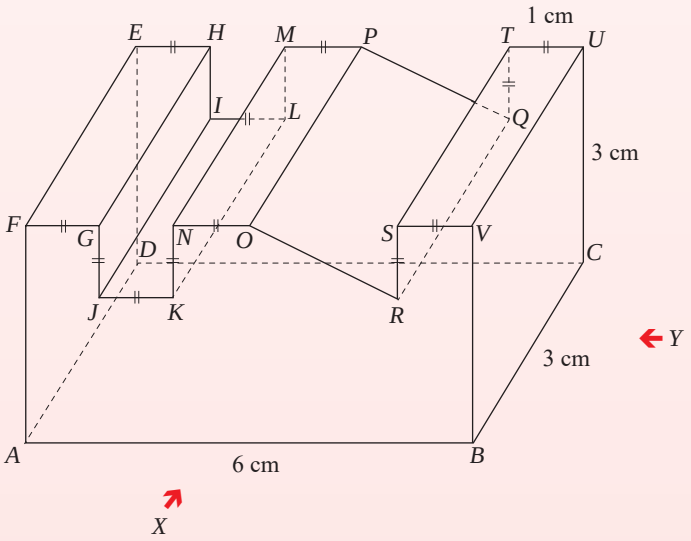


2. The diagram below shows a block where rectangle *ABCD* lies on a horizontal plane. *ABVSRONKJGF* is a uniform cross section of the block. *AF*, *JG*, *KN*, *RS* and *BV* are vertical. Draw to full scale,

(a) the plan of the object

(b) the elevation of the object as viewed from *X*

(c) the elevation of the object as viewed from *Y*

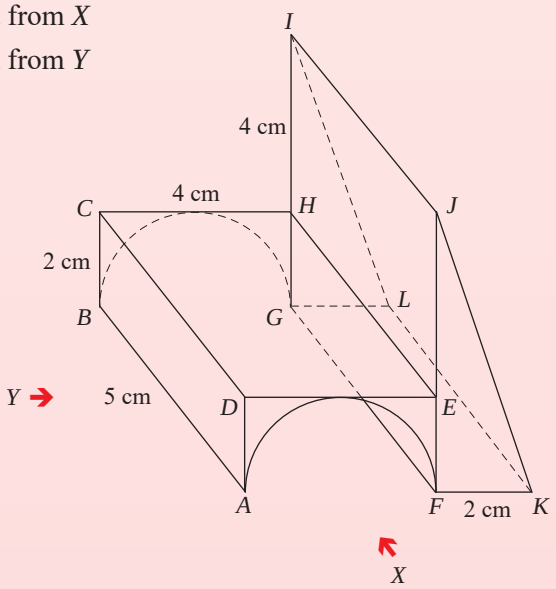


3. The diagram below shows a combination of a cuboid and a right prism placed on a horizontal plane. A semi-cylinder is removed from the cuboid. *ADEJKF* is a uniform cross section of the object. *AD* and *FEJ* are vertical. Draw to full scale,

(a) the plan of the object

(b) the elevation of the object as viewed from *X*

(c) the elevation of the object as viewed from *Y*



How do you synthesise plan and elevations of an object and sketch the object?

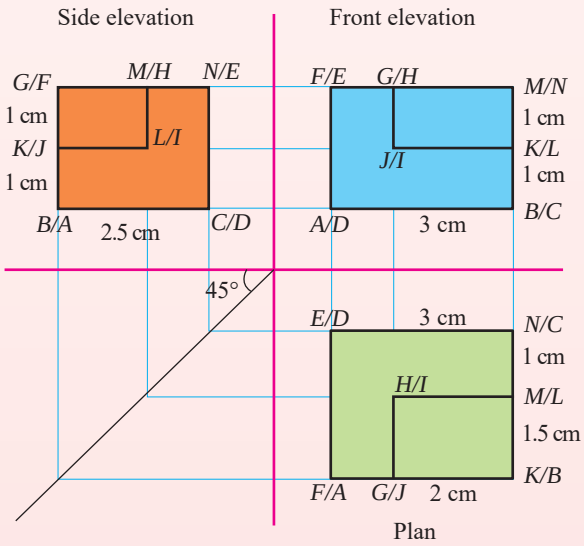
LEARNING STANDARD

Synthesise plan and elevations of an object and sketch the object.

The drawings of plan and elevations on four quadrants are connected to each other and can be used to sketch the three-dimensional shape of an object with ease.

Example 9

The diagram on the right shows the plan, front elevation and side elevation of a right prism with a rectangular base. A cuboid-shaped block has been removed from the prism. Sketch the three-dimensional shape of the prism.

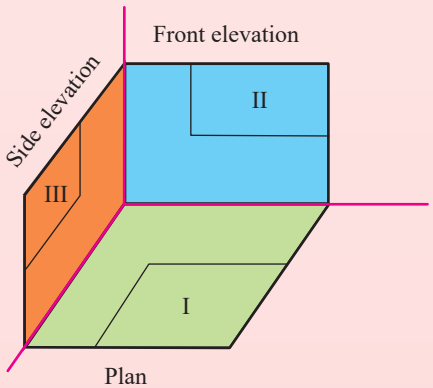


Solution:

The position of the side elevation is on the second quadrant. Thus, the view of the side elevation is from the right.

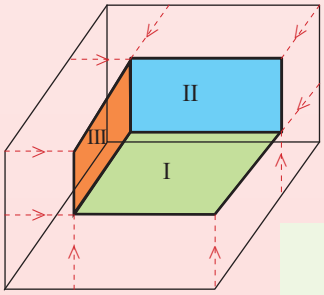
Step 1

Sketch the three orthogonal projections given on the planes using the measurements given. Surfaces marked I, II and III are surfaces of the cuboid block.



Step 2

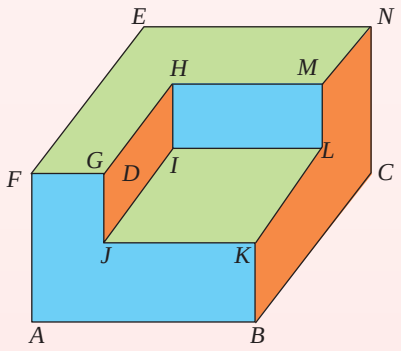
Project the surfaces I, II and III so that they meet as shown in the diagram below.



Scan QR Code or browse http://yakin-pelajar.com/Bab%207%video/ to watch a video about orthographic projection drawings using dynamic software.

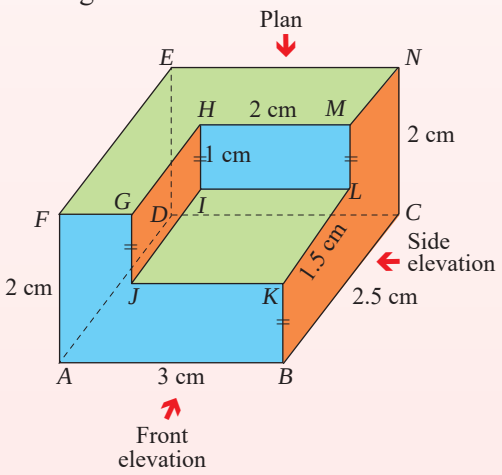
Step 3

Sketch the object and label the vertices with the letters in the orthogonal projections using the colours as the guide.



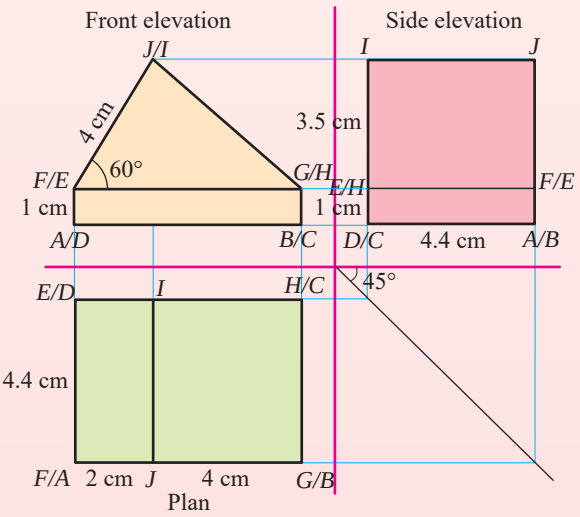
Step 4

Complete the sketched object by labelling the length of sides.



Example 10

The diagram on the right shows the plan, front elevation and side elevation of a combination of a cuboid and a right prism. Sketch the three-dimensional shape of the object.

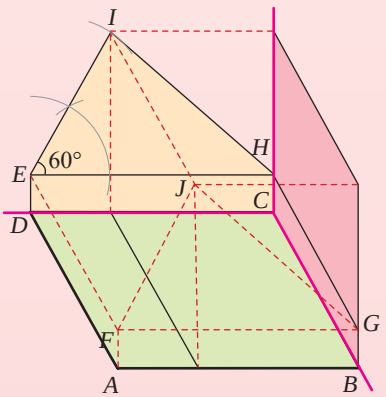


Solution:

The position of side elevation is in the first quadrant. Thus, the view of side elevation is from left to right.

Step 1

Sketch the three orthogonal projections given on the planes using the measurements given. This object contains an angle of 60° on a triangular surface. Thus, the angle of 60° must be built with the correct method.

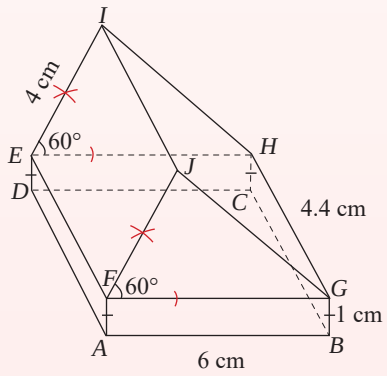


Step 2

Connect the vertices to create a combined object. Label the vertices according to the projections.

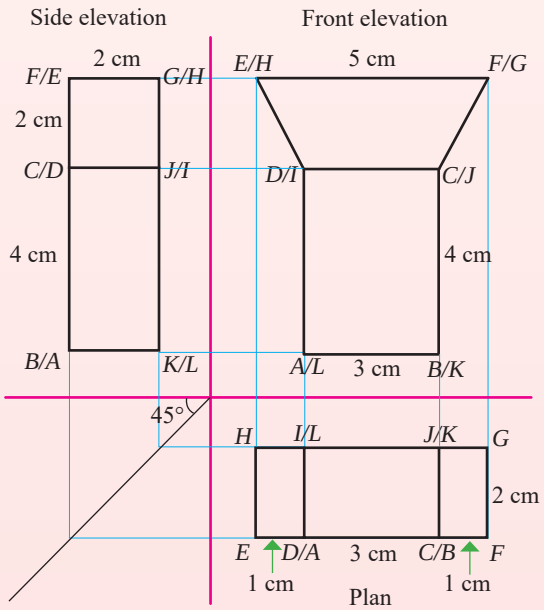
Step 3

Draw the combined object and label the vertices and the length of the sides.

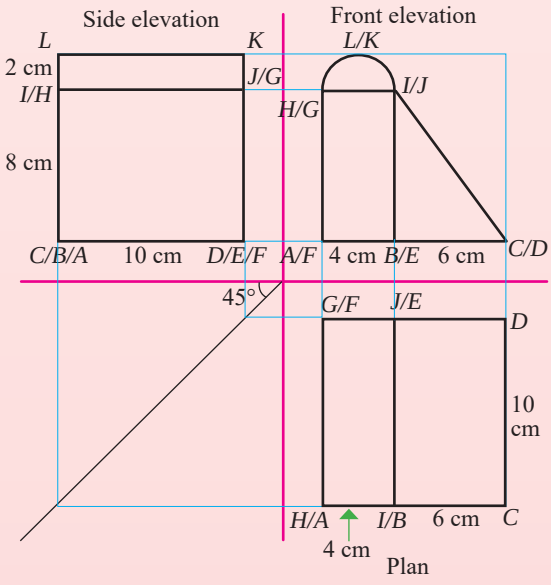


MIND TEST 7.2b

1. The diagram on the right shows the plan, front elevation and side elevation of a combination of a cuboid and a right prism. Sketch the three-dimensional shape of the combined object.



2. The diagram on the right shows the plan, front elevation and side elevation of a combination of a cuboid, a right prism and a semi-cylinder. Sketch the three-dimensional shape of the combined object.



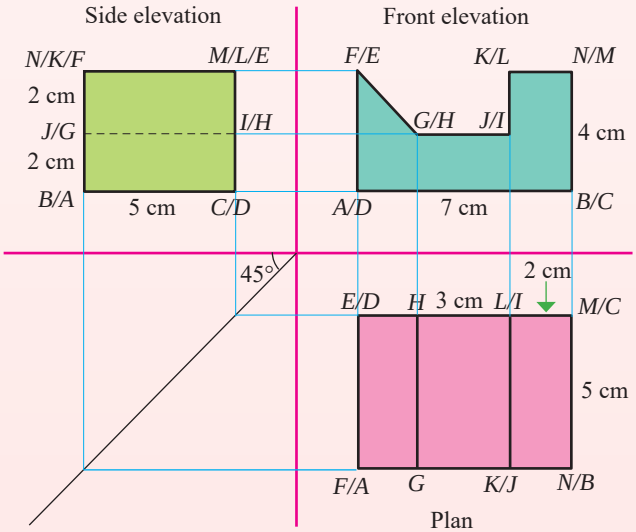
How do you solve problems involving plans and elevations?

LEARNING STANDARD

Solve problems involving plans and elevations.

Example 11

The diagram below shows the plan, front elevation and side elevation of a right prism.



(a) Draw the right prism to full scale.

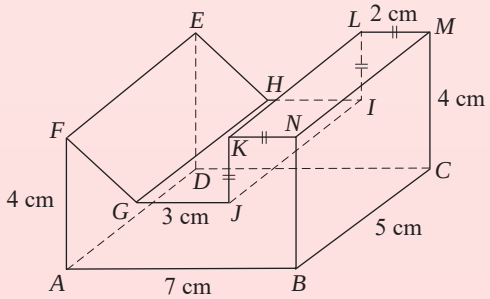
(b) State the length of FG, in cm, correct to one decimal place.

(c) Originally the prism was a cuboid of size 7 cm × 5 cm × 4 cm. Calculate the volume of the right prism $EFGJKLIH$, in cm^3, which was removed from the cuboid.

(d) State the ratio of the volume of the right prism that was removed to the volume of the right prism you drew in question (a).

Solution:

(a)



(b) $FG = 2.8$ cm

(c) The volume of the removed prism

$= \frac{1}{2}(2\text{ cm})(3 + 5)\text{ cm} \times 5\text{ cm}$

$= 40\text{ cm}^3$

(d) The volume of the projected right prism
= the volume of the cuboid – the volume of the prism $EFGJKLIH$
$= (7\text{ cm} \times 5\text{ cm} \times 4\text{ cm}) - 40\text{ cm}^3$
$= 140\text{ cm}^3 - 40\text{ cm}^3$
$= 100\text{ cm}^3$

Thus, the ratio is
40 : 100
2 : 5

MIND TEST 7.2c

1. The diagram below shows a right prism with square *ABCD* on a horizontal plane. *ABNKJGF* is a uniform cross section of the prism.

(a) Draw to full scale,

(i) the plan of the prism.

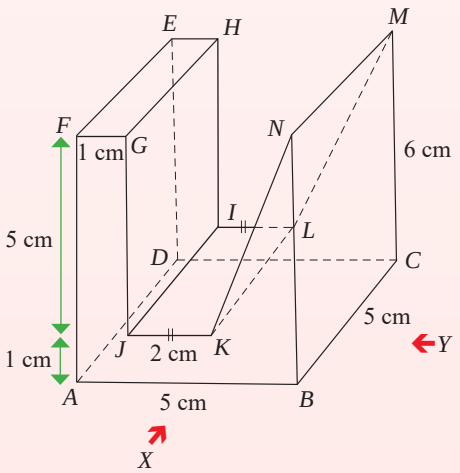
(ii) the elevation on a vertical plane parallel to *AB* as viewed from *X*.

(iii) the elevation on a vertical plane parallel to *BC* as viewed from *Y*.

(b) This right prism was originally a cuboid with a dimension of 5 cm × 5 cm × 6 cm. A right prism *GJKNMLIH* has been removed from the cuboid. Calculate

(i) the volume of the removed prism

(ii) the ratio of the volume of the right prism *GJKNMLIH* to the volume of the remaining right prism



2. The diagram below shows a combination of a right prism and a triangular pyramid on a horizontal plane. *AF* and *BG* are vertical.

(a) Draw to full scale,

(i) the plan of the combined prism

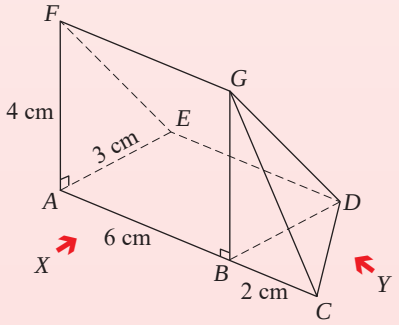
(ii) the elevation on a vertical plane parallel to *AC* as viewed from *X*

(iii) the elevation on a vertical plane parallel to *BD* as viewed from *Y*

(b) Measure the lengths of *CD*, *CG* and *DG* on the plan, elevation as viewed from *X* and elevation as viewed from *Y*.

(c) Use another way to calculate the lengths *CD*, *CG* and *DG* of the original object. Is your answer the same as the answer in question (b)? Explain.

(d) Which orthogonal projections show the actual values of ∠*AEF*, ∠*AFE*, ∠*BCG*, ∠*BGC*, ∠*BCD* and ∠*BDC*?



Test Yourself

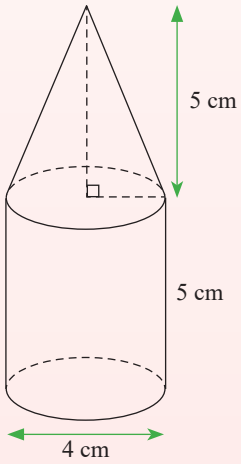
1. The diagram on the right shows a combination of a cylinder and a cone placed on a horizontal table. State whether the following statements are true or false regarding the orthogonal projections of the combined object.

(a) The plan is a circle of diameter 4 cm with a dot in the centre of the circle.

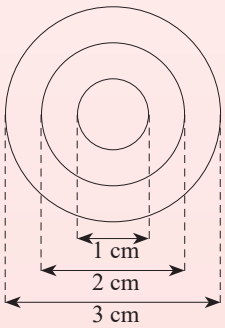
(b) The elevations of the orthogonal projections from all directions are congruent.

(c) The length of the hypotenuse of the cone on the side elevation is less than 5 cm.

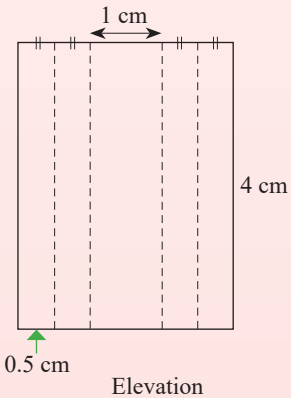
(d) There is no curved surface on the front elevation.



2. The diagram below shows the plan and the elevation of a combined object. Describe the original design of the combined object.



Plan



Elevation

Skills Enhancement

1. The diagram on the right shows a right prism with square *ABHG* placed on a horizontal plane. *ABCD* is a uniform cross section of the prism.

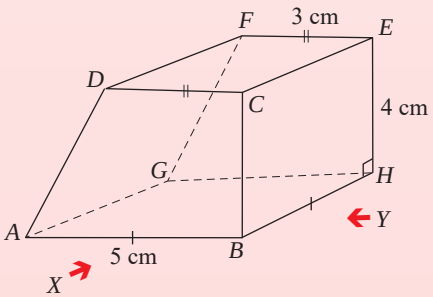
(a) Draw to full scale,

(i) the plan of the prism

(ii) the elevation as viewed from *X*

(iii) the elevation as viewed from *Y*

(b) Measure the length of *AD* and size of $\angle ADC$ on the projection plane which is the uniform cross section of the prism.



2. The diagram on the right shows a combination of a right prism and a cuboid placed on a horizontal plane. *AD*, *FG*, *BC* and *KJ* are vertical.

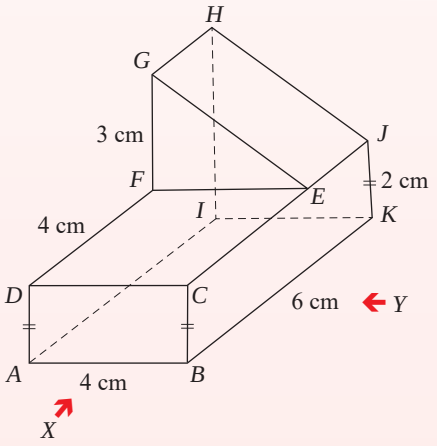
(a) Draw to full scale,

(i) the plan of the object

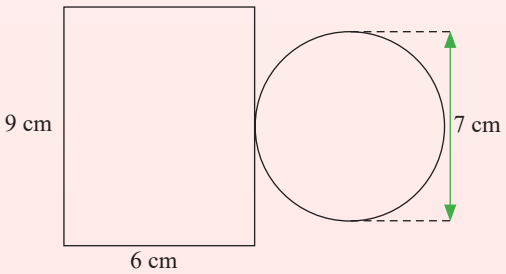
(ii) the elevation as viewed from *X*

(iii) the elevation as viewed from *Y*

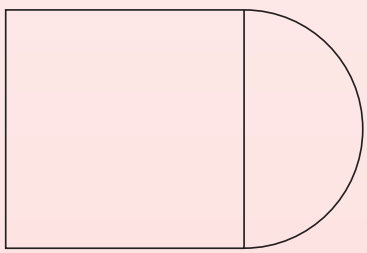
(b) Calculate the volume, in cm^3, of the combined solid.



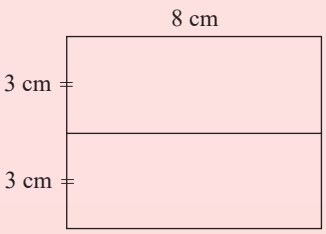
3. The diagram on the right shows the plan of a combination of a cuboid and a right cylinder. If the heights of the cuboid and the cylinder are 5 cm, calculate the volume of the combined solid in cm^3.



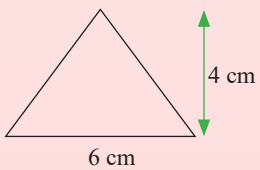
4. The diagram on the right shows the plan of a combination of a cube and a semi-cylinder. Given that the circumference of the plan of the semi-cylinder is 11 cm and the height of the semi-cylinder is equal to the length of the side of the cube, calculate the volume of the combined solid, in cm^3.



5. The diagram below shows the plan and the front elevation of a prism. Calculate the volume of the prism in cm^3.



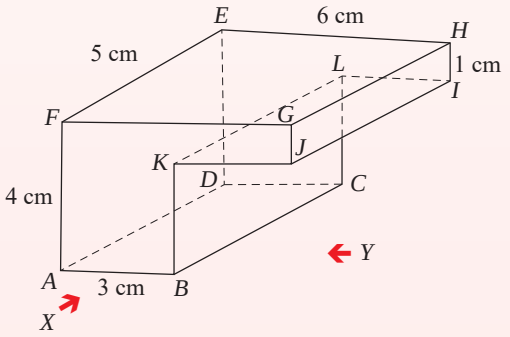
Plan



Front elevation

Self Mastery

1. The diagram on the right shows a right prism with rectangle *ABCD* placed on a horizontal plane. *AF*, *BK* and *JG* are vertical.



(a) Draw to full scale,

(i) the plan of the prism

(ii) the elevation on a vertical plane parallel to *AB* as viewed from *X*

(iii) the elevation on a vertical plane parallel to *BC* as viewed from *Y*

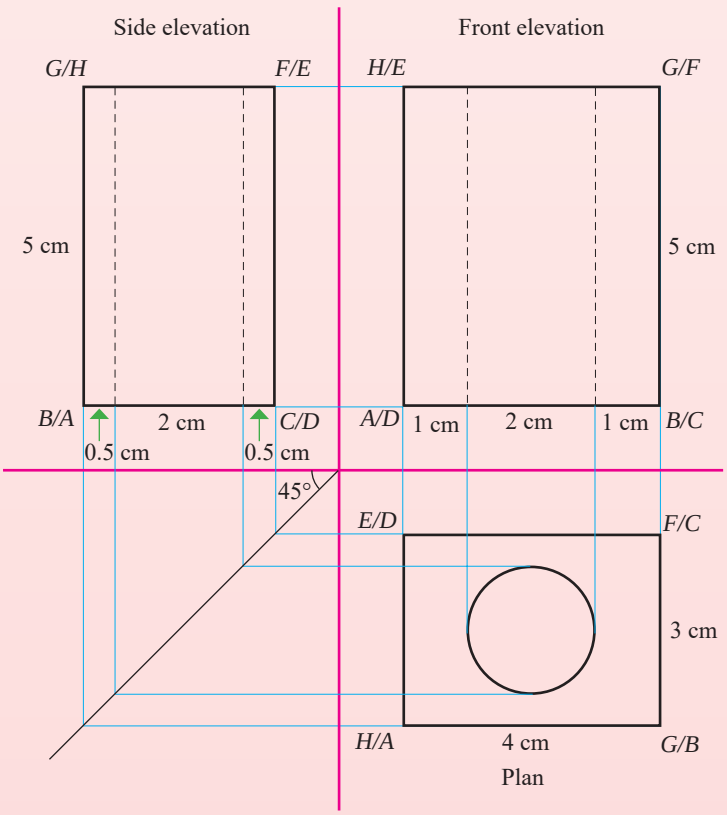
(b) The object should be reinforced so that the base of the object is equal to the shape of the plan. What is the volume of the new object to be added?

(c) If the cost of 1 cm^3 of the new object is RM2.20, calculate the total cost to build the entire combined object.

2. The diagram below shows the plan, front elevation and side elevation of a hollow cuboid. The hollowed section is a right cylinder.

(a) Sketch the three-dimensional shape of the object.

(b) Calculate the volume of the object.



STEM

Aim: To build a study hut.

Instructions:

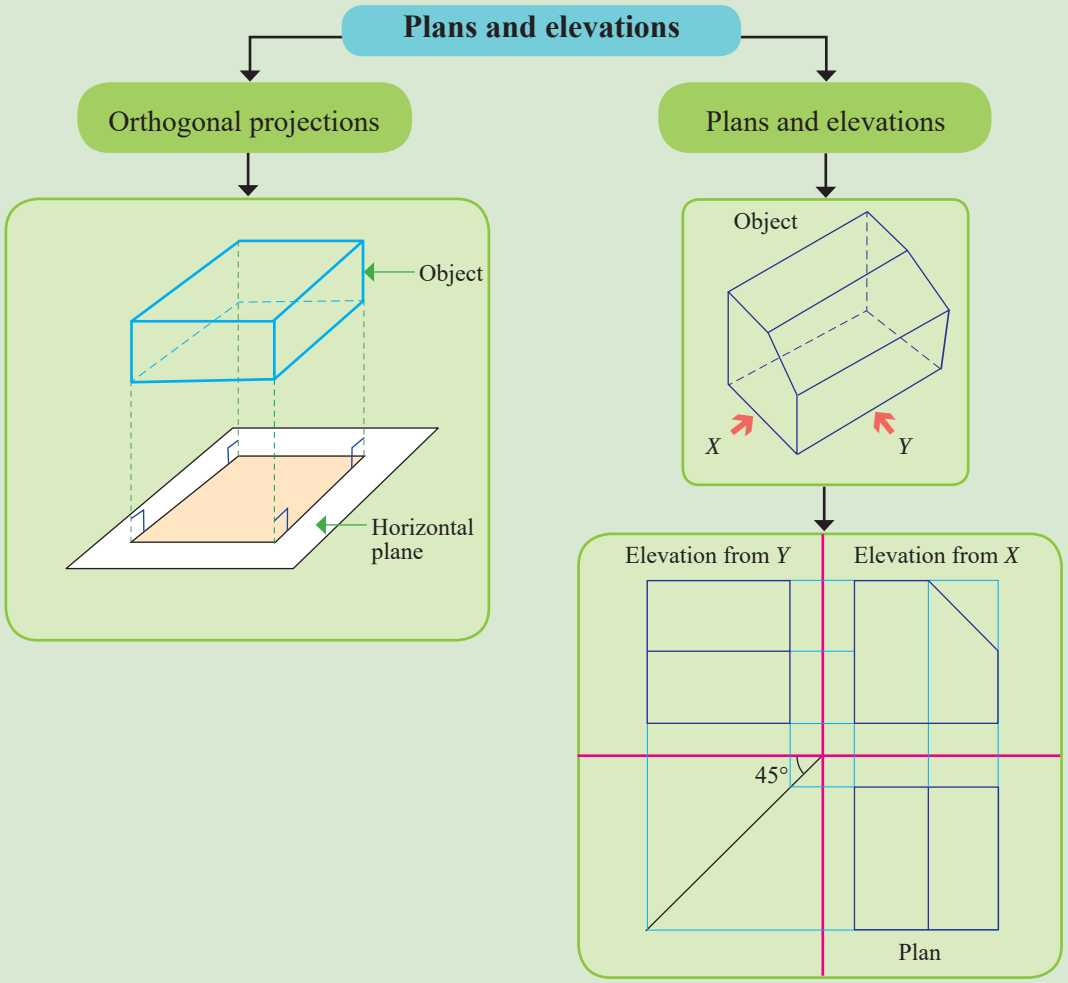
1. Carry out this activity in groups.
2. Propose the construction of a covered study hut in an area of 5 m × 5 m.
3. Your proposal should consider the following criteria:
   (a) Maximum use of sunlight during the day.
   (b) Good air circulation.
   (c) Eco-friendly and conducive.
   (d) Minimum construction cost.
4. Prepare a report using multimedia applications.

PROJECT

My dream house

1. Draw your dream house with an appropriate scale using scale drawing.
2. Draw the plan, front elevation and side elevation of the house.
3. Build a model of your dream house based on the scale drawing, the plan and the elevations drawn.
4. Decide the building materials required from various sources based on the size of the house that you want to build.
5. Calculate the estimated cost to build your dream house.
6. Show your model house and present your project.

CONCEPT MAP



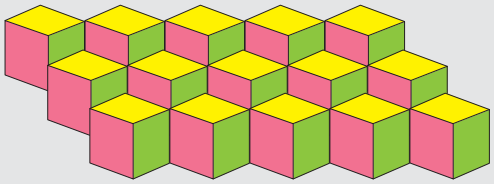
SELF-REFLECT

At the end of this chapter, I can:

1.	Draw orthogonal projections.		
2.	Compare and contrast between objects and the corresponding orthogonal projections.		
3.	Draw the plan and elevations of an object to scale.		
4.	Synthesise the plan and elevations of an object and sketch the object.		
5.	Solve problems involving plans and elevations.		

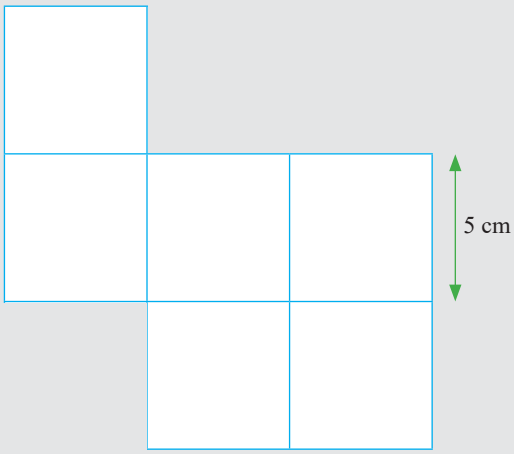
EXPLORING MATHEMATICS

1. Prepare 15 pieces of cubes with the side of 5 cm as shown in the diagram. You can also use the Rubik's cube.



2. Use the cubes to form a combined object according to your creativity.

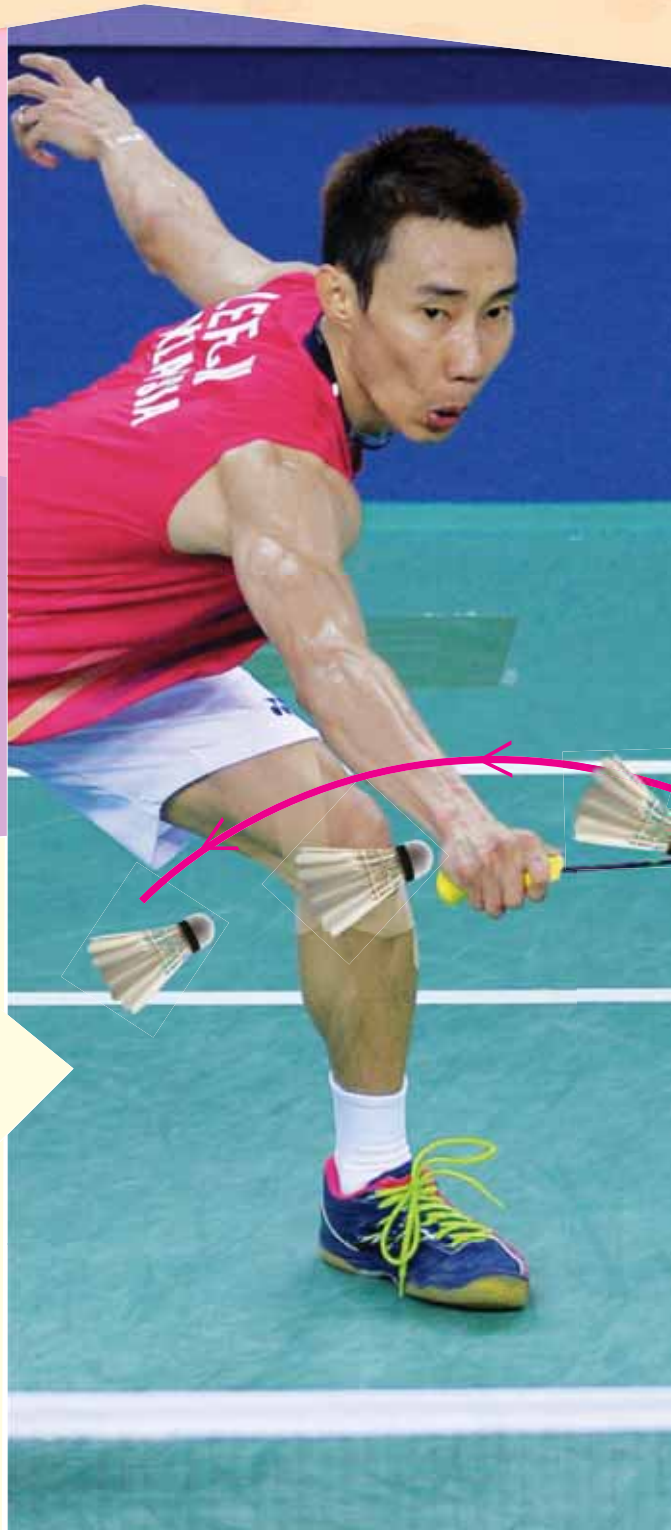
3. Draw the plan and elevations of the combined object that you have created.



4. The group that builds the most creative combined object is the winner.

CHAPTER 8

Loci in Two Dimensions



What will you learn?

8.1 Loci

8.2 Loci in Two Dimensions

Why do you learn this chapter?

- Knowledge about loci allows one to estimate or predict the path of the moving points based on certain conditions.
- The concept of loci is used in construction, engineering drawings, aviation, satellite movements and so on.

National badminton champion Datuk Lee Chong Wei currently holds the record for the fastest smash since September 2015 when he did the shot with a speed of 408 kilometres per hour (km/h). He won the 2015 Hong Kong Open Badminton Championships which was held at the Hung Hom Coliseum. According to Badminton World Federation (BWF), the speed of the shot made by Chong Wei was recorded and measured using Hawk Eye technology that has been adopted in several major tournaments since September 2015. Do you know that the movement of a shuttlecock follows certain conditions?

Exploring Era

Apollonius (260 – 190 BC) was an ancient Greek mathematician who was very interested in studying problems of loci. He had conducted research on various forms of loci such as the straight lines and certain curves. However, the most outstanding Greek mathematician in studying loci was Pappus (290 AD – 350 AD). Pappus's loci materials are still being researched by mathematicians today.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter8.pdf

WORD BANK

• equidistant	• *berjarak sama*
• circle	• *bulatan*
• arc	• *lengkok*
• curve	• *lengkung*
• locus	• *lokus*
• loci	• *lokus-lokus*
• perpendicular bisector	• *pembahagi dua sama serenjang*
• angle bisector	• *pembahagi dua sudut*

8.1 Locus

What do you understand about loci?

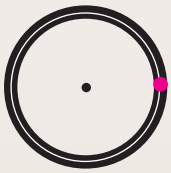
LEARNING STANDARD

Recognise loci in real life situations and hence explain the meaning of locus.

In the picture on the right, a piece of coloured sticker is pasted on the tyre of a bicycle.

What is the shape generated by the sticker when the bicycle is pedalled?

The shape formed by the sticker is a circle as shown in the diagram on the right. Does this shape comply with certain conditions?



The picture below shows a ball being kicked by a football player. The movement of a point on the ball yields a curve.

BULLETIN

Malaysia's football fans will always remember 'Super Mokh', the late Datuk Mokhtar Dahari, who fired a solid 40-metre shot against England Squad 3 in 1978.

The picture on the right shows a rocket being launched. The movement of a point on the rocket will produce a straight line.

A locus is a trace or trajectory formed by a set of points in a plane or three-dimensional space that satisfies certain conditions.

Brainstorming 1

In groups

Aim: To identify two-dimensional loci in daily life situations.

Materials: Situation cards.

Steps:

1. Each group is given several situation cards that show activities involving movements in daily activities as shown below.

Situation A

Throwing a ball into the net.

Situation B

A durian falling from a tree.

Situation C

An airplane landing.

Situation D

The moving tip of the wiper on the windshield.

2. Discuss in the group and sketch the locus of a point on the object involved in the given situations.
3. Present the loci sketch and compare your answers with other groups.

Discussion:
Discuss five other movements in daily activities that can be categorised as loci.

From Brainstorming 1, it is found that:

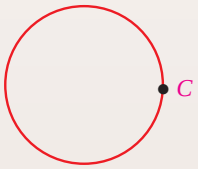
The shapes of two-dimensional loci can be seen in the form of straight lines, arcs and curves.

Example 1

Point C is drawn on a blade of a revolving fan as shown in the diagram. Elaborate and sketch the locus of point C.

Solution:

This locus is a circle.



MIND TEST 8.1a

1. Explain and sketch the locus of point C on each object in the following diagrams.

(a) A ball centred at C rolling along an inclined plane.

(b) Point C on a swinging pendulum.

(c) Point C on a spinning yo-yo.

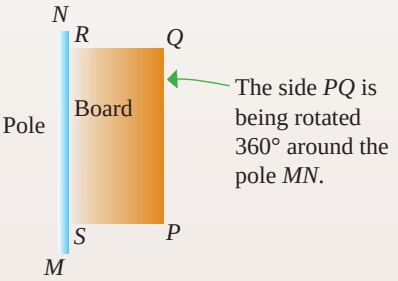
(d) Point C on the shoe of a child who is playing on a slide at the playground.

2. State and sketch the locus of a point on

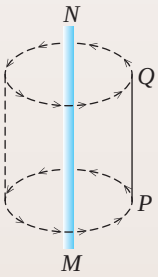
(a) a coconut falling from a tree
(b) a moving car on a straight road
(c) a leaping frog

Example 2

The diagram on the right shows a pole *MN*. A rectangular board *PQRS* is attached to the pole where *PQRS* is movable. If the side *PQ* is rotated 360° around *MN*, what is the three-dimensional shape formed?



Solution:



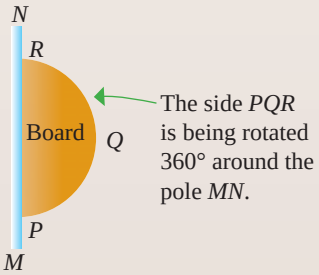
The shape formed when the side *PQ* is rotated 360° around pole *MN* is a right **cylinder**.

FLASHBACK

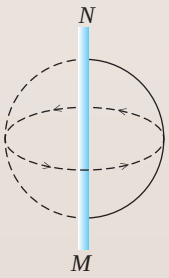
Cylinders, spheres, cones, prisms and pyramids are examples of **three-dimensional shapes**.

Example 3

The diagram on the right shows a pole *MN*. A semicircular board *PQR* is attached to the pole where *PQR* is movable. If *PQR* is rotated 360° around *MN*, what is the three-dimensional shape formed?



Solution:



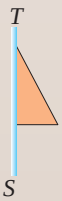
The shape formed when the semicircular board is rotated 360° around pole *MN* is a **sphere**.

BULLETIN

This three-dimensional shape is known as a frustum.

MIND TEST 8.1b

1. Sketch the three-dimensional loci when the two-dimensional shaded shapes are rotated 360° around pole *ST*.

(a) 

(b) 

(c) 

(d) 

8.2 Loci in Two Dimensions

What is the locus of points that are of constant distance from a fixed point?

Brainstorming 2

In pairs

LEARNING STANDARD

Describe the locus of points that are of constant distance from a fixed point.

Aim: To determine the locus of points that are of constant distance from a fixed point.

Materials: Blank paper, a pencil and a ruler.

Steps:

1. Mark a fixed point O on a sheet of paper (Diagram 1).
2. Measure 5 cm from the point O and mark ×.
3. Repeat step 2 as many times as possible (Diagram 2).

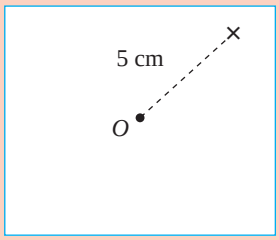


Diagram 1

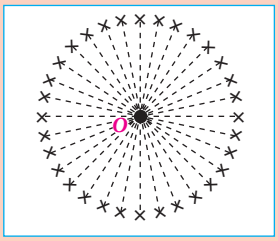


Diagram 2

4. Note the location of the points marked with × (Diagram 2).
5. Repeat steps 1 to 3 with different distances from the fixed point O.
6. Are the resulting geometric shapes the same as the shape obtained in step 4? Explain.

Discussion:

What is the geometric shape generated by the location of the dots ×? Explain.

From Brainstorming 2, it is found that:

Points marked at the same distance from a fixed point O forms a circle.

In general,

The locus of a point that is equidistant from a **fixed point** is a **circle centred at that fixed point**.

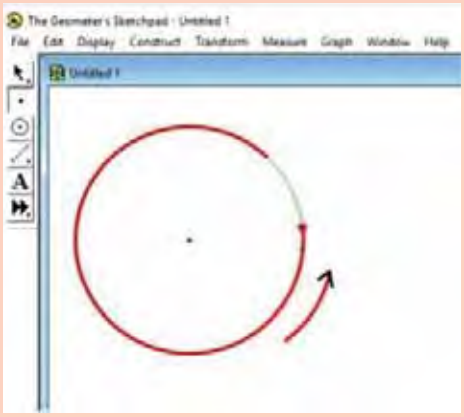
Brainstorming 3

In pairs

Aim: To construct locus of points that are of constant distance from a fixed point.

Materials: Dynamic software

Steps:



1. Start with *New Sketch*.
2. Select *Compass Tool* and draw a circle.
3. Select *Point Tool* and mark.
4. Open *Display* menu and select *Trace Point* followed by *Animate Point*.
5. Observe the animation of the movement generated.

Discussion:

What is the geometric shape generated from the movement of the marked point?

From Brainstorming 3, it is found that:

A point that always moves at the same distance from a fixed point forms a circle.

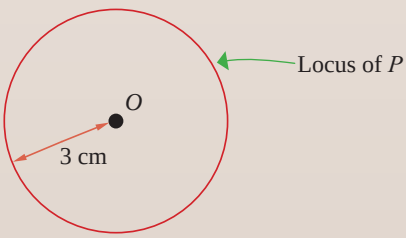
How do you construct a locus of points that are of constant distance from a fixed point?

Example 4

Construct a locus of point P which is always 3 cm from a fixed point O.

Solution:

1. Mark point O.
2. Set the gap of the compasses at 3 cm.
3. Construct a circle of radius 3 cm centred at the point O.



What is the locus of points that are equidistant from two fixed points?

Brainstorming 4

In pairs

LEARNING STANDARD

Describe the locus of points that are of equidistant from two fixed points.

Aim: To determine the locus of points that are equidistant from two fixed points.

Materials: Plain paper, a compasses, a ruler and a pencil.

Steps:

1. Mark two fixed points P and Q which are 8 cm apart (Diagram 1).
2. Using the compasses, mark the intersection, 4.5 cm from point P and point Q (Diagram 2).
3. Repeat step 2 with distances more than 4.5 cm from point P and point Q (Diagram 3).

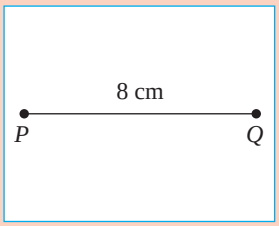


Diagram 1

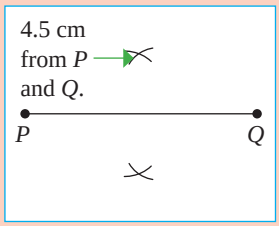


Diagram 2

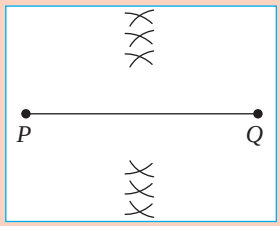


Diagram 3

4. Note the location of the intersecting marks in Diagram 3.
5. Repeat steps 1 to 3 with different distances between point P and point Q.
   Are your answers the same as the answer in step 4?

Discussion:

What is the shape formed by the location of the intersecting marks? Explain.

Scan the QR code or visit http://bukutekskssm.my/Mathematics/F3/Chapter8LocusfromTwoFixedPoint.mp4 to watch a video that describes the locus of the points that are equidistant from two fixed points.

From Brainstorming 4, it is found that:

The location of the intersecting marks that are equidistant from fixed points P and Q form a straight line through the midpoint of PQ.

In general,

The locus of a point that is equidistant from **two fixed points** is the **perpendicular bisector** of the line connecting the two fixed points.

Brainstorming 5

In pairs

Aim: To construct locus of points that are equidistant from two fixed points.

Materials: Dynamic software

Steps:

1. Start with *New Sketch*.
2. Select *Straightedge Tool* to draw a line segment. Select *Text Tool* to label point *A* and point *B*.
3. Select *Construct* menu to construct the midpoint of the line segment.
4. Mark both lines and midpoint segments with *Selection Arrow Tool*.
5. Select *Construct* menu to construct a perpendicular line (Diagram 1).

Discussion:

What is the geometric shape formed? Explain.

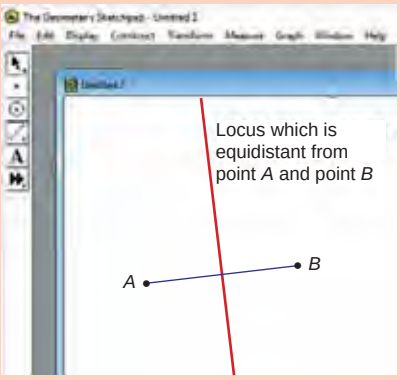
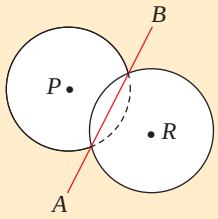


Diagram 1

FLASHBACK

The line *AB* is known as a bisector.



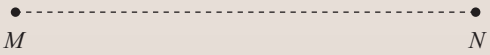
From Brainstorming 5, it is found that:

The **locus that is equidistant** from two fixed points *A* and *B* is **a straight line perpendicular** to the straight line *AB* and it passes through the midpoint of *AB*.

How do you construct the locus of points that are equidistant from two fixed points?

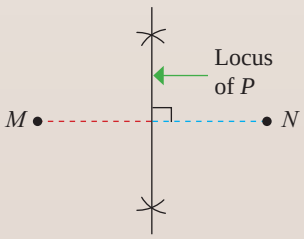
Example 5

Construct the locus of point *P* that is equidistant from two fixed points *M* and *N*.



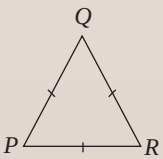
Solution:

1. Mark two small arcs using a pair of compasses with the gap set at more than half of the length of *MN* from the point *M*.
2. With the compasses set at the same gap, mark the intersecting arcs of point *N*.
3. Connect the two points of intersection with a straight line.



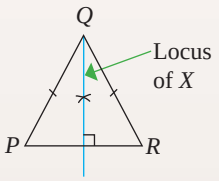
Example 6

The diagram on the right shows an equilateral triangle *PQR*. Determine the locus of point *X* that is equidistant from point *P* and point *R*.



Solution:

Locus of point X that is equidistant from point P and point R is the perpendicular bisector of the line connecting point P and point R.



What is the locus of points that are of constant distance from a straight line?

Brainstorming 6

In pairs

LEARNING STANDARD

Describe the locus of points that are of constant distance from a straight line.

Aim: To determine the locus of points that are of constant distance from a straight line.

Materials: Square grid paper, a ruler, a pencil.

Steps:

1. Draw a straight line MN (Diagram 1).
2. Mark a point ×, which is 3 units from the line MN (Diagram 2).

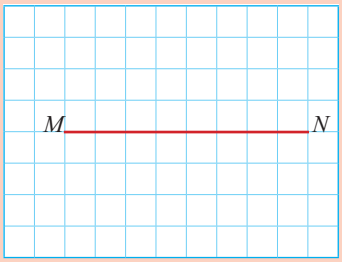


Diagram 1

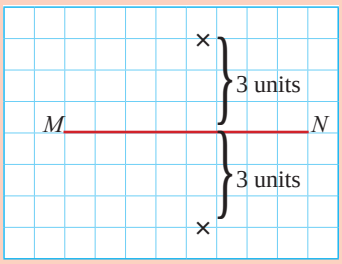


Diagram 2

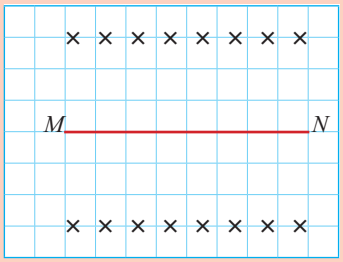


Diagram 3

3. Repeat step 2 with as many × points as possible (Diagram 3).
4. Note the location of the × points in Diagram 3. What do you think about the location of the × points?
5. Repeat steps 1 through 4 with a different unit distance.
6. Repeat steps 1 through 4 with the straight line MN drawn vertically.

Discussion:

What is your conclusion about the location of the points marked equidistantly from the straight line?

From Brainstorming 6, it is found that:

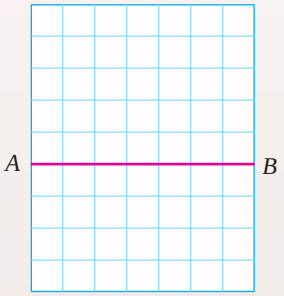
The locus of points that are equidistant from the line MN is a pair of straight lines parallel to MN.

In general,

The locus of points that are of constant distance **from a straight line** are **straight lines parallel** to that straight line.

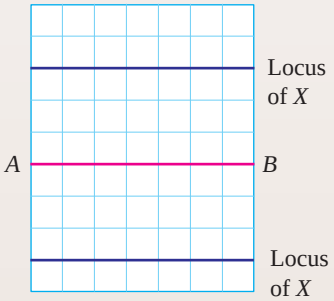
Example 7

The diagram on the right shows a line AB drawn on a square grid with sides of 1 unit. On the diagram, draw the locus of the point X which always moves at 3 units from the line AB.



Solution:

The locus of point X moving 3 units from the line AB is a pair of lines parallel to AB and 3 units from AB.



What is the locus of points that are equidistant from two parallel lines?

Brainstorming 7

In groups

LEARNING STANDARD

Describe the locus of points that are equidistant from two parallel lines.

Aim: To determine the locus of points that are equidistant from two parallel lines.

Materials: Plain paper, compasses, a ruler and a pencil.

Steps:

1. Draw two straight lines PQ and MN that are parallel (Diagram 1).
2. Using compasses, mark the point of intersection from point P and point M.
3. Repeat steps 2 for point Q and point N (Diagram 2).
4. Connect all the intersection points marked by drawing a straight line (Diagram 3).

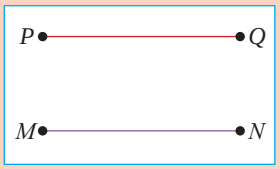


Diagram 1

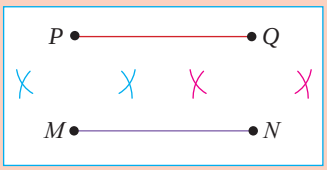


Diagram 2

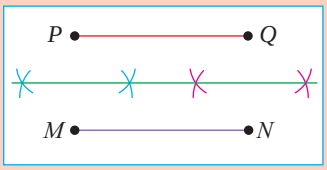


Diagram 3

5. Describe the nature of the straight line that connects all the points of intersection (Diagram 3).

Discussion:

1. Repeat steps 1 to 4 by drawing two vertical straight lines and two inclined straight lines. Ensure that each pair of lines is parallel.
2. Do you get the same result as in step 4?

From Brainstorming 7, it is found that:

(a) The locus of points that are equidistant from two parallel lines PQ and MN is a straight line.

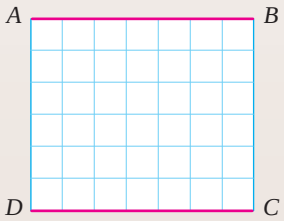
(b) The locus is parallel to the straight lines PQ and MN and it passes through the midpoints of the lines PQ and MN.

In general,

The locus of points that are **equidistant from two parallel lines** is **a straight line parallel to and passes through the midpoints** of the pair of parallel lines.

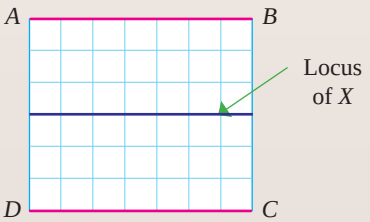
Example 8

The diagram on the right shows the rectangle, $ABCD$ drawn on a square grid with sides of 1 unit. Describe and draw the locus of X which is equidistant from the lines AB and DC.



Solution:

The locus of point X that is equidistant from the line AB and DC is a line parallel to AB and DC and is 3 units from the lines AB and DC.



What is the locus of points that are equidistant from two intersecting lines?

Brainstorming 8

In groups

LEARNING STANDARD

Describe the locus of points that are of equidistant from two intersecting lines.

Aim: To determine the locus of points that are equidistant from two intersecting lines.

Materials: A square grid paper, a ruler, a pencil and a protractor.

Steps:

1. Draw x-axis and y-axis on a Cartesian plane on the grid paper (Diagram 1).

2. Mark the coordinates of equal value pairs. For example, (0, 0), (–2, –2), (4, 4) and so on (Diagram 2).

3. Connect all the points with a straight line. Measure $\angle a$, $\angle b$, $\angle c$ and $\angle d$ using a protractor (Diagram 3).

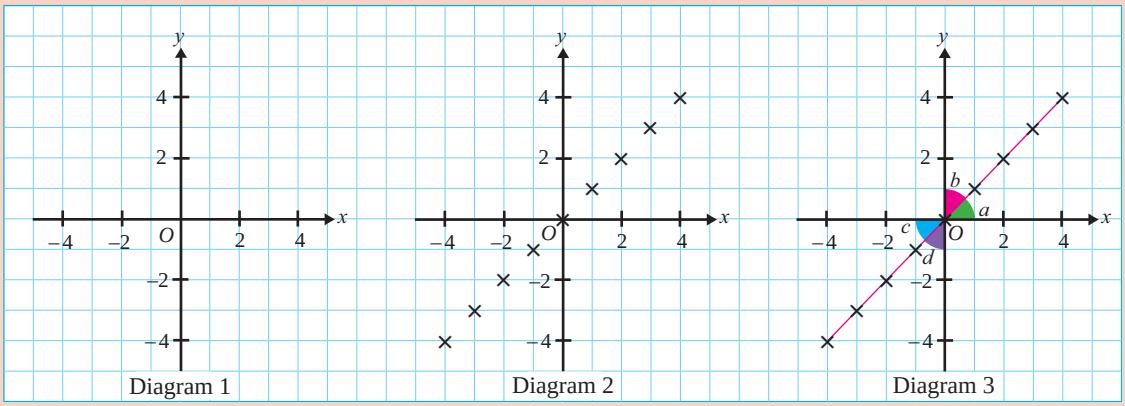


Diagram 1 Diagram 2 Diagram 3

Discussion:

1. What is your conclusion about the values of $\angle a$, $\angle b$, $\angle c$ and $\angle d$ which are the angles formed at the intersection of the x-axis and y-axis?
2. What is the relationship between the straight line that connects equal value pairs of coordinates to the values of $\angle a$, $\angle b$, $\angle c$ and $\angle d$?

From Brainstorming 8, it is found that:

(a) $\angle a = \angle b = \angle c = \angle d = 45°$.

(b) The straight line that connects equal value pairs of coordinates bisects the angle of intersection between the x-axis and y-axis.

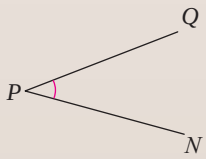
In general,

The locus of points that are equidistant from **two intersecting lines** is the **angle bisector** of the angles formed by the intersecting lines.

How do you construct a locus of points that are equidistant from two intersecting lines?

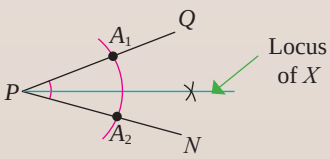
Example 9

Construct the locus of point X that is equidistant from two straight lines PQ and PN intersecting at P.



Solution:

1. By using a pair of compasses, draw an arc from the point P which cuts through the straight lines PQ and PN.
2. Mark the points of intersection between the arc and the straight lines PQ and PN as A_1 and A_2 respectively.
3. Construct intersecting mark from A_1 and A_2.
4. Draw a straight line joining the intersecting mark constructed in step 3 and the point P.



Brainstorming 9

In pairs

Aim: To construct locus of a point that is equidistant from two intersecting straight lines.

Materials: Dynamic software

Steps:

1. Start with *New Sketch*.
2. Select *Straightedge Tool* to draw lines *AB* and *BC* intersecting at point *B*.
3. Use *Text Tool* to label point *A*, followed by point *B* and then point *C* (point of intersection must be marked on the second turn).
4. Mark all three points *A*, *B* and *C* with *Selection Arrow Tool*. (Diagram 1)
5. Select the *Construct* menu to construct the bisector of the angle (*Angle bisector*) between the two intersecting lines. (Diagram 2)

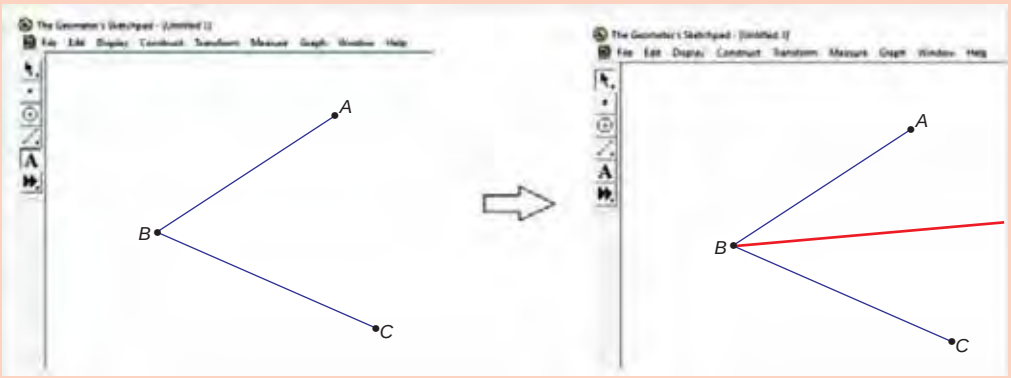


Diagram 1 Diagram 2

Discussion:

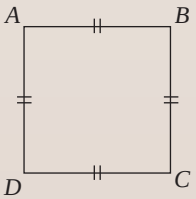
What is your conclusion about the locus of points that are equidistant from two intersecting lines?

From Brainstorming 9, it is found that:

The locus of a point that is equidistant from the two straight lines *AB* and *BC* intersecting at the point *B* is a straight line that bisects $\angle ABC$.

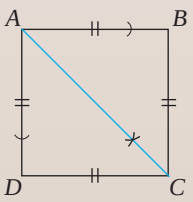
Example 10

The diagram on the right shows a square *ABCD*. Describe and draw the locus of a point which moves at the same distance from the straight lines *AB* and *AD*.



Solution:

The locus of a point which moves at the same distance from the line *AB* and *AD* is a straight line which bisects the angle *BAD*.



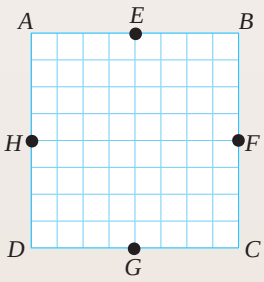
MIND TEST 8.2a

1. The diagram shows a straight line PQ of 5 cm.

(a) X is a point that is always 3 cm from point P. Describe the locus of point X completely.

(b) Y is a point that is always 4 cm from point Q. Describe the locus of point Y completely.

2. The diagram on the right shows a square $ABCD$ drawn on a square grid with sides of 1 unit. P, Q, R, S and T are five points that move in the square $ABCD$. Using the letters in the diagram, state the locus of point



(a) P moving equidistantly from points A and D

(b) Q moving equidistantly from points B and D

(c) R moving such that it is always 4 units from the line BC

(d) S moving equidistantly from the straight lines AB and BC

(e) T moving such that it is always 4 units from the line EG

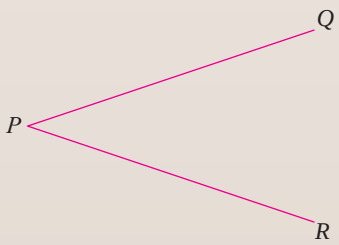
3. The diagram on the right shows the straight line, CD which is 6 cm long. T is a point that is always 1.5 cm from the straight line, CD.

(a) Draw the locus of point T.

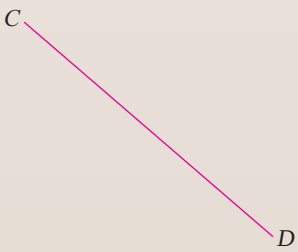
(b) Describe completely, the locus of point T.

4. Construct the locus of point Y for a given situation.

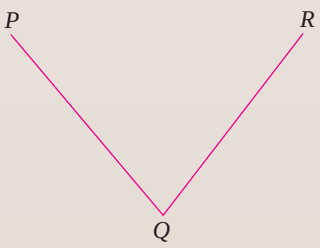
(a) It is equidistant from the straight lines PQ and PR.



(b) $YC = YD$



(c) $\angle PQY = \angle RQY$



5. The picture on the right shows a running track. An athlete always practises by running two lanes away from lane 4 of the track. Draw the locus of the athlete's run.

How do you determine the locus that satisfies two or more conditions?

The intersection of two or more loci can be determined by constructing each specified locus in the same diagram.

LEARNING STANDARD

Determine the locus that satisfies two or more conditions.

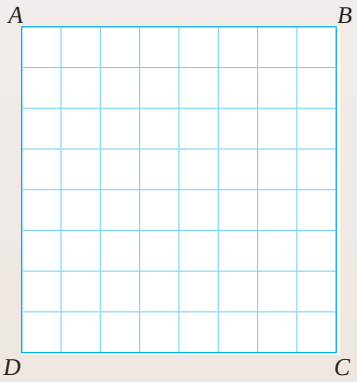
Example 11

The grid on the right shows a square *ABCD* drawn on a square grid with sides of 1 unit. Points *X* and *Y* are two points that move inside the square *ABCD*. On the grid,

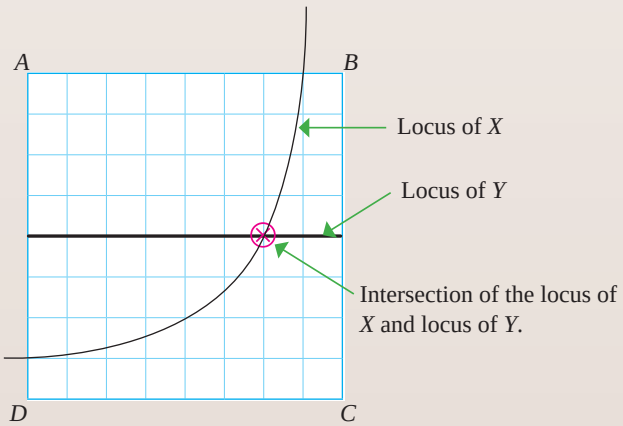
(a) draw the locus of a moving point *X* which is constantly 7 units from *A*

(b) draw the locus of a moving point *Y* which is equidistant from the lines *AB* and *CD*

(c) mark all points of intersection of locus of *X* and locus of *Y* with the symbol ⊗



Solution:



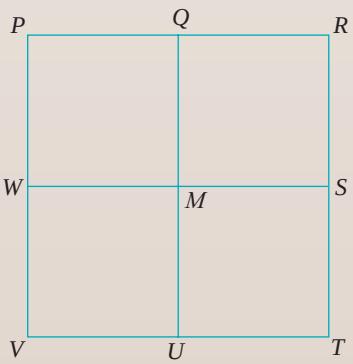
Example 12

The diagram on the right shows four combined squares with sides of 2 cm. *X* and *Y* are two moving points inside the square *PRTV*. On the diagram,

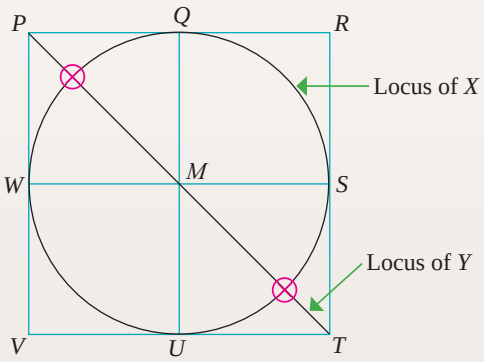
(a) draw the locus of a moving point *X* which is always 2 cm from point *M*

(b) draw the locus of a moving point *Y* which is equidistant from line *PR* and line *PV*

(c) mark all points of intersection of locus of *X* and locus of *Y* with the symbol ⊗



Solution:



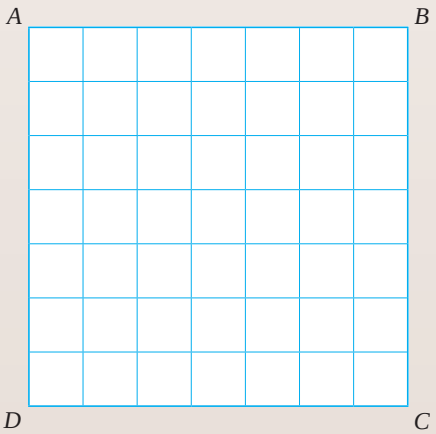
MIND TEST 8.2b

1. In the grid on the right, the rectangle *ABCD* represents a part of a lake. *ABCD* is drawn on a square grid with sides of 1 unit. Points *V* and *W* represent the trips of boat *V* and boat *W*. On the grid,

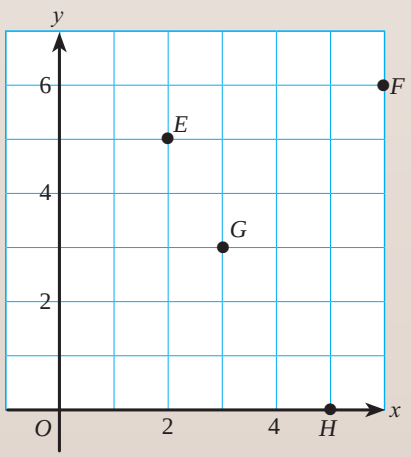
(a) draw the locus of boat *V* which always moves 5 units from point *D*

(b) draw the locus of boat *W* which is 3 units from line *BC*

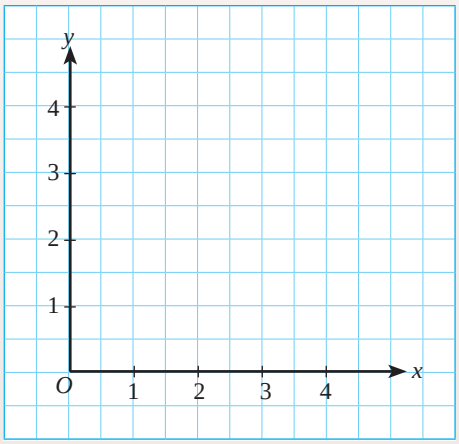
(c) mark the intersection of the paths of boat *V* and boat *W* with the symbol ⊗



2. The diagram on the right shows the Cartesian plane marked with four points *E*, *F*, *G* and *H*. Faruk is at the same distance from *x*-axis and *y*-axis. Faruk's location is also less than 5 units from the centre of *O*. Which of the points *E*, *F*, *G* and *H* is Faruk's location?



3. The diagram on the right shows the Cartesian plane. Point F always moves 3 units from the x-axis while point G always moves 4 units from the origin. Mark all the points of intersection between the locus of F and the locus of G with the symbol ⊗.



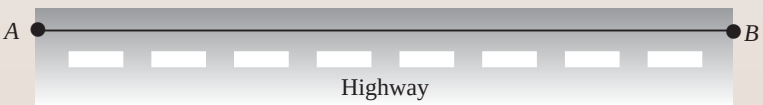
How do you solve problems involving loci?

Example 13

A clinic will be built in a village. The clinic should be equidistant from house P and house Q, as well as 600 metres away from the highway AB. Determine the possible location of the clinic.
(scale 1 cm = 600 metres)

LEARNING STANDARD

Solve problems involving loci.



Solution:

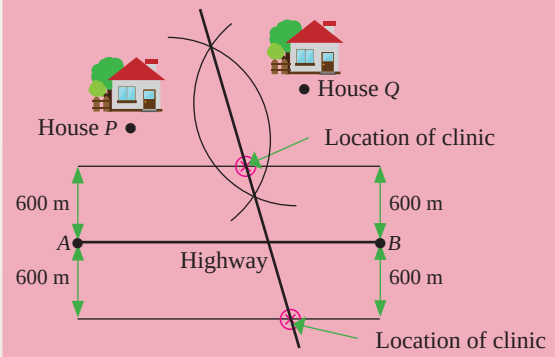
Understanding the problem

The clinic is equidistant from P and Q. Therefore the locus is the bisector of the straight line connecting points P and Q. The clinic is 600 metres from the highway AB. There are two lines parallel to the highway AB.

Planning a strategy

To draw using a pair of compasses and a ruler.

Implementing the strategy



Making a conclusion

Two locations that are marked with the symbol ⊗ satisfy the requirements of the location to build the clinic.

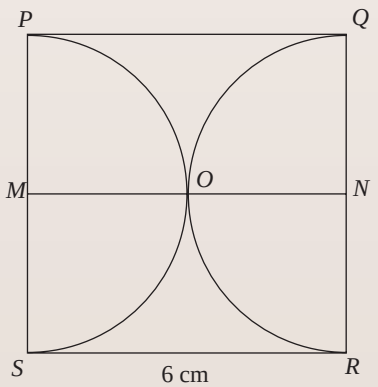
MIND TEST 8.2c

1. The diagram on the right shows square *PQRS* with sides of 6 cm. Two semicircles with centres *M* and *N* are drawn inside square *PQRS*. *M* and *N* are the midpoints of *PS* and *QR*. On the diagram, shade the region that satisfies the following conditions.

(a) The locus of point *X* which moves such that $XM \leqslant 3$ cm and more than 3 cm from line *SR*.

(b) The locus of *Y* which moves such that $YM \geqslant 3$ cm and $YN \geqslant 3$cm.

(c) Describe the intersection between locus of *X* and locus of *Y*.



2. The diagram on the right shows a rectangular fenced-up grass field *PQRS* measuring 6 m × 8 m. A goat is tied at point *Q* with a 7-metre long rope.

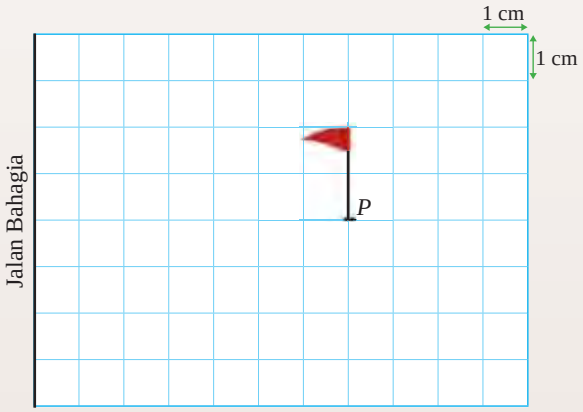
Shade the region that is reachable by the goat.

3. Khalid draws the plan for a treasure hunt on a square grid with a scale of 1 cm to 1 metre.
On the diagram, draw

(a) the location of the treasure if it is 3 m away from the flagpole *P*

(b) the location of the treasure if it is 5 m from Jalan Bahagia

Then, mark the possible locations of the treasure with the symbol ⊗.



Dynamic Challenge

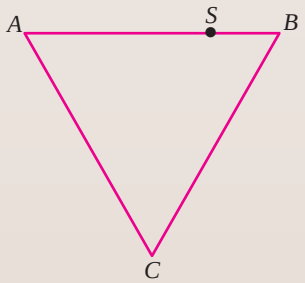
Test Yourself

1. The diagram below shows an equilateral triangle *ABC*. *S* is a point on line *AB*. *X* and *Y* are two moving points in the diagram. On the diagram,

(a) draw the locus of point *X* such that *AX* = *AS*

(b) draw the locus of point *Y* such that *Y* is equidistant from *AC* and *BC*

(c) mark all the intersection points for locus of *X* and locus of *Y* with the symbol ⊗

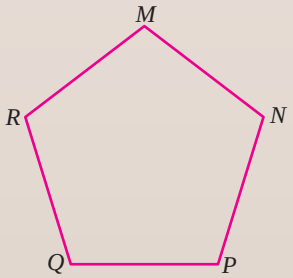


2. The diagram below shows a regular pentagon *MNPQR*. *X* and *Y* are two moving points inside the pentagon. On the diagram,

(a) draw the locus of point *X* such that *RX* = *XN*

(b) draw the locus of point *Y* such that *RY* = *RQ*

(c) mark all the intersection points for locus of *X* and locus of *Y* with the symbol ⊗

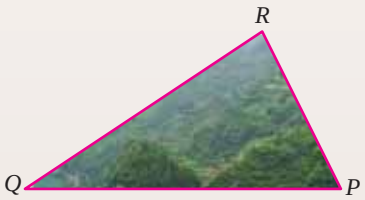


3. The picture below shows the triangular forest area *PQR*. Point *X* and point *Y* are two loci that describe the location of a helicopter that crashed. On the diagram,

(a) draw the locus of point *X* such that it is equidistant from lines *QR* and *QP*

(b) draw the locus of point *Y* such that *YP* = *PR*

(c) mark the possible location for the helicopter with the symbol ⊗



Skills Enhancement

1. The diagram on the right is drawn on a square grid with sides of 1 unit. Point *X*, point *Y* and point *Z* are three points which move in the square.

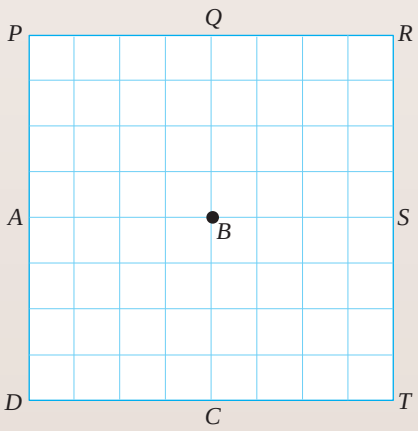
(a) *X* is a point which moves such that it is equidistant from points *Q* and *C*. Using the letters in the diagram, state the locus of point *X*.

(b) On the diagram,

(i) draw the locus of point *Y* which moves such that it is equidistant from the straight lines *PD* and *DT*

(ii) draw the locus of point *Z* which moves such that it is always 5 units from point *S*

(c) Mark the location of all the intersection points for locus of *Y* and locus of *Z* with the symbol ⊗.

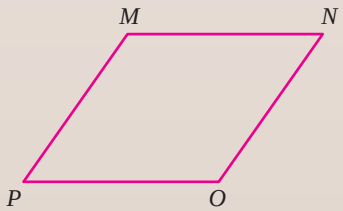


2. The diagram below shows a rhombus *MNOP*. Point *X* and point *Y* are two points that move within the rhombus. On the diagram,

(a) draw the locus of point *X* which moves equidistantly from the straight lines *PM* and *PO*

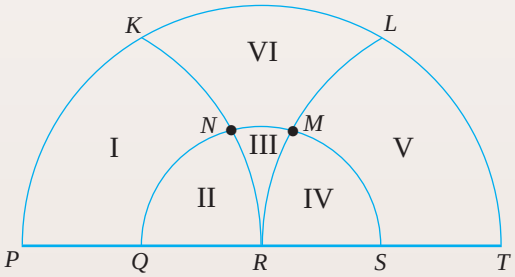
(b) draw the locus of point *Y* which moves such that *YP* = *PO*

(c) mark the location of all the intersection points for locus of *X* and locus of *Y* with the symbol ⊗



Self Mastery

1. The diagram below shows two semicircles, *PKLT* and *QNMS* centred at *R*, with diameters of 8 cm and 4 cm respectively. *KNR* and *RML* are arcs of circles centred at *P* and *T* respectively.



Based on the diagram above, state

(a) the point which is 2 cm from *R* and 4 cm from *P*

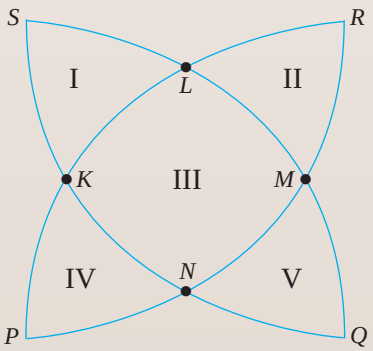
(b) the point which is more than 2 cm from *R* and 4 cm from *T*

(c) the location of a moving point *X* in the diagram such that it is less than 4 cm from *P* and more than 2 cm from *R*

(d) the location of a moving point *Y* in the diagram such that $YR < 2$ cm and $YP < 4$ cm.

(e) the location of a moving point *Z* in the diagram such that $ZT > 4$ cm, $ZP > 4$ cm and $ZR > 2$ cm

2. In the diagram below, *SLMQ*, *PKLR*, *QNKS* and *RMNP* are arcs of circles with radii of 4 cm and centred at *P*, *Q*, *R* and *S* respectively.



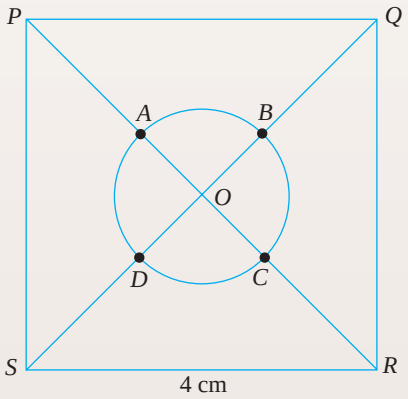
Based on the diagram above, state

(a) the location of a moving point *X* in the diagram such that $XS < 4$ cm, $XP < 4$ cm and $XQ > 4$ cm

(b) the location of a moving point *Y* in the diagram such that $YR > YP$

(c) the location of a moving point *Z* in the diagram such that $ZP < 4$ cm, $ZQ < 4$ cm, $ZR < 4$ cm and $ZS < 4$ cm

3. The diagram below shows a square *PQRS* with sides of 4 cm and a circle centred at *O* with radius of 1 cm. Point *X* and point *Y* are two points that always move inside the square *PQRS*.



Describe the possible movement of the loci of point *X* and point *Y* for the following points of intersection:

(a) *B* and *D*

(b) *A* and *C*

PROJECT

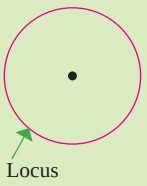
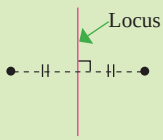
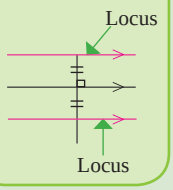
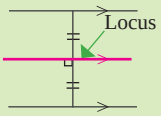
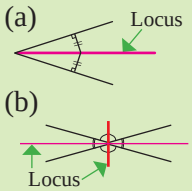
When we look at a clock to tell the time, we can see that the tip of the hour hand always moves with the same pattern, that is always equidistant from the centre of the clock.

Why is the shape of a circle selected to represent the movement of time on a clock? Gather information about the relationship between hours, minutes and seconds and the shape of a circle. Create a report with illustrations using multimedia applications.

CONCEPT MAP

Loci in Two Dimensions

A set of points whose locations satisfy certain conditions.

The locus of points that are of constant distance from a fixed point.	The locus of points that are equidistant from two fixed points.	The locus of points that are of constant distance from a straight line.	The locus of points that are equidistant from two parallel lines.	The locus of points that are equidistant from two intersecting lines.
Circle 	Perpendicular bisector 	A pair of parallel lines 	A straight line 	Angle bisector 

SELF-REFLECT

At the end of this chapter, I can:

1.	Recognise loci in real life situations and hence explain the meaning of locus.		
2.	Describe the locus of points that are of constant distance from a fixed point.		
3.	Describe the locus of points that are equidistant from two fixed points.		
4.	Describe the locus of points that are of constant distance from a straight line.		
5.	Describe the locus of points that are equidistant from two parallel lines.		
6.	Describe the locus of points that are equidistant from two intersecting lines.		
7.	Determine the locus that satisfies two or more conditions.		
8.	Solve problems involving loci.		

EXPLORING MATHEMATICS

We can sketch an ellipse using the following steps:

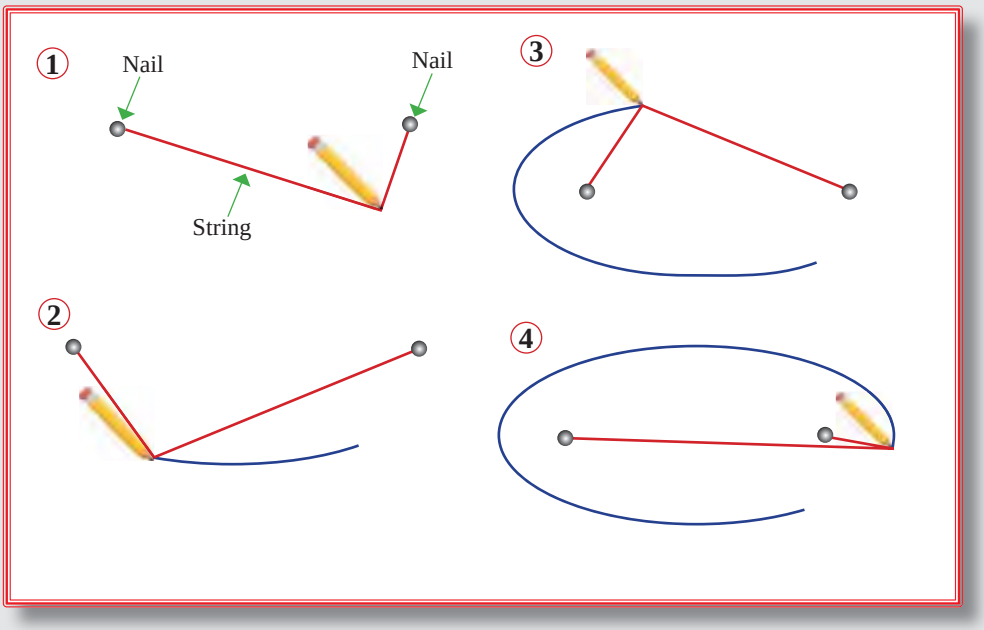
1. Tie two nails with a string (one nail on each end of the string).

2. Place a sheet of paper on a flat piece of board.

3. Fix the two nails onto the piece of board but do not pull the string too tightly. The two nails are called the foci.

4. We can begin to sketch an elliptical shape by using the tip of a pencil to pull the string tightly and draw on the paper, first from one nail to the second nail, and then from the second nail back to the first nail. The curve drawn by the pencil is an ellipse.

5. Observe the elliptical shape formed.



SMART MIND

Why is an ellipse also known as a locus?

CHAPTER 9 Straight Lines

What will you learn?

9.1 Straight Lines

Why do you learn this chapter?

- The concept of straight lines is widely used in the construction of various geometric shapes such as squares, triangles and kites.
- The concept of straight lines is used in engineering, architecture, construction, mapping, sciences, sports and so on.

Normally, every building is built vertically. Some buildings such as the Leaning Tower of Teluk Intan which was built in 1885, became inclined due to the soil structure. Although inclined and over 100 years old, the Leaning Tower of Teluk Intan is still standing strong and is a landmark of Teluk Intan. The leaning tower was declared a national heritage in 2015.

Exploring Era

Euclid was a Greek mathematician. He had conducted a lot of research about straight lines and geometry such that he was known as the founder of geometry.

The field of geometry was named Euclidean Geometry to commemorate Euclid's contributions to the fundamental principles of geometry.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter9.pdf

WORD BANK

• straight line	• *garis lurus*
• parallel line	• *garis selari*
• vertical distance	• *jarak mencancang*
• horizontal distance	• *jarak mengufuk*
• gradient	• *kecerunan*
• axis	• *paksi*
• intercept	• *pintasan*
• simultaneous equation	• *persamaan serentak*
• intersection point	• *titik persilangan*

9.1 Straight Lines

What is the equation of a straight line?

LEARNING STANDARD

Make connection between the equation, $y = mx + c$, and the gradient and y-intercept, and hence make generalisation about the equation of a straight line.

In Form 2, you have learnt how to draw the graph of linear function and non-linear functions by constructing a table of values of related functions.

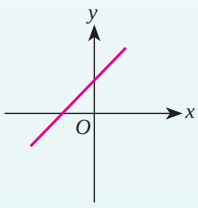


Diagram 1

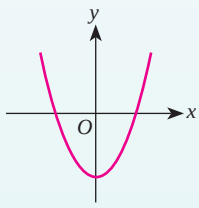


Diagram 2

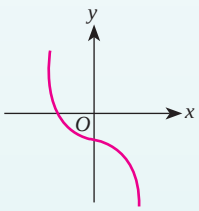


Diagram 3

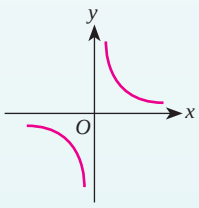


Diagram 4

Each of the above graph is drawn based on a specific function. The function is also an equation for the related graph.

Can you differentiate a graph of linear function and a graph of non-linear function? Discuss.

FLASHBACK

The gradient, m, of a straight line that connects two points (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

or

$$m = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

Brainstorming 1

In groups

Aim: To determine the relationship between equation $y = mx + c$ with gradient and y-intercept.

Materials: Graph paper, linear function cards.

Steps:

1. Get into four groups.
2. Each group is given a card written with two linear functions.

Group 1	*Group 2*	*Group 3*	*Group 4*
$y = 3x + 6$ $y = -2x - 4$	$y = 2x + 6$ $y = -4x + 8$	$y = 5x - 10$ $y = -3x + 9$	$y = 4x - 8$ $y = -2x + 2$

3. Complete the table of values below for each given function.

x	–3	–2	–1	0	1	2	3
y							

4. Based on the table of values, draw the graphs of the functions.

5. From the graph of the function, calculate the gradient and state the *y*-intercept.
6. Compare the values of gradient and y-intercept from the graph with the values in the function card.

Discussion:

1. Compare your findings in step 6 with linear function $y = mx + c$. What is your conclusion?
2. Present your findings. Are your findings the same as the other groups' findings?

From Brainstorming 1, it is found that:

(a) For a linear function, $y = mx + c$, m is the gradient and c is the *y*-intercept of the straight line.
(b) The graph of linear function, $y = mx + c$ is a straight line.

In general,

Equation of straight line

$y = mx + c$

m: Gradient; c: *y*-intercept

Brainstorming 2

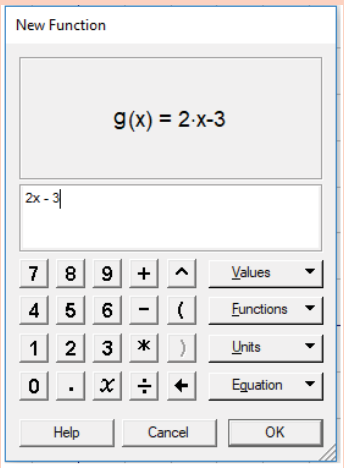
In pairs

Aim: To produce a graph of linear function.

Materials: Dynamic software

Steps:

1. Start with *New sketch.*
2. Select *graph* icon.
3. Select *plot new function* and enter the required equation of straight line (Diagram 1).



The first graph of straight line: $y = 2x - 3$

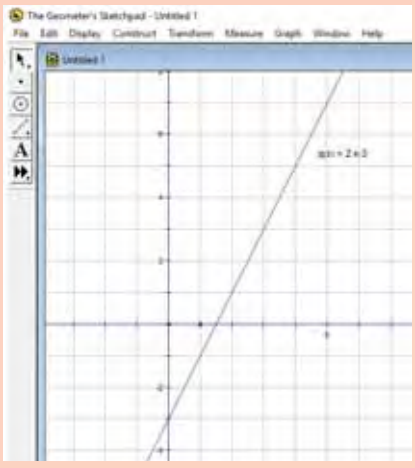


Diagram 1

4. Click *straightedge tool* and mark two points on the constructed graph of straight line.

5. Click *measure* and then click *slope* (Diagram 2).
The gradient value will be displayed (Diagram 3).

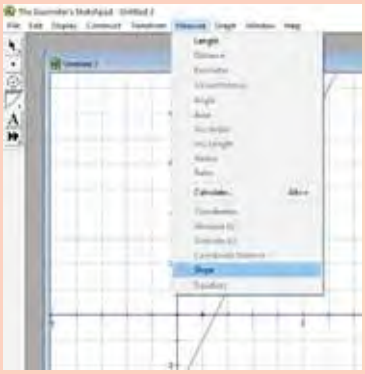


Diagram 2

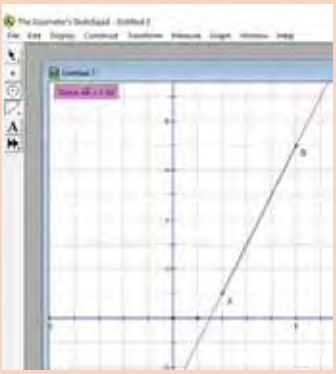
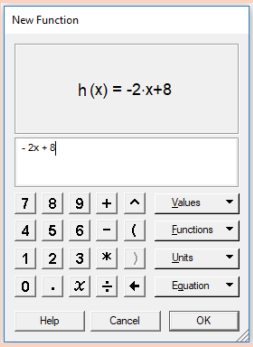


Diagram 3

6. Repeat steps 2 to 5 to draw and determine the gradient of the graph of straight line for function $y = -2x + 8$.
(Diagram 4)



The second graph of straight line: $y = -2x + 8$

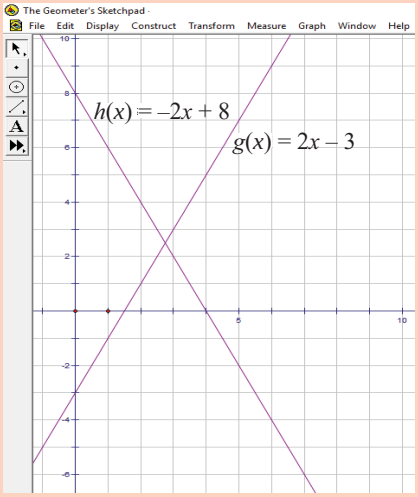
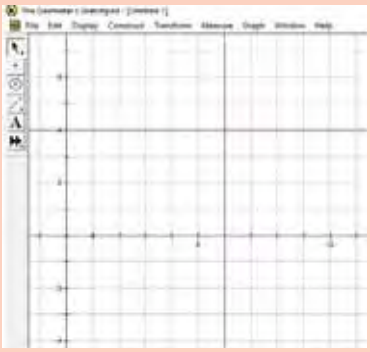


Diagram 4

7. **Straight lines that are parallel to the *x*-axis and *y*-axis.**

A displayed example of straight lines such as
(a) $y = 4$ (b) $x = 6$

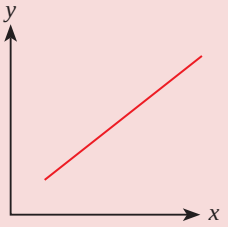


$y = 4$

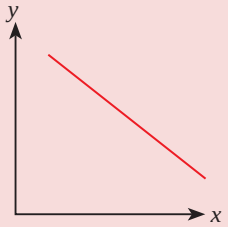
$x = 6$

TIPS

Relationship between the value m and the form of a straight line graph.
If $m > 0$



If $m < 0$



Discussion:

1. Compare the forms of graph resulting from dynamic software with the forms of graph from Brainstorming 1.
2. Make a conclusion for the values of m and c of the equation of straight line in the form $y = mx + c$. Discuss the shape of the graph when
   (a) m is positive (b) m is negative
   (c) parallel to x-axis (d) parallel to y-axis

From Brainstorming 2, it is found that:

(a) The graph of linear function $y = mx + c$ is a straight line.
(b) The graph of function $y = h$ is a straight line parallel to x-axis.
(c) The graph of function $x = h$ is a straight line parallel to y-axis.

Example 1

Determine the gradient and y-intercept of the straight line

(a) $y = 2x + 9$ (b) $3y = -2x + 12$

SMART MIND

What is the y-intercept of a straight line that passes through the origin?

Solution:

(a) Compare $y = 2x + 9$ with $y = mx + c$;
$m = 2$ and $c = 9$
Thus, gradient = 2 and y-intercept = 9

(b) Given $3y = -2x + 12$

÷3 $\frac{3y}{3} = -\frac{2x}{3} + \frac{12}{3}$

Divide by 3 so that the coefficient of y is +1.

$y = -\frac{2}{3}x + 4$

Compare $y = -\frac{2}{3}x + 4$ with $y = mx + c$;

$m = -\frac{2}{3}$ and $c = 4$

Thus, gradient $= -\frac{2}{3}$ and y-intercept = 4.

QUIZ

What is the value of the gradient for the straight line
(a) $y = x$
(b) $y = -x$

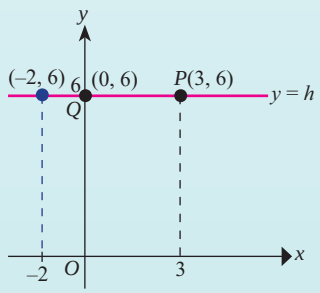
BULLETIN

In the equation $y = mx + c$, the coefficient of y is +1.

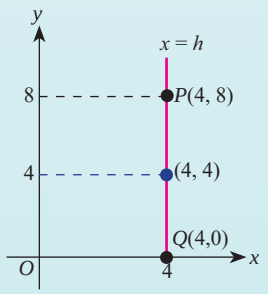
Example 2

State the value of h for the straight line graph below. State reasons for your answer.

(a)



(b)



Solution:

(a) $h = 6$ because the straight line $y = 6$ is always 6 units from the x-axis

(b) $h = 4$ because the straight line $x = 4$ is always 4 units from the y-axis.

MIND TEST 9.1a

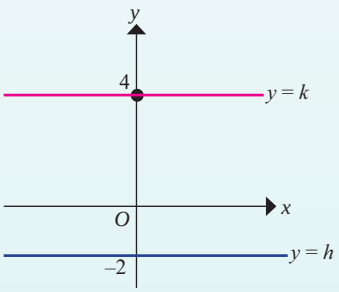
1. Determine the gradient and y-intercept of the following straight lines.

(a) $y = 3x + 5$ (b) $y = 2x - 7$ (c) $y = -x + 4$

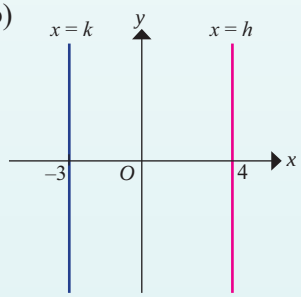
(d) $2y = 8x + 6$ (e) $3y = -x + 18$ (f) $-4y = -2x + 5$

2. State the value of k and of h for each straight line graph given.

(a)



(b)



What is the relationship between the equations of straight lines in the form $ax + by = c$, $\frac{x}{a} + \frac{y}{b} = 1$ and $y = mx + c$?

LEARNING STANDARD

Investigate and interpret the equations of straight lines in other forms such as $ax + by = c$ and $\frac{x}{a} + \frac{y}{b} = 1$, and change to the form of $y = mx + c$, and vice versa.

Brainstorming 3

In groups

Aim: To determine the relationship between the equations of straight lines in the form of $ax + by = c$, $\frac{x}{a} + \frac{y}{b} = 1$ and $y = mx + c$.

Materials: Graph paper, straight line equation cards

Steps:

1. Get into four groups.

2. Each group is given a card with three equations of a straight line written on it.

Group 1	*Group 2*	*Group 3*	*Group 4*
$2x + 3y = 6$	$4x - 2y = -8$	$-3x + 4y = -12$	$-x - 4y = 4$
$\frac{x}{3} + \frac{y}{2} = 1$	$\frac{x}{(-2)} + \frac{y}{4} = 1$	$\frac{x}{4} + \frac{y}{(-3)} = 1$	$\frac{x}{(-4)} + \frac{y}{(-1)} = 1$
$y = -\frac{2}{3}x + 2$	$y = 2x + 4$	$y = \frac{3}{4}x - 3$	$y = -\frac{1}{4}x - 1$

3. Determine the corresponding value of y when $x = 0$ and the corresponding value of x when $y = 0$ for each equation.

Example:

x	0	3
y	2	0

$2x + 3y = 6$

When $x = 0$:
$2(0) + 3y = 6$
$3y = 6$
$y = 2$

When $y = 0$:
$2x + 3(0) = 6$
$2x = 6$
$x = 3$

TIPS

A straight line graph can be drawn by plotting at least two points.

4. Draw a straight line graph for each equation.

5. From the graph, state the x-intercept and y-intercept and determine the gradient of the graph.

TIPS

Gradient $= -\dfrac{y\text{-intercept}}{x\text{-intercept}}$

Discussion:

1. What is your conclusion about the relationship between the x-intercept with the y- intercept and the gradients of the three straight line graphs?

2. What is your conclusion about the relationship between the equations of straight line in different forms?

From Brainstorming 3, it is found that:

(a) The values of x-intercept and y-intercept and the gradient for these three straight lines are the same.

(b) Equations of straight line in the forms of $ax + by = c$, $\dfrac{x}{a} + \dfrac{y}{b} = 1$ and $y = mx + c$ produce the same straight line graph if the values of x-intercept and y-intercept are the same.

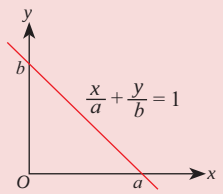
In general,

Straight line equation can also be written in the form of $\boldsymbol{ax + by = c}$ and $\boldsymbol{\dfrac{x}{a} + \dfrac{y}{b} = 1}$; $a \neq 0$ and $b \neq 0$

How do you change the equation of straight line in the form of $ax + by = c$, $\dfrac{x}{a} + \dfrac{y}{b} = 1$ to the form of $y = mx + c$ and vice versa?

TIPS

For the straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$,
a = x-intercept
b = y -intercept



Example 3

Change the equation of straight line below to the form of $\dfrac{x}{a} + \dfrac{y}{b} = 1$ and $y = mx + c$.

(a) $2x + 3y = 12$ (b) $3x - 5y = 15$

Solution:

(a) $2x + 3y = 12$

(i) $2x + 3y = 12$

$\div 12 \rightarrow \dfrac{2x}{12} + \dfrac{3y}{12} = \dfrac{12}{12}$

$\dfrac{x}{6} + \dfrac{y}{4} = 1$

Divide by 12 to get a value of 1.

(ii) $2x + 3y = 12$

$3y = -2x + 12$

$\div 3 \rightarrow \dfrac{3y}{3} = \dfrac{-2x}{3} + \dfrac{12}{3}$

$y = -\dfrac{2}{3}x + 4$

Divide by 3 so that the coefficient of y is +1.

(b) $3x - 5y = 15$

(i) $3x - 5y = 15$

$\div 15 \rightarrow \dfrac{3x}{15} - \dfrac{5y}{15} = \dfrac{15}{15}$

$\dfrac{x}{5} - \dfrac{y}{3} = 1$

(ii) $3x - 5y = 15$

$-5y = -3x + 15$

$\div (-5) \rightarrow \dfrac{-5y}{(-5)} = \dfrac{-3x}{(-5)} + \dfrac{15}{(-5)}$

$y = \dfrac{3}{5}x - 3$

DISCUSSION CORNER

Among the three forms of equation of straight line that you have learned, which is the easiest form to know the gradient value, the y-intercept value and the x-intercept value of a straight line? Discuss.

Example 4

Change the equation of straight line below to the form of $ax + by = c$ and $y = mx + c$.

(a) $\dfrac{x}{6} + \dfrac{y}{3} = 1$ (b) $-\dfrac{x}{2} + \dfrac{y}{4} = 1$

Solution:

(a) $\dfrac{x}{6} + \dfrac{y}{3} = 1$

(i) $\dfrac{x}{6} + \dfrac{y}{3} = 1$

$\dfrac{3x + 6y}{6(3)} = 1$ → Equate the denominator.

$3x + 6y = 1(18)$

$3x + 6y = 18$

$x + 2y = 6$

(ii) $\dfrac{x}{6} + \dfrac{y}{3} = 1$

$\dfrac{y}{3} = -\dfrac{x}{6} + 1$

$\times 3 \rightarrow \dfrac{3y}{3} = \dfrac{-x(3)}{6} + 1(3)$

$y = -\dfrac{1}{2}x + 3$

(b) $-\dfrac{x}{2} + \dfrac{y}{4} = 1$

(i) $-\dfrac{x}{2} + \dfrac{y}{4} = 1$

$\dfrac{-4x + 2y}{2(4)} = 1$

$-4x + 2y = 1(8)$

$-4x + 2y = 8$

$-2x + y = 4$

(ii) $-\dfrac{x}{2} + \dfrac{y}{4} = 1$

$\dfrac{y}{4} = \dfrac{x}{2} + 1$

$\times 4 \rightarrow \dfrac{4y}{4} = \dfrac{x(4)}{2} + 1(4)$

$y = 2x + 4$

Multiply by 4 so that the coefficient y is +1.

TIPS

$-4x + 2y = 8$ can also be written as $4x - 2y = -8$

QUIZ

What is the value of the gradient for the straight line $-\dfrac{x}{2} - \dfrac{y}{2} = 1$?

Example 5

Change the equation of the following straight lines to the form of $ax + by = c$ and $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

(a) $y = -2x + 8$ (b) $y = 3x + 6$

Solution:

(a) $y = -2x + 8$

(i) $y = -2x + 8$

$2x + y = 8$

(ii) $y = -2x + 8$

$2x + y = 8$

$\div 8 \rightarrow \dfrac{2x}{8} + \dfrac{y}{8} = \dfrac{8}{8}$

$\dfrac{x}{4} + \dfrac{y}{8} = 1$

(b) $y = 3x + 6$

(i) $y = 3x + 6$

$-3x + y = 6$

(ii) $y = 3x + 6$

$-3x + y = 6$

$\div 6 \rightarrow \dfrac{-3x}{6} + \dfrac{y}{6} = \dfrac{6}{6}$

$-\dfrac{x}{2} + \dfrac{y}{6} = 1$

MIND TEST 9.1b

1. Write the equation of the following straight lines in the form of $\frac{x}{a} + \frac{y}{b} = 1$ and $y = mx + c$.

(a) $3x - 4y = 24$ (b) $7x + 2y = 28$ (c) $5x - 3y = 15$ (d) $-2x + 3y = 9$

2. Write the equation of the following straight lines in the form of $ax + by = c$ and $y = mx + c$.

(a) $\frac{x}{4} + \frac{y}{3} = 1$ (b) $-\frac{x}{3} + \frac{y}{6} = 1$ (c) $\frac{3x}{2} + \frac{y}{6} = 1$ (d) $\frac{2x}{3} - \frac{y}{4} = 1$

3. Write the equation of the following straight lines in the form of $ax + by = c$ and $\frac{x}{a} + \frac{y}{b} = 1$.

(a) $y = 2x + 6$ (b) $y = 3x - 12$ (c) $y = -x + 5$ (d) $y = -2x - 4$

What is the relationship between the points on a straight line and the equation of the line?

LEARNING STANDARD

Investigate and make inference about the relationship between the points on a straight line and the equation of the line.

Diagram 1 and Diagram 2 show two straight lines drawn on a Cartesian plane based on the equation of straight lines $x + 2y = 4$ and $x - y = -3$.

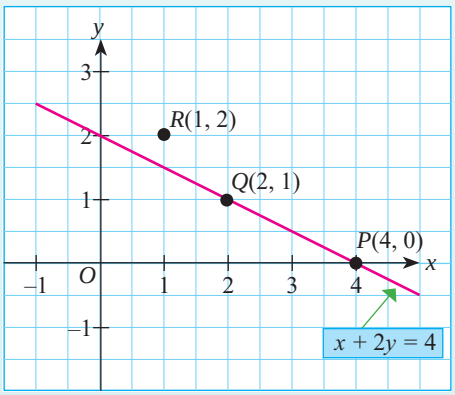


Diagram 1

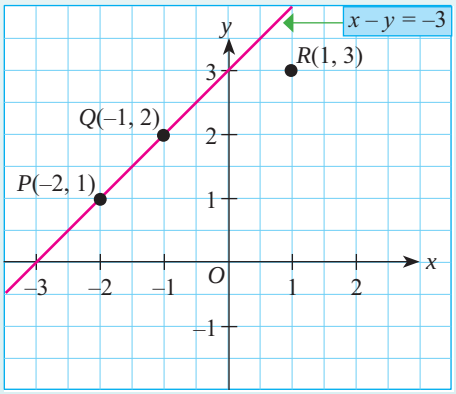


Diagram 2

Examine the position of points P, Q and R in Diagram 1 and Diagram 2. What can you say about the points P, Q and R and the straight line drawn?

(a) Diagram 1

$\overbrace{x}^{\text{Left}} + \overbrace{2y}^{\text{Right}} = 4$

(i) Substitute point $P(4, 0)$

Left: $x + 2y = 4 + 2(0) = 4$ ← Equal ← Right: $= 4$

(ii) Substitute point $Q(2, 1)$

Left: $x + 2y = 2 + 2(1) = 4$ ← Equal ← Right: $= 4$

(iii) Substitute point $R(1, 2)$

Left: $x + 2y = 1 + 2(2) = 5$ ← Not Equal ← Right: $= 4$

(b) Diagram 2

$\overbrace{x-y}^{\text{Left}} = \overbrace{-3}^{\text{Right}}$

(i) Substitute point $P(-2, 1)$

Left: $x - y = -2 - 1 = -3$ — Right: $= -3$ — Equal

(ii) Substitute point $Q(-1, 2)$

Left: $x - y = -1 - 2 = -3$ — Right: $= -3$ — Equal

(iii) Substitute point $R(1, 3)$

Left: $x - y = 1 - 3 = -2$ — Right: $= -3$ — Not Equal

From the above activity, it is found that:

(a) Points on a straight line or points that the straight line passes through will satisfy the equation of a straight line.

(b) Points that do not lie on a straight line will not satisfy the equation.

Example 6

1. Determine whether point P lies on the given straight line.

(a) $y = 3x + 2$, $P(2, 8)$

(b) $3x - 2y = 12$, $P(-4, 2)$

(c) $\frac{x}{3} + \frac{y}{2} = 1$, $P(6, -2)$

(d) $2y = -5x - 7$, $P(4, 3)$

Solution:

(a) $\underbrace{y}_{\text{Left}} = \underbrace{3x + 2}_{\text{Right}}$, $P(2, 8)$

Left: $= 8$ — Right: $3x + 2 = 3(2) + 2 = 8$ — Equal

Thus, $P(2, 8)$ lies on the straight line $y = 3x + 2$.

(b) $\underbrace{3x - 2y}_{\text{Left}} = \underbrace{12}_{\text{Right}}$, $P(-4, 2)$

Left: $3x - 2y = 3(-4) - 2(2) = -16$ — Right: $= 12$ — Not Equal

Thus, $P(-4, 2)$ does not lie on the straight line $3x - 2y = 12$

(c) $\underbrace{\frac{x}{3} + \frac{y}{2}}_{\text{Left}} = \underbrace{1}_{\text{Right}}$, $P(6, -2)$

Left: $\frac{x}{3} + \frac{y}{2} = \frac{(6)}{3} + \frac{(-2)}{2} = 1$ — Right: $= 1$ — Equal

Thus, $P(6, -2)$ lies on the straight line $\frac{x}{3} + \frac{y}{2} = 1$.

(d) $\underbrace{2y}_{\text{Left}} = \underbrace{-5x - 7}_{\text{Right}}$, $P(4, 3)$

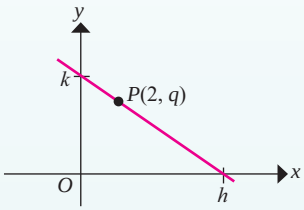
Left: $2y = 2(3) = 6$ — Right: $-5x - 7 = -5(4) - 7 = -27$ — Not Equal

Thus, $P(4, 3)$ does not lie on the straight line $2y = -5x - 7$.

Example 7

The diagram shows a straight line $3x + 5y = 15$. Given that O is the origin, determine the value of

(a) h (b) k (c) q (d) gradient of the straight line $3x + 5y = 15$



Solution:

(a) h is the x-intercept.
Thus, $y = 0$
$3x + 5y = 15$
$3(h) + 5(0) = 15$
$3h = 15$
$h = \frac{15}{3}$
$h = 5$

(b) k is the y-intercept.
Thus, $x = 0$
$3x + 5y = 15$
$3(0) + 5(k) = 15$
$5k = 15$
$k = \frac{15}{5}$
$k = 3$

(c) $P(2, q)$ is a point on the straight line $3x + 5y = 15$.
Thus,
$3x + 5y = 15$
$3(2) + 5(q) = 15$
$6 + 5q = 15$
$5q = 15 - 6$
$5q = 9$
$q = \frac{9}{5}$

(d) Gradient of the straight line $3x + 5y = 15$

$m = -\frac{y\text{-intercept}}{x\text{-intercept}}$

Gradient $= -\frac{3}{5}$

TIPS

- For points that lie on x-axis its value of y-coordinate is zero.
- For points that lie on y-axis its value of x-coordinate is zero.

FLASHBACK

Gradient, m

$m = -\frac{y\text{-intercept}}{x\text{-intercept}}$

SMART MIND

Do the coordinates $(-3, -3)$ lie on the straight line $y = x$?

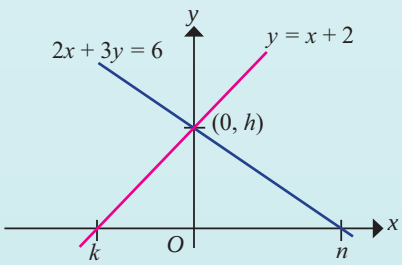
MIND TEST 9.1c

1. Determine whether the following points lie on the straight line $y = 2x + 16$.
(a) $M(-4, 3)$ (b) $N(1, 18)$ (c) $P(-8, 0)$ (d) $Q(-5, 8)$

2. Determine whether the following points lie on the straight line $2x + 3y = 12$.
(a) $M(0, 4)$ (b) $N(3, -2)$ (c) $P(15, -6)$ (d) $Q(-4, 8)$

3. Determine whether the following points lie on the straight line $\frac{x}{2} + \frac{y}{3} = 1$.
(a) $M(2, 0)$ (b) $N(-2, 12)$ (c) $P(4, -3)$ (d) $Q(0, 6)$

4. The diagram shows two straight lines, $y = x + 2$ and $2x + 3y = 6$. Given that O is the origin, determine the value of
(a) h (b) k (c) n



What do you understand about the gradients of parallel lines?

LEARNING STANDARD

Investigate and make inference about the gradients of parallel lines.

You have learnt that the gradient of a straight line is the ratio of vertical distance to horizontal distance, and the corresponding angles of the parallel lines are equal.

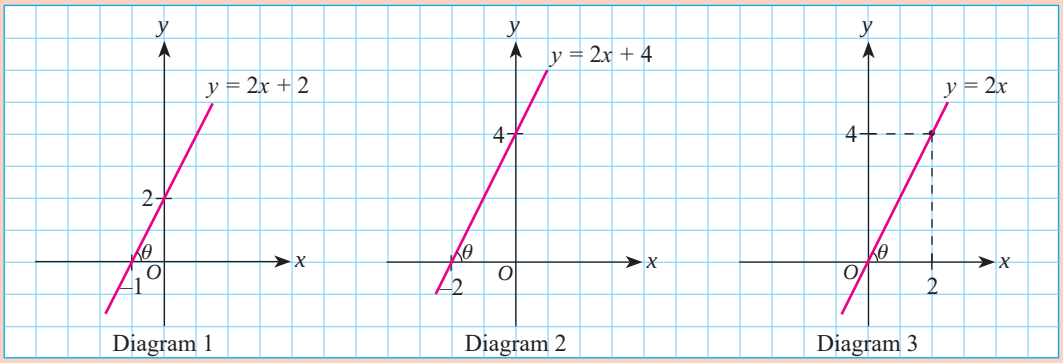
Brainstorming 4

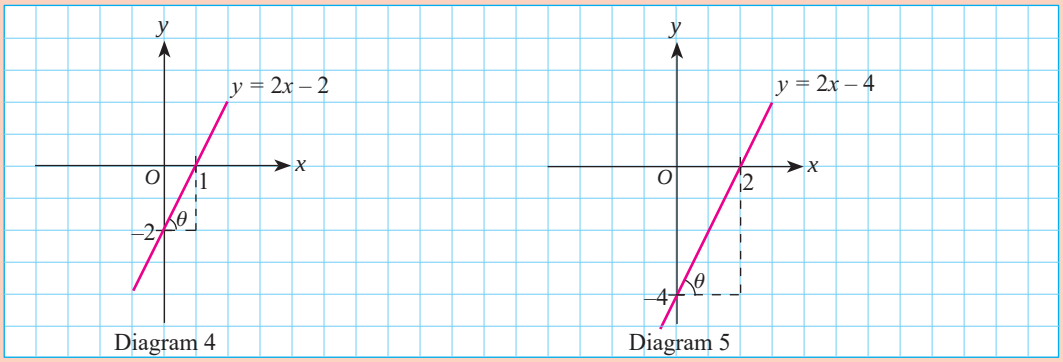
In pairs

Aim: To determine the relationship between gradients of straight lines with parallel lines.

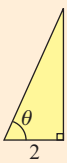
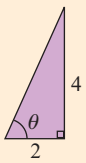
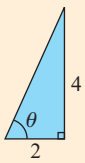
Steps:

1. Examine the straight lines below that were drawn based on the equation of a straight line with the same gradient of $m = 2$.



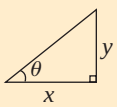


2. Based on Diagram 1 to Diagram 5, calculate the value θ.

 Diagram 1	 Diagram 2	 Diagram 3	 Diagram 4	 Diagram 5
$\tan\theta = \frac{2}{1}$ $\theta = 63.43°$				

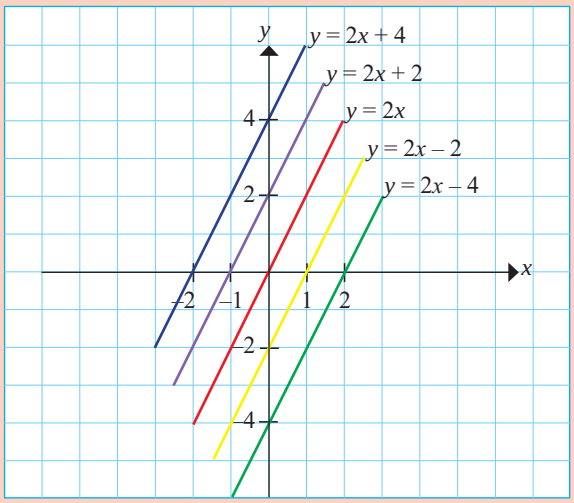
3. Are the values of θ for the five diagrams the same?

FLASHBACK



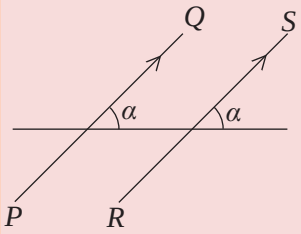
$\tan\theta = \frac{y}{x}$

4. The graphs in Diagram 1 to Diagram 5 are combined as below.



TIPS

PQ and *RS* are parallel lines because they have corresponding angles.



Discussion:

1. What is the connection between the values of θ with the five straight lines above?
2. Are the straight lines $y = 2x + 4$, $y = 2x + 2$, $y = 2x$, $y = 2x - 2$ and $y = 2x - 4$ parallel? Why?
3. What are the connections between the gradients and the parallel lines?
4. Are your findings the same as those of the other groups?

From Brainstorming 4, it is found that:

The straight lines $y = 2x + 4$, $y = 2x + 2$, $y = 2x$, $y = 2x - 2$ and $y = 2x - 4$ are parallel because they have the same gradient, that is $m = 2$ and the same corresponding angle, that is 63.43°.

In general, **Straight lines** that have the same gradients are parallel.

Example 8

Determine whether the straight line $y = 3x + 5$ is parallel to the straight line $6x - 2y = 9$.

TIPS

To determine the gradient value of a straight line, change the equation of the given straight line to the form $y = mx + c$.

Solution:

$y = 3x + 5$

Compare with $y = mx + c$

Gradient = 3

$$6x - 2y = 9$$
$$-2y = -6x + 9$$
$$\frac{-2y}{-2} = \frac{-6x}{(-2)} + \frac{9}{(-2)}$$
$$y = 3x - \frac{9}{2}$$

Gradient = 3

Equal

The gradients of both straight lines are equal, thus $y = 3x + 5$ is parallel to $6x - 2y = 9$.

Example 9

Determine whether the straight line $y = 3x + 8$ is parallel to the straight line $6y = 3x - 9$.

Solution:

$y = 3x + 8$

Compare with $y = mx + c$

Gradient = 3

$6y = 3x - 9$

$y = \frac{3x}{6} - \frac{9}{6}$

$y = \frac{1}{2}x - \frac{3}{2}$

Gradient $= \frac{1}{2}$

Not equal

DISCUSSION CORNER

Will two parallel lines intersect? Discuss.

The gradients of both straight lines are not equal. So, $y = 3x + 8$ is not parallel to $6y = 3x - 9$.

Example 10

Given that the straight line $4x + 3y = 18$ is parallel to the straight line $2x + hy = 2$, calculate the value of h.

Solution:

If both straight lines are parallel, then the gradients are equal.

For $4x + 3y = 18$

$3y = -4x + 18$

$y = -\frac{4}{3}x + 6$

Gradient $= -\frac{4}{3}$

For $2x + hy = 20$

$hy = -2x + 20$

$y = -\frac{2}{h}x + \frac{20}{h}$

Gradient $= -\frac{2}{h}$

For,

$-\frac{4}{3} = -\frac{2}{h}$

$h = 2 \times \frac{3}{4}$

$h = \frac{3}{2}$

MIND TEST 9.1d

1. Determine whether the following pairs of straight lines are parallel.

(a) $3y = -6x + 3$ and $y + 2x = 14$

(b) $2x + 3y = 3$ and $2x + 6y = 12$

(c) $y = 2x + 1$ and $8x - 4y = 5$

(d) $y = -3x + 4$ and $9x + 2y = 12$

2. Determine the value of k for each of the following pairs of parallel lines.

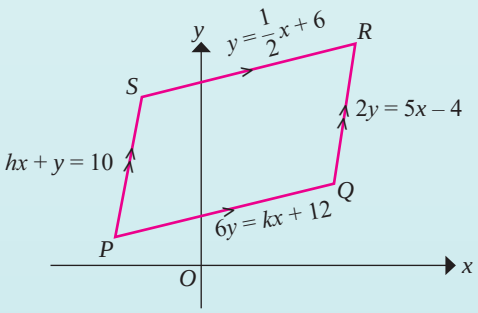
(a) $y = -3x + 4$ and $y + kx = 14$

(b) $kx + 2y = 7$ and $6x + 2y = 15$

(c) $8y = 5x + 1$ and $kx - 3y = 8$

(d) $3x + ky = 4$ and $2x + y = 3$

3. The diagram on the right shows a parallelogram $PQRS$. Given that the straight line PQ is parallel to SR and the straight line PS is parallel to QR, and O is the origin, calculate the values of h and k.



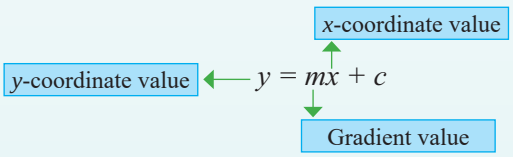
How do you determine the equation of a straight line?

LEARNING STANDARD

Determine the equation of a straight line.

The equation of a straight line $y = mx + c$ can be determined by the following steps:

1. Determine the value of gradient, m.
2. Determine a point which the straight line passes through or a point which lies on the straight line.
3. Substitute the gradient, m, the x-coordinate and y-coordinate from the point into the equation $y = mx + c$ to determine the value of c, that is, the y-intercept.



4. Substitute the gradient value and y-intercept value specified in the equation of the straight line $y = mx + c$.

Determine the equation of a straight line when the gradient and a point on the straight line are given

Example 11

Determine the equation of a straight line with a gradient of $\frac{1}{2}$ and passes through point $P(6, 8)$.

Solution:

$m = \frac{1}{2}$ Given $P(6, 8)$, thus $x = 6$, $y = 8$

Substitute the values of m, x and y into $y = mx + c$ to determine the value of c.

$8 = \frac{1}{2}(6) + c$

$8 = 3 + c$

$c = 8 - 3$

$c = 5$

Therefore, the equation of the straight line is $y = \frac{1}{2}x + 5$.

QUIZ

Determine the equation of a straight line with a gradient of 0 and passes through point $P(1, 5)$.

MIND TEST 9.1e

1. Determine the equation of a straight line with the given gradient and passes through point P given.

(a) Gradient = 2, $P(3, 7)$

(b) Gradient = -3, $P(-6, 4)$

(c) Gradient = $\frac{2}{3}$, $P(12, 5)$

(d) Gradient = $-\frac{1}{2}$, $P(4, -6)$

Determine the equation of a straight line that passes through two points

When two points on a straight line are given, the gradient of the straight line can be calculated. Hence the equation of the straight line can be determined.

Example 12

Determine the equation of a straight line that passes through point $P(-1, 5)$ and point $Q(2, -7)$.

Solution:

$$m = \frac{-7-5}{2-(-1)} = \frac{-12}{2+1} = \frac{-12}{3} = -4$$

For point $P(-1, 5)$, $x = -1$, $y = 5$.

Substitute the value of m, x and y into $y = mx + c$ to determine the value of c.

$5 = (-4)(-1) + c$

$5 = 4 + c$

$c = 5 - 4$

$c = 1$

Therefore, the equation of the straight line is $y = -4x + 1$.

TIPS

You can also substitute the value of point Q, where $x = 2$ and $y = -7$ and $m = -4$ into $y = mx + c$ to calculate the value of c and thus determine the equation of the straight line.

QUIZ

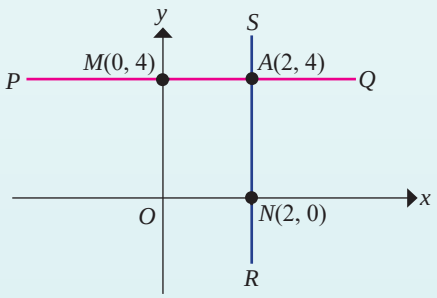
Determine the equation of a straight line that passes through the points $P(-4, 4)$ and $Q(5, -5)$.

Example 13

The diagram on the right shows straight lines PQ and RS. Given that straight line PQ is parallel to the x-axis and straight line RS is parallel to the y-axis, determine

(a) the equation of the straight line PQ

(b) the equation of the straight line RS



Solution:

(a) Gradient of straight line PQ with $A(2, 4)$ and $M(0, 4)$

$$m = \frac{4-4}{2-0} = \frac{0}{2} = 0$$

y-intercept $= 4$

Thus, the equation of the straight line PQ is

$y = 0(x) + 4$

$y = 4$

(b) Gradient of straight line RS with $A(2, 4)$ and $N(2, 0)$.

$$m = \frac{4-0}{2-2} = \frac{4}{0} = \text{Undefined}$$

The gradient of the straight line RS is undefined and is always 2 units from the y-axis.

Hence, the equation of the straight line RS is $x = 2$.

MIND TEST 9.1e

1. Determine the equation of the straight line that passes through the given pair of points.

(a) $K(0, 2)$, $L(6, 0)$

(b) $R(-2, 0)$, $S(0, 8)$

(c) $T(3, -1)$, $U(5, 7)$

(d) $G(-4, -2)$, $H(8, 6)$

(e) $M(-1, 3)$, $N(1, 5)$

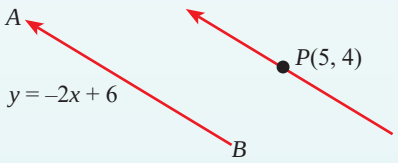
(f) $P(-5, 3)$, $Q(4, -6)$

Determine the equation of a straight line which passes through a point and is parallel to a given straight line

By now you would know that if two straight lines are parallel, then the gradients of both lines are equal.

Example 14

The diagram below shows the straight line AB with equation $y = -2x + 6$. Determine the equation of a straight line parallel to AB and passes through point $P(5, 4)$.



Solution:

The equation of the straight line AB is $y = -2x + 6$, thus the gradient of AB is -2.
The straight line which passes through point P is parallel to AB, thus the gradient m of that line is -2.
Substitute the values of m, x and y into $y = mx + c$ to determine the value of c.

$4 = (-2)(5) + c$ ← Given $P(5, 4)$, thus $x = 5$ and $y = 4$.
$4 = -10 + c$
$c = 4 + 10$
$c = 14$

Thus, the equation of the straight line parallel to AB and passes through the point P is $y = -2x + 14$.

Example 15

Determine the equation of a straight line parallel to the straight line $2x + 3y = 12$ and passes through point $G(6, 8)$.

Solution:

Given the equation of straight line $2x + 3y = 12$.
Thus, $3y = -2x + 12$

$$y = -\frac{2}{3}x + 4$$

Gradient of the straight line $= -\frac{2}{3}$.

The straight line which passes through point G is parallel to the straight line $2x + 3y = 12$.

Hence, the gradient of the straight line is $-\frac{2}{3}$.

Substitute the values of m, x and y into $y = mx + c$, and determine the value of c.

Thus, $8 = (-\frac{2}{3})(6) + c$ ← Given $Q(6, 8)$, thus $x = 6$ and $y = 8$.
$8 = -4 + c$
$c = 8 + 4$
$c = 12$

Hence, the equation of the straight line parallel to $2x + 3y = 12$ and passes through point G is $y = -\frac{2}{3}x + 12$.

MIND TEST 9.1g

1. Determine the equation of a straight line that is parallel to the given straight line and passes through point P.

(a) $y = 3x + 9$, $P(2, 7)$

(b) $y = -2x + 7$, $P(-3, 4)$

(c) $3x + 2y = 4$, $P(2, 6)$

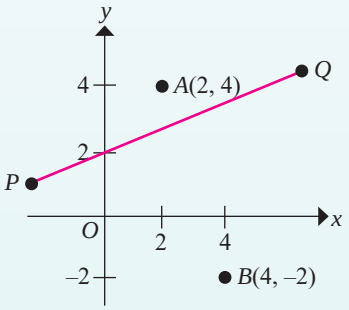
(d) $\frac{x}{2} + \frac{y}{3} = 1$, $P(-12, 9)$

2. The diagram on the right shows a straight line PQ. Given that the equation of the straight line PQ is $y = \frac{1}{3}x + 2$ and O is the origin, determine the equation of a straight line parallel to PQ and passes through

(a) point $A(2, 4)$

(b) point $B(4, -2)$

(c) the origin



How do you determine the point of intersection of two straight lines?

LEARNING STANDARD

Determine the point of intersection of two straight lines.

The point of intersection of two straight lines can be determined by the following methods:

1. Drawing both straight line graphs on the same Cartesian plane and determine the point of intersection from the graphs.

2. Solving simultaneous equations using

(a) substitution method

(b) elimination method

REMINDER

The calculator should only be used for checking answers.

Example 16

Determine the point of intersection of the straight lines $2x + y = 5$ and $x + 2y = 1$.

Graphical method

(a) $2x + y = 5$

$y = -2x + 5$

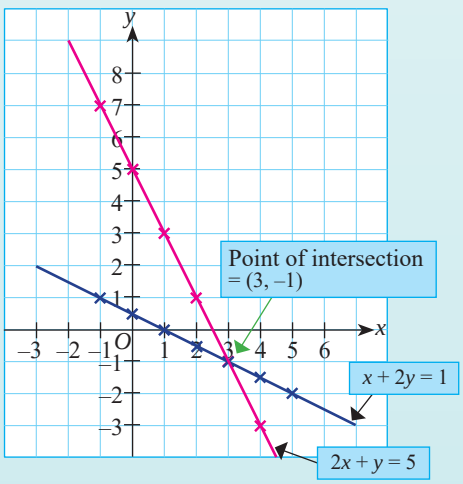
x	–1	0	1	2	3	4
y	7	5	3	1	–1	–3

(b) $x + 2y = 1$

$2y = -x + 1$

$y = -\frac{1}{2}x + \frac{1}{2}$

x	–1	0	1	2	3	4	5
y	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	–1	$-\frac{3}{2}$	–2



From the graph, it is found that the point of intersection of the straight lines $2x + y = 5$ and $x + 2y = 1$ is $(3, -1)$.

Substitution method

$2x + y = 5$ ----------(1)

$x + 2y = 1$ ----------(2)

From (1), $y = 5 - 2x$--(3)

Substitute $y = 5 - 2x$ in (2),

$x + 2(5 - 2x) = 1$

$x + 10 - 4x = 1$

$x - 4x = 1 - 10$

$-3x = -9$

$x = 3$

Substitute $x = 3$ in (3),

$y = 5 - 2(3)$

$y = 5 - 6$

$y = -1$

Thus, the point of intersection is $(3, -1)$.

Elimination method

$2x + y = 5$ ----------(1)

$x + 2y = 1$ ----------(2)

(1) × 2 $4x + 2y = 10$ ---------(3)

$x + 2y = 1$ ----------(2) (minus)

$3x = 9$

$x = 3$

Substitute $x = 3$ in (1),

$2(3) + y = 5$

$6 + y = 5$

$y = 5 - 6$

$y = -1$

Thus, the point of intersection is $(3, -1)$.

Brainstorming 5

In pairs

Aim: To determine the coordinates of the intersection of two straight lines.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click *Graph* next click *Show Grid.*
2. Click *Graph* again and select *Plot New Function* (Diagram 1).
3. Use *Plot New Function* to plot the intersection of the two straight lines.
4. Example: $y = x + 3$ and $y = -x + 5$.
5. Use *Arrow Tool* to select both straight line graphs. Click *Construct* and select *Intersection.*
6. Click *Measure* and select *Coordinates*. The intersection point A (1.00, 4.00) will be displayed (Diagram 2).
7. Repeat steps 1 to 6 for intersection of the other two straight lines.
   (a) $y = x + 2$ and $y = 2x + 4$ (Diagram 3)
   (b) $y = 4$ and $y = 3x - 2$ (Diagram 4)

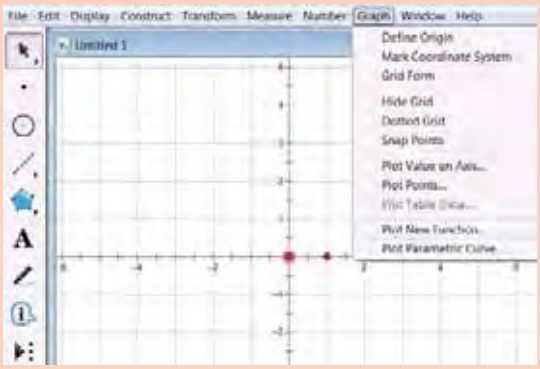


Diagram 1

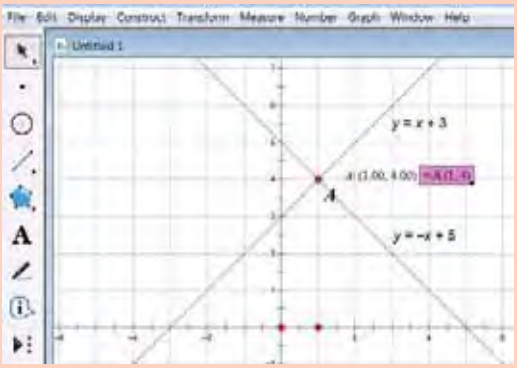


Diagram 2

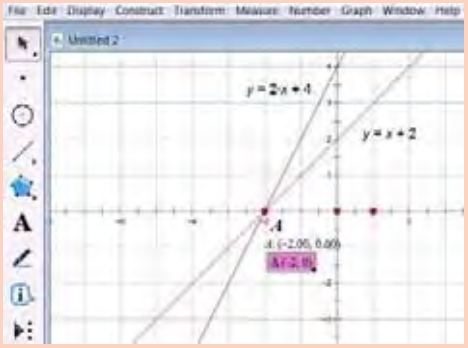


Diagram 3

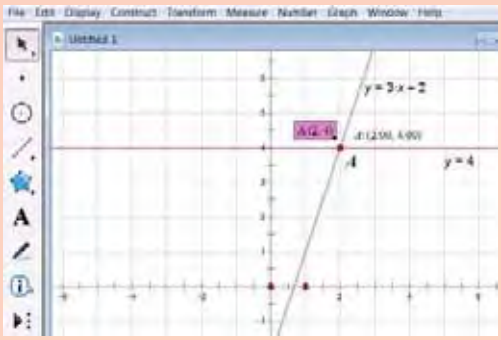


Diagram 4

Discussion:

What can you conclude from the results above?

From Brainstorming 5, it is found that:

(a) The point of intersection of two straight lines can be determined by plotting both straight lines on the Cartesian plane.

(b) Two straight lines that are not parallel intersect at only one point.

MIND TEST 9.1h

1. Determine the point of intersection of the following pairs of straight lines using the substitution method.

(a) $x = 3, 2x + y = 10$

(b) $y = 4, 3x - 2y = 7$

(c) $x + y = 5, 2x - y = 4$

(d) $2x + y = 3, 3x - 2y = 8$

2. Determine the point of intersection of the following pairs of straight lines using the elimination method.

(a) $x + y = 1, 2x + y = -1$

(b) $x - y = -4, 3x + y = 4$

(c) $x - y = -5, 2x + 3y = -10$

(d) $2x - 3y = 5, 3x + 2y = 14$

How do you solve problems involving straight lines?

LEARNING STANDARD

Solve problems involving straight lines.

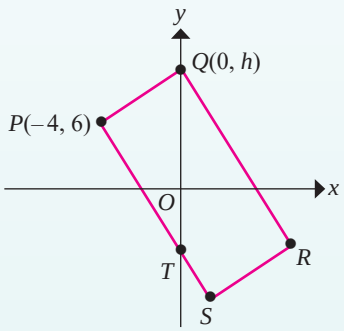
Example 17

The diagram on the right shows a parallelogram $PQRS$. Given that the gradient of SR is $\frac{1}{2}$ and y-intercept of straight line PS is -4, determine

(a) the value of h

(b) equation of straight line PS

(c) x-intercept for straight line PS



Solution:

Understanding the problem

- $PQRS$ is a parallelogram.
- Gradient of PQ = gradient of SR = $\frac{1}{2}$
- y-intercept of PS is -4.

Planning a strategy

- The value of h can be determined by using the gradient that is, gradient of PQ = gradient of $SR = \frac{1}{2}$.
- The y-intercept of the straight line PS is -4. Thus, the coordinates of T are $(0, -4)$.
- The x-intercept of the straight line PS can be determined by substituting $y = 0$ into equation PS.

Implementing the strategy

(a) Gradient PQ = Gradient $SR = \frac{1}{2}$

$$\frac{h-6}{0-(-4)} = \frac{1}{2}$$

$$\frac{h-6}{4} = \frac{1}{2}$$

$$h - 6 = 2$$

$$h = 2 + 6$$

$$h = 8.$$

(b) Straight line PS passes through point $T(0, -4)$

Gradient $PS = \frac{-4-6}{0-(-4)} = \frac{-10}{4} = -\frac{5}{2}$

y-intercept of straight line PS is -4

Thus, equation of straight line PS is $y = -\frac{5}{2}x - 4$.

(c) Equation of straight line PS is $y = -\frac{5}{2}x - 4$

When $y = 0$

$$0 = -\frac{5}{2}x - 4$$

$$\frac{5}{2}x = -4$$

$$x = -\frac{8}{5}$$

x-intercept of the straight line PS is $-\frac{8}{5}$.

Making a conclusion

(a) The value of h is 8.

(b) The equation of the straight line PS is $y = -\frac{5}{2}x - 4$.

(c) x-intercept of the straight line PS is $-\frac{8}{5}$.

Example 18

Given straight lines $y = -\frac{1}{3}x + 3$ and $2x - y = 4$ intersect at point A, determine the coordinates of point A using the graphical method.

TIPS

A straight line can be drawn if its x-intercept and y-intercept are known.

Solution:

For the straight line $y = -\frac{1}{3}x + 3$,

(a) when $x = 0$,

$y = -\frac{1}{3}(0) + 3$

$y = 3$

y-intercept $= 3$

(b) when $y = 0$,

$0 = -\frac{1}{3}(x) + 3$

$\frac{1}{3}x = 3$

$x = 9$

x-intercept $= 9$

For the straight line $2x - y = 4$,

(a) when $x = 0$,

$2(0) - y = 4$

$-y = 4$

$y = -4$

y-intercept $= -4$

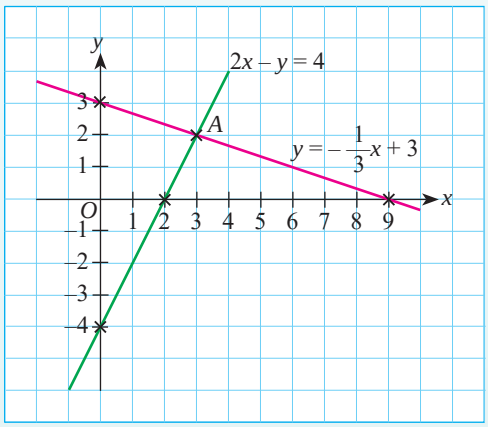
(b) when $y = 0$,

$2x - (0) = 4$

$2x = 4$

$x = 2$

x-intercept $= 2$



From the graph, it is found that the coordinates of A are $(3, 2)$.

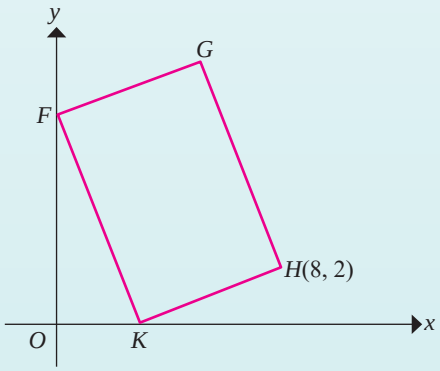
MIND TEST 9.1i

1. The diagram on the right shows a parallelogram $FGHK$. Given that O is the origin and point K is located on the x-axis, the equation of straight line FG is $2y = x + 20$, determine

(a) the gradient of straight line FG

(b) the y-intercept of straight line HK

(c) the equation of straight line HK

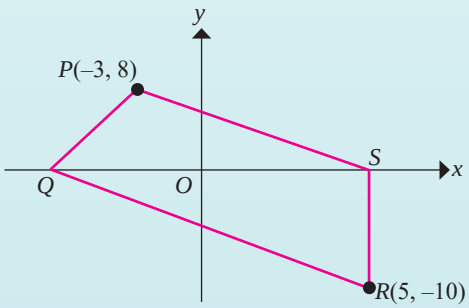


2. In the diagram on the right, O is the origin and $PQRS$ is a trapezium where PS and QR are parallel. The straight line RS is parallel to the y-axis, and points Q and S are on the x-axis. Determine

(a) the coordinates of S

(b) the equation of straight line QR

(c) the x-intercept of straight line QR



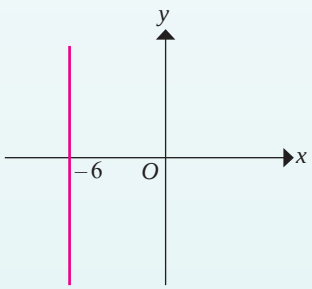
Dynamic Challenge

Test Yourself

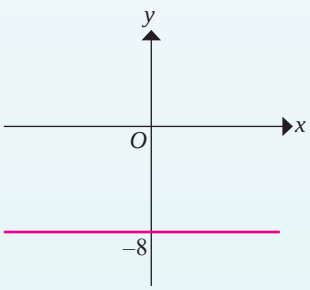
1. Given that $2x + 5y = 30$ is an equation of a straight line, determine
(a) the x-intercept (b) the y-intercept (c) the gradient

2. State the equation of the straight line for each of the following diagrams.

(a)



(b)



3. Determine the equation of a straight line that has a gradient of 3 and passes through point $R(-4, 6)$.

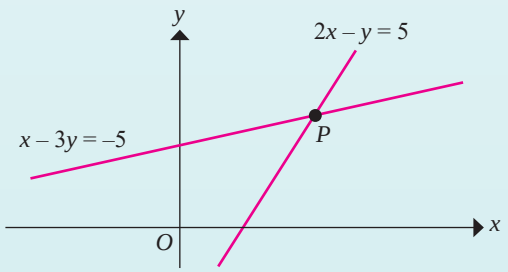
4. Determine the equation of a straight line that passes through point $P(-1, -2)$ and point $Q(3, 14)$.

5. Determine the equation of a straight line that passes through point $M(-3, 5)$ and is parallel to the straight line $6x + 2y = 18$.

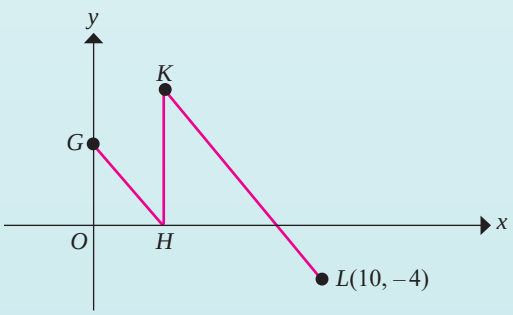
6. Determine the point of intersection of the straight lines $y = -8$ and $y = -4x + 12$.

Skills Enhancement

1. The diagram on the right shows two straight lines intersecting at point P. Given that O is the origin, determine the coordinates of P.



2. In the diagram on the right, GH, HK and KL are straight lines. Point H lies on the x-axis, GH is parallel to KL, and HK is parallel to the y-axis.
Given the equation of GH is $2x + y = 6$,
(a) state the equation of straight line HK
(b) determine the equation of straight line KL and then state the x-intercept of KL

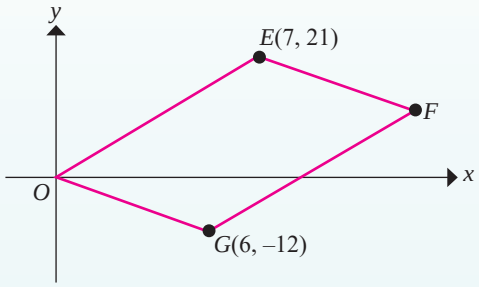


3. The diagram on the right shows a parallelogram $OEFG$. Given O is the origin, determine

(a) the equation of straight line OG

(b) the equation of straight line EF

(c) the x-intercept of straight line EF

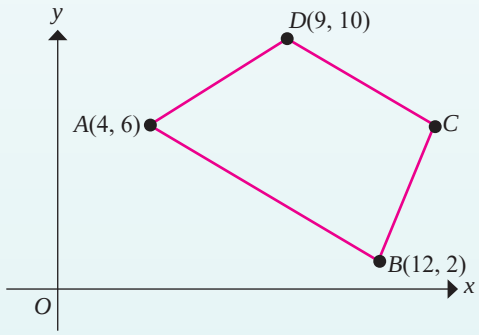


4. The diagram on the right shows a trapezium $ABCD$ drawn on the Cartesian plane. Given AB is parallel to DC, determine

(a) the equation of straight line AB

(b) the equation of straight line CD

(c) if the straight lines AB and CD intersect. State the reasons for your answer.



Self Mastery

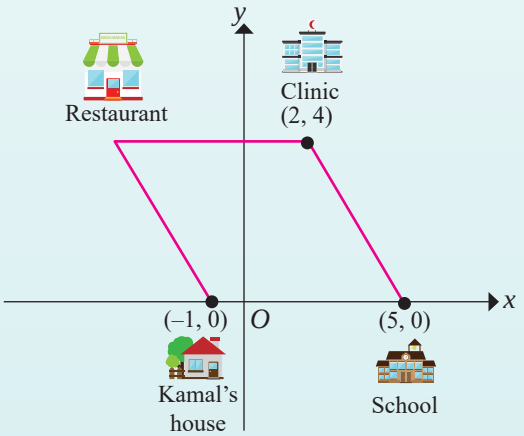
1. The diagram on the right shows a parallelogram drawn on a Cartesian plane and it represents the locations of Kamal's house, the school, the clinic and the restaurant. Given that the scale is 1 unit = 1 km,

(a) calculate the distance, in km, between Kamal's house and the school

(b) determine the coordinates of the restaurant

(c) calculate the distance, in km, between Kamal's house and the restaurant

(d) determine the equation of the straight line connecting the school and the clinic



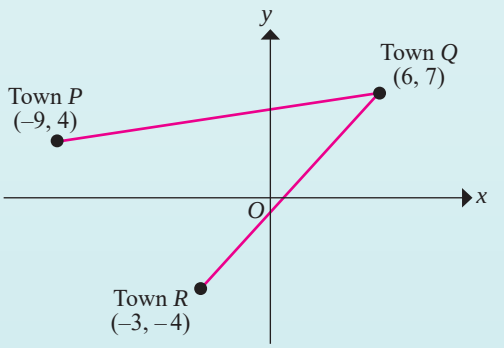
2. The diagram on the right shows the positions of town P, town Q and town R drawn on a Cartesian plane. Given that the scale is 1 unit = 2 km,

(a) calculate the distance in km, between town R and the origin O

(b) determine the equation of straight line connecting town P and town Q

(c) calculate the nearest distance, in km, between town P and town R

(d) calculate the time taken, in minutes, by Encik Mazlan to arrive at town Q if he drives from town R to town Q using the shortest route at an average speed of 50 km h^{-1}

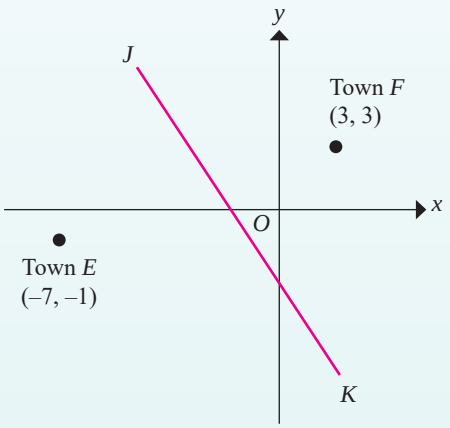


3. The original height of a plant F is 9 cm. Its height is y cm after x days and is represented by the equation $y = \frac{3}{16}x + 9$. Plant G has the same growth rate as plant F. Plant G reaches a height of 15 cm after 8 days. Determine the equation to represent the height of plant G. Then, state the original height, in cm, of plant G.

4. JK is a straight road that passes through the midpoint between town E and town F.

(a) The equation for the straight road JK is $y = -2x + k$, where k is a constant. Determine the value of k.

(b) Another straight road GH with the equation $y = 2x + 17$ will be constructed. A traffic light will be installed at the junction of both roads JK and GH. Determine the coordinates of the traffic light.



TIPS

Solution by scale drawing is not accepted.

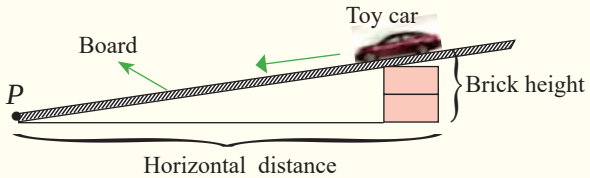
PROJECT

Title: Gradient and speed.

Material: Toy car, board, brick, long ruler and stopwatch.

Steps:

1. Place a board over two bricks as in the diagram below.



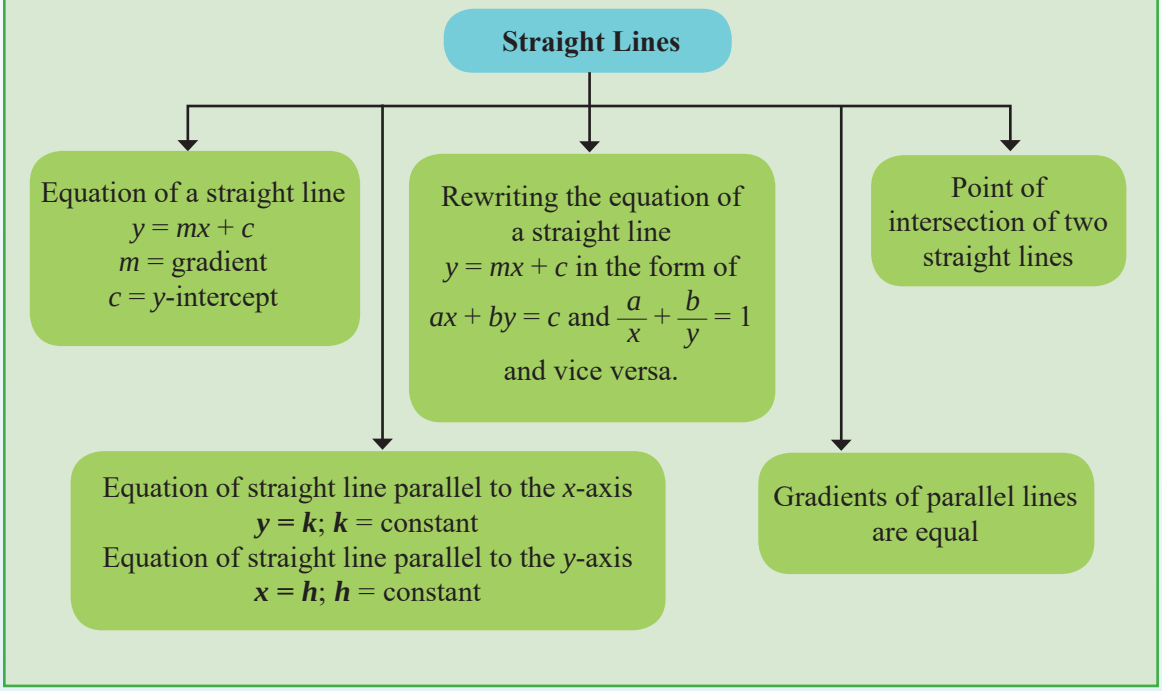
2. Measure the horizontal distance (fixed) and the height of the car from the ground. Calculate the gradient of the board and record it.

3. Release the toy car. Record the time, in seconds, for the toy car to reach point P.

4. Add the bricks one by one. Repeat steps 2 and 3.

5. What can you conclude about the relationship between the gradient of the board and the speed of the car?

CONCEPT MAP



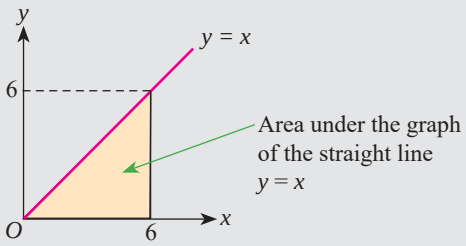
SELF-REFLECT

At the end of this chapter, I am able to:

1.	Determine the gradient and y-intercept when the equation of the straight line in the form of $y = mx + c$ is given.		
2.	Determine the gradient and y-intercept when the equation of the straight line in the form of $ax + by = c$ is given.		
3.	Determine the gradient and y-intercept when the equation of the straight line in the form of $\frac{x}{a} + \frac{y}{b} = 1$ is given.		
4.	Determine whether a point lies on a given straight line.		
5.	Determine whether two straight lines are parallel.		
6.	Determine the equation of a straight line.		
7.	Determine the point of intersection of two straight lines.		
8.	Solve problems involving straight lines.		

EXPLORING MATHEMATICS

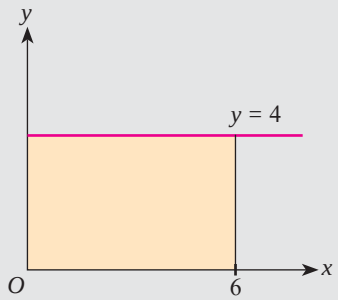
The area under a straight line can be determined if enough information is given.
For example, the area under the graph of the straight line $y = x$ for the range $0 \leqslant x \leqslant 6$ in the diagram on the right can be determined as follows:



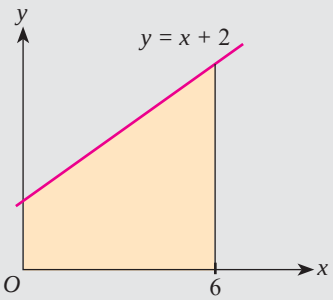
$$\text{The area under the graph} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 6 \text{ units} \times 6 \text{ units}$$
$$= 18 \text{ unit}^2$$

Worksheet

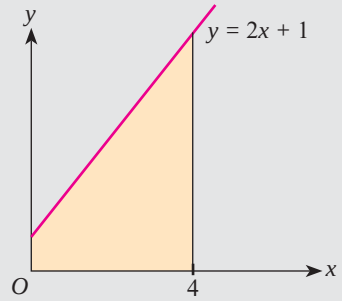
1.



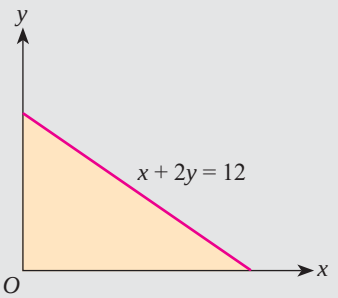
2.



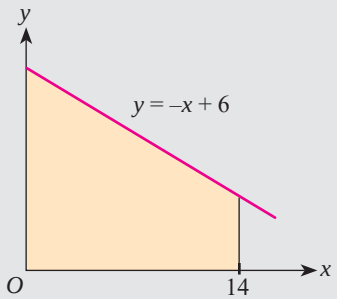
3.



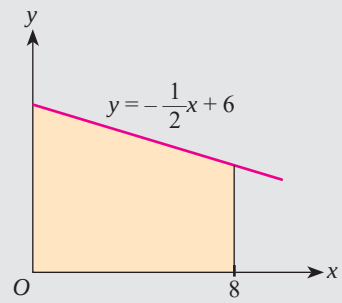
4.



5.



6.



Steps:

1. Get into groups.
2. Calculate the area under each graph of straight line provided.
3. Present your group's findings.
4. Propose at least two other ways to determine the area under the graph of a straight line.

Answers

CHAPTER 1 Indices

MIND TEST 1.1a

1.

Base	Index
5	3
-4	7
$\frac{1}{2}$	10
m	6
n	0
0.2	9
$-\frac{3}{7}$	4
x	20
$2\frac{1}{3}$	2
8	1

2. (a) 6^6
(b) $(0.5)^7$
(c) $\left(\frac{1}{2}\right)^4$
(d) $(-m)^5$
(e) $\left(1\frac{2}{3}\right)^3$
(f) $\left(-\frac{1}{n}\right)^6$

3. (a) $(-3) \times (-3) \times (-3)$
(b) $2.5 \times 2.5 \times 2.5 \times 2.5$
(c) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$
(d) $(-2\frac{1}{4}) \times (-2\frac{1}{4}) \times (-2\frac{1}{4})$
(e) $k \times k \times k \times k \times k \times k$
(f) $(-p) \times (-p) \times (-p) \times (-p) \times (-p) \times (-p) \times (-p)$
(g) $\frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m}$
(h) $(3n) \times (3n) \times (3n) \times (3n) \times (3n)$

MIND TEST 1.1b

1. (a) 3^4 (b) 5^6 (c) $\left(\frac{4}{5}\right)^3$
(d) $(0.2)^5$ (e) $(-4)^7$ (f) $\left(-\frac{1}{4}\right)^2$

MIND TEST 1.1c

1. (a) 6 561 (b) $-1\,024$ (c) 15.625
(d) -32.768 (e) $\frac{243}{32\,768}$ (f) $\frac{1}{1\,296}$
(g) $2\frac{7}{9}$ (h) $-12\frac{19}{27}$

MIND TEST 1.2a

1. (a) 3^7 (b) $(-0.4)^8$ (c) $\left(\frac{4}{7}\right)^9$
(d) $\left(-1\frac{2}{5}\right)^{10}$ (e) $-6m^9$ (f) $\frac{n^{12}}{5}$
(g) $-15x^7$ (h) y^{12}

MIND TEST 1.2b

1. (a) $5^5 \times 9^5$ (b) $(0.4)^3 \times (1.2)^9$
(c) $4x^6y^7$ (d) $-\frac{3}{2}k^6p^{11}$

MIND TEST 1.2c

1. (a) 4 (b) 7^2 (c) m^4n^5
(d) $3xy^3$ (e) m (f) $-5h$

2. (a) $8^{\boxed{8}} \div 8^4 \div 8^3 = 8$
(b) $m^4n^{\boxed{6}} \div m^{\boxed{2}}n^5 = m^2n$
(c) $\frac{m^{10}n^4 \times m^{\boxed{2}}n^2}{m^7n} = m^5n^{\boxed{5}}$
(d) $\frac{27x^3y^6 \times xy^{\boxed{2}}}{\boxed{9}x^2y^3} = 3x^{\boxed{2}}y^5$

3. 8

MIND TEST 1.2d

1. (a) 12^{10} (b) 3^{20} (c) 7^6 (d) $(-4)^{21}$
(e) k^{24} (f) g^{26} (g) $(-m)^{12}$ (h) $(-c)^{21}$

2. (a) True (b) False (c) False (d) False

MIND TEST 1.2e

1. (a) $2^2 \times 3^8$ (b) $11^9 \times 9^{15}$ (c) $13^6 \div 7^{12}$
(d) $5^{15} \times 3^{20}$ (e) $m^{15}n^{20}p^{10}$ (f) $16w^8x^{12}$
(g) $\frac{729a^{30}}{b^{24}}$ (h) $\frac{8a^{15}}{27b^{12}}$

2. (a) $11^2 \times 4^4$ (b) $3^3 \times 6^2$ (c) $\frac{4^4}{6^6}$
(d) $(-4)^6 \times (-5)^4$ (e) x^4y^4
(f) $h^{10}k^6$ (g) $m^{11}n^{15}$ (h) b^2d^6

3. (a) $6mn^8$ (b) $10x^8y^3$ (c) de

MIND TEST 1.2f

1. (a) $\frac{1}{5^3}$ (b) $\frac{1}{8^4}$ (c) $\frac{1}{x^8}$ (d) $\frac{1}{y^{16}}$
(e) a^4 (f) 20^2 (g) $\frac{3}{n^4}$ (h) $-\frac{5}{n^6}$
(i) $\frac{2}{7m^5}$ (j) $-\frac{3}{8m^4}$ (k) $\left(\frac{5}{2}\right)^{12}$ (l) $\left(-\frac{7}{3}\right)^{14}$
(m) $\left(\frac{y}{x}\right)^{10}$ (n) $\left(\frac{3y}{2x}\right)^4$ (o) $(2x)^5$

2. (a) 5^{-4} (b) 8^{-3} (c) m^{-7} (d) n^{-9}
(e) $\frac{1}{10^{-2}}$ (f) $\frac{1}{(-4)^{-3}}$ (g) $\frac{1}{m^{-12}}$ (h) $\frac{1}{n^{-16}}$
(i) $\left(\frac{7}{4}\right)^{-9}$ (j) $\left(\frac{y}{x}\right)^{-10}$

3. (a) $\frac{1}{4}$ (b) $\frac{2^4}{3^{14}}$ (c) $2^6 \times 5^2$
(d) $\frac{1}{3m^3n^7}$ (e) $\frac{1}{8m^8}$ (f) $\frac{m^6n}{18}$

MIND TEST 1.2g

1. (a) $125^{\frac{1}{3}}$ (b) $2\,187^{\frac{1}{7}}$ (c) $(-1\,024)^{\frac{1}{5}}$ (d) $n^{\frac{1}{10}}$

2. (a) $\sqrt{4}$ (b) $\sqrt[5]{32}$ (c) $\sqrt[3]{-729}$ (d) $\sqrt[15]{n}$

3. (a) 7 (b) -6 (c) 8 (d) -8

MIND TEST 1.2h

$a^{\frac{m}{n}}$	$729^{\frac{5}{6}}$	$121^{\frac{3}{2}}$	$w^{\frac{3}{7}}$	$x^{\frac{2}{5}}$	$\left(\frac{16}{81}\right)^{\frac{3}{4}}$	$\left(\frac{h}{k}\right)^{\frac{2}{3}}$
$(a^m)^{\frac{1}{n}}$	$(729^5)^{\frac{1}{6}}$	$(121^3)^{\frac{1}{2}}$	$(w^3)^{\frac{1}{7}}$	$(x^2)^{\frac{1}{5}}$	$\left[\left(\frac{16}{81}\right)^3\right]^{\frac{1}{4}}$	$\left[\left(\frac{h}{k}\right)^2\right]^{\frac{1}{3}}$
$(a^{\frac{1}{n}})^m$	$(729^{\frac{1}{6}})^5$	$(121^{\frac{1}{2}})^3$	$(w^{\frac{1}{7}})^3$	$(x^{\frac{1}{5}})^2$	$\left[\left(\frac{16}{81}\right)^{\frac{1}{4}}\right]^3$	$\left[\left(\frac{h}{k}\right)^{\frac{1}{3}}\right]^2$
$\sqrt[n]{a^m}$	$\sqrt[6]{729^5}$	$\sqrt{121^3}$	$\sqrt[7]{w^3}$	$\sqrt[5]{x^2}$	$\sqrt[4]{\left(\frac{16}{81}\right)^3}$	$\sqrt[3]{\left(\frac{h}{k}\right)^2}$
$(\sqrt[n]{a})^m$	$(\sqrt[6]{729})^5$	$(\sqrt{121})^3$	$(\sqrt[7]{w})^3$	$(\sqrt[5]{x})^2$	$\left(\sqrt[4]{\frac{16}{81}}\right)^3$	$\left(\sqrt[3]{\frac{h}{k}}\right)^2$

MIND TEST 1.2i

1. (a) 9 (b) 4 (c) 4 (d) 8
(e) 256 (f) 16 (g) 216 (h) 343
(i) 7 (j) 1 331 (k) 169 (l) 1 000

2. (a) $\sqrt[2]{6\ 561}^{[1]}$; $3^{[4]}$; $9^{[2]}$; $81^{[1]}$; $243^{[5]}$, $27^{[3]}$
(b) $25^{[2]}$, $125^{[1]}$; $625^{[4]}$, $\sqrt[2]{15\ 625}^{[1]}$; $3\ 125^{[5]}$, $5^{[3]}$

MIND TEST 1.2j

1. (a) $\frac{c^7}{de}$ (b) mn^6 (c) $\frac{10x}{3z^2}$

2. (a) $\frac{1}{2\ 401}$ (b) 648 (c) 86 400
(d) $\frac{7}{54}$ (e) 81 (f) $\frac{125}{8}$

3. 3 456 **4.** 48

Dynamic Challenge

Test Yourself

1. (a) True (b) False (25) (c) False (1)
(d) False $(32x^{15})$ (e) True (f) False $\left(\frac{2}{a^4}\right)$
(g) False $[(\sqrt[5]{32})^2]$ (h) True (i) False $\left(\frac{1}{625\text{ m}}\right)$

2.

$5^{[4]} \times 5^5$	$5^{3([3])}$
$5^{12} \div 5^{[3]}$	$(\sqrt{25})^{[9]}$
$\left(\frac{1}{5}\right)^{[-9]}$	$(\sqrt[3]{125})^{[9]}$
$(5^{[6]})^{\frac{3}{2}}$	$\frac{5^6 \times 5^{[5]}}{5^2}$
$\frac{1}{5^{[-9]}}$	$\left(\frac{1}{5^{[-3]}}\right)^3$

3.

2^0	as	$\frac{1}{3^{-4}}$	as	$\left(\frac{3}{5}\right)^{-2}$	as	$7^2 \times 5^{-3}$	as	$(5^{-1} \times \sqrt{25})^3$
1		3^4		$(\frac{5}{3})^2$		$\frac{7^2}{5^3}$		1

Skills Enhancement

1. (a) $\frac{n^7}{m}$ (b) $\frac{x^4y^7}{2}$ (c) xy^2

2. (a) $\frac{4}{125}$ (b) $\frac{25}{7}$ (c) 1 (d) 2 (e) 7 (f) 1

3. (a) 3 (b) 0 (c) –8
(d) –5 (e) 5 (f) 2
(g) 2 (h) –1 (i) 1

Self Mastery

1. (a) 1 000 (b) 500 000 (c) 50

2. (a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) 15

3. (a) –1, 6 (b) 1, –7 (c) –1, 4

4. (a) $x = \frac{1}{6}, y = 2$ (b) $x = 1, y = -\frac{2}{3}$

5. 12°C **6.** RM27 130 **7.** RM61 462.77

CHAPTER 2 Standard Form

MIND TEST 2.1a

1. (a) 2 s.f. (b) 5 s.f. (c) 5 s.f. (d) 4 s.f.
(e) 2 s.f. (f) 5 s.f. (g) 4 s.f. (h) 6 s.f.

MIND TEST 2.1b

1.

(a)	47 200	47 000	50 000
(b)	5 260	5 300	5 000
(c)	306	310	300
(d)	20.7	21	20
(e)	8.60	8.6	9
(f)	5.90	5.9	6
(g)	0.694	0.69	0.7
(h)	0.0918	0.092	0.09
(i)	0.00571	0.0057	0.006

2. (a) 12.02 (b) 2.83 (c) 11.1
(d) 24 (e) 6.61 (f) 13
(g) 20 (h) 36.0

MIND TEST 2.2a

1. (a) 3.5×10^1 (b) 4.81×10^2
(c) 5.075×10^3 (d) 9.725×10^1
(e) 3.1243×10^3 (f) 9.0×10^{-1}
(g) 2.3×10^{-1} (h) 3.75×10^{-2}

2. (a) 2.5 (b) 37.5 (c) 423
(d) 5 070 (e) 91 000 (f) 0.62
(g) 0.0729 (h) 0.001034 (i) 0.0008504

3. (a) 1.05×10^6 metres (b) 2.16×10^{11} bytes
(c) 7.5×10^{11} litres (d) 9.5×10^{-5} metres
(e) 1.23×10^{-7} metres (f) 8.9×10^{-17} metres

MIND TEST 2.2b

1. (a) 5.97×10^4 (b) 3.93×10^6
(c) 1.021×10^8 (d) 1.574×10^5
(e) 5.46×10^8 (f) 8.59×10^4
(g) 5.77×10^4 (h) 1.08×10^{-3}
(i) 6.09×10^{-5} (j) 9.91×10^{-3}
(k) 7.68×10^{-4} (l) 8.685×10^{-6}

MIND TEST 2.2c

1. (a) 1.48×10^8 (b) 3.75×10^{-8}
(c) 2.52×10^8 (d) 2.12×10^3
(e) 4.5×10^{-3} (f) 6.4×10^3
(g) 2.95×10^3 (h) 8.6×10^8

2. 3.126×10^3 **3.** 63 **4.** 10^3 micrometres

MIND TEST 2.2d

1. $2.02 \times 10^5\ m^3$

2. (a) 9.17×10^7 km (b) 4.44×10^9 km (c) 4.35×10^9 km

Dynamic Challenge

Test Yourself

1. (a) 24 000 (b) 54 300 (c) 9 000 (d) 300 000
(e) 5 000 (f) 5.00 (g) 0.28 (h) 40
(i) 420 (j) 10 (k) 1.04 (l) 502

2. (a) 3.48×10^8 (b) 5.75×10^4 (c) 5.11×10^4
(d) 2.96×10^9 (e) 8.84×10^{-2} (f) 3.31×10^{-4}
(g) 9.77×10^{-8} (h) 5.43×10^4

3. (a) –2, 0.025, 0.025, 1.35, 1.375
(b) –3, 0.0034, 5.74, 0.0034, 5.7434
(c) –3, 0.0042, 1.75, 0.0042, 1.7458
(d) –3, 0.0043, 3.7, 0.0043, 3.657

4. (a) 1.2×10^4 (b) RM214 **5.** 97 people

Skills Enhancement

1. (a) $5.57 \times 10^2\ m^2$ (b) RM10 824

2. (a) (i) $70.9\ km\ h^{-1}$ (ii) $47.1\ km\ h^{-1}$ (iii) $68.4\ km\ h^{-1}$

Self Mastery

1. (a) Mercury = $7.48 \times 10^7\ km^2$
Neptune = $7.62 \times 10^9\ km^2$
Jupiter = $6.14 \times 10^{10}\ km^2$
(b) $6.133 \times 10^{10}\ km^2$

2. (a) 4.37 g (b) 4.99 g

CHAPTER 3 Consumer Mathematics: Savings and Investments, Credit and Debt

MIND TEST 3.1a

1.
- For a well-planned life in the future
- As an additional income
- For emergency use

2.
- Open a Fixed Deposit Account
- This is because the money will not be used for a given period
- Higher interest rates are also offered

3. Cheques are commonly used by businessmen/businesswomen for payments in large amounts while most people only make daily payments in small amounts.

MIND TEST 3.1b

1. RM610.10 **2.** RM1 159.70 **3.** RM106.17

MIND TEST 3.1c

1. Return on investment is the value of return of the investment.

2. (a) RM2 000
(b) RM24 000 + RM230 000 = RM254 000

3. RM320

MIND TEST 3.1d

1. The higher the risk, the higher the return.

2. Bank Negara Malaysia guarantees deposits in the bank.

3. It can be cashed immediately.

4. Real estate price usually increases but rarely falls.

5. (a) Real estate
(b) Risk potential = Low
Return = High
Liquidity = Low
(c) Encik Osman's action is wise because our country focuses on the tourism sector. Therefore, it is appropriate to set up the homestay. Besides, the investment in the homestay has low risk.

MIND TEST 3.1e

1. Purchase of shares every month or periodically but not at a lump sum.

2. (a) Investor 2. This is because the purchase of 2 shares on a regular basis allows him to purchase many units of shares and the average cost per unit can be reduced.
(b) RM1.80. 13 268 units of shares
(c)
- The average cost per unit share can be reduced
- Reduce the risk of loss

MIND TEST 3.1f

1. (a) Mr Rasamanie – Real estates (Low Risk)
Encik Nik Izwan – Savings (Low Risk)
Real estates (Low Risk)
Shares (High Risk)
(b) Encik Nik Izwan. This is because if there is a loss in one of the investments, it can be covered by other investments.
(c) Economic factor and political factor of the location of the real estate.

2. 25.74%

MIND TEST 3.2a

1. Personal loans are short term loans for consumer use.

2.
- Prepare your budget
- Plan your expenses

3. Credit card – He is not required to pay interest if his debts are settled in interest-free period as compared to loan.

Dynamic Challenge

Test Yourself

1. Savings is the balance after deducting mandatory expenditure from salary.

2.
- High interest rate.
- Savings period is subjected to a specified time.

3. RM8 640

Skills Enhancement

1. Increase the number of shares purchased and the average cost per unit will be lower as compared to if the units are purchased all at once.

2. Purchase of land lots, houses, factories and so on.

3. (a) Dividend (b) Capital gain (c) Bonus share

4. (a) Lee Chong needs to have the knowledge to assess and select shares while Mokhtar's investment is assisted by a professional company.
(b) Lee Chong's risk is higher compared to Mokhtar's.

5. RM300 **6.** (a) RM360 (b) 3 000 units (c) 9 000 units

7. RM1 000, 3%, 3 years **8.** RM634.12

Self Mastery

1. RM3 750
2. 8.85%
3. RM7 000
4. RM400
5. RM233.33
6. RM52.87
7. (a) Masnah Rasam's view is not recommended because she has to pay interest.
   (b) RM320, 8%
   (c) Cash, because no interest needs to be paid.
8. RM15 000
9. 4%
10. RM900

CHAPTER 4 Scale Drawings

MIND TEST 4.1a

1. Diagram 1, Diagram 2, Diagram 4

MIND TEST 4.1b

1. (a) $1 : \frac{1}{2}$ (b) $1 : 3$ (c) $1 : \frac{1}{2}$ (d) $1 : \frac{2}{3}$
2. Length = 6 cm Width = 2 cm
3. 10 km
4. 6 cm

MIND TEST 4.1c

2. (b) (i) $1 : \frac{1}{2}$ (ii) $1 : 2$

MIND TEST 4.1d

1. 1 944 cm^2
2. 34.8 cm
3. 560 m^2
4. 20 cm
5. (a) 7 200 m^2
   (b) 2 hours 24 minutes

Dynamic Challenge

Test Yourself

1. $1 : \frac{1}{5}$
2. (a) I and III
   (b) I = 1 : 2
   III = $1 : \frac{1}{2}$
   (c) (i) I = 1.5 cm^2
   III = 24 cm^2
   (ii) I = 1 : 4
   III = $1 : \frac{1}{4}$
   The ratio of area is not proportional to the scale of the scale drawings.
3. (a) 17.0 cm
   (b) 203.5 m^2

Skills Enhancement

1. 540 km h^{-1}
2. 50 cm × 50 cm tile. RM633.20 can be saved.
3. (a) 2 829 m^2
   (b) 4 : 13
   (c) 1 971 m^2
   (d) RM3 960

Self Mastery

1. (a) 48 m^2 (b) 8 : 1 (c) 1 440 m^3
2. (a) 8 400 m^2
   (b) 1 : 500. The most relevant value for scale.
   (c) (i) 60 pieces (ii) RM31 500

CHAPTER 5 Trigonometric Ratios

MIND TEST 5.1a

Angle	Hypotenuse	Opposite side	Adjacent side
$\angle QPR$	PR	QR	PQ
$\angle PRQ$	PR	PQ	QR
$\angle MNK$	KN	KM	MN
$\angle MKN$	KN	MN	KM
$\angle FEG$	EG	FG	EF
$\angle EGF$	EG	EF	FG
$\angle BAE$	AE	BE	AB
$\angle AEB$	AE	AB	BE
$\angle BCD$	CD	BD	BC
$\angle BDC$	CD	BC	BD

MIND TEST 5.1b

1. ΔDEF
   $\sin x = \frac{EF}{DF}$ $\cos x = \frac{DE}{DF}$ $\tan x = \frac{EF}{DE}$
   $\sin y = \frac{DE}{DF}$ $\cos y = \frac{EF}{DF}$ $\tan y = \frac{DE}{EF}$
   ΔKLM
   $\sin x = \frac{KL}{KM}$ $\cos x = \frac{LM}{KM}$ $\tan x = \frac{KL}{LM}$
   $\sin y = \frac{LM}{KM}$ $\cos y = \frac{KL}{KM}$ $\tan y = \frac{LM}{KL}$
   ΔPQR
   $\sin x = \frac{QS}{PQ}$ $\cos x = \frac{PS}{PQ}$ $\tan x = \frac{QS}{PS}$
   $\sin y = \frac{QS}{QR}$ $\cos y = \frac{RS}{QR}$ $\tan y = \frac{QS}{RS}$

MIND TEST 5.1c

1. Trigonometric ratios of angle x and angle y are the same. This is because all lengths of side are reduced by the same rate.
2. (a) (i) $\frac{38}{145}$ (ii) $\frac{28}{29}$ (iii) $\frac{19}{70}$
   (iv) $\frac{1}{2}$ (v) $\frac{7}{8}$ (vi) $\frac{4}{7}$
   (b) No

MIND TEST 5.1d

1. (a) $\sin\theta = \frac{15}{39}$ $\cos\theta = \frac{12}{13}$ $\tan\theta = \frac{15}{36}$
   (b) $\sin\theta = \frac{24}{25}$ $\cos\theta = \frac{7}{25}$ $\tan\theta = \frac{24}{7}$
   (c) $\sin\theta = \frac{15}{17}$ $\cos\theta = \frac{8}{17}$ $\tan\theta = \frac{15}{8}$
   (d) $\sin\theta = \frac{5}{13}$ $\cos\theta = \frac{12}{13}$ $\tan\theta = \frac{5}{12}$
   (e) $\sin\theta = \frac{15}{17}$ $\cos\theta = \frac{8}{17}$ $\tan\theta = \frac{15}{8}$
   (f) $\sin\theta = 0.6$ $\cos\theta = 0.8$ $\tan\theta = 0.75$

2. (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{39}}{8}$ (d) $\frac{4\sqrt{2}}{9}$

3. (a) 3 m (b) 21 m (c) 25 mm

4. (a) 10 cm (b) 15 cm (c) 30 mm

5. (a) 18 cm (b) 20 cm (c) 9 mm

6. (a) 15 cm (b) 20 cm

7. 51.61 cm

MIND TEST 5.1e

1. (a) 2 (b) 3.5 (c) 2.5 (d) 0.5
(e) –0.5 (f) 3 (g) $\frac{5\sqrt{3}}{2}$ (h) $\frac{9\sqrt{3}}{2}$
(i) 10 (j) 9

MIND TEST 5.1f

1. (a) 37°48' (b) 74°36' (c) 58°6'
(d) 60°12' (e) 41°30' (f) 16°54'
(g) 5°24' (h) 72°18'

2. (a) 65.9° (b) 47.7° (c) 18.2°
(d) 69.4° (e) 70.1° (f) 36.6°
(g) 35.5° (h) 20.3°

MIND TEST 5.1g

1. (a) 0.6947 (b) 0.2840 (c) 2.6746
(d) 0.7815 (e) 0.8630 (f) 1.5051

MIND TEST 5.1h

1. (a) 12.2° (b) 54° (c) 24°
(d) 65.8° (e) 14.4° (f) 75.3°
(g) 55.9° (h) 8.7° (i) 35.8°
(j) 78.3° (k) 45.3° (l) 84.3°

MIND TEST 5.1i

1. 2.15 m **2.** 83.2 m **3.** 173.9 m

4. (a) 13 cm (b) 67.4°

Dynamic Challenge

Test Yourself

1. (a) 28° 4' (b) $\frac{15}{17}$ (c) $\frac{8}{17}$

2. (a) 39 cm (b) $\frac{12}{5}$ (c) 22.6°

3. (a) 27 cm (b) 39°

4. (a) 6 (b) 39° 48'

Skills Enhancement

1. (a) $\sqrt{3}$ (b) 4 (c) $4\sqrt{6}$

2. (a) 12 cm (b) 35 cm (c) 45°

3. 8.66 m

Self Mastery

1. (a) $\frac{7}{12}$ (b) 15.56 cm (c) 26° 45'

2. (a) 90° (b) 30° (c) 10.4 m (d) 1 : 2

3. (a) $4\sqrt{5}$ cm (b) 63° 26'
(c) Not true. The actual ratio is 3 : 5

CHAPTER 6 Angles and Tangents for Circles

MIND TEST 6.1a

1. (a) 35° (b) 25° (c) 30° (d) 35°

2. (a) 40° (b) 35° (c) 70° (d) 105°

3. (a) 40° (b) 30° (c) 10° (d) 80°

4. (a) 24° (b) 25°

MIND TEST 6.1b

1. (a) 40° (b) 30° (c) 3.6 cm (d) 10.4 cm

2. (a) 70° (b) 30°

3. (a) 22° (b) 114° (c) 40°

MIND TEST 6.1c

1. (a) 40° (b) 80° (c) 50°

2. (a) 50° (b) 65° (c) 50°

3. (a) 110° (b) 55° (c) 125°

4. (a) 124° (b) 34° (c) 54°

MIND TEST 6.1d

1. (a) 45° (b) 5 cm (c) 10 cm (d) 55°

2. (a) 40° (b) $\angle ORQ$ and $\angle OQR$

3. (a) 40° (b) 10 cm

MIND TEST 6.1e

1. (a) 55° (b) 25° (c) 27.5° (d) 30°

2. 216° **3.** 90°

MIND TEST 6.1f

1. (a) 110° (b) 10.3 cm **2.** 176° **3.** 132°

MIND TEST 6.2a

1. (a) (i) No – vertex P does not lie on the circumference
(ii) Yes – $DEFG$
(iii) Yes – $KNPQ$ and $KLMN$
(iv) Yes – $ABDE$

(b) (i) None
(ii) $\angle D$ and $\angle F$, $\angle DEF$ and $\angle DGF$
(iii) $\angle KQP$ and $\angle KNP$, $\angle NPQ$ and $\angle NKQ$, $\angle KLM$ and $\angle KNM$, $\angle LMN$ and $\angle LKN$
(iv) $\angle BAE$ and $\angle BDE$, $\angle ABD$ and $\angle AED$

MIND TEST 6.2b

1. (a) 30° (b) 20° (c) 120°

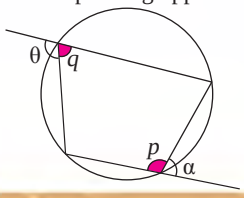
2. 50° **3.** 40°

4. (a) 125° (b) 117.5°

MIND TEST 6.2c

1. Exterior angle = a
Corresponding opposite interior angle = d
Exterior angle = e
Corresponding opposite interior angle = b

2.



MIND TEST 6.2d

1. 97° **2.** 38° **3.** 79° **4.** 99° **5.** 108°

MIND TEST 6.3a

1. (a) (i) *RS* and *ST* – touching the circle at only one point.
(ii) *X* and *Y*.
(iii) *PQ* – passes through 2 points on the circle.
(iv) *A* and *B*.
(b) (i) *BC* and *BD* – touching the circle at only one point.
(ii) *H* and *E*.
(iii) *BF* – passes through 2 points on the circle.
(iv) *F* and *G*.

MIND TEST 6.3b

1. 34°
2. (a) 120° (b) 60° (c) 30° **3.** 114°

MIND TEST 6.3c

1. (a) 60° (b) 30° (c) 8.66 cm (d) 10 cm
2. (a) 40° (b) 3.575 cm (c) 7.667 cm

MIND TEST 6.3d

1. (a) $\angle y = \angle z$, $\angle x = \angle a$ (b) $\angle x = \angle b$, $\angle y = \angle a$ (c) $\angle x = \angle y$, $\angle f = \angle e$, $\angle z = \angle a$
2. 27° **3.** 52° **4.** 44°

MIND TEST 6.3e

1. 50° **2.** $x = 26°34'$, $y = 31°43'$
3. (a) 130°
(b) (i) 12.87 cm (ii) 8.578 cm (iii) 23.66 cm
4. (a) 4 cm (b) 3.87 cm (c) 11.61 cm

MIND TEST 6.4a

1. (a) 8.49 cm (b) 38.21 cm^2
2. (a) 35° (b) 55° (c) 11.31 cm

Dynamic Challenge

Test Yourself

1. $x = 40°$, $y = 150°$ **2.** 100°
3. $x = 30°$, $y = 60°$ **4.** 230°
5. $x + y = 180°$ **6.** 86°

Skills Enhancement

1. 30° **2.** 130° **3.** 114° **4.** 60°

Self Mastery

1. (a) 61° (b) 80° **2.** 64.8 cm^2
3. (a) 36°52' (b) 3.6 cm
4. (a) 5 cm (b) 13 cm (c) 30 cm^2

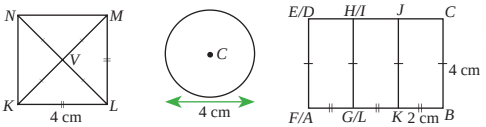
CHAPTER 7 Plans and Elevations

MIND TEST 7.1a

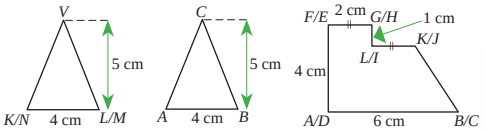
1. (a) Yes (b) Yes (c) No (d) Yes
2. (a) Correct (b) Wrong

MIND TEST 7.1b

1. (a) (i) (ii) (iii)

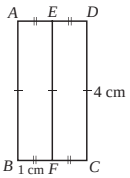


(b) (i) (ii) (iii)

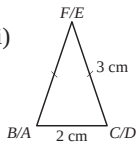


MIND TEST 7.1c

1. Diagram 1
(a) (i)

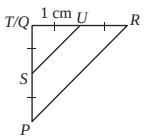


(a) (ii)

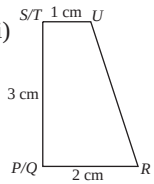


(b) As viewed from *Z*:
Changed – the lengths of sides *AE*, *ED*, *BF* and *FC*.
Unchanged – the lengths of sides *EF*, *AB*, *DC*, *AD*, *BC* and all sizes of angles.
As viewed from *X*:
No changes in length of sides and size of angles.

Diagram 2
(a) (i)



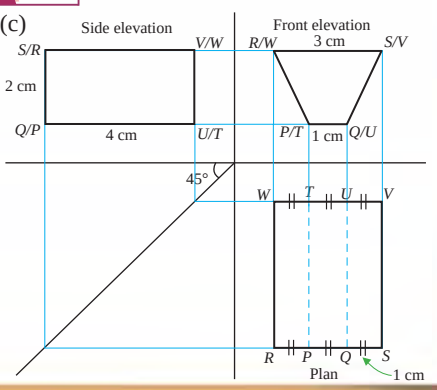
(a) (ii)



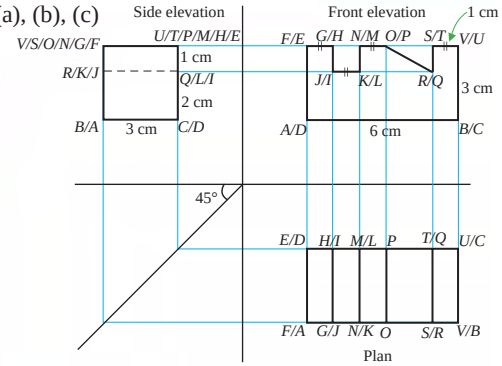
(b) As viewed from *Z*:
Changed – the lengths of sides *SP* and *UR*.
Unchanged – the lengths of sides *ST*, *TU*, *PQ*, *QR* and all sizes of angles.
As viewed from *X*:
Changed – the lengths of sides *SP*, *SU* and *PR*.
Unchanged – the lengths of sides *TQ*, *QR*, *TU*, *UR* and all sizes of angles.

MIND TEST 7.2a

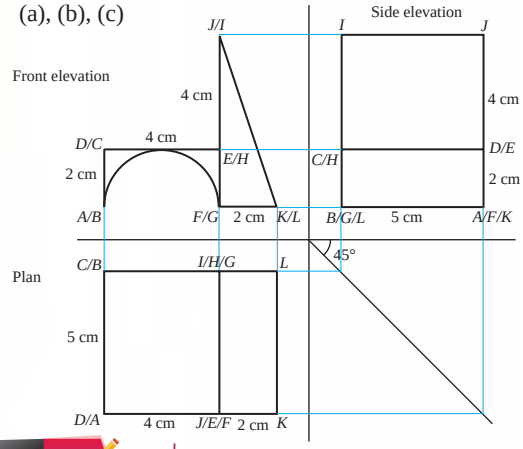
1. (a), (b), (c)



2. (a), (b), (c)

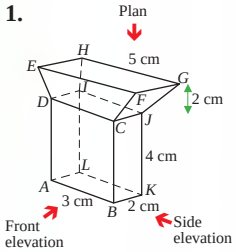


3. (a), (b), (c)

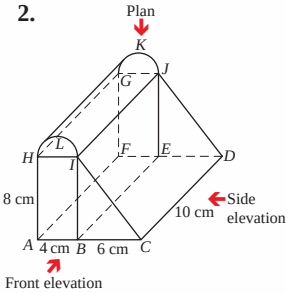


MIND TEST 7.2b

1.

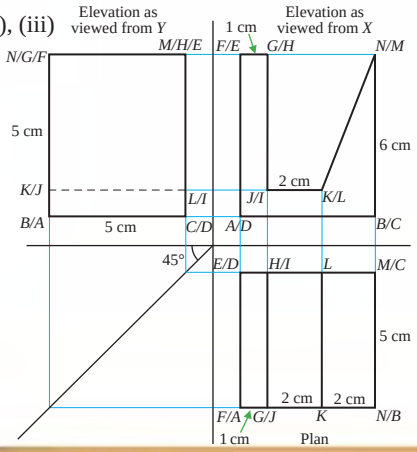


2.



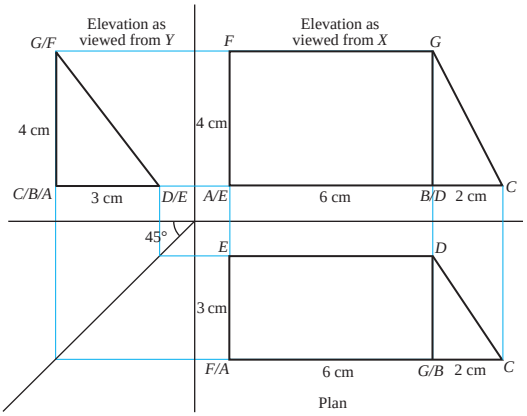
MIND TEST 7.2c

1. (a) (i), (ii), (iii)



(b) (i) 75 cm^3 (ii) 1 : 1

2. (a) (i), (ii), (iii)



(b)

	CD	*CG*	*DG*
Plan	3.6 cm	2 cm	3 cm
Elevation as viewed from *X*	2 cm	4.5 cm	4 cm
Elevation as viewed from *Y*	3 cm	4 cm	5 cm

(c) Original object
CD = 3.61 cm, *CG* = 4.47 cm, *DG* = 5 cm.

(d) Elevation as viewed from *X* = ∠*BCG*, ∠*BGC*
Elevation as viewed from *Y* = ∠*AEF*, ∠*AFE*
Plan = ∠*BCD*, ∠*BDC*.

Dynamic Challenge

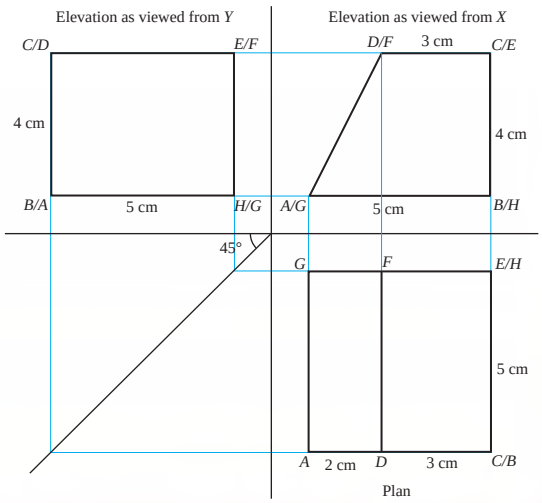
Test Yourself

1. (a) True (b) True (c) False (d) True

2. Three vertical cylinders with diameters 1 cm, 2 cm and 3 cm. The height of all cylinders is 4 cm. The three cylinders are arranged symmetrically from all directions.

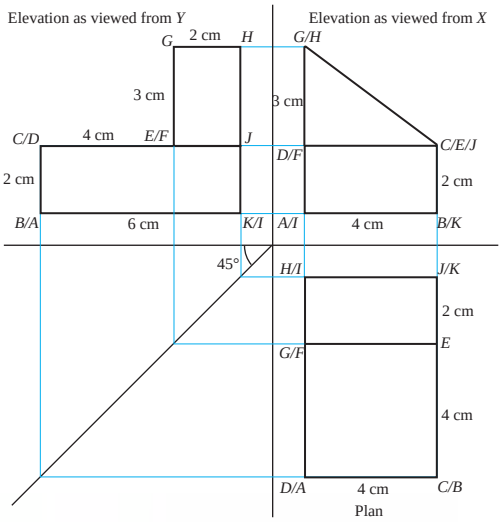
Skills Enhancement

1. (a) (i), (ii), (iii)



(b) *AD* = 4.5 cm, ∠*ADC* = 116°

2. (a) (i), (ii), (iii)

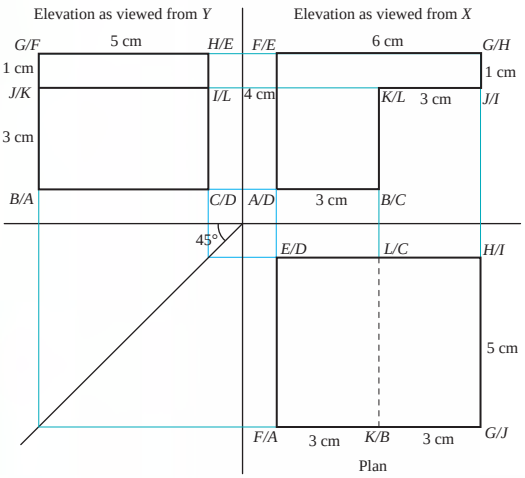


(b) 60 cm^3

3. 462.5 cm^3 **4.** 477.75 cm^3 **5.** 96 cm^3

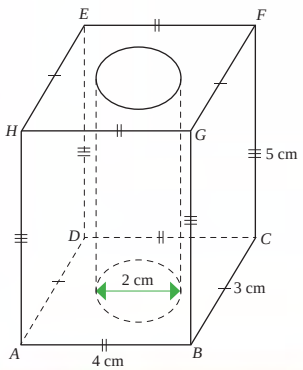
Self Mastery

1. (a) (i), (ii), (iii)



(b) 45 cm^3 (c) RM264

2. (a)



(b) $44\frac{2}{7}$ cm^3

CHAPTER 8 Loci in Two Dimensions

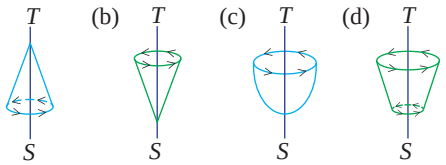
MIND TEST 8.1a

1. (a) A straight line parallel to the inclined plane
(b) A curve
(c) A vertical straight line
(d) A straight line parallel to slide

2. (a) A vertical straight line (c) A curve
(b) A horizontal straight line

MIND TEST 8.1b

1. (a) (b) (c) (d)

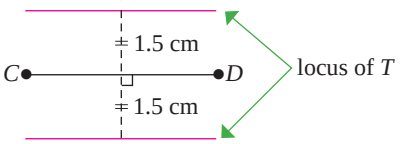


MIND TEST 8.2a

1. (a) Locus of X is a circle centred at P with radius 3 cm.
(b) Locus of Y is a circle centred at Q with radius 4 cm.

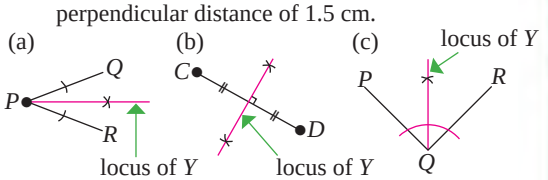
2. (a) HF (b) AC (c) EG
(d) BD (e) AD and BC

3. (a)



(b) The locus of point T is a pair of straight lines of 6 cm parallel to the straight line CD with a perpendicular distance of 1.5 cm.

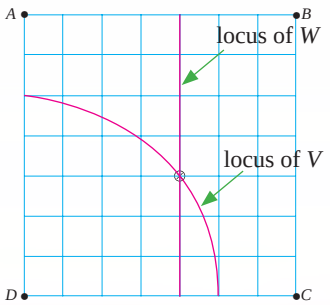
4. (a) (b) (c)



5.

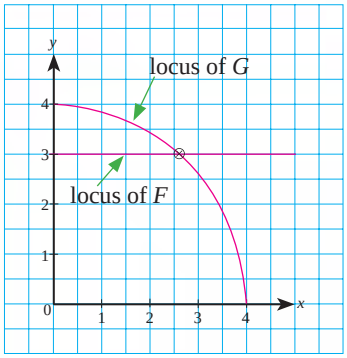
MIND TEST 8.2b

1.



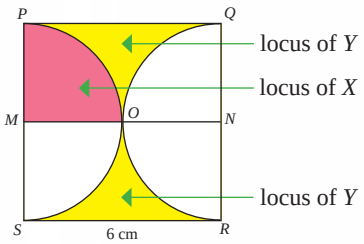
2. Point G

3.



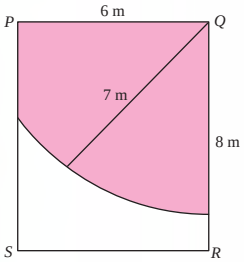
MIND TEST 8.2c

1. (a), (b)

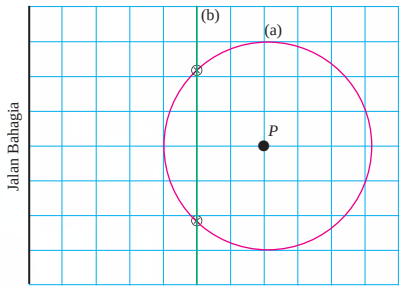


(c) The intersection between the locus of X and locus of Y is arc OP.

2.



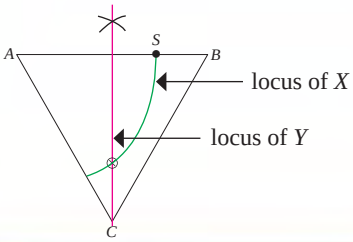
3.



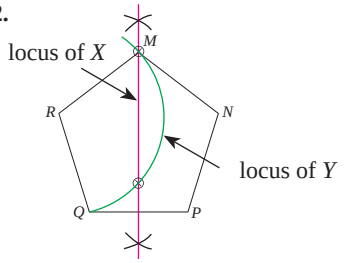
Dynamic Challenge

Test Yourself

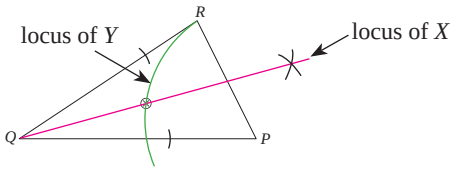
1.



2.



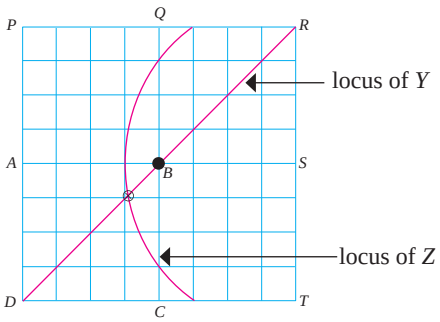
3.



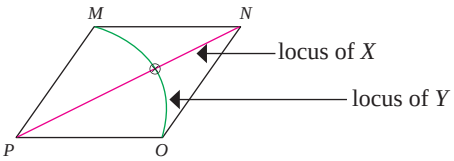
Skills Enhancement

1. (a) ABS

(b)



2.



Self Mastery

1. (a) N (b) L (c) I (d) II (e) VI

2. (a) I (b) IV (c) III

3. (a) locus of X – moving constantly 1 cm from O.
locus of Y – equidistant from P and R.

(b) locus of X – moving constantly 1 cm from O.
locus of Y – equidistant from Q and S.

CHAPTER 9 Straight Lines

MIND TEST 9.1a

1. (a) gradient = 3, y-intercept = 5

(b) gradient = 2, y-intercept = –7

(c) gradient = –1, y-intercept = 4

(d) gradient = 4, y-intercept = 3

(e) gradient = $-\frac{1}{3}$, y-intercept = 6

(f) gradient = $\frac{1}{2}$, y-intercept = $-\frac{5}{4}$

2. (a) $h = -2$, $k = 4$ (b) $h = 4$, $k = -3$

MIND TEST 9.1b

1. (a) $\frac{x}{8}-\frac{y}{6}=1$, $y=\frac{3}{4}x-6$ (b) $\frac{x}{4}+\frac{y}{14}=1$, $y=-\frac{7}{2}x+14$

(c) $\frac{x}{3}-\frac{y}{5}=1$, $y=\frac{5}{3}x-5$ (d) $-\frac{2x}{9}+\frac{y}{3}=1$, $y=\frac{2}{3}x+3$

2. (a) $3x+4y=12$, $y=-\frac{3}{4}x+3$ (b) $-6x+3y=18$, $y=2x+6$

(c) $9x+y=6$, $y=-9x+6$ (d) $8x-3y=12$, $y=\frac{8}{3}x-4$

3. (a) $-2x+y=6$ (b) $3x-y=12$, $\frac{x}{4}-\frac{y}{12}=1$

(c) $x+y=5$, $\frac{x}{5}+\frac{y}{5}=1$ (d) $2x+y=-4$, $-\frac{x}{2}-\frac{y}{4}=1$

MIND TEST 9.1c

1. (a) No (b) Yes (c) Yes (d) No
2. (a) Yes (b) No (c) Yes (d) No
3. (a) Yes (b) No (c) Yes (d) No
4. (a) $h=2$ (b) $k=-2$ (c) $n=3$

MIND TEST 9.1d

1. (a) Parallel (b) Not parallel
(c) Parallel (d) Not parallel
2. (a) $k=3$ (b) $k=6$
(c) $k=\frac{15}{8}$ (d) $\frac{3}{2}$
3. $h=-\frac{5}{2}$, $k=3$

MIND TEST 9.1e

1. (a) $y=2x+1$ (b) $y=-3x-14$
(c) $y=\frac{2}{3}x-3$ (d) $y=-\frac{1}{2}x-4$

MIND TEST 9.1f

1. (a) $y=-\frac{1}{3}x+2$ (b) $y=4x+8$
(c) $y=4x-13$ (d) $y=\frac{2}{3}x+\frac{2}{3}$
(e) $y=x+4$ (f) $y=-x-2$

MIND TEST 9.1g

1. (a) $y=3x+1$ (b) $y=-2x-2$
(c) $y=-\frac{3}{2}x+9$ (d) $y=-\frac{3}{2}x-9$
2. (a) $y=\frac{1}{3}x+\frac{10}{3}$ (b) $y=\frac{1}{3}x-\frac{10}{3}$
(c) $y=\frac{1}{3}x$

MIND TEST 9.1h

1. (a) (3, 4) (b) (5, 4) (c) (3, 2) (d) (2, –1)
2. (a) (–2, 3) (b) (0, 4) (c) (–5, 0) (d) (4, 1)

MIND TEST 9.1i

1. (a) $\frac{1}{2}$ (b) –2 (c) $y=\frac{1}{2}x-2$
2. (a) (5, 0) (b) $y=-x-5$
(c) x-intercept = –5

Dynamic Challenge

Test Yourself

1. (a) x-intercept = 15 (b) y-intercept = 6
(c) gradient $=-\frac{2}{5}$
2. (a) $x=-6$ (b) $y=-8$
3. $y=3x+18$ **4.** $y=4x+2$
5. $y=-3x-4$ **6.** (5, –8)

Skills Enhancement

1. (4, 3)
2. (a) $x=3$
(b) $y=-2x+16$, x-intercept = 8
3. (a) $y=-2x$
(b) $y=-2x+35$
(c) x-intercept $=\frac{35}{2}$
4. (a) $y=-\frac{1}{2}x+8$
(b) $y=-\frac{1}{2}x+\frac{29}{2}$ or $x+2y=29$
(c) No, because the two straight lines are parallel.

Self Mastery

1. (a) 6 km (b) (–4, 4)
(c) 5 km (d) $4x+3y=20$
2. (a) 10 km (b) $y=\frac{x}{5}+\frac{29}{5}$
(c) 20 km (d) 34.11 minutes
3. $y=\frac{3}{16}x+\frac{27}{2}$, 13.5 cm
4. (a) $k=-3$ (b) (–5, 7)

Glossary

Accuracy
The degree of approximation of the measured value to the actual value.

Arc
A curved line connecting any two points on the circumference of a circle.

Bisector
A line that divides an angle into two angles of the same size.

Centre of a circle
The given point from which all points on the circumference are of the same distance.

Chord
A line segment connecting any two points on a curve.

Circumference
A closed curve which is the boundary of a circular shape.

Cyclic quadrilateral
A quadrilateral inscribed in a circle where all vertices lie on the circumference of the circle.

Common tangent
A straight line that touches two circles, each at one point only.

Corresponding angles
A pair of angles formed when a line cuts two parallel lines. These two angles are equal.

Cosine
The ratio of the lengths of the adjacent side to the hypotenuse of an angle in a right-angled triangle. Its abbreviation is cos.

Degree
A unit of measurement for angles. The degree symbol is shown as °.

Diameter
A straight line connecting two points on the circumference of the circle and passes through the centre of the circle.

Elevation
A vertical sketch of an object viewed from a certain side.

Elimination method
A method for solving simultaneous equations by eliminating one of the variables.

Factor
Numbers, terms or algebraic expressions that divides exactly the given number, term or algebraic expression.

Front elevation
The orthogonal projection of an object to a vertical plane as viewed from the front and is a uniform cross section of the object.

Gradient
The ratio of the vertical distance to the horizontal distance.

Grid
A set of straight lines that cross each other and are usually in the form of squares or equilateral triangles.

Hypotenuse
The opposite side of the right angle in a right-angled triangle.

Index
A number that states the power. Generally, a^n with n is the index for a.

Linear function
A function of the form , where a and b are constants and a $\neq 0$. The graph of linear function is of straight line form.

Locus
The path formed by a set of points in a plane or three-dimensional space that meets one or more conditions.

Normal to a plane
A line that is perpendicular or at right angle to the corresponding plane.

Origin
The point of intersection of the horizontal axis and vertical axis. The coordinates of origin are (0, 0).

Orthogonal projection
The image formed on a plane as a result of the line projection of an object perpendicular to the plane.

Parallel line
Straight lines that are on the same plane and do not intersect each other. The perpendicular distance between the parallel lines are always the same.

Perpendicular bisector
A line perpendicular to a line segment and divides the segment into two equal parts.

Plan
A sketch of an object on a horizontal plane and is viewed from above.

Plane
A flat surface in all directions and is two-dimensional.

Proportion
A mathematical statement that indicates the relationship between two quantities or values in the same ratio.

Scale
The ratio of the size of the drawing to the size of the actual object.

Scale drawing
A drawing that represents the actual object according to a certain scale. Scale drawing will be larger or smaller or of equal size as the actual object.

Side elevation
The orthogonal projection of an object to a vertical plane as viewed from the side.

Significant figure
The digits in a number specified exactly to a degree of accuracy required.

Simultaneous equation
Two equations with two variables that can be solved simultaneously.

Sinus
The ratio of the length of the opposite side to the hypotenuse of an angle in a right-angled triangle. Its abbreviation is sin.

Standard form
The standard method to write a real number. Through scientific notation, all real numbers are written in the form $A \times 10^n$, with $1 \leqslant A < 10$ and n is an integer.

Substitution method
A method for solving simultaneous equations by substituting one of the variables.

Subtend
To include an angle at the circumference or centre of the circle, opposite a particular arc.

Tangent
The ratio of the length of the opposite side to the adjacent side of an angle in a right-angled triangle. Its abbreviation is tan.

Tangent to circle
The straight line touches the circle at only one point without cutting it.

Three-dimensional shape
Shapes that have length, width, height and volume.

Trigonometric ratio
A ratio describes the relationship between the sides in a right-angled triangle.

Trigonometry
A mathematical branch related to the relationship between the sides and angles of a triangle and their applications.

Two-dimensional shape
Shapes that are of two measurements, namely length and width.

Uniform cross section
The cross section resulting from cutting a solid, which is of same size and shape as the base.

***x*-intercept**
The point where a straight line or curve cuts the x-axis.

***y*-intercept**
The point where a straight line or curve cuts the y-axis.

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dengan baiknya dan bertanggungjawab atas kehilangannya
serta mengembalikannya kepada pihak sekolah pada
tarikh yang ditetapkan

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